

Two-Period Version of Gertler-Karadi, Gertler-Kiyotaki Financial Friction Model

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Summary of Christiano-Ikeda, 2012, 'Government Policy, Credit Markets and Economic Activity,' in Federal Reserve Bank of Atlanta conference volume,

A Return to Jekyll Island: the Origins, History, and Future of the Federal Reserve, Cambridge University Press.

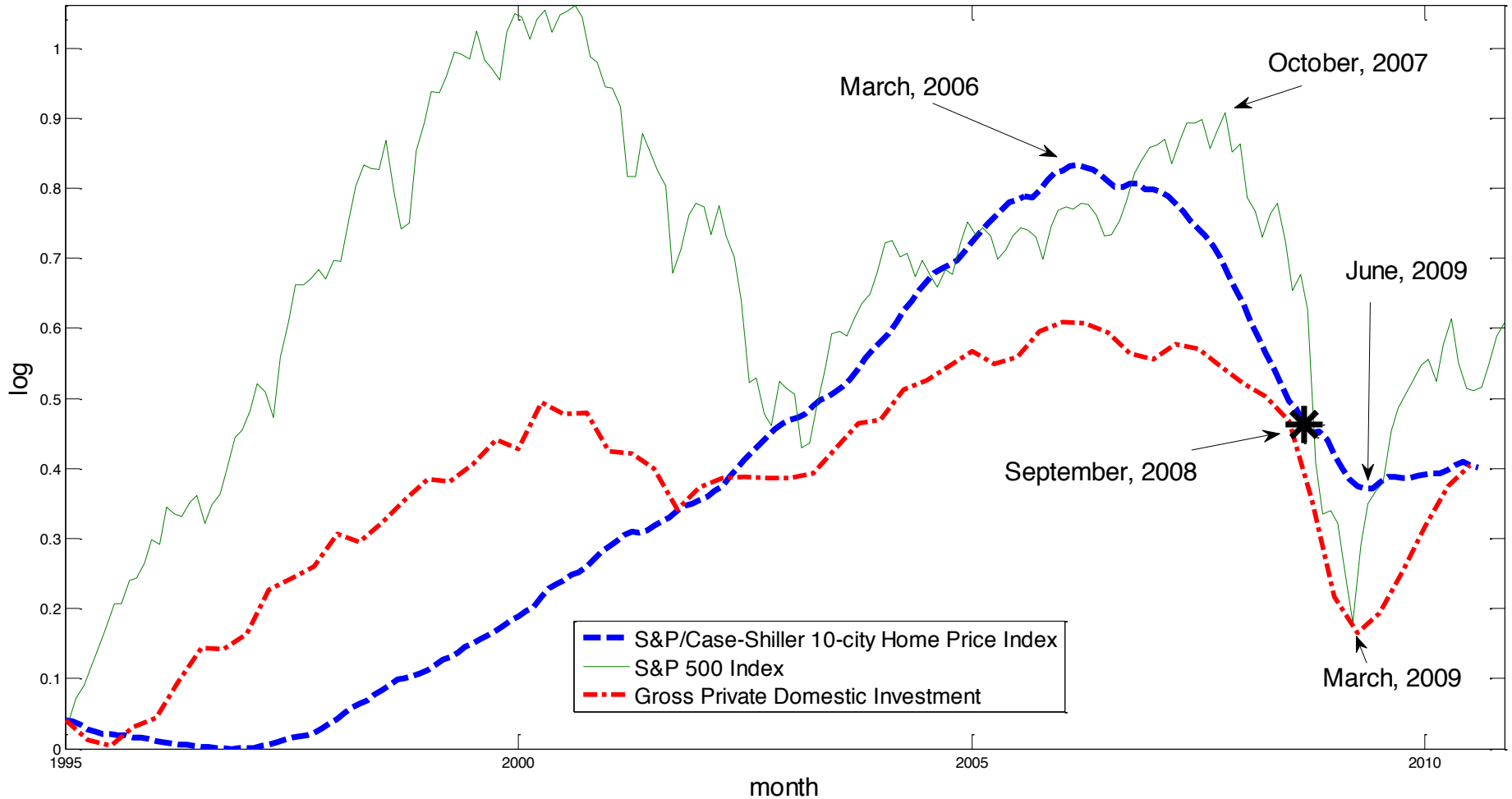
SAIF, December 2014

Motivation

- Beginning in 2007 and then accelerating in 2008:
 - Asset values (particularly for banks) collapsed.
 - Intermediation slowed and investment/output fell.
 - Interest rates spreads over what the US Treasury and highly safe private firms had to pay, jumped.
 - US central bank initiated unconventional measures (loans to financial and non-financial firms, very low interest rates for banks, etc.)
- In 2009 – the worst parts of 2007-2008 began to turn around.

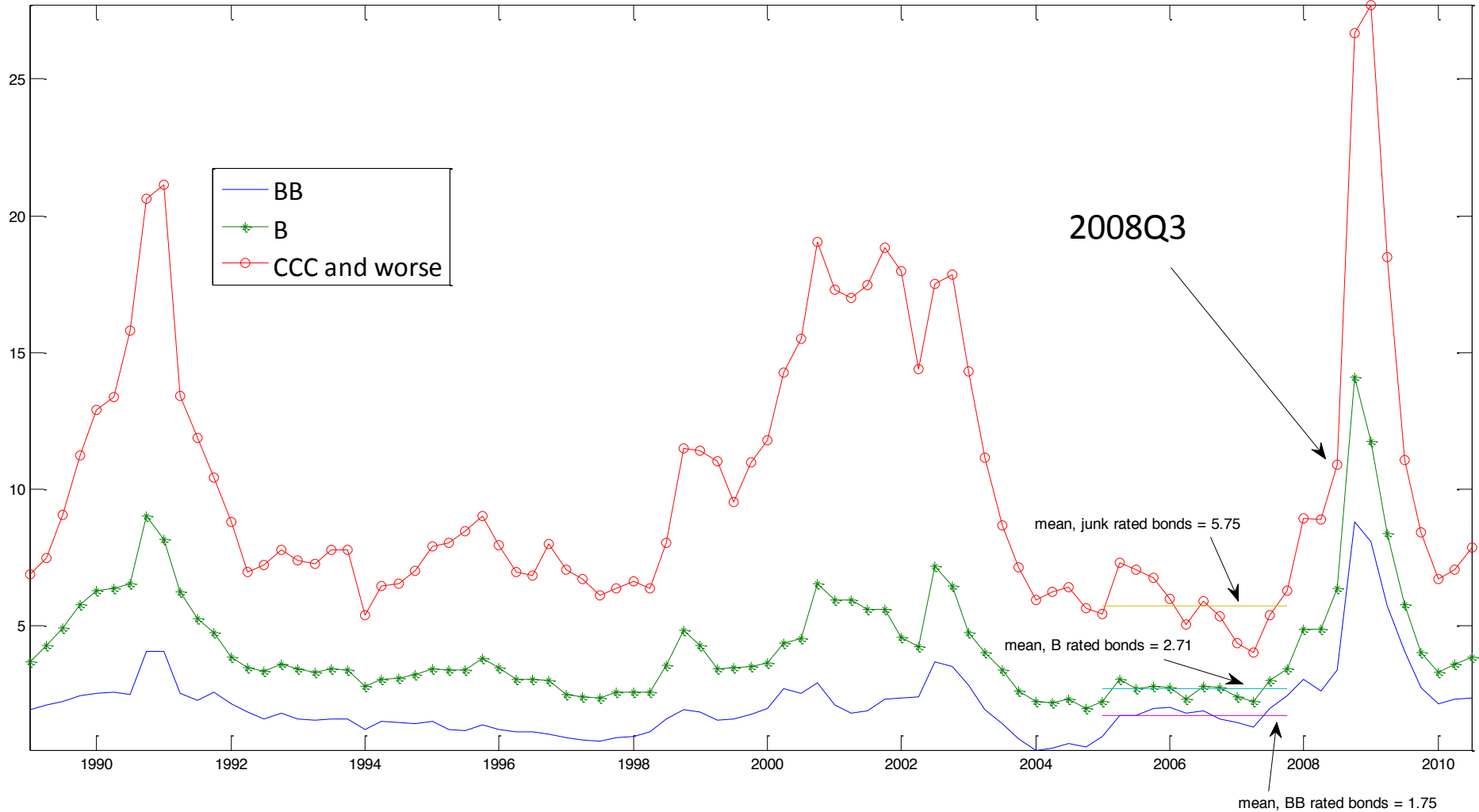
Collapse in Asset Values and Investment

Log, real Stock Market Index, real Housing Prices and real Investment



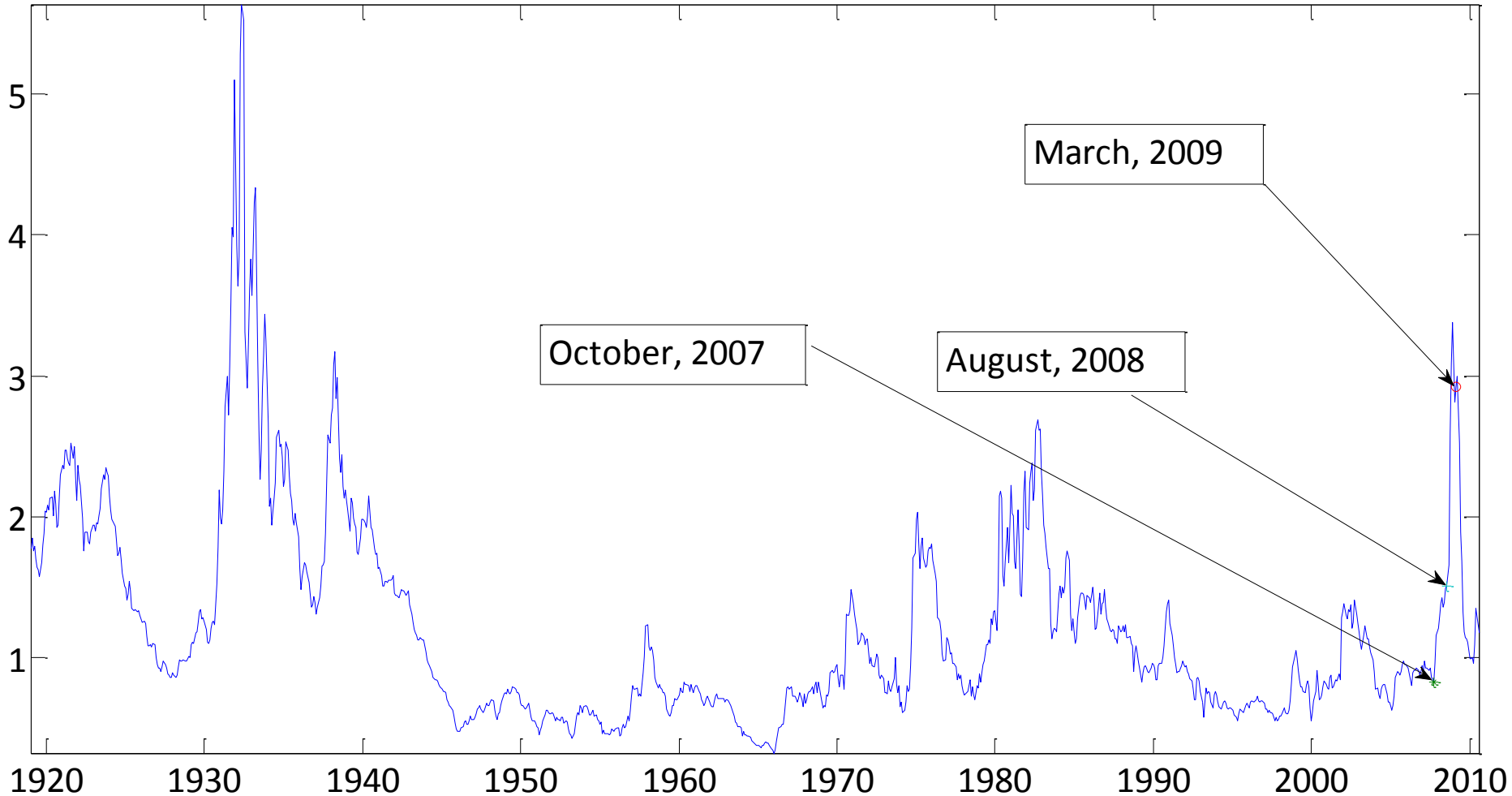
Spreads for 'Risky' Firms Shot Up in Late 2008

Interest Rate Spread on Corporate Bonds of Various Ratings Over Rate on AAA Corporate Bonds

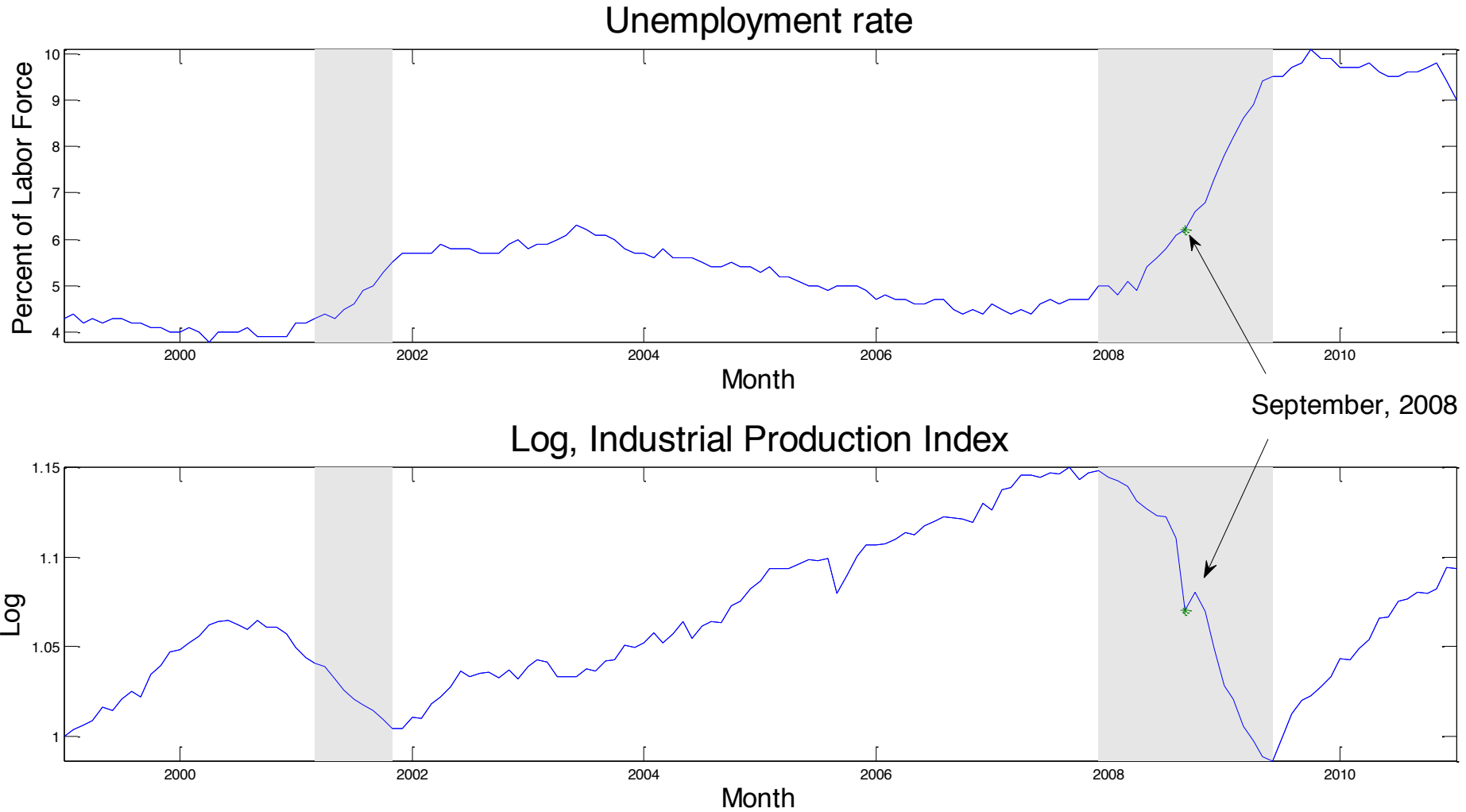


Must Go Back to Great Depression to See Spreads as Large as the Recent Ones

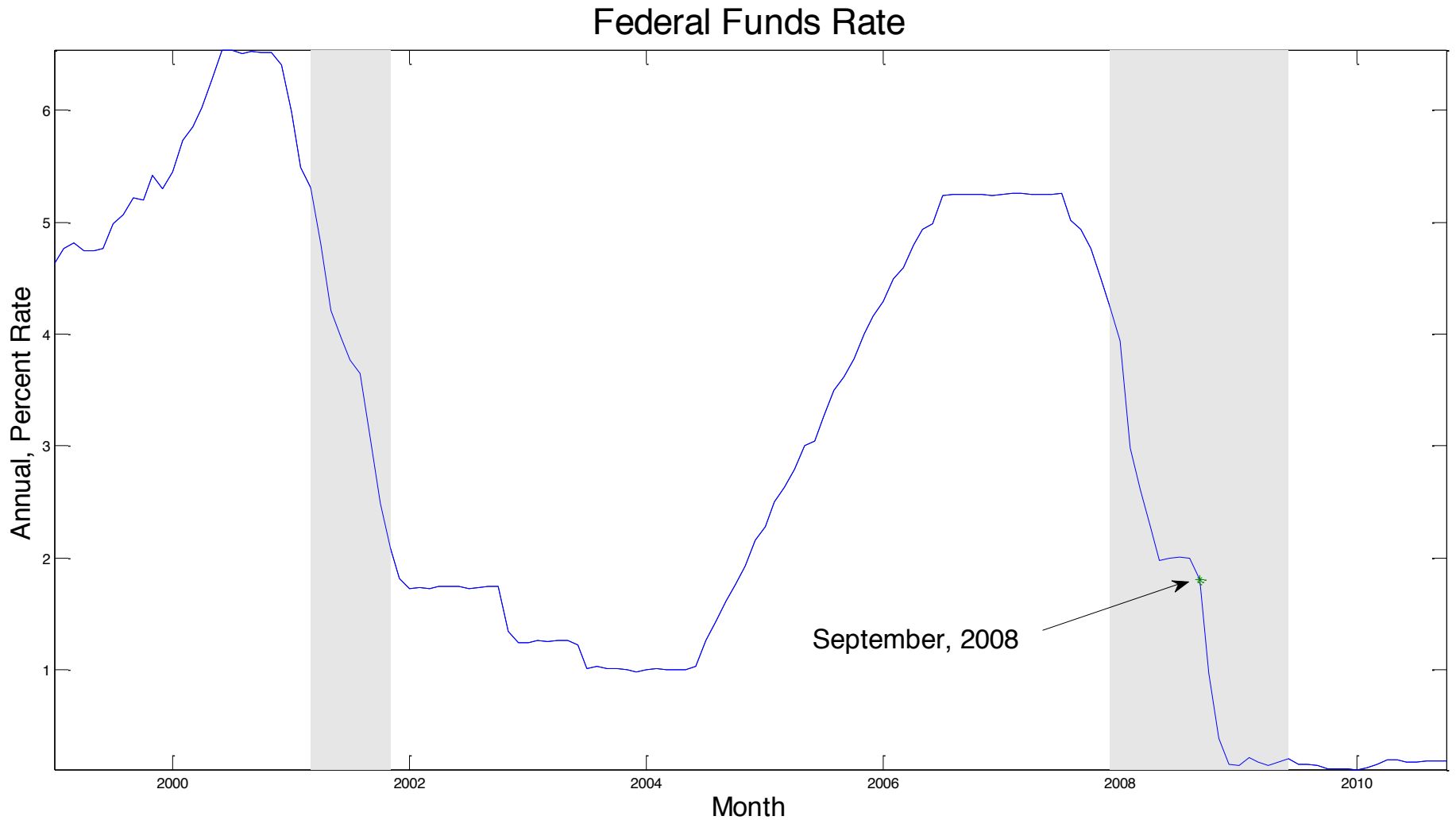
Spread, BAA versus AAA bonds



Economic Activity Shows (anemic!) Signs of Recovery June, 2009



Banks' Cost of Funds Low



Characterization of Crisis to be Explored Here

- Bank Asset Values Fell.
- Banking System Became ‘Dysfunctional’
 - Interest rate spreads rose.
 - Intermediation and economy slowed.
- Monetary authority:
 - Transferred funds on various terms to private companies and to banks.
 - Sharply reduced cost of funds to banks.
- Economy in (tentative) recovery.
- Seek to construct models that links these observations together.

Objective

- Keep analysis simple and on point by:
 - Two periods
 - Minimize complications from agent heterogeneity.
 - Leave out endogeneity of employment.
 - Leave out nominal variables: just look ‘behind the veil of monetary economics’
- Models:
 - Gertler-Kiyotaki/Gertler-Karadi
 - In two-period setting easy to study an interesting nonlinearity that is possible:
 - Participation constraint may be binding in a crisis and not binding in normal times.

Two-period Version of GK Model

- Many identical households, each with a unit measure of members:
 - Some members are ‘bankers’
 - Some members are ‘workers’
 - Perfect insurance inside households...everyone consumes same amount.
- Period 1
 - Workers endowed with y goods, household makes deposits, d , in a bank
 - Bankers endowed with N goods, take deposits and purchase securities, d , from a firm.
 - Firm issues securities, s , to produce sR^k in period 2.
- Period 2
 - Household consumes earnings from deposits plus profits, π , from banker.
 - Goods consumed are produced by the firm.

Problem of the Household		
	period 1	period 2
budget constraint	$c + d \leq y$	$C \leq R^d d + \pi$
problem	$\max_{c,C,d} [u(c) + \beta u(C)]$	

Solution to Household Problem	
$\frac{u'(c)}{\beta u'(C)} = R^d$	$c + \frac{C}{R^d} = y + \frac{\pi}{R^d}$

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$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$	$c = \frac{y + \frac{\pi}{R^d}}{1 + \frac{(\beta R^d)^{\frac{1}{\gamma}}}{R^d}}$

Household budget constraint when gov't buys private assets using tax receipts, T , and gov't gets the same rate of return, R^d , as households:

$$c + \frac{C}{R^d} = y - T + \frac{\pi + TR^d}{R^d} = y + \frac{\pi}{R^d}$$

No change!
(Ricardian-Wallace
Irrelevance)

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Household Supply of Deposits

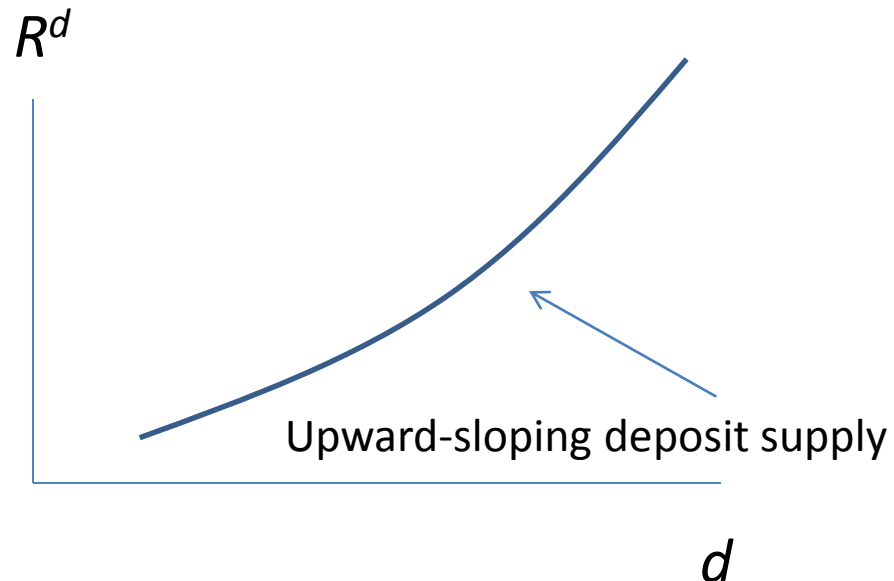
- For given π , d rises or falls with R^d , depending on parameter values.
- But, in equilibrium $\pi = R^k(N+d) - R^d d$.
- Substituting into the expression for c and solving for d :

$$d = \frac{(\beta R^d)^{\frac{1}{\gamma}} - \frac{N}{y} R^k}{(\beta R^d)^{\frac{1}{\gamma}} + R^k} y$$

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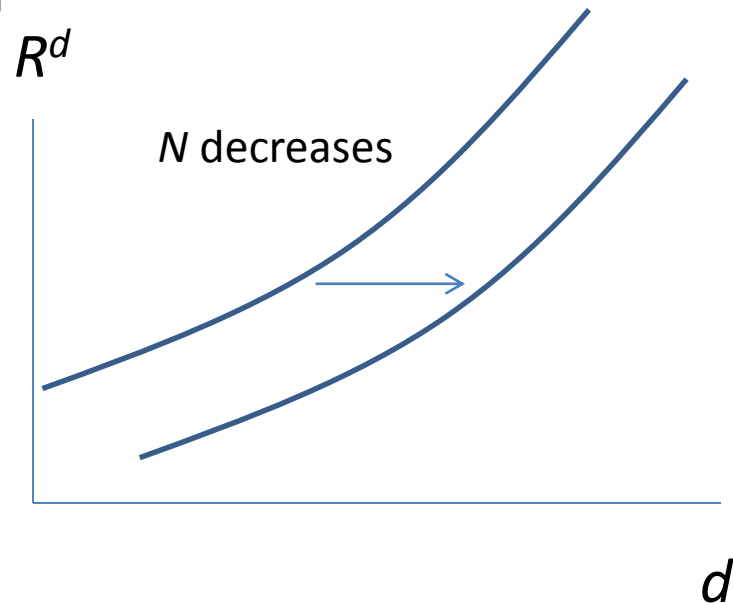
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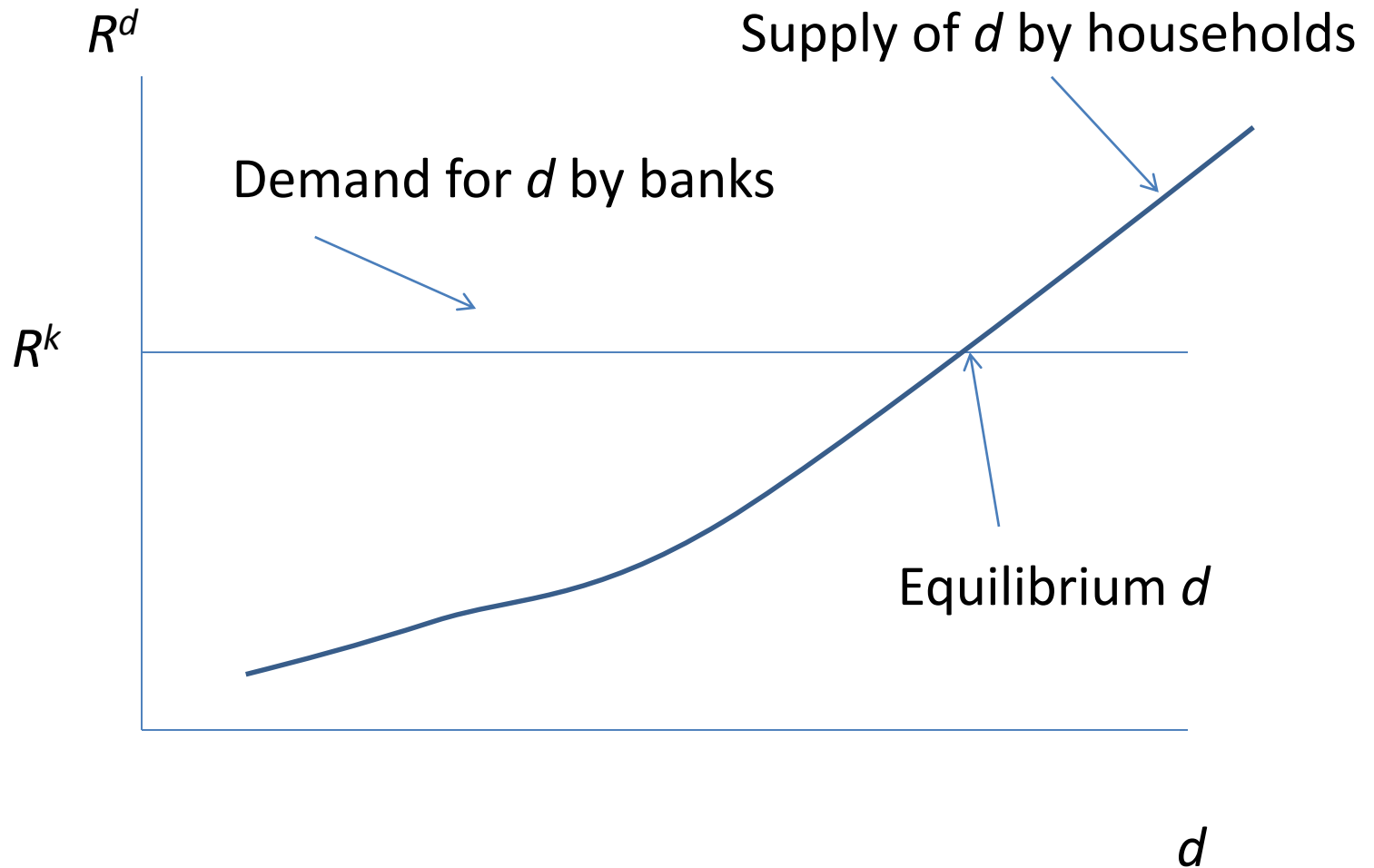
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Efficient Benchmark

Problem of the Bank	
period 1	period 2
take deposits, d	pay dR^d to households
buy securities, $s = N + d$	receive sR^k from firms
problem: $\max_d [sR^k - R^d d]$	

Bank demand for d



Equilibrium in Absence of Frictions

Interior Equilibrium: R^d, π, d, c, C

(i) $c, d, C > 0$

(ii) household problem is solved

(iii) bank problem is solved

(iv) goods and financial markets clear

- Properties:

- Household faces true social rate of return on saving:

$$R^k = R^d$$

- Equilibrium is ‘first best’, i.e., solves

$$\max_{c, C, k} u(c) + \beta u(C)$$

$$c + k \leq y + N, \quad C \leq kR^k$$

Friction

- bank combines deposits, d , with net worth, N , to purchase $N+d$ securities from firms.
- bank has two options:
 - ('no-default') wait until next period when $(N + d)R^k$ arrives and pay off depositors, $R^d d$, for profit:

$$(N + d)R^k - R^d d$$

- ('default') take $\theta(N + d)$ securities, refuse to pay depositors and wait until next period when securities pay off:
$$\theta(N + d)R^k$$
- Bank must announce what value of d it will choose at the beginning of a period.

Incentive Constraint

- Recall, banks maximize profits
- Choose 'no default' iff

$$\overbrace{(N + d)R^k - R^d d}^{\text{no default}} \geq \overbrace{\theta(N + d)R^k}^{\text{default}}$$

- Next: derive banking system's demand for deposits in presence of financial frictions.

Result for a no-default equilibrium:

- Consider an individual bank that contemplates defaulting.
- It sets a d that implies default,

$$R^k(N + d) - R^d d < \theta R^k(d + N) \quad , \text{ or}$$

what the household gets in the other banks

$$\overbrace{R^d} > \frac{\overbrace{(1 - \theta)R^k(d + N)}}{d}$$

what the household gets in the defaulting bank

- A deviating bank will in fact receive no deposits.
- **An optimizing bank would never default**

Problem of the bank in no-default, interior equilibrium

- Maximize, by choice of d ,

$$R^k(N + d) - R^d d$$

If interest rate is REALLY low, then bank has no incentive to default because it makes lots of profits not defaulting

subject to:

$$R^k(N + d) - R^d d - R^k \theta(N + d) \geq 0,$$

or,

$$(1 - \theta)R^k N - [R^d - (1 - \theta)R^k]d \geq 0.$$

- Note that $0 < d < \infty$ requires

if not, then $d = \infty$

if not, then $d = 0$

$(1 - \theta)R^k$

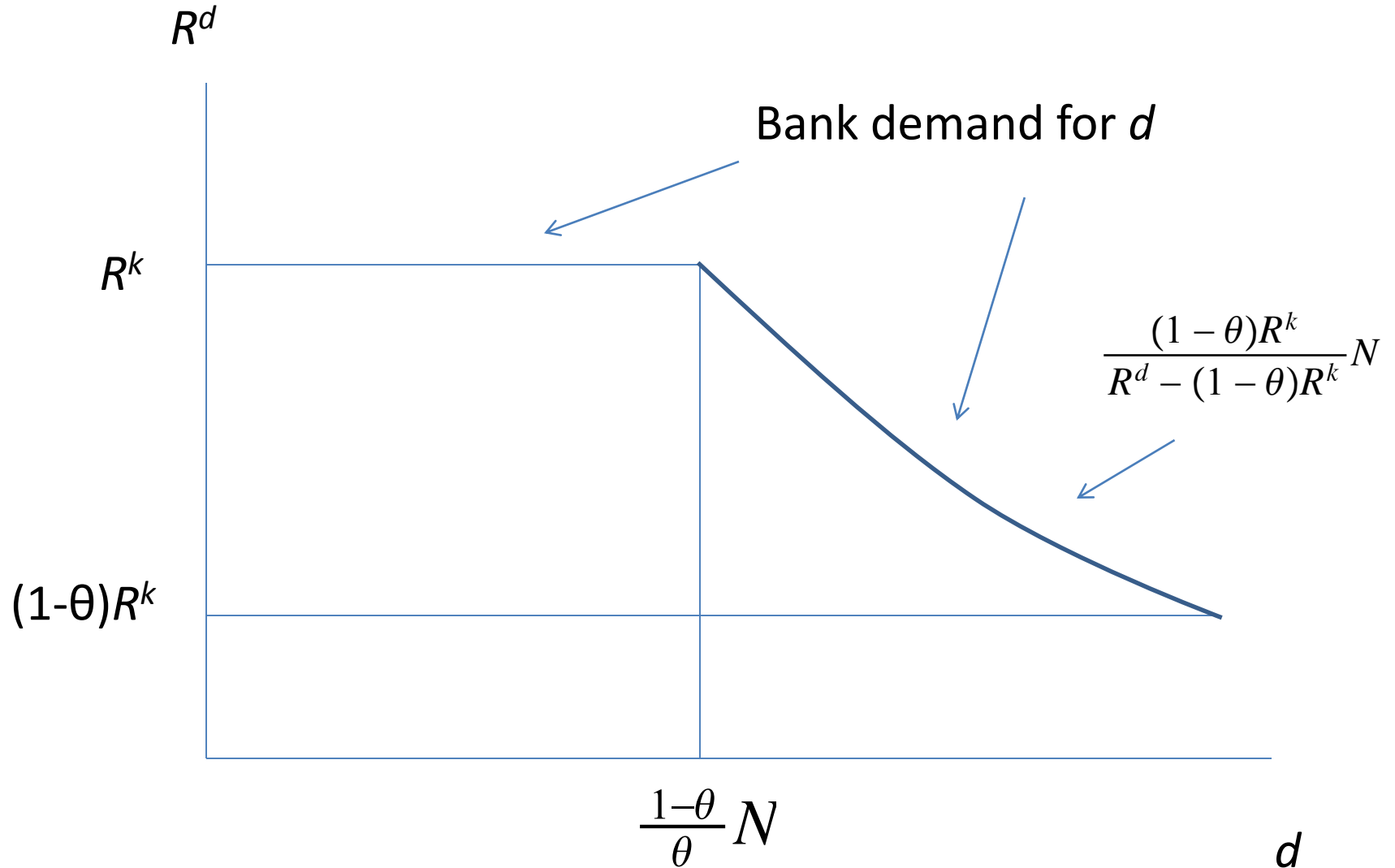
 $\underbrace{\hspace{1em}}_{<}$
 R^d

 $\underbrace{\hspace{1em}}_{\leq}$
 R^k

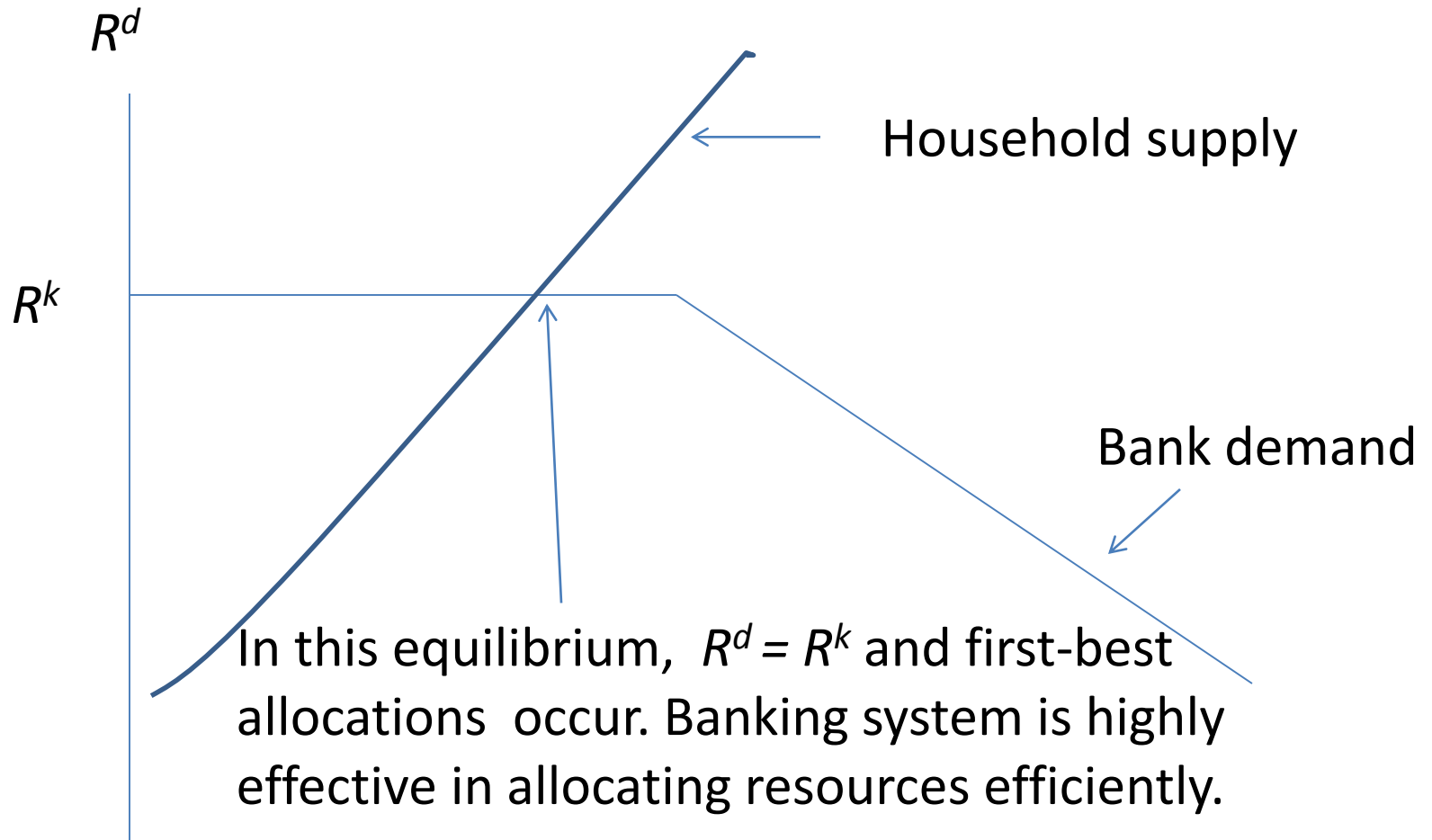
Problem of the bank in no-default, interior equilibrium, cnt'd

- For $R^d = R^k$
 - a bank makes no profits on d so – absent default considerations - it is indifferent over all values of $0 \leq d$
 - Taking into account default, a bank is indifferent over $0 \leq d \leq N(1-\theta)/\theta$
- For $(1-\theta)R^k < R^d < R^k$
 - Bank wants d as large as possible, subject to incentive constraint.
 - So, $d = R^k N(1-\theta) / (R^d - (1-\theta)R^k)$

Bank demand for d



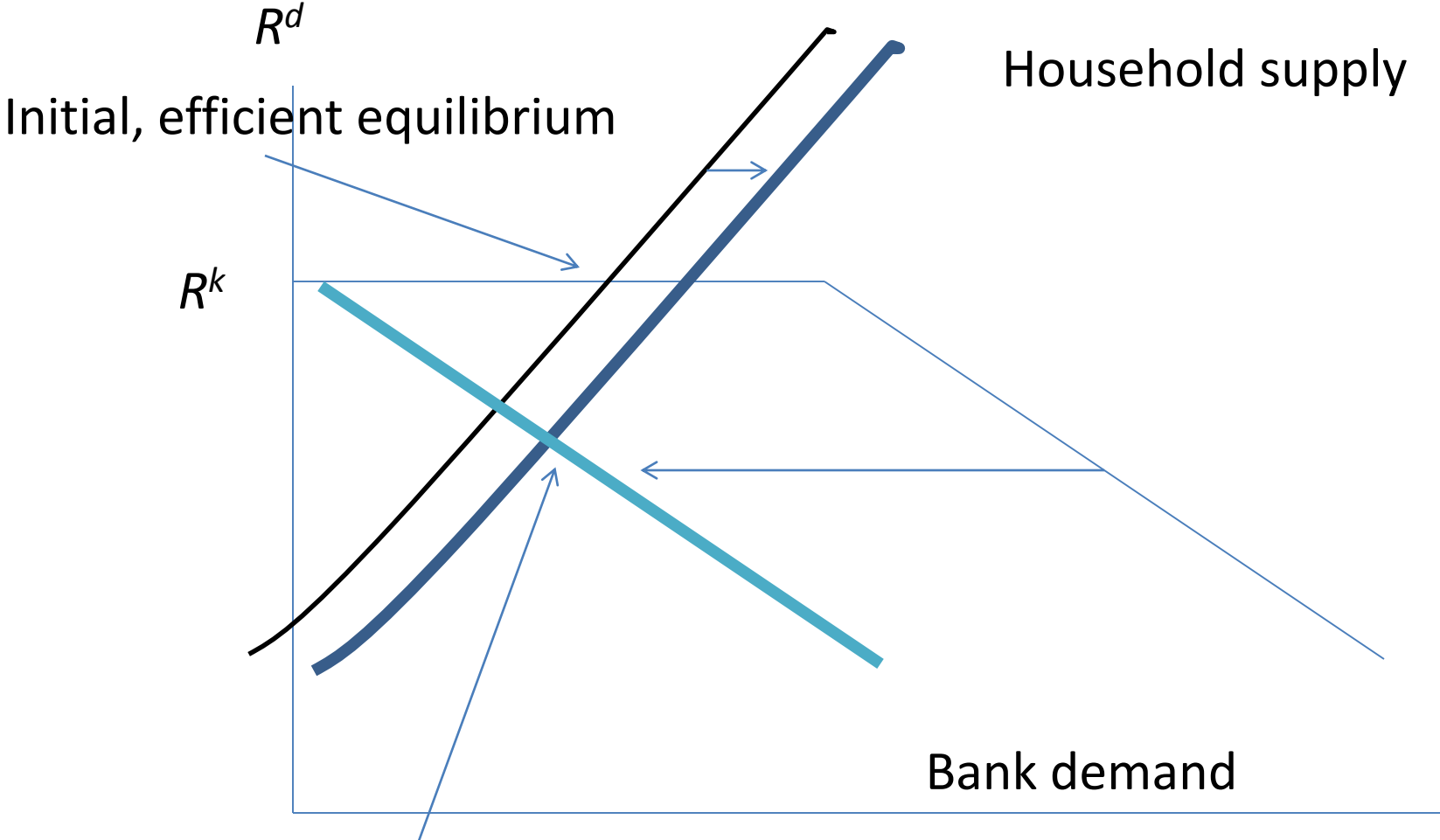
Interior, no default equilibrium



Collapse in Bank Net Worth

- Suppose that the economy is represented by a sequence of repeated versions of the above model.
- In the periods before the 2007-2008 crisis, net worth was high and the equilibrium was like it is on the previous slide: efficient, with zero interest rate spreads.
 - In practice, spreads are always positive, but that reflects various banking costs that are left out of this model.
- With the crisis, N dropped a lot, shifting demand to the right and supply to the left.

Effect of Substantial Drop in Bank Net Worth



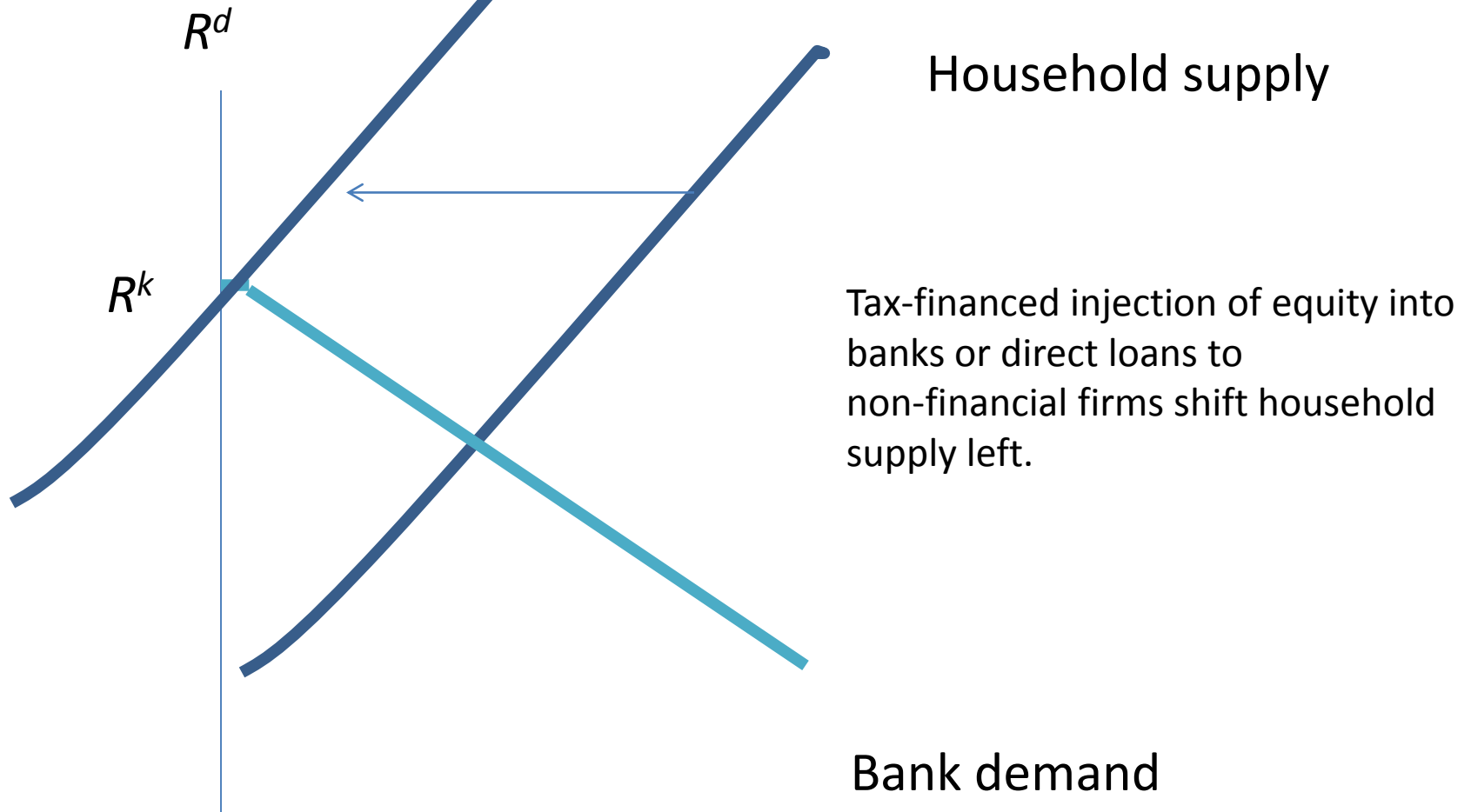
Equilibrium after N drops is inefficient because $R^d < R^k$.

d

Government Intervention

- Equity injection.
 - Government raises T in period 1, provides proceeds to banks and demands $R^k T$ in return at start of period 2.
 - Rebates earnings to households in 2.
- Has no impact on demand for deposits by banks (no impact on default incentive or profits).
- Reduces supply of deposits by households.
 - $d+T$ rises when T rises (even though d falls) because R^d rises.
- Direct, tax-financed government loans to firms work in the same way.
- An interest rate subsidy to banks will shift their demand for deposits to the right....it will also shift supply to the left.

Equity Injection and Drop in N



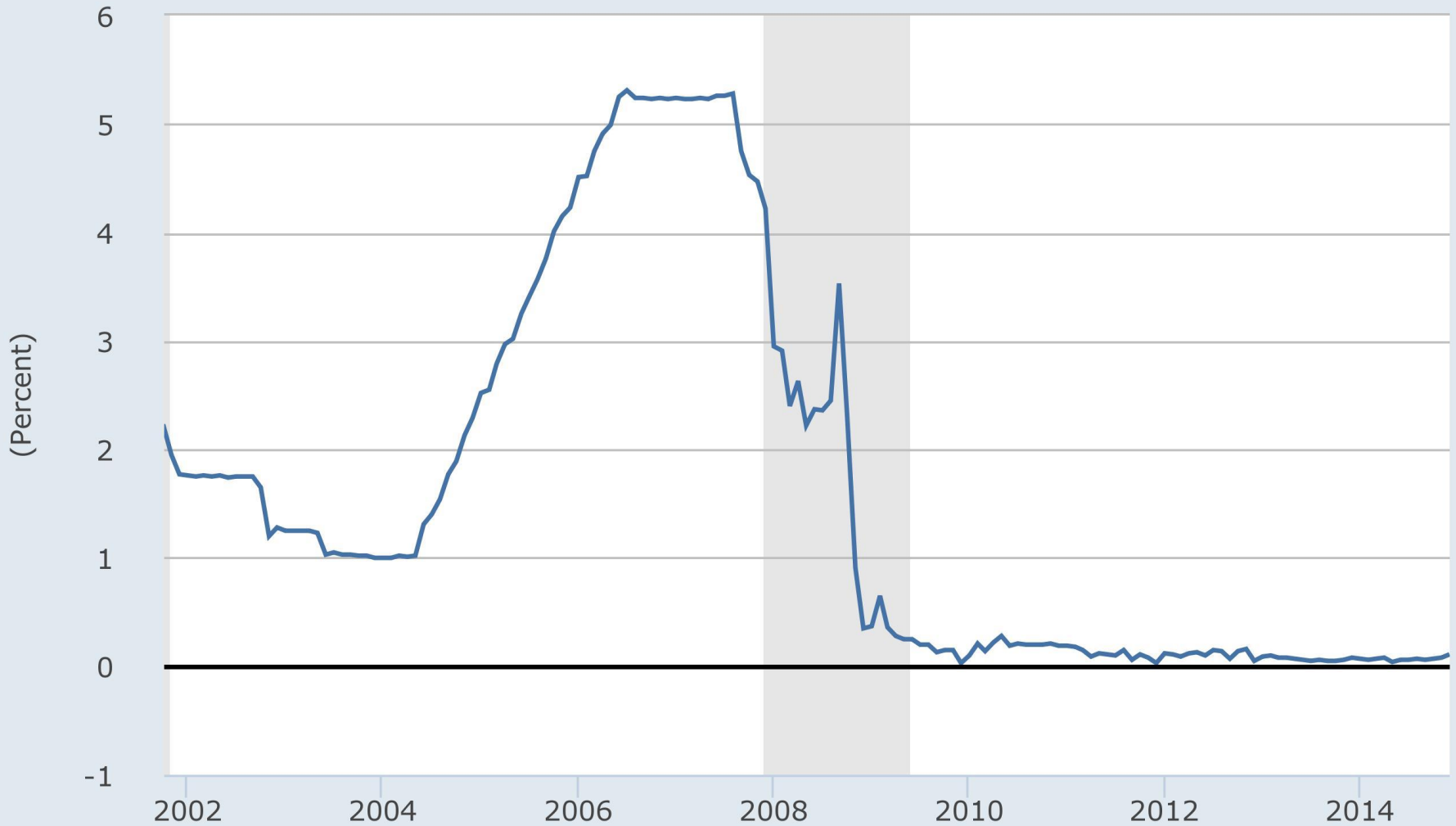
Recap

- Basic idea:
 - Bankers can run away with a fraction of bank assets.
 - If banker net worth is high relative to deposits, friction not a factor and banking system efficient.
 - If banker net worth falls below a certain cutoff, then banker must restrict the deposits.
 - Bankers fear (correctly) that otherwise depositors would lose confidence and take their business to another bank.
 - Reduction in banker demand for deposits:
 - makes deposit interest rates fall and so spreads rise.
 - Reduced intermediation means investment drops, output drops.
 - Equity injections by the government can revive the banking system.

Is the Model Narrative Consistent with the Evidence?

- Model says that reduced intermediation of funds through the financial system reflected reduced demand for credit by financial institutions.
- Prediction: interest rate to financial institutions fall.

— 1-Month AA Financial Commercial Paper Rate



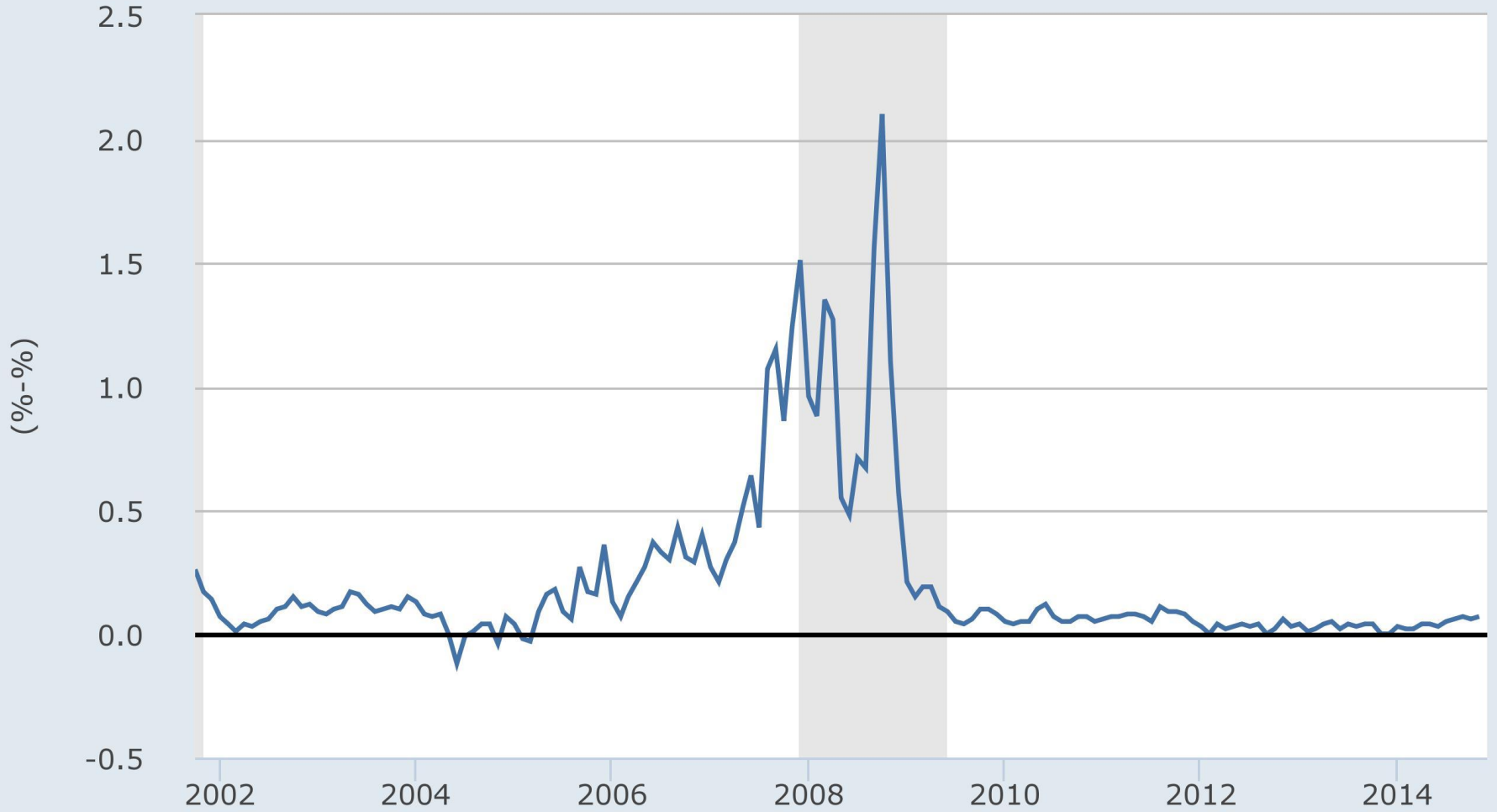
Source: Board of Governors of the Federal Reserve System (US)

Shaded areas indicate US recessions - 2014 research.stlouisfed.org

- Model prediction for decline in cost of funds to financial institutions seems verified.
- But, other 'risk free' interest rates fell even more.
 - Interest rates on US government debt fell more than interest rate on financial firm commercial paper.



— 1-Month AA Financial Commercial Paper Rate-3-Month Treasury Bill: Secondary Market Rate



Shaded areas indicate US recessions - 2014 research.stlouisfed.org

Assessment

- Fact that interest rates on US government debt went down more than cost of funds to financial institutions suggests that a complete picture of financial crisis may require two additional features:
 - Risky Banks:
 - Banks in the model are risk free. Default only occurs out of equilibrium.
 - Increased actual riskiness of banks is perhaps also an important part of the picture.
 - Liquidity:
 - Low interest rates on US government debt consistent with idea that high demand for liquidity played an important role in the crisis.

Macro Prudential Policy

- In recent years there has been increased concern that banks may have a tendency to take on too much debt.
- Has accelerated thinking about debt restrictions on banks.
- There are several models of financial frictions in banks, but they do not necessarily provide a foundation for thinking about debt restrictions on banks.
 - A CSV model of banks implies they issue too *little* debt. (See Christiano-Ikeda).
 - The ‘running away’ model of banks does *not* rationalize debt restrictions. (See next).

Optimal Debt Restriction in Two-Period Running Away Banking Model

- Debt restriction on banks:

$$d \leq \bar{d}$$

- What is the socially optimal level of \bar{d} ?
- To answer this, must take into account structure of private economy
 - The way households choose debt in competitive markets
 - The fact that banks will not choose a debt level that violates incentive constraints.

Social Welfare Function

$$u(c) + \beta u(C)$$

$$= u\left(\underbrace{y-d}_{c}\right) + \beta u\left(\underbrace{\overbrace{R^d d}^{\text{=earnings on deposits}} + \overbrace{+R^k(N+d)-R^d d}^{\text{=bank profits}}}_{C}\right)$$

$$= u(y - d) + \beta u(R^k (N + d)).$$

Household Saving

- Optimization:

$$u'(y - d) = R^d u'(C)$$

plus budget constraint and definition of profits
(see above) implies:

$$d = \frac{(\beta R^d)^{\frac{1}{\gamma}} - \frac{N}{y} R^k}{(\beta R^d)^{\frac{1}{\gamma}} + R^k}.$$

or

$$R^d = \frac{1}{\beta} \left[R^k \left(\frac{\frac{N}{y} + d}{1-d} \right) \right]^{\gamma} \equiv f(d).$$

Implementability Constraint

- Let d^* denote the value of deposits that a benevolent planner wishes the banks would choose.
- Planner must take into account:
 - banks will not choose a level of d which implies a violation of the incentive constraint.
 - market arrangement in which households make their deposit supply decision.
 - these considerations restrict d as follows:

$$(1 - \theta)(N + d)R^k - f(d)d \geq 0$$

Planning Problem

- d^* is solution to the following problem:

$$\max_d u(y - d) + u(R^k(N + d)) + \mu[(1 - \theta)(N + d)R^k - f(d)d]$$

- Fonc

$=u'(y-d)/R^d$ by households

$$\overbrace{u'(C)} \times R^k - u'(y - d) + \mu[(1 - \theta)R^k - f'(d)d - f(d)] = 0$$

$$\mu \geq 0, [(1 - \theta)(N + d)R^k - f(d)d] \geq 0, \mu[(1 - \theta)(N + d)R^k - f(d)d] = 0 \quad .$$

Planning Problem

- First order conditions:

$$u'(y - d) \left[\frac{R^k}{f(d)} - 1 \right] + \mu [(1 - \theta)R^k - f'(d)d - f(d)] = 0$$

Complementary Slackness

$$\mu \geq 0, [(1 - \theta)(N + d)R^k - f(d)d] \geq 0, \mu [(1 - \theta)(N + d)R^k - f(d)d] = 0$$

- Solving the problem:

- Try $\mu = 0$ and solve ('savings supply crosses horizontal line at R^k) $R^k = f(d)$

- Check incentive constraint. If satisfied, $R^k = f(d^*)$

- Otherwise, conclude $\mu > 0$ and

$$(1 - \theta)(N + d^*)R^k - f(d^*)d^* = 0$$

- ('Savings supply crosses incentive constraint').

No Borrowing Restrictions Desired

- Deposits selected by government coincide with equilibrium deposits when there is no borrowing restriction.
- So, according to the model, restriction on bank borrowing not necessary.
- Model is not a good laboratory for thinking about leverage restrictions on banks, if you're firmly convinced that leverage restrictions are required.