Simple New Keynesian Model without Capital, but With Networks

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Objectives

- Provide a rigorous development of the basic New Keynesian model without capital.
  - Previous exposure to the model is helpful, but not absolutely necessary.

- Present a version of the model that incorporates a simple formulation of the 'network' nature of production.
  - In standard model, all production is sold directly to final purchasers.
  - In fact (see, e.g., Basu AER1996) about 1/2 of gross production by firms is sold to other firms.
    - See Christiano, Trabandt and Walentin (Handbook of Monetary Economics, 2011) for an extended discussion of the approach to networks developed here.

Implications of thinking about networks

- Obtain a quantitatively important theory of the cost of inflation.
- Raise questions about the effectiveness of inflation targeting as a device for stabilizing inflation and the macroeconomy.
- Flatten the slope of the Phillips curve because of strategic complementarities in price setting.

Background Readings on Networks

- Basu, Susanto, 1995, 'Intermediate goods and business cycles: Implications for productivity and welfare,' *American Economic Review*, 85 (3), 512–531.
- Rotemberg, J., and M. Woodford, 1995, 'Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets,' in, T. Cooley, ed., *Frontiers of Business Cycle Research,* Princeton University Press (also, NBER wp 4502).
- Nakamura, Emi and Jon Steinsson, 2010, 'Monetary Non-Neutrality in a Multisector Menu Cost Model,' *The Quarterly Journal of Economics*, August.
- Jones, Chad, 2013, 'Misallocation, Economic Growth, and Input-Output Economics,' in D. Acemoglu, M. Arellano, and E. Dekel, *Advances in Economics and Econometrics*, Tenth World Congress, Volume II, Cambridge University Press.
- Daron Acemoglu, Ufuk Akcigit, William Kerr, 'Networks and the Macroeconomy: An Empirical Exploration,' NBER Macroeconomics Annual 2015.

Households

- Problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}$$

$$\text{s.t. } P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + \text{Profits net of taxes}_t$$

- First order conditions:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5)$$

$$\exp(\tau_t) C_t N_t^{\varphi} = \frac{W_t}{P_t}.$$

Goods Production

- A homogeneous final good is produced using the following (Dixit-Stiglitz) production function:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

- Each intermediate good, $Y_{i,t}$, is produced as follows:

$$\begin{aligned} Y_{i,t} &= \exp(a_t) N_{i,t}^{\gamma} I_{i,t}^{1-\gamma},\ a_t \text{ ~exogenous shock to technology,} \\ 0 &< \gamma \leq 1. \end{aligned}$$

- $I_{i,t}$ ~'materials' these are purchases of the homogeneous output good (Basu's simplified way of capturing that firms buy goods from other firms).
- Before discussing the firms that operate these production functions, we briefly investigate the socially efficient ('First Best') allocation of resources across i.
  - simplify the discussion with $\gamma = 1$ (no materials).

Efficient Sectoral Allocation of Resources Across Sectors

- With Dixit-Stiglitz final good production function, there is a socially optimal allocation of resources to all the intermediate activities, $Y_{i,t}$
  - It is optimal to run them all at the same rate, *i.e.*, $Y_{i,t} = Y_{j,t}$ for all $i, j \in [0,1]$.
- For given N_t, it is optimal to set $N_{i,t} = N_{j,t}$, for all $i, j \in [0,1]$
- In this case, final output is given by

$$Y_t = e^{a_t} N_t.$$

- Best way to see this is to suppose that labor is *not* allocated equally to all activities.
  - Explore one simple deviation from $N_{i,t} = N_{j,t}$ for all $i, j \in [0,1]$.

Suppose Labor *Not* Allocated Equally

- Example:

$$N_{it} = \begin{cases} 2\alpha N_t & i \in \left[0, \frac{1}{2}\right] \\ 2(1-\alpha)N_t & i \in \left[\frac{1}{2}, 1\right] \end{cases}, \; 0 \leq \alpha \leq 1.$$

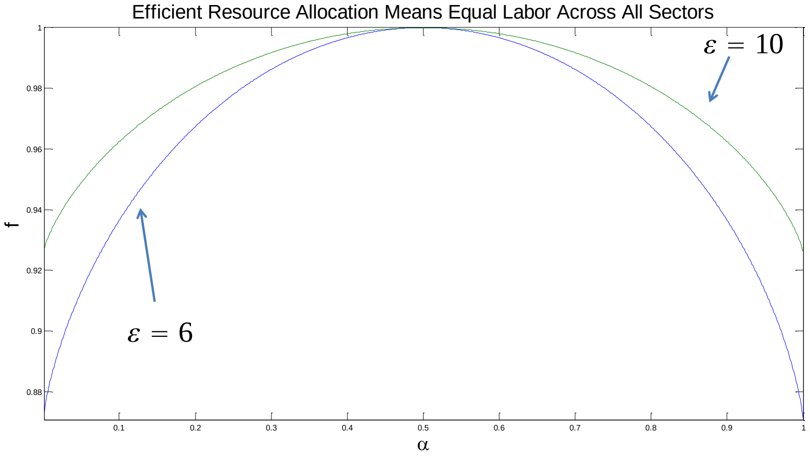
- Note that this is a particular distribution of labor across activities:

$$\int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1-\alpha)N_t = N_t$$

Labor *Not* Allocated Equally, cnt'd

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\int_0^{\frac{1}{2}} Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} (2\alpha N_t)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha)N_t)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\int_0^{\frac{1}{2}} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\frac{1}{2} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t f(\alpha)
\end{aligned}
$$

$$f(\alpha) = \left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$



Homogeneous Goods Production

- Competitive firms:
  - maximize profits:

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,$$

subject to:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} .$$

  - Foncs:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon} \rightarrow \overbrace{P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}}^{\text{"cross price restrictions"}}$$

Intermediate Goods Production

- Demand curve for i^{th} monopolist:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon}.$$

- Production function:

$$\begin{aligned} Y_{i,t} &= \exp(a_t) N_{i,t}^{\gamma} I_{i,t}^{1-\gamma},\ a_t \text{ ~exogenous shock to technology,} \\ 0 &< \gamma \leq 1. \end{aligned}$$

- $I_{i,t}$ ~'materials' these are purchases of the homogeneous output good (Basu's simplified way of capturing that firms buy goods from other firms).
- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}.$$

Cost Minimization Problem

- Price setting by intermediate good firms is discussed later.
  - The intermediate good firm must produce the quantity demanded, $Y_{i,t}$, at the price that it sets.
  - Right now we take $Y_{i,t}$ as given and we investigate the cost minimization problem that determines the firm's choice of inputs.

Cost minimization problem:

$$\min_{N_{i,t},I_{i,t}} \bar{W}_t N_{i,t} + \bar{P}_t I_{i,t} + \overbrace{\lambda_{i,t}}^{\text{marginal cost (money terms)}} \left[Y_{i,t} - A_t N_{i,t}^{\gamma} I_{i,t}^{1-\gamma} \right]$$

with resource costs:

$$\begin{aligned} \bar{W}_t &= \overbrace{(1-\nu)}^{\text{subsidy, if } \nu>0} \times \overbrace{(1-\psi+\psi R_t)\, W_t}^{\text{cost, including finance, of a unit of labor}} \\ \bar{P}_t &= (1-\nu) \times \overbrace{(1-\psi+\psi R_t)\, P_t}^{\text{cost, including finance, of a unit of materials}} . \end{aligned}$$

Cost Minimization Problem

- Problem:

$$\min_{N_{i,t}, I_{i,t}} \bar{W}_t N_{i,t} + \bar{P}_t I_{i,t} + \lambda_{i,t} \left[Y_{i,t} - A_t N_{i,t}^{\gamma} I_{i,t}^{1-\gamma} \right]$$

- First order conditions:

$$\bar{P}_t I_{i,t} = (1-\gamma)\,\lambda_{i,t} Y_{i,t},\ \bar{W}_t N_{i,t} = \gamma \lambda_{i,t} Y_{i,t},$$

so that,

$$\begin{aligned} \frac{I_{it}}{N_{it}} &= \frac{1-\gamma}{\gamma} \frac{\bar{W}_t}{\bar{P}_t} = \frac{1-\gamma}{\gamma} \exp(\tau_t)\, C_t N_t^{\varphi} \\ &\rightarrow \frac{I_{it}}{N_{it}} = \frac{I_t}{N_t},\ \text{for all } i. \end{aligned}$$

Cost Minimization Problem

- Firm first order conditions imply

$$\lambda_{i,t} = \left(\frac{\bar{P}_t}{1-\gamma} \right)^{1-\gamma} \left(\frac{\bar{W}_t}{\gamma} \right)^{\gamma} \frac{1}{A_t}.$$

- Divide marginal cost by P_t :

$$\begin{aligned} s_t \;\; &\equiv \;\; \frac{\lambda_{i,t}}{P_t} = (1-\nu)\,(1-\psi+\psi R_t) \left(\frac{1}{1-\gamma} \right)^{1-\gamma} \\ &\times \left(\frac{1}{\gamma} \exp\left(\tau_t\right) C_t N_t^{\varphi} \right)^{\gamma} \frac{1}{A_t} \text{ (9)}, \end{aligned}$$

  after substituting out for $\bar{P}_t$ and $\bar{W}_t$ and using the household's labor first order condition.

- Note from (9) that i^{th} firm's marginal cost, s_t, is independent of i and $Y_{it,}$.

Share of Materials in Intermediate Good Output

- Firm i materials proportional to $Y_{i,t}$:

$$I_{i,t} = \frac{(1-\gamma)\,\lambda_{i_it} Y_{i,t}}{\bar{P}_t} = \mu_t Y_{i,t},$$

where

$$\mu_t = \frac{(1-\gamma)\,s_t}{(1-\nu)\,(1-\psi+\psi R_t)} \text{ (10).}$$

- "Share of materials in firm-level gross output", μ_t.

Decision By Firm that Can Change Its Price

- i^{th} intermediate good firm's objective:

$$E_t^i \sum_{j=0}^{\infty} \beta^j \ v_{t+j} \overbrace{\left[\overbrace{P_{i,t+j} Y_{i,t+j}}^{\text{revenues}} - \overbrace{P_{t+j} s_{t+j} Y_{i,t+j}}^{\text{total cost}} \right]}^{\text{period } t+j \text{ profits sent to household}}$$

v_{t+j} - Lagrange multiplier on household budget constraint

- Firm that gets to reoptimize its price is concerned only with future states in which it does not change its price:

$$\begin{aligned} & E_t^i \sum_{j=0}^{\infty} \beta^j v_{t+j} \left[P_{i,t+j} Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right] \\ = \ & E_t \sum_{j=0}^{\infty} (\beta\theta)^j v_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right] + X_t, \end{aligned}$$

where $\tilde{P}_t$ denotes a firm's price-setting choice at time t and X_t not a function of $\tilde{P}_t$.

Decision By Firm that Can Change Its Price

- Substitute out demand curve:

$$
\begin{aligned}
& E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j}\right] \\
= \;\; & E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[\tilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon}\right].
\end{aligned}
$$

- Differentiate with respect to $\tilde{P}_t$:

$$
E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[(1-\varepsilon) \left(\tilde{P}_t\right)^{-\varepsilon} + \varepsilon P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon-1}\right] = 0,
$$

or,

$$
E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j}\right] = 0.
$$

- When $\theta = 0$, get standard result - price is fixed markup over marginal cost.

Decision By Firm that Can Change Its Price

- Substitute out the multiplier:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \overbrace{\frac{u'\left(C_{t+j}\right)}{P_{t+j}}}^{=v_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon-1} s_{t+j}\right] = 0.$$

- Using assumed log-form of utility,

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon-1} s_{t+j}\right] = 0,$$

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t},\ \bar{\pi}_t \equiv \frac{P_t}{P_{t-1}},\ X_{t,j} = \begin{cases} \frac{1}{\bar{\pi}_{t+j}\bar{\pi}_{t+j-1}\cdots\bar{\pi}_{t+1}},\ j \geq 1 \\ 1,\ j = 0. \end{cases},$$

'recursive property': $X_{t,j} = X_{t+1,j-1}\dfrac{1}{\bar{\pi}_{t+1}},\ j > 0$

Decision By Firm that Can Change Its Price

- Want $\tilde{p}_t$ in:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j}\right] = 0$$

- Solving for $\tilde{p}_t$, we conclude that prices are set as follows:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+1}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_t}{F_t}.$$

- Need convenient expressions for K_t, F_t.

Simplifying Numerator

$$
\begin{aligned}
K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j} \\
&= \frac{\varepsilon}{\varepsilon-1} \frac{Y_t}{C_t} s_t \\
&\quad + \beta\theta E_t \sum_{j=1}^{\infty} (\beta\theta)^{j-1} \frac{Y_{t+j}}{C_{t+j}} \left(\overbrace{\frac{1}{\bar{\pi}_{t+1}} X_{t+1,j-1}}^{=X_{t,j},\ \text{recursive property}} \right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j} \\
&= \frac{\varepsilon}{\varepsilon-1} \frac{Y_t}{C_t} s_t + \mathcal{Z}_t,
\end{aligned}
$$

where

$$
\mathcal{Z}_t = \beta\theta E_t \sum_{j=1}^{\infty} (\beta\theta)^{j-1} \frac{Y_{t+j}}{C_{t+j}} \left(\frac{1}{\bar{\pi}_{t+1}} X_{t+1,j-1} \right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}
$$

Simplifying Numerator, cnt'd

$$K_t = E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+j} = \frac{\varepsilon}{\varepsilon - 1} \frac{Y_t}{C_t} s_t + \mathcal{Z}_t$$

$$\begin{aligned}
\mathcal{Z}_t &= \beta\theta E_t \sum_{j=1}^{\infty} (\beta\theta)^{j-1} \frac{Y_{t+j}}{C_{t+j}} \left(\frac{1}{\bar{\pi}_{t+1}} X_{t+1,j-1} \right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \\
&= \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}} \right)^{-\varepsilon} \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j+1}}{C_{t+j+1}} X_{t+1,j}^{-\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+1+j} \\
&= \beta\theta \overbrace{E_t E_{t+1}}^{=E_t \text{ by LIME}} \left(\frac{1}{\bar{\pi}_{t+1}} \right)^{-\varepsilon} \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j+1}}{C_{t+j+1}} X_{t+1,j}^{-\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+1+j} \\
&= \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}} \right)^{-\varepsilon} E_{t+1} \overbrace{\sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j+1}}{C_{t+j+1}} X_{t+1,j}^{-\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+1+j}}^{\text{exactly } K_{t+1}!}
\end{aligned}$$

Decision By Firm that Can Change Its Price

- Recall,

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+1}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_t}{F_t}$$

- We have shown that the numerator has the following simple representation:

$$\begin{aligned} K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j} \\ &= \frac{\varepsilon}{\varepsilon-1} \frac{Y_t}{C_t} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1} \text{ (1)} \end{aligned}$$

- Similarly,

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{1-\varepsilon} F_{t+1} \text{ (2)}$$

Interpretation of Price Formula

- Note,

$$\frac{1}{P_{t+j}} = \frac{1}{P_t} X_{t,j},\ s_{t+j} = \frac{\lambda_{t+j}}{P_{t+j}} = \frac{\lambda_{t+j}}{P_t} X_{t,j},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t}.$$

Multiply both sides of the expression for $\tilde{p}_t$ by P_t :

$$\tilde{P}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}} \frac{\varepsilon}{\varepsilon-1} \lambda_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}} = \frac{\varepsilon}{\varepsilon-1} \sum_{j=0}^{\infty} E_t \omega_{t+j} \lambda_{t+j}$$

where

$$\omega_{t+j} = \frac{(\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}},\ \sum_{j=0}^{\infty} E_t \omega_{t+j} = 1.$$

Evidently, price is set as a markup over a weighted average of future marginal cost, where the weights are shifted into the future depending on how big θ is.

Moving On to Aggregates

- Aggregate price level.
- Aggregate measures of production.
  - Value added.
  - Gross output.

Aggregate Price Index

- Rewrite the aggregate price index.
  - let $p \in (0, \infty)$ the set of logically possible prices for intermediate good producers.
  - let $g_t(p) \geq 0$ denote the measure (e.g., 'number') of producers that have price, p, in t
  - let $g_{t-1,t}(p) \geq 0$, denote the measure of producers that had price, p, in $t-1$ and could not reoptimize in t
- Then,

$$P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}} = \left(\int_0^\infty g_t(p)\, p^{(1-\varepsilon)} dp \right)^{\frac{1}{1-\varepsilon}}.$$

- Note:

$$P_t = \left((1-\theta)\, \tilde{P}_t^{1-\varepsilon} + \int_0^\infty g_{t-1,t}(p)\, p^{(1-\varepsilon)} dp \right)^{\frac{1}{1-\varepsilon}}.$$

Aggregate Price Index

- Calvo randomization assumption:

$$\begin{aligned}\overbrace{g_{t-1,t}(p)}^{\text{measure of firms that had price, } p\text{, in } t-1 \text{ and could not change}} \\ = \theta \times \overbrace{g_{t-1}(p)}^{\text{measure of firms that had price } p \text{ in } t-1}\end{aligned}$$

- Then,

$$\begin{aligned} P_t &= \left((1-\theta)\tilde{P}_t^{1-\varepsilon} + \int_0^\infty g_{t-1,t}(p)\, p^{(1-\varepsilon)} dp \right)^{\frac{1}{1-\varepsilon}} \\ &= \left((1-\theta)\tilde{P}_t^{1-\varepsilon} + \theta \overbrace{\int_0^\infty g_{t-1}(p)\, p^{(1-\varepsilon)} dp}^{=P_{t-1}^{1-\varepsilon}} \right)^{\frac{1}{1-\varepsilon}} \end{aligned}$$

Restriction Between Aggregate and Intermediate Good Prices

- 'Calvo result':

$$P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}} = \left[(1-\theta) \tilde{P}_t^{(1-\varepsilon)} + \theta P_{t-1}^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}.$$

- Divide by P_t :

$$1 = \left[(1-\theta) \tilde{p}_t^{(1-\varepsilon)} + \theta \left(\frac{1}{\bar{\pi}_t} \right)^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}.$$

- Rearrange:

$$\tilde{p}_t = \left[\frac{1-\theta}{1-\theta \bar{\pi}_t^{(\varepsilon-1)}} \right]^{\frac{1}{\varepsilon-1}}$$

Aggregate inputs and outputs

- *Gross output* of firm i :

$$Y_{i,t} = \exp\left(a_t\right) N_{i,t}^{\gamma} I_{i,t}^{1-\gamma}.$$

  – Net output or *value-added* would subtract out the materials that were bought from other firms.

- Economy-wide *gross output*: sum of value of $Y_{i,t}$ across all firms:

$$\begin{aligned}\int_0^1 P_{i,t} Y_{i,t} di &= \int_0^1 P_t \left(\frac{Y_t}{Y_{i,t}}\right)^{\frac{1}{\varepsilon}} Y_{i,t} di \\ &= P_t Y_t^{\frac{1}{\varepsilon}} \overbrace{\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di}^{=Y_t^{\frac{\varepsilon-1}{\varepsilon}}} = P_t Y_t.\end{aligned}$$

- *Gross output production function*: relation between Y_t and non-produced inputs, N_t.

Aggregate inputs and outputs, cnt'd

- Gross output, Y_t, is not a good measure of economic output, because it double counts.
  - Some of the output that firm i 'produced' is materials produced by another firm, which is counted in that firm's output.
  - If wheat is used to make bread, wrong to measure production by adding all wheat and all bread. That double counts the wheat.
- Want aggregate *value-added*: sum of firm-level gross output, minus purchases of materials from other firms.
- *Value-added production function*: expression relating aggregate value-added in period t to inputs not produced in period t.
  - capital and labor.

Gross Output vs Agg Materials and Labor

- Approach developed by Tack Yun (JME, 1996).
- Define Y_t^*:

$$
\begin{aligned}
Y_t^* &\equiv \int_0^1 Y_{i,t} di \\
&\overbrace{=}^{\text{demand curve}} \quad Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} di = Y_t P_t^{\varepsilon} \int_0^1 \left(P_{i,t} \right)^{-\varepsilon} di \\
&= Y_t P_t^{\varepsilon} \left(P_t^* \right)^{-\varepsilon}
\end{aligned}
$$

where, using ‘Calvo result’:

$$
P_t^* \equiv \left[\int_0^1 P_{i,t}^{-\varepsilon} di \right]^{\frac{-1}{\varepsilon}} = \left[(1-\theta) \tilde{P}_t^{-\varepsilon} + \theta \left(P_{t-1}^* \right)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}
$$

- Then

$$
Y_t = p_t^* Y_t^*, \; p_t^* = \left(\frac{P_t^*}{P_t} \right)^{\varepsilon}.
$$

Gross Output vs Agg Materials and Labor

- Relationship between aggregate inputs and outputs:

$$\begin{aligned} Y_t &= p_t^* Y_t^* = p_t^* \int_0^1 Y_{i,t} di \\ &= p_t^* A_t \int_0^1 N_{i,t}^{\gamma} I_{i,t}^{1-\gamma} di = p_t^* A_t \int_0^1 \left(\frac{N_{i,t}}{I_{i,t}} \right)^{\gamma} I_{i,t} di, \\ &= p_t^* A_t \left(\frac{N_t}{I_t} \right)^{\gamma} I_t, \end{aligned}$$

  or,

$$Y_t = p_t^* A_t N_t^{\gamma} I_t^{1-\gamma} \text{ (6)}.$$

- Note that p_t^* is a function of the ratio of two averages (with different weights) of $P_{i,t}$, $i \in (0,1)$
  - So, when $P_{i,t} = P_{j,t}$ for all $i, j \in (0,1)$, then $p_t^* = 1$.
  - But, what is p_t^* when $P_{i,t} \neq P_{j,t}$ for some $i, j \in (0,1)$?

Tack Yun Distortion

- Consider the object,

$$p_t^* = \left(\frac{P_t^*}{P_t}\right)^{\varepsilon},$$

where

$$P_t^* = \left(\int_0^1 P_{i,t}^{-\varepsilon} di\right)^{\frac{-1}{\varepsilon}},\ P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{1}{1-\varepsilon}}$$

- In following slide, use Jensen's inequality to show:

$$p_t^* \leq 1.$$

Tack Yun Distortion

- Let $f(x) = x^4$, a convex function. Then,

$$\text{convexity: } \alpha x_1^4 + (1-\alpha) x_2^4 > (\alpha x_1 + (1-\alpha) x_2)^4$$

  for $x_1 \neq x_2$, $0 < \alpha < 1$.

- Applying this idea to prices:

$$\text{convexity: } \int_0^1 \left(P_{i,t}^{(1-\varepsilon)}\right)^{\frac{\varepsilon}{\varepsilon-1}} di \;\geq\; \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\iff \left(\int_0^1 P_{i,t}^{-\varepsilon} di\right) \;\geq\; \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\iff \overbrace{\left(\int_0^1 P_{i,t}^{-\varepsilon} di\right)^{\frac{-1}{\varepsilon}}}^{P_t^*} \;\leq\; \overbrace{\left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{1}{1-\varepsilon}}}^{P_t}$$

Law of Motion of Tack Yun Distortion

- We have

$$P_t^* = \left[(1-\theta) \tilde{P}_t^{-\varepsilon} + \theta \left(P_{t-1}^* \right)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}$$

- Then,

$$\begin{aligned} p_t^* &\equiv \left(\frac{P_t^*}{P_t} \right)^{\varepsilon} = \left[(1-\theta) \tilde{p}_t^{-\varepsilon} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \\ &= \left[(1-\theta) \left(\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \quad (4) \end{aligned}$$

using the restriction between $\tilde{p}_t$ and aggregate inflation developed earlier.

Gross Output Production Function

- Recall

$$I_{i,t} = \mu_t Y_{i,t},$$

so,

$$I_t \equiv \int_0^1 I_{i,t} di = \mu_t \int_0^1 Y_{i,t} d = \mu_t Y_t^* = \frac{\mu_t}{p_t^*} Y_t.$$

- Then, the gross output production function is:

$$\begin{aligned} Y_t &= p_t^* A_t N_t^{\gamma} I_t^{1-\gamma} \\ &= p_t^* A_t N_t^{\gamma} \left(\frac{\mu_t}{p_t^*} Y_t \right)^{1-\gamma} \\ \longrightarrow & \quad Y_t = \left(p_t^* A_t \left(\frac{\mu_t}{p_t^*} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} N_t \end{aligned}$$

Value Added (GDP) Production Function

- We have

$$
\begin{aligned}
GDP_t &= Y_t - I_t = \left(1 - \frac{\mu_t}{p_t^*}\right) Y_t \\
&= \left(1 - \frac{\mu_t}{p_t^*}\right) \left(p_t^* A_t \left(\frac{\mu_t}{p_t^*}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}} N_t \\
&= \overbrace{\left(p_t^* A_t \left(1 - \frac{\mu_t}{p_t^*}\right)^{\gamma} \left(\frac{\mu_t}{p_t^*}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}^{=\text{Total Factor Productivity (TFP)}} N_t
\end{aligned}
$$

- Note how an increase in technology at the firm level, by A_t, gives rise to a bigger increase in TFP by $A_t^{1/\gamma}$.
  - In the literature on networks, $1/\gamma$ is referred to as a 'multiplier effect' (see Jones, 2011).
- The Tack Yun distortion, p_t^*, is associated with the same multiplier phenomenon.

Decomposition for TFP

- To maximize GDP for given aggregate N_t and A_t :

$$\max_{0<p_t^*\le 1,\, 0\le\lambda_t\le 1} \left(p_t^* A_t \left(1-\lambda_t\right)^{\gamma}\left(\lambda_t\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}$$

$$\rightarrow \quad \lambda_t = 1-\gamma,\ p_t^* = 1.$$

- So,

$$TFP_t \quad = \quad \overbrace{\left(p_t^* \left(\frac{1-\frac{\mu_t}{p_t^*}}{\gamma}\right)^{\gamma}\left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}^{\text{Component due to market distortions}\equiv\chi_t}$$

$$\times \quad \overbrace{\left(A_t \left(\gamma\right)^{\gamma}\left(1-\gamma\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}^{\text{Exogenous, technology component}\equiv\tilde{A}_t}$$

Evaluating the Distortions

- The equations characterizing the TFP distortion, χ_t :

$$\chi_t = \left(p_t^* \left(\frac{1 - \frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}}$$

$$p_t^* = \left[(1-\theta) \left(\frac{1 - \theta \bar{\pi}_t^{(\varepsilon - 1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon - 1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1}.$$

- Potentially, NK model provides an 'endogenous theory of TFP'.
- Standard practice in NK literature is to set $\chi_t = 1$ for all t.
  - Set $\gamma = 1$ and linearize around $\bar{\pi}_t = p_t^* = 1$.
  - With $\gamma = 1$, $\chi_t = p_t^*$, and first order expansion of p_t^* around $\bar{\pi}_t = p_t^* = 1$ is:

$$p_t^* = p^* + 0 \times \bar{\pi}_t + \theta \left(p_{t-1}^* - p^* \right), \text{ with } p^* = 1,$$

so $p_t^* \rightarrow 1$ and is invariant to shocks.

Empirical Assessment of the Distortions

- First, do 'back of the envelope' calculations in a steady state when inflation is constant and p^* is constant.

$$p^* = \left[(1-\theta)\left(\frac{1-\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}^{\varepsilon}}{p^*} \right]^{-1}$$

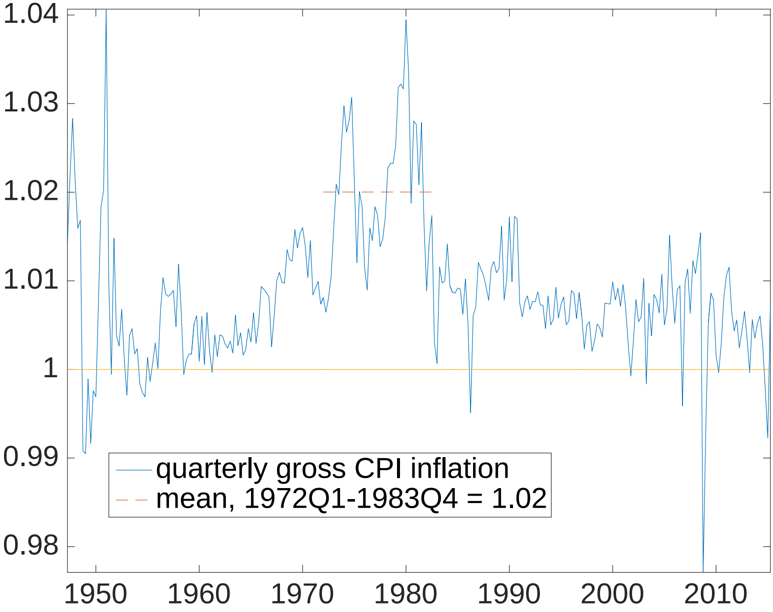
$$\rightarrow p^* = \frac{1-\theta\bar{\pi}^{\varepsilon}}{1-\theta}\left(\frac{1-\theta}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

- Approximate TFP distortion, χ :

$$\chi_t = \left(p_t^* \left(\frac{1-\frac{\mu_t}{p_t^*}}{\gamma}\right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma}\right)^{1-\gamma} \right)^{\frac{1}{\gamma}} \overbrace{\simeq}^{\text{more on this later}} (p^*)^{1/\gamma}$$

Three Inflation Rates:

- Average inflation in the 1970s, 8 percent APR.
- Several people have suggested that the US raise its inflation target to 4 percent to raise the nominal rate of interest and thereby reduce the likelihood of the zero lower bound on the interest rate becoming binding again.
  - http://www.voxeu.org/article/case-4-inflation
- Two percent inflation is the average in the recent (pre-2008) low inflation environment.


1.04
1.03
1.02
1.01
1
0.99
0.98
1950
1960
1970
1980
1990
2000
2010
quarterly gross CPI inflation
mean, 1972Q1-1983Q4 = 1.02

Cost of Three Alternative Permanent Levels of Inflation

$$p^* = \frac{1-\theta\bar{\pi}^{\varepsilon}}{1-\theta}\left(\frac{1-\theta}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\right)^{\frac{\varepsilon}{\varepsilon-1}},\ \chi = (p^*)^{1/\gamma}$$

Table 1: Percent of GDP Lost[1] Due to Inflation, 100$(1-\chi_t)$	
Without networks ($\gamma = 1$)	With networks ($\gamma = 1/2$)
a: Steady state inflation: 8 percent per year	
2.41[2] (3.92) [10.85]	4.76 (7.68) [20.53]
b: Steady state inflation: 4 percent per year	
0.46 (0.64) [1.13]	0.91 (1.27) [2.25]
c: Steady state inflation: 2 percent per year	
0.10 (0.13) [0.21]	0.20 (0.27) [0.42]
Note: number not in parentheses assumes a markup of 20 percent; number in parentheses: 15 percent; number in square brackets: 10 percent	

Next: Assess Costs of Inflation Using Non-Steady State Formulas

Figure 1a: Percent loss of GDP due to Inflation, assumed markup is 1.2

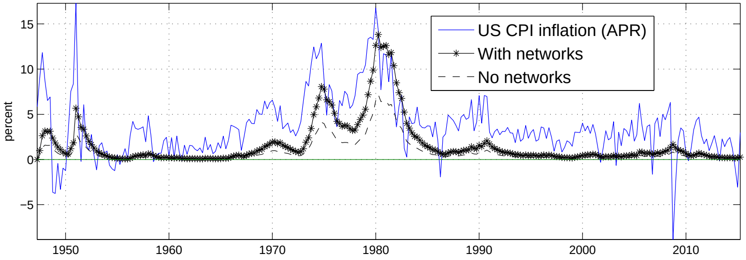
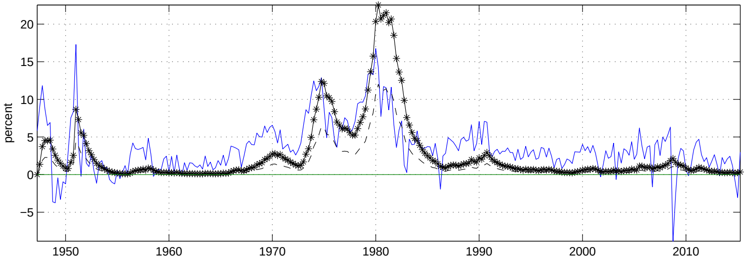


Figure 1b: Percent loss of GDP due to Inflation, assumed markup is 1.15



Inflation Distortions Displayed are Big

- With $\varepsilon = 6$,
  - mean$(\chi_t) = 0.98$, a 2% loss of GDP.
  - frequency, $\chi_t < 0.955$, is 10% (i.e., 10% of the time, the output loss is greater than 4.5 percent).
- With more competition (i.e., ε higher), the losses are greater.
  - with higher elasticity of demand, given movements in inflation imply much greater substitution away from high priced items, thus greater misallocation (caveat: this intuition is incomplete since with greater ε the consequences of a given amount of misallocation are smaller).
- Distortions with $\gamma = 1/2$ are roughly twice the size of distortions in standard case, $\gamma = 1$.
  - To see this, note

$$1 - \chi_t \simeq 1 - (p^*)^{\frac{1}{\gamma}} \overbrace{\simeq}^{\text{Taylor series expansion about } p^*=1} \frac{1}{\gamma}(1 - p^*).$$

Comparison of Steady State and Dynamic Costs of Inflation in 1970s

- Results

Table 1: Fraction of GDP Lost, $100(1-\chi)$, During High Inflation		
	No networks, $\gamma = 1$	Networks, $\gamma = 2$
Steady state lost output	2.41 (3.92)*	4.76 (7.68)
Mean, 1972Q1-1982Q4	3.13 (5.22)	6.26 (10.44)
Note * number not in parentheses - markup of 20 percent (i.e., $\varepsilon = 6$)		
number in parentheses - markup of 15 percent. (i.e., $\varepsilon = 7.7$)		

- Evidently, distortions increase rapidly in inflation,

$$E\left[distortion\left(\text{inflation}\right)\right] > distortion\left(E\text{inflation}\right)$$

Next

- Collect the equilibrium conditions.
  - For careful comparison of NK model with RBC model, see http://faculty.wcas.northwestern.edu/~lchrist/course/China_Chengdu_2016/NewKeynesian_model_handout.pdf
  - In RBC model, markets obtain socially efficient allocations independent of monetary policy.
  - In NK model, markets don't necessarily work well and good monetary policy essential.

- Solve the model.

Summarizing the Equilibrium Conditions

- Break up the equilibrium conditions into three sets:
  1. Conditions (1)-(4) for prices: $K_t, F_t, \bar{\pi}_t, p_t^*, s_t$
  2. Conditions (6)-(10) for: $C_t, Y_t, N_t, I_t, \mu_t$
  3. Conditions (5) and (11) for R_t and χ_t.
- We have 11 equilibrium conditions for 12 variables: system is underdetermined.
  - Not surprising: have said nothing about monetary policy.

Equilibrium Conditions Associated with Price Setting

$$K_t = \frac{\varepsilon}{\varepsilon - 1}\frac{Y_t}{C_t}s_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \text{ (1)}$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \text{ (2)}$$

$$\frac{K_t}{F_t} = \left[\frac{1 - \theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \quad \text{(3)}$$

$$p_t^* = \left[(1-\theta)\left(\frac{1 - \theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1} \quad \text{(4)}$$

Equilibrium Conditions Associated With Gross Output

- Equations:

$$
\begin{aligned}
Y_t &= p_t^* A_t N_t^{\gamma} I_t^{1-\gamma} \text{ (6)}, \; C_t + I_t = Y_t \text{ (7)}, \; I_t = \mu_t \frac{Y_t}{p_t^*} \text{ (8)} \\
s_t &= (1-\nu)(1-\psi+\psi R_t)\left(\frac{1}{1-\gamma}\right)^{1-\gamma} \\
&\quad \times \left(\frac{1}{\gamma} \overbrace{\exp(\tau_t) C_t N_t^{\varphi}}^{\text{used household Euler equation to substitute out } W_t/P_t}\right)^{\gamma} \frac{1}{A_t} \\
\mu_t &= \frac{(1-\gamma) s_t}{(1-\nu)(1-\psi+\psi R_t)} \text{ (10)},
\end{aligned}
$$

Other Equilibrium Conditions

- Allocative distortion:

$$\chi_t = \left(p_t^* \left(\frac{1 - \frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} \quad (11)$$

- Intertemporal equation

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad \text{(5)}$$

- On way to close the system: specify a monetary policy rule:

$$R_t/R = (R_{t-1}/R)^{\rho} \exp\left[(1-\rho)\,\phi_{\pi}(\bar{\pi}_t - \bar{\pi}) + u_t\right] \quad (12)$$

- Smoothing parameter: ρ.
  - Bigger is ρ the more persistent are policy-induced changes in the interest rate.

- Monetary policy shock: u_t.

Conclusion About Networks

- Networks alter the New Keynesian model's implications for inflation.
  - Doubles the cost of inflation.
  - Phillips curve is flatter because of strategic complementarities (when there are price frictions, this makes materials prices inertial which makes marginal costs inertial, which reduces firms' interest in changing prices).
- For the result on the Taylor principle, see my 2011 handbook chapter and Christiano (2015).
  - When the smoothing parameter in Taylor rule is set to zero and $\psi = 1$, then the model has indeterminacy, even when the coefficient on inflation is 1.5.
  - So, the likelihood of the Taylor principle breaking down goes up when γ is reduced, consistent with intuition.
  - When the smoothing parameter is at its empirically plausible value of 0.8, then the solution of the model does not display indeterminacy.