

Consensus New Keynesian DSGE Model

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Overview

- A consensus has emerged about the rough outlines of a model for the analysis of monetary policy.
 - Consensus influenced heavily by estimated impulse response functions from Structural Vector Autoregression (SVARs)
- Construct the consensus models based on SVAR results.
 - Christiano, Eichenbaum and Evans JPE (2005)
 - Smets and Wouters, AER (2007)

- Very brief review of SVARs.

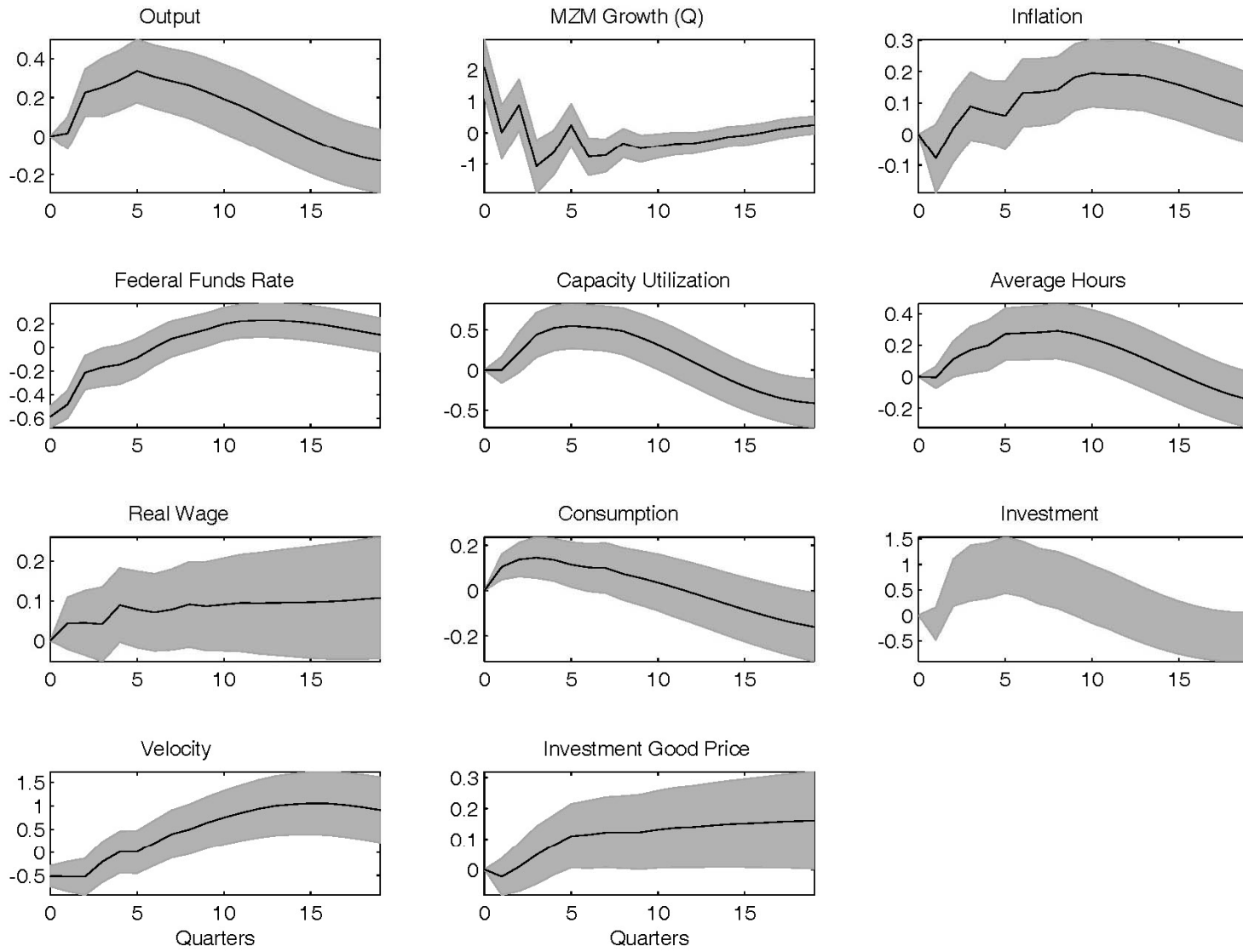
Identifying Monetary Policy Shocks

- Rule that relates Fed's actions to state of the economy.

$$R_t = f(\Omega_t) + e_t^R$$

- f is a linear function
- Ω_t : set of variables that Fed looks at.
- e_t^R : time t policy shock, orthogonal to Ω_t

Response to a monetary policy shock



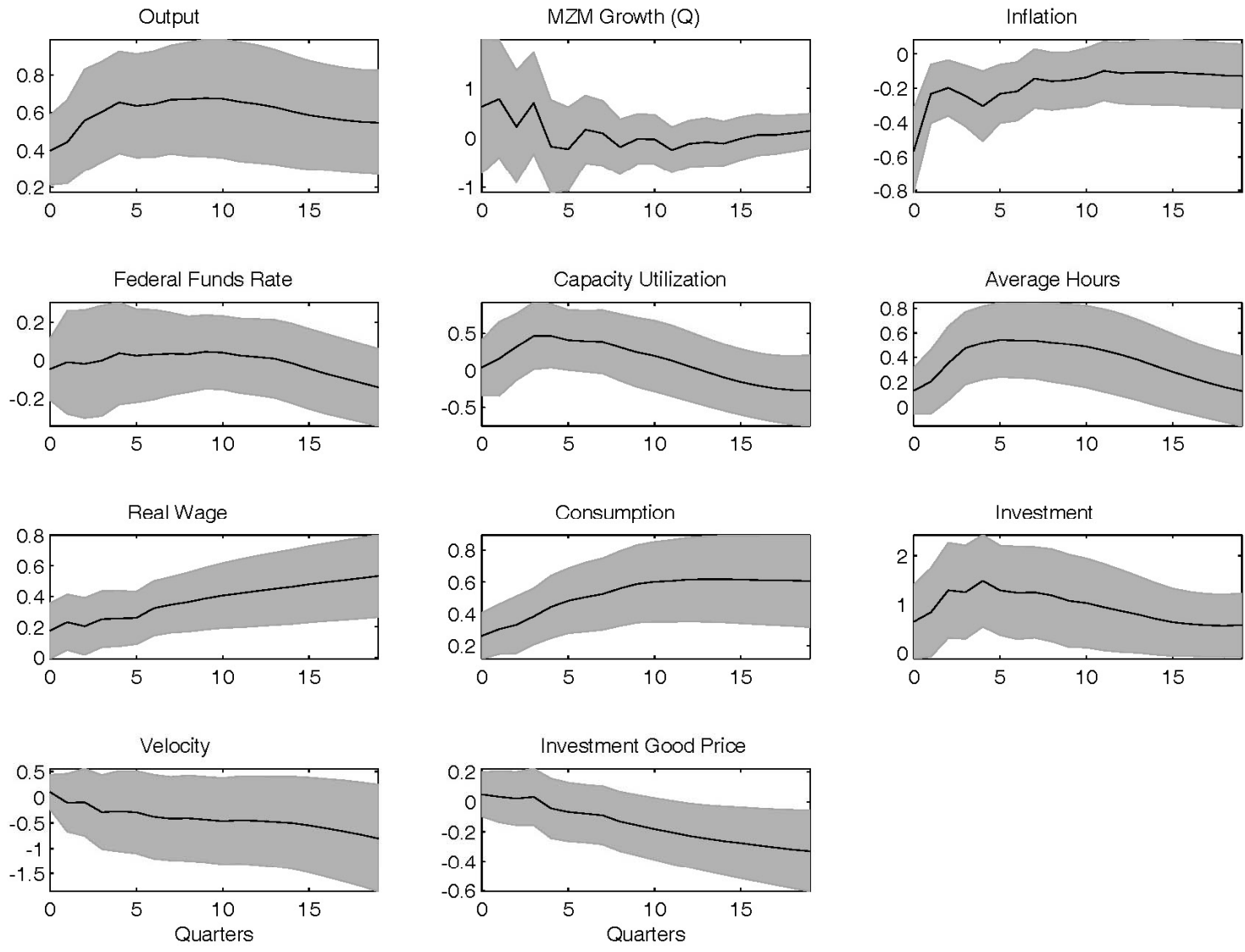
Interesting Properties of Monetary Policy Shocks

- Plenty of endogenous persistence:
 - money growth and interest rate over in 1 year, but other variables keep going....
- Inflation slow to get off the ground: peaks in roughly two years
 - It has been conjectured that explaining this is a major challenge for economics
 - Chari-Kehoe-McGrattan (*Econometrica*), Mankiw.
 - Kills models in which movements in P are key to monetary transmission mechanism (Lucas misperception model, pure sticky wage model)
 - Has been at the heart of the recent emphasis on sticky prices.
- Output, consumption, investment, hours worked and capacity utilization hump-shaped
- Velocity comoves with the interest rate

Identification of Technology Shocks

- Two technology shocks:
 - One perturbs price of investment goods
 - One perturbs total factor productivity
- Identification assumptions:
 - They are the only two shocks that affect labor productivity in the long run
 - Only the shock to investment good prices have an impact on investment good prices in the long run.

Response to a neutral technology shock



Observations on Neutral Shock

- Generally, results are ‘noisy’, as one expects.
 - Interest, money growth, velocity responses not pinned down.
- Interestingly, inflation response is immediate and *precisely* estimated.
- Does this raise a question about the conventional interpretation of the response of inflation to a monetary shock?
- Alternative possibility: information confusion stories.
 - A variant of recent work by Rhys Mendes that builds on Guido Lorenzoni’s work.

Importance of Three Shocks

- According to VAR analysis, they account for a large part of economic fluctuations.

Variance Decomposition

Variable	BP(8,32)
Output	86 [18]
Money Growth	23 [11]
Inflation	33 [17]
Fed Funds	52 [16]
Capacity Util.	51 [16]
Avg. Hours	76 [17]
Real Wage	44 [16]
Consumption	89 [21]
Investment	69 [16]
Velocity	29 [16]
Price of investment goods	11 [16]

Next

- Use Impulse Responses to Estimate a DSGE Model
 - Motivate the Basic Model Features.
 - Model Estimation.
- Determine if there is a conflict regarding price behavior between micro and macro data.
 - Macro Evidence:
 - Inflation responds slowly to monetary shock
 - Single equation estimates of slope of Phillips curve produce small slope coefficients.
 - Micro Evidence:
 - Bils-Klenow, Nakamura-Steinsson report evidence on frequency of price change at micro level: 5-11 months.
- Finding: no micro macro puzzle, as long as we suppose that capital used by firms is ‘firm-specific’.

Outline

- Model
- Econometric Estimation of Model
 - Fitting Model to Impulse Response Functions
- Model Estimation Results (is there a micro/macro puzzle?)

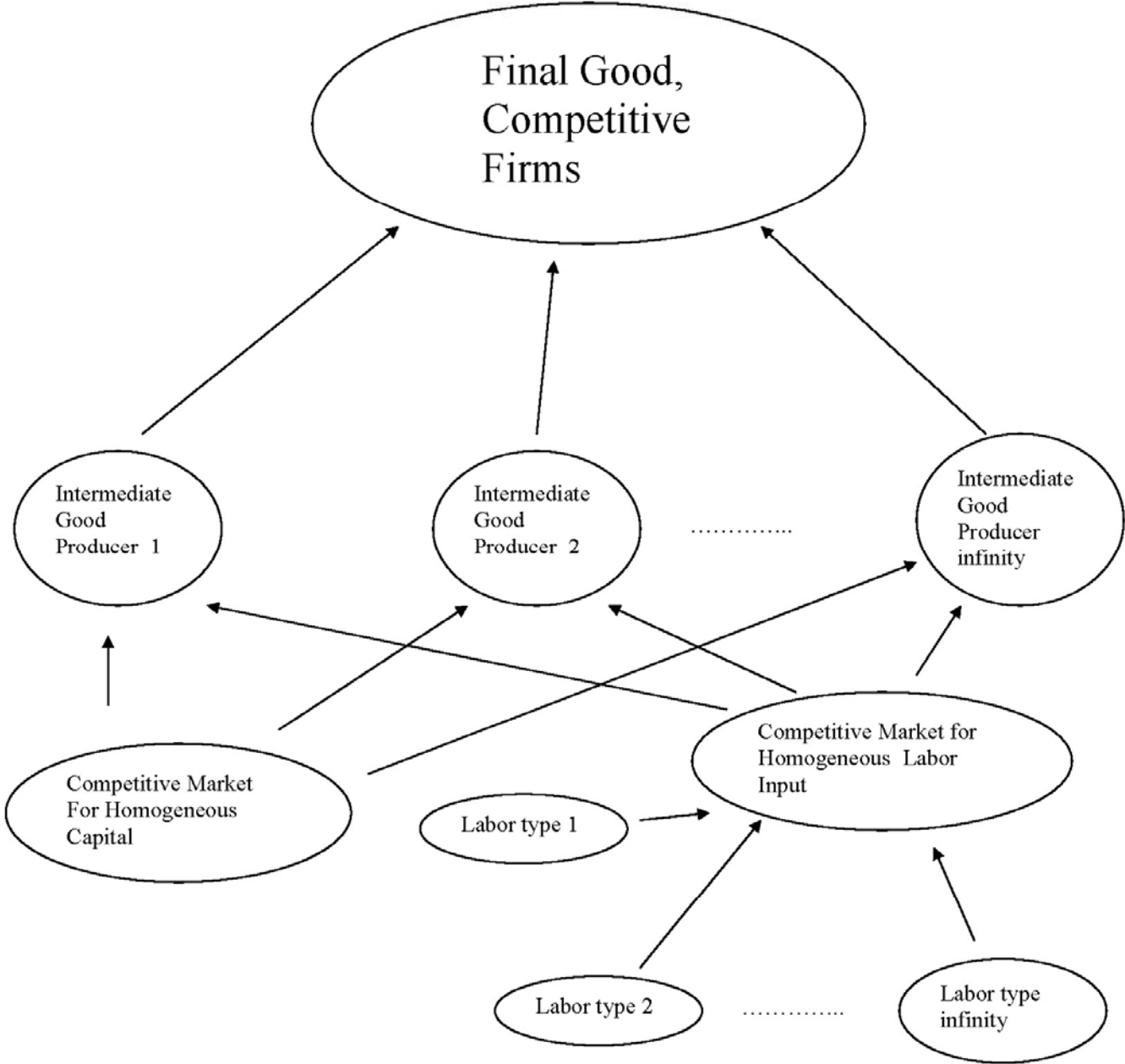
Description of Model

- Timing Assumptions
- Firms
- Households
- Monetary Authority
- Goods Market Clearing and Equilibrium

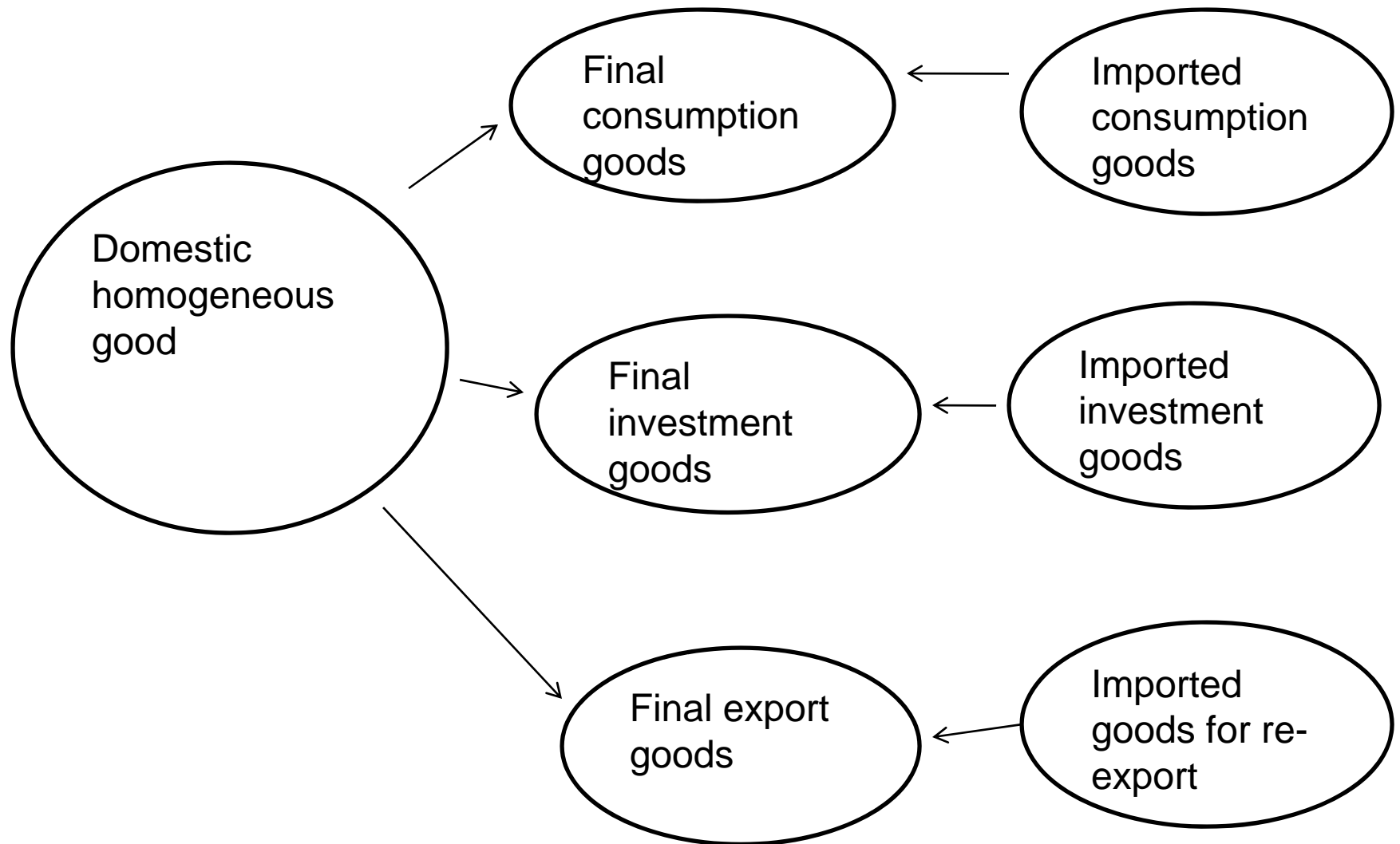
Timing

- Technology Shocks Realized.
- Agents Make Price/Wage Setting, Consumption, Investment, Capital Utilization Decisions.
- Monetary Policy Shock Realized.
- Household Money Demand Decision Made.
- Production, Employment, Purchases Occur, and Markets Clear.
- Note: Wages, Prices and Output Predetermined Relative to Policy Shock.

Firm Sector



Extension to small open economy (Christiano, Trabandt, Walentin (2009))



Firms

- Final good firms

- Technology:

$$Y_t = \left[\int_0^1 y_{it}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f}, \quad 1 \leq \lambda_f < \infty$$

- Objective:

$$\max_{Y_t, \{y_{it}, 0 \leq i \leq 1\}} P_t Y_t - \int_0^1 P_{it} y_{it} di$$

- Focals and prices:

$$\left(\frac{P_t}{P_{it}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{y_{it}}{Y_t}, \quad P_t = \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f}$$

Firms, cont'd

- Intermediate good firms

- Each y_{it} produced by a monopolist with demand curve:

$$y_{it} = \left(\frac{P_t}{P_{it}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} Y_t$$

- Technology:

$$y_{it} = K_{it}^\alpha (z_t L_{it})^{1-\alpha}, \quad 0 < \alpha < 1$$

- Law of motion of technology shock:

$$\mu_{z,t} \equiv \log z_t - \log z_{t-1}, \quad \hat{\mu}_{z,t} \equiv \frac{\mu_{z,t} - \mu_z}{\mu_z}, \quad \mu_z = E\mu_{z,t}$$

$$\hat{\mu}_{z,t} = \rho_{\mu_z} \hat{\mu}_{z,t-1} + \varepsilon_{\mu_{z,t}}$$

- consistent with identifying assumption on technology.

Firms, cnt'd

Nominal wage

Real rental rate of capital services

- Intermediate good firm marginal cost

$$MC\$ = [\psi + (1 - \psi)R_t] \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{P_t r_t^k}{\alpha} \right)^\alpha \frac{1}{z_t^{1-\alpha}}$$

Fraction of wage and capital rental bill that must be borrowed in advance at gross nominal rate of interest, R

$\psi < 1$ creates 'working capital channel' for the interest rate, R , on the supply side of the economy.

Helps keep prices from rising after monetary injection (actually, may Even help explain the 'price puzzle'.

Firms, cnt'd

- Intermediate good firm marginal cost

$$MC\$ = [\psi + (1 - \psi)R_t] \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{P_t r_t^k}{\alpha} \right)^\alpha \frac{1}{z_t^{1-\alpha}}$$

- Marginal cost divided by final good price:

$$s_t \equiv \frac{MC\$}{P_t} = [\psi + (1 - \psi)R_t] \left(\frac{W_t/P_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_t^k}{\alpha} \right)^\alpha \frac{1}{z_t^{1-\alpha}}$$

Calvo price frictions in intermediate good firms

- With probability, $1 - \xi_p$, firms may optimize price:

$$P_{it} = \tilde{P}_t$$

- With probability, ξ_p ,

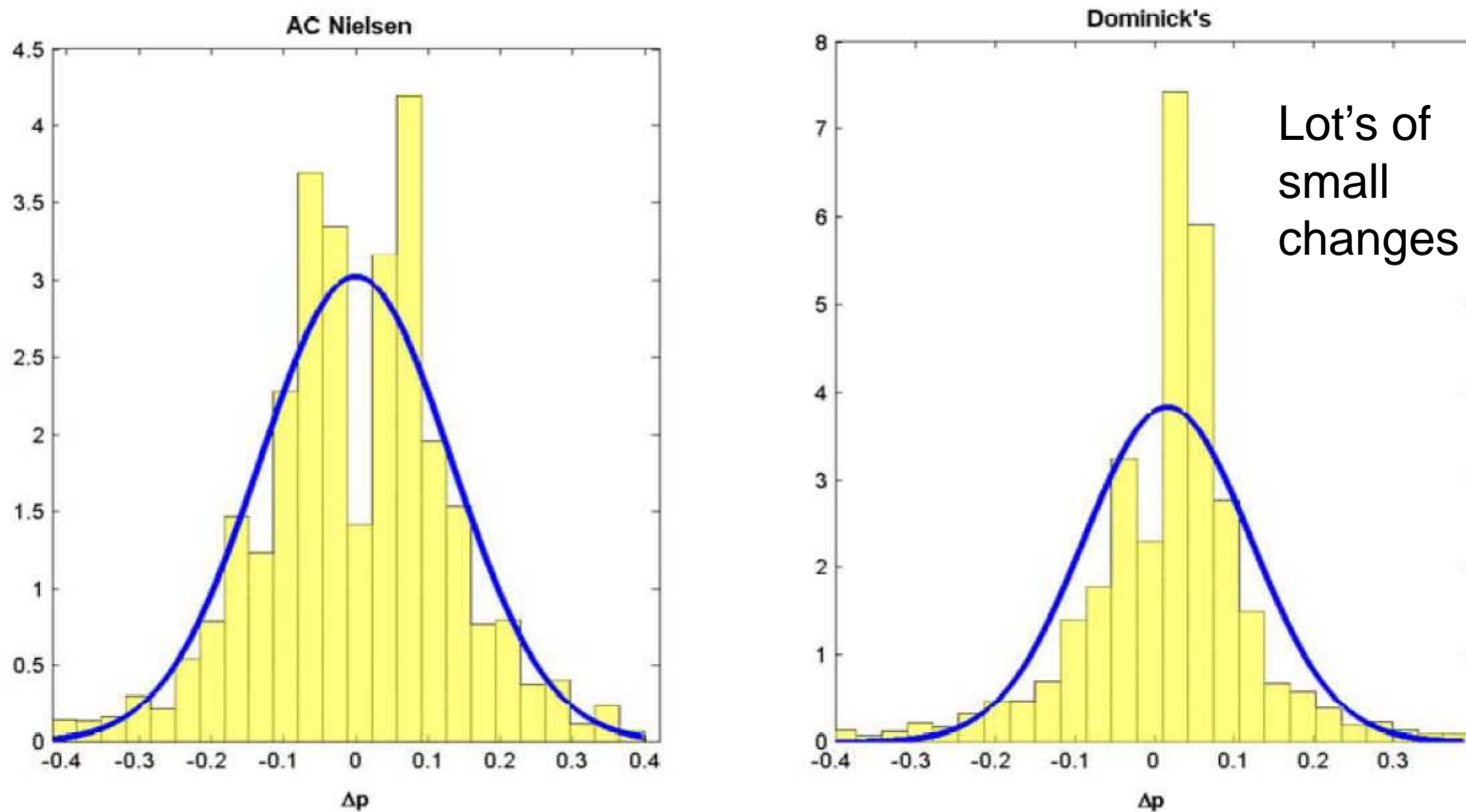
$$P_{it} = \bar{\pi}^v \pi_{t-1}^{1-v} P_{i,t-1}, \quad 0 < v < 1$$

- Alternative is that with probability ξ_p ,

$$P_{it} = P_{i,t-1}$$

Evidence from Midrigan, 'Menu Costs, Multi-Product Firms, and Aggregate Fluctuations'

Figure 1: Distribution of price changes conditional on adjustment



Note: superimposed is the pdf of a Gaussian distribution with equal mean and variance

Histograms of $\log(P_t/P_{t-1})$, conditional on price adjustment, for two data sets pooled across all goods/stores/months in sample.

- Combining Optimal Price and Aggregate Price Relation:

$$\Delta \hat{\pi}_t = \beta E_t \Delta \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} E_t \hat{s}_t, \quad \mathbf{v} = \mathbf{0}$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} E_t \hat{s}_t. \quad \mathbf{v} = \mathbf{1}$$

Households: Sequence of Events

- Technology shock realized.
- Decisions: Consumption, Capital accumulation, Capital Utilization.
- Wage rate set.
- Monetary policy shock realized.
- Household allocates beginning of period cash between deposits at financial intermediary and cash to be used in consumption transactions.

Households

- Each household is identical
- Each household supplies each of many different varieties of labor, $j \in (0, 1)$
 - Quantity of j -type labor: $h_{j,t}$
- Quantity of consumption: C_t
- Household preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - bC_{t-1}) - \frac{\psi_L}{1+\sigma} \int_0^1 h_{j,t}^{1+\sigma} dj \right]$$

Household and Labor Market

Erceg-Henderson-Levin Model

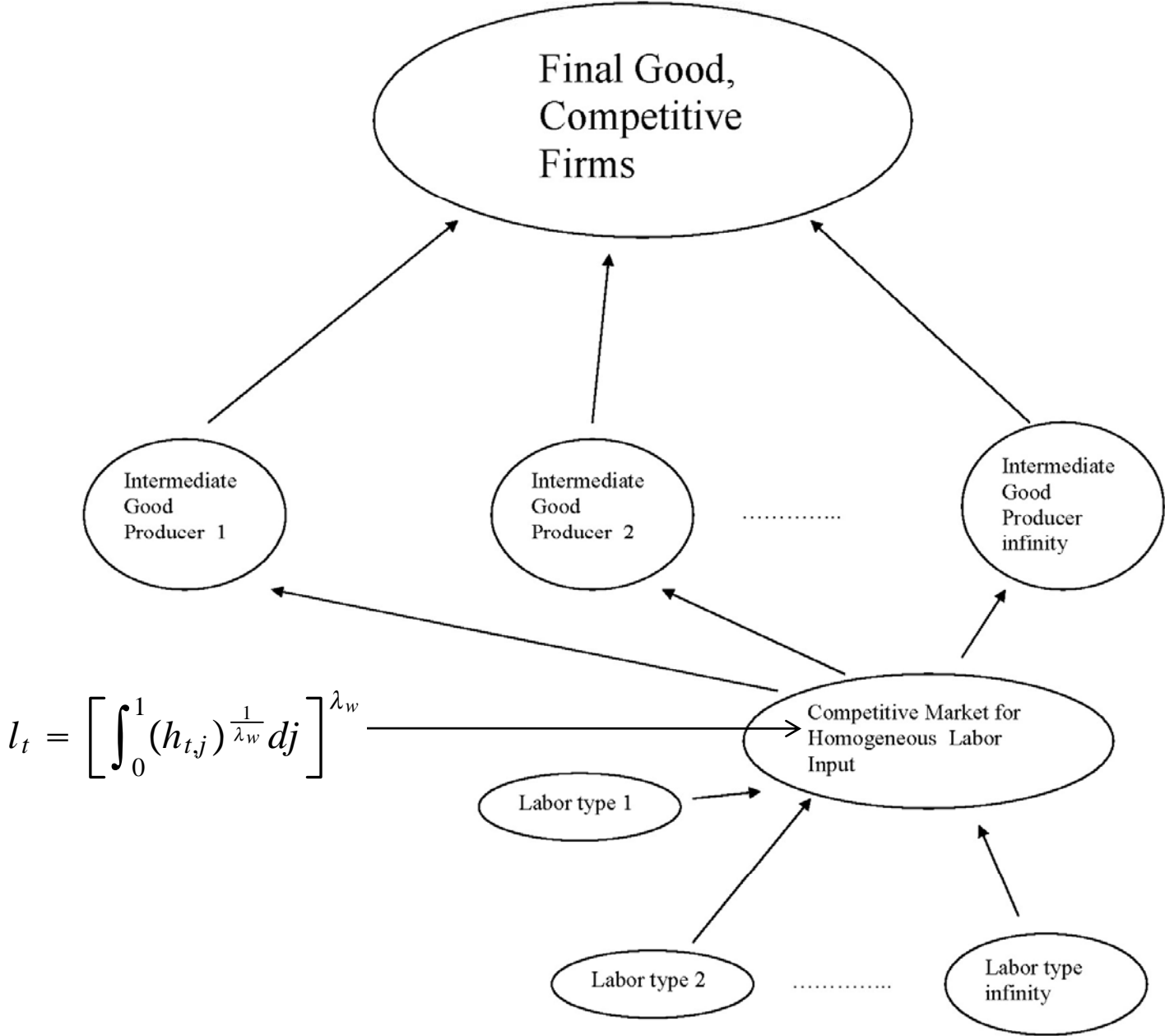
- Each type of labor, j , in the household joins a union of all j -type labor from all other households.
- The union for j -type labor behaves as a monopolist on behalf of its members, setting the wage $W_{j,t}$ subject to a demand curve for j -type labor.
- With probability ξ_w the union may not reoptimize the wage, and with probability $1 - \xi_w$ it may reoptimize.

Labor market, cnt'd

- Given the specified wage, j -type workers supply whatever quantity of labor is demanded.
- Labor is demanded by competitive 'labor contractors', who aggregate different labor services into a homogeneous labor input that they rent to intermediate good producers.
- Labor contractors use the following technology:

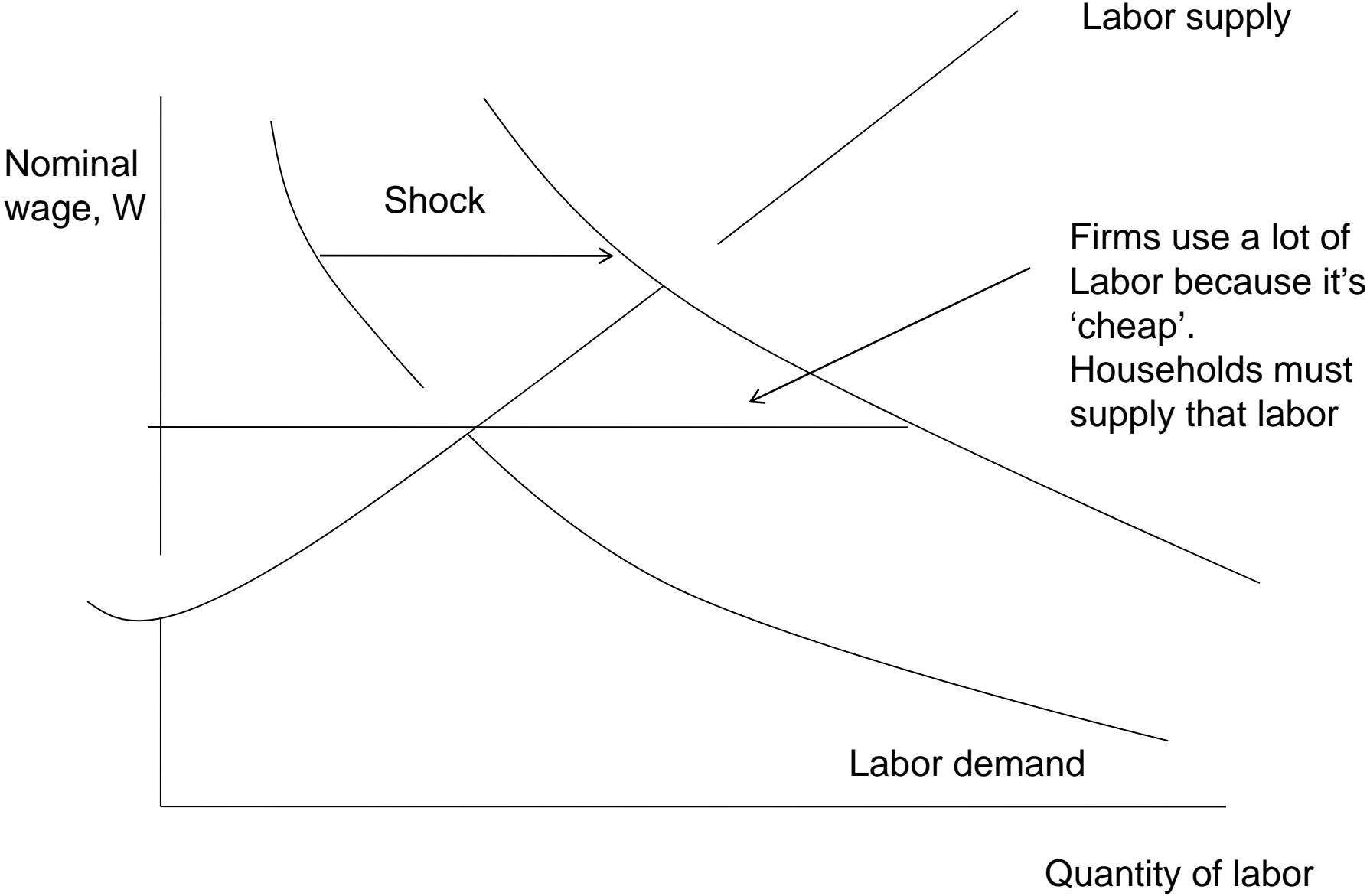
$$l_t = \left[\int_0^1 (h_{t,j})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \quad 1 \leq \lambda_w < \infty.$$

Firm Sector



What's the point of the wage setting frictions?

- They help the model account for the response of inflation and output to a monetary policy shock.
 - Sticky wage in effect makes labor supply highly elastic.
 - Positive monetary policy shock leads to:
 - Big increase in employment and output.
 - Small increase in cost and, hence, inflation.



Extensions of Labor Market

- Jordi Gali (2009) shows how to derive a theory of unemployment from the EHL model.
- Christiano-Trabandt-Walentin (2010) extend the model to obtain 'involuntary' unemployment.
- Gertler-Trigari, Gertler-Sala-Trigari show how to introduce Mortensen-Pissarides-style search and matching approach
 - see Christiano-Illut-Motto-Rostagno and Christiano-Trabandt-Walentin for empirical applications to closed and small open economies.

Why Habit Persistence in Preferences?

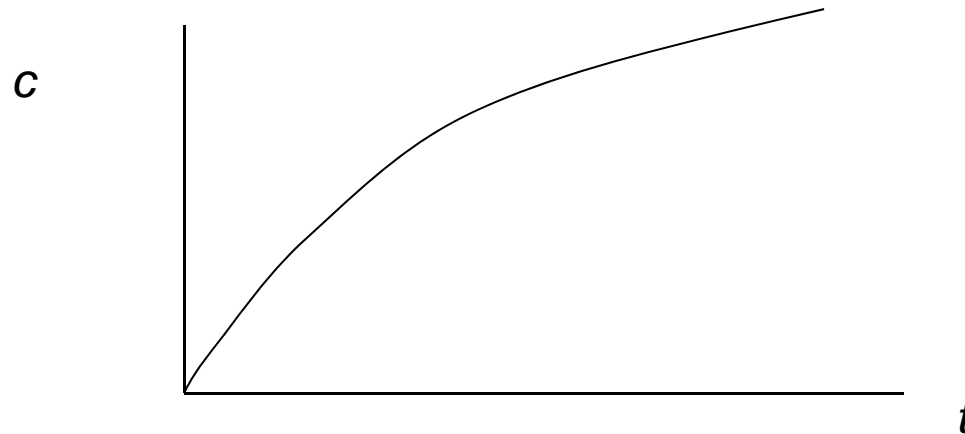
- They help resolve the ‘consumption puzzle’ in monetary economics.....
- With standard preferences, hard to understand the way consumption responds to monetary policy shock.

Consumption 'Puzzle'

- In Estimated Impulse Responses:
 - Real Interest Rate Falls

$$R_t / \pi_{t+1}$$

- Consumption Rises in Hump-Shape Pattern:



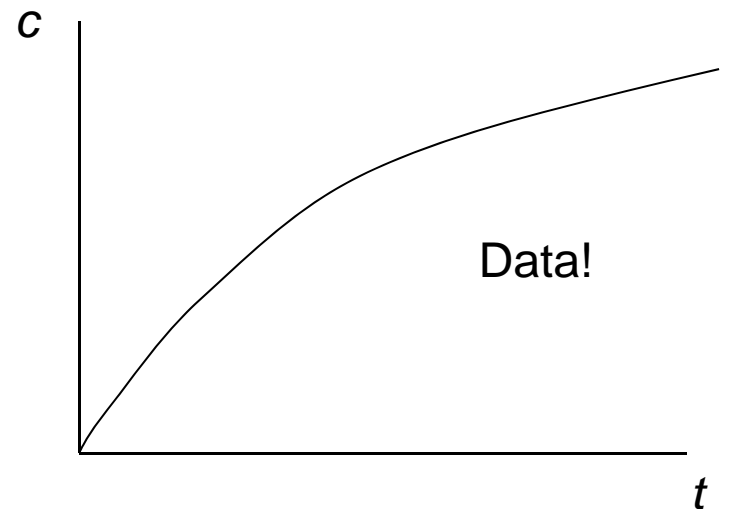
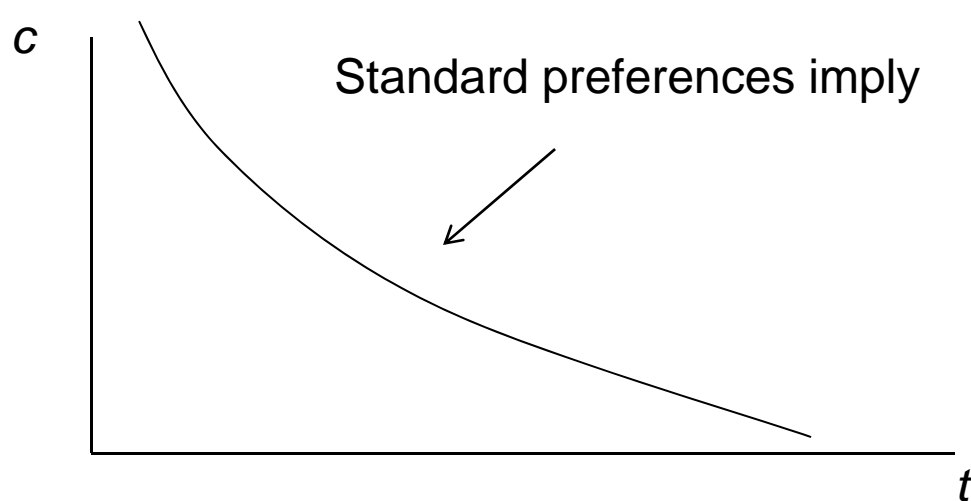
- Standard preferences inconsistent with above

Consumption 'Puzzle'

- Intertemporal First Order Condition:

'Standard' Preferences \longrightarrow

$$\frac{c_{t+1}}{\beta c_t} = \frac{MU_{c,t}}{\beta MU_{c,t+1}} \approx R_t / \pi_{t+1}$$



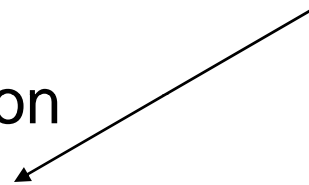
A Solution to the Consumption Puzzle

- Concave Consumption Response Displays:
 - Rising Consumption (problem)
 - Falling Slope of Consumption

- Habit Persistence in Consumption

$$U(c) = \log(c - b \times c_{-1})$$

Habit parameter



- Marginal Utility Function of *Slope* of Consumption
 - Hump-Shape Consumption Response Not a Puzzle
- Econometric Estimation Strategy Given the Option, $b > 0$

Households...

- Asset Evolution Equation:

$$M_{t+1} = R_t[M_t - Q_t + (x_t - 1)M_t^a] + Q_t + \int_0^1 W_{j,t}h_{j,t}dj$$

$$+ P_t r_t^k u_t \bar{K}_t + D_t - P_t \left[(1 + \eta(V_t))C_t + \frac{1}{\Upsilon_t} (I_t + a(u_t)\bar{K}_t) \right]$$

- M_t : Beginning of Period Base Money; Q_t : Transactions Balances
- x_t : Growth Rate of Base; u_t : Utilization Rate of Capital
 - * $u_t = 1$ in steady state, $a(1) = 0$, $a'(1) > 0$, $\sigma_a = a''(1)/a'(1)$.
- Υ_t^{-1} : (Real) Price of investment goods, $\mu_{\Upsilon,t} = \Upsilon_t/\Upsilon_{t-1}$,

$$\hat{\mu}_{\Upsilon,t} = \rho_{\mu_{\Upsilon}} \hat{\mu}_{\Upsilon,t-1} + \varepsilon_{\mu_{\Upsilon,t}}$$

- Velocity:

$$V_t = \frac{P_t C_t}{Q_t},$$

Money Demand

- Asset Evolution Equation:

$$M_{t+1} = R_t[M_t - Q_t + (x_t - 1)M_t^a] + Q_t + \int_0^1 W_{j,t}h_{j,t}dj$$

$$+ P_t r_t^k u_t \bar{K}_t + D_t - P_t \left[(1 + \eta(V_t))C_t + \frac{1}{Y_t} (I_t + a(u_t)\bar{K}_t) \right]$$

- Increase in Q_t :
 - Marginal Cost of Interest Foregone: R_t
 - Marginal Benefit:

$$1 - P_t \eta'(V_t) C_t \frac{dV_t}{dQ_t}$$

$$= \underbrace{1}_{\text{additional cash available at end of period}} + \overbrace{\eta' \left(\frac{P_t C_t}{Q_t} \right) \left(\frac{P_t C_t}{Q_t} \right)^2}_{\text{reduction in transactions costs due to extra cash}}$$

Money Demand ...

- Money Demand: Equate Marginal Benefits and Costs of Q_t —

$$R_t = 1 + \eta' \left(\frac{P_t C_t}{Q_t} \right) \left(\frac{P_t C_t}{Q_t} \right)^2 .$$

- Properties of Money Demand:

- Unit Consumption Elasticity of Money Demand

- * Increase C_t 1 percent and Hold R_t, P_t Fixed \Rightarrow Desired Q_t increases 1 percent

- $R_t \uparrow$ Implies $Q_t \downarrow$

- * To Induce Households to Hold Additional Q , Must Have Lower R

- * Money Demand Elasticity is Bigger, the Bigger is η''

Money Demand ...

- Quantitative Analysis of Money Demand
 - Consider the Following Parametric Function for η

$$\eta = AV_t + \frac{B}{V_t} - 2\sqrt{AB}$$

\Rightarrow

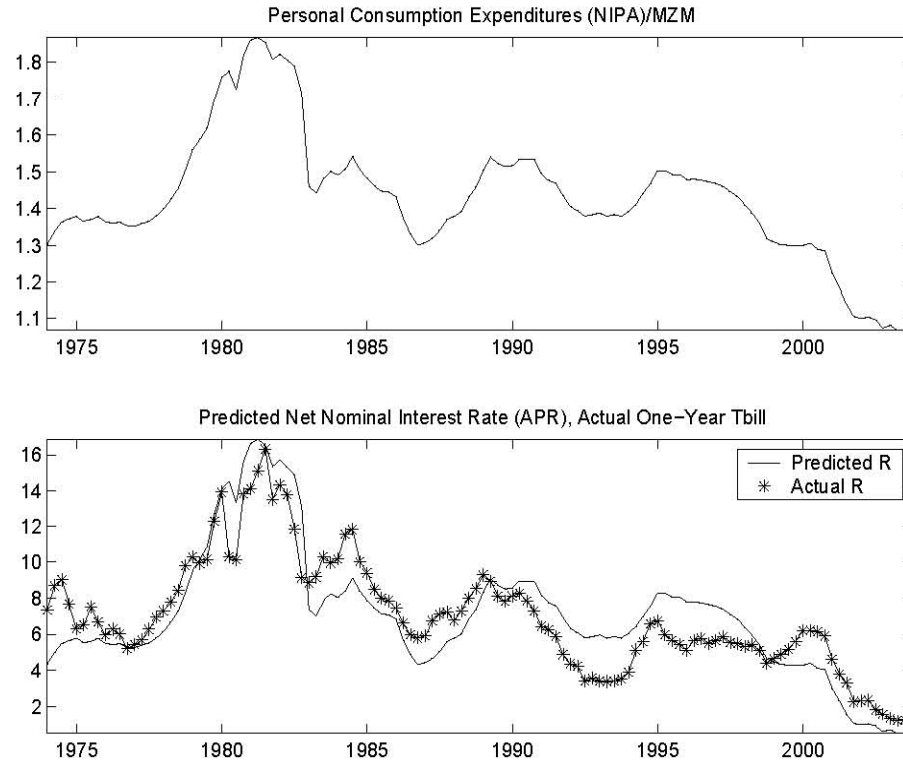
$$R = 1 + \eta'(V) \times V^2 = 1 + [A - BV^{-2}] V^2 = 1 - B + AV^2$$

– Data:

- * Money - St. Louis Fed's MZM, 1974-2004
- * Consumption - NIPA Consumption of Services and Nondurables
- * Interest Rate - One Year T-Bills.
- * OLS Regression of V^2 on $R \Rightarrow A = 0.0174$ and $B = 0.0187$

Money Demand ...

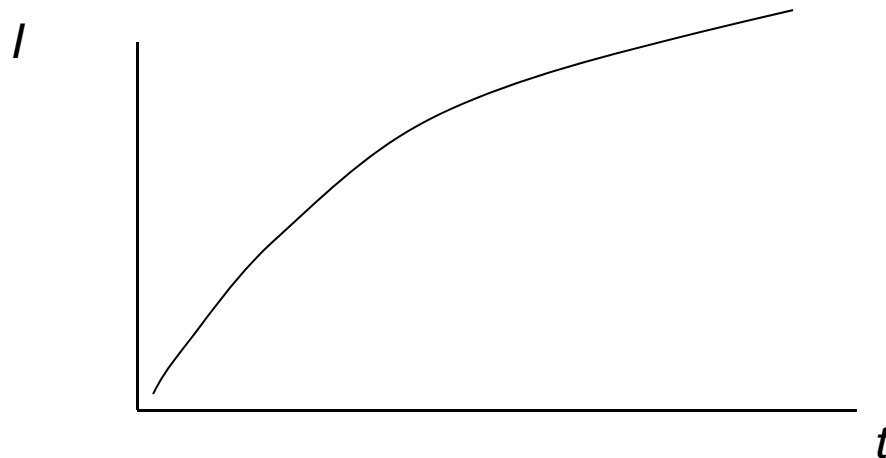
- Top Graph: Velocity of Money
- Bottom Graph: Actual and Predicted Interest Rate



- Findings: Static Money Demand Equation Fits the Data Well!

Dynamic Response of Investment to Monetary Policy Shock

- In Estimated Impulse Responses:
 - Investment Rises in Hump-Shaped Pattern:



Investment 'Puzzle'

- Rate of Return on Capital

$$R_t^k = \frac{MP_{t+1}^k + P_{k',t+1}(1-\delta)}{P_{k',t}},$$

$P_{k',t} \sim$ consumption price of installed capital

$MP_t^k \sim$ marginal product of capital

$\delta \in (0, 1) \sim$ depreciation rate.

- Rough 'Arbitrage' Condition:

$$\frac{R_t}{\pi_{t+1}} \approx R_t^k.$$

- Positive Money Shock Drives Real Rate:

$$R_t^k \downarrow$$

- Problem: Burst of Investment!

Investment Puzzle: a failed approach

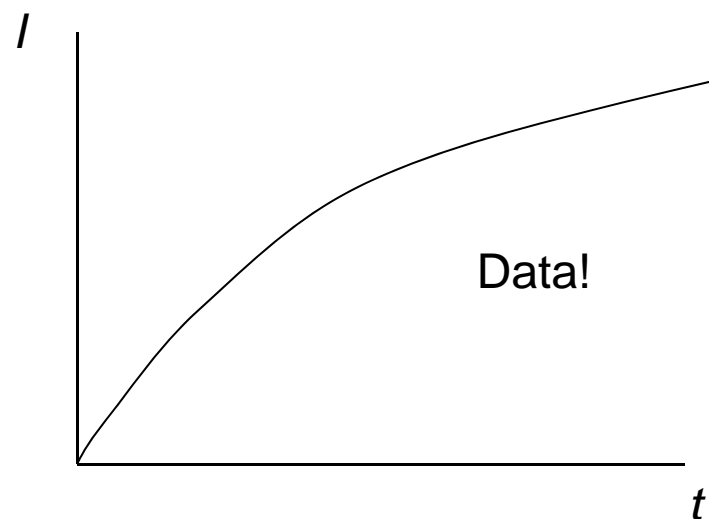
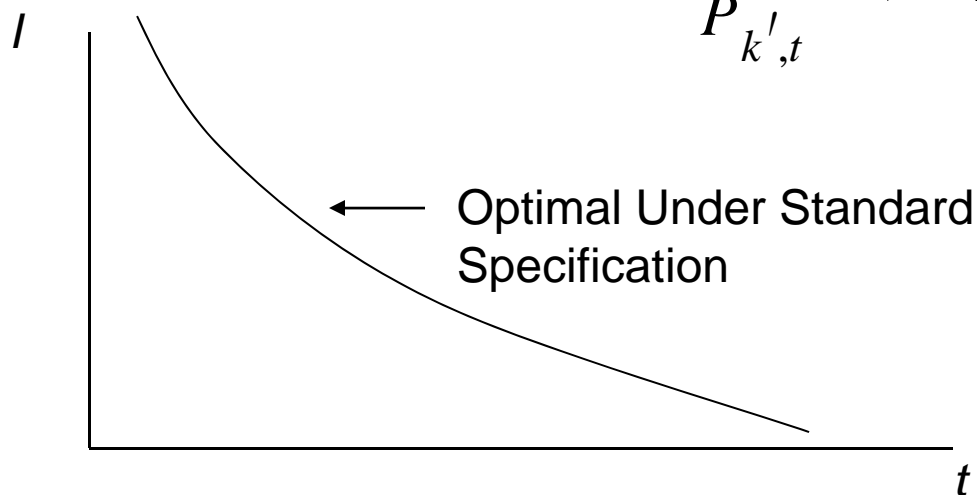
- Adjustment Costs in Investment
 - Standard Model (Lucas-Prescott)

$$k' = (1 - \delta)k + F\left(\frac{I}{k}\right)I.$$

– Problem:

- Hump-Shape Response Creates Anticipated Capital Gains

$$\frac{P_{k',t+1}}{P_{k',t}} > 1$$



A Solution to the Investment Puzzle

- Cost-of-Change Adjustment Costs:

$$k' = (1 - \delta)k + F\left(\frac{I}{I_{-1}}\right)I$$

- This Does Produce a Hump-Shape Investment Response
 - Other Evidence Favors This Specification
 - Empirical: Matsuyama, Sherwin Rosen
 - Theoretical: Matsuyama, David Lucca

Monetary and Fiscal Policy

$$x_t = M_t/M_{t-1}$$

$$\hat{x}_{M,t} = \rho_M \hat{x}_{M,t-1} + \varepsilon_{M,t}$$

$$\hat{x}_{z,t} = \rho_{xz} \hat{x}_{z,t-1} + c_z \varepsilon_{z,t} + c_z^p \varepsilon_{z,t-1}$$

$$\hat{x}_{\Upsilon,t} = \rho_{x\Upsilon} \hat{x}_{\Upsilon,t-1} + c_{\Upsilon} \varepsilon_{\Upsilon,t} + c_{\Upsilon}^p \varepsilon_{\Upsilon,t-1}$$

- $\hat{x}_{M,t}$: response of monetary policy to a monetary policy shock, $\varepsilon_{M,t}$
- $\hat{x}_{z,t}$: response of monetary policy to an innovation in neutral technology, $\varepsilon_{z,t}$.
- $\hat{x}_{\Upsilon,t}$: response of monetary policy to an innovation in capital embodied technology, $\varepsilon_{\Upsilon,t}$.
- Government has access to lump sum taxes, pursues a Ricardian fiscal policy.

Loan Market and Final Good Market Clearing Conditions, Equilibrium

- Financial intermediaries receive $M_t - Q_t + (x_t - 1) M_t$ from the household.
 - Lend all of their money to intermediate good firms, which use the funds to pay for H_t .

- Loan market clearing

$$W_t H_t = x_t M_t - Q_t.$$

- The aggregate resource constraint is

$$(1 + \eta(V_t)) C_t + \Upsilon_t^{-1} [I_t + a(u_t) \bar{K}_t] \leq Y_t.$$

- We adopt a standard sequence-of-markets equilibrium concept.

Econometric Methodology

- Variant of limited information strategy used in CEE (2004).
 - Impose a subset of assumptions made in equilibrium model to estimate impulse response functions of ten key macroeconomic variables to the three shocks in our model.
 - Neutral technology shocks, capital embodied technology shocks and monetary policy shocks.
- Choose values for key parameters of structural model to minimize difference between estimated impulse response functions and analogous objects in model.

Estimating Parameters in the Model

- Partition Parameters into Three Groups.

- Parameters set a priori (e.g., β, δ, \dots)

- ζ_1 : remaining parameters pertaining to the nonstochastic part of model

We estimate γ , the slope of the Phillips curve, rather than ξ_p .

$$\zeta_1 = [\xi_w, \gamma, \sigma_a, b, S'', \epsilon]$$

- ζ_2 : parameters pertaining to stochastic part of the model

- Number of parameters, $\zeta = (\zeta_1, \zeta_2)$, to be estimated - 18

- Estimation Criterion

- $\Psi(\zeta)$: mapping from ζ to model impulse responses

- $\hat{\Psi}$: 592 impulse responses estimated using VAR

- Estimation Strategy:

$$\hat{\zeta} = \arg \min_{\zeta} \left(\hat{\Psi} - \Psi(\zeta) \right)' V^{-1} \left(\hat{\Psi} - \Psi(\zeta) \right).$$

- V : diagonal matrix with sample variances of $\hat{\Psi}$ along the diagonal.

Classical Perspective

- Impulse response functions have the following asymptotic distribution:

$$\sqrt{T} (\hat{\Psi} - \Psi^0) \stackrel{a}{\sim} N(0, \tilde{V})$$

– or,

$$\hat{\Psi} \stackrel{a}{\sim} N\left(\Psi^0, \frac{\tilde{V}}{T}\right) = \left(\frac{1}{2\pi}\right)^{\frac{n}{2}} \left|\frac{\tilde{V}}{T}\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2} (\hat{\Psi} - \Psi^0)' \left(\frac{\tilde{V}}{T}\right)^{-1} (\hat{\Psi} - \Psi^0)\right]$$

- Estimation criterion:

$$L(\zeta, \hat{\Psi}) \equiv (\hat{\Psi} - \Psi(\zeta))' V^{-1} (\hat{\Psi} - \Psi(\zeta))$$

- Estimator: $L_1(\hat{\zeta}, \hat{\Psi}) = 0 \rightarrow \hat{\zeta} = f(\hat{\Psi})$

- Asymptotic distribution (delta function method):

$$\sqrt{T} (\hat{\zeta} - \zeta^0) \stackrel{a}{\sim} N\left(0, f'(\Psi^0) \tilde{V} [f'(\Psi^0)]^{\text{transpose}}\right)$$

Bayesian Perspective

- Suppose that the estimation criterion used the actual asymptotic variance-covariance of $\hat{\Psi}$, \tilde{V}/T :

$$L(\zeta, \hat{\Psi}) \equiv -\frac{1}{2} (\hat{\Psi} - \Psi(\zeta))' \left(\frac{\tilde{V}}{T} \right)^{-1} (\hat{\Psi} - \Psi(\zeta))$$

- Suppose that the model is true, with parameter values, ζ .

- Then, the likelihood of the observed impulse response functions, conditional on ζ is (for large T):

$$\text{likelihood}(\hat{\Psi}|\zeta) \propto e^{L(\zeta, \hat{\Psi})}$$

- Bayesian posterior of model parameters

$$\text{posterior}(\zeta|\hat{\Psi}) \propto e^{L(\zeta, \hat{\Psi})} \times \text{prior}(\zeta)$$

- Parameter estimates

Estimated Parameter Values, ζ_1

Model	λ_f	ξ_w	γ	σ_a	b	S''
Benchmark	1.01	0.78 (0.08)	0.014 (0.007)	11.42 (6.86)	0.76 (0.08)	1.50 (0.83)

↑

Markup parameter goes to unity in estimation, and estimation criterion is very flat.

- Parameter estimates

Estimated Parameter Values, ζ_1

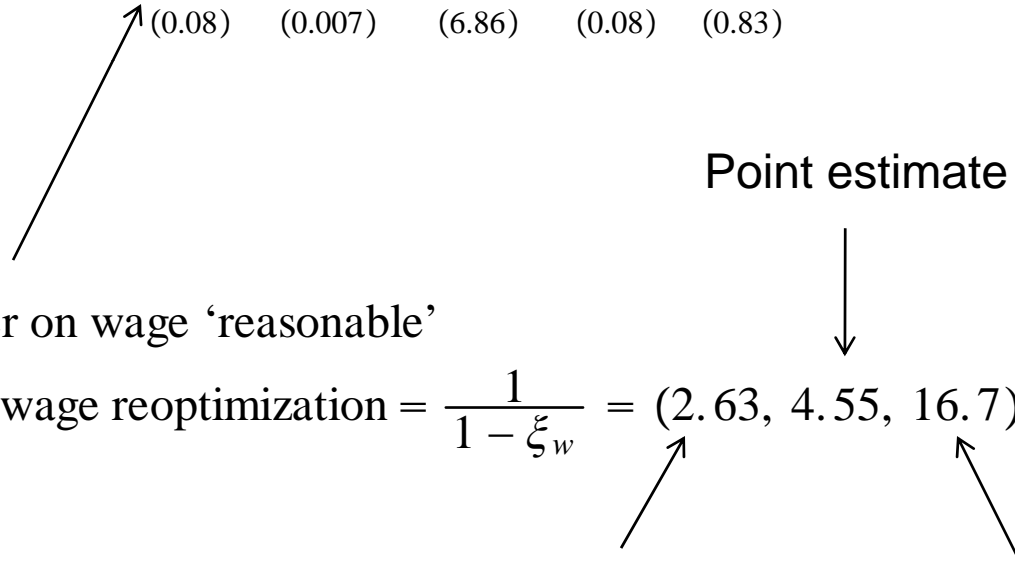
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Point estimate

Calvo parameter on wage 'reasonable'

Mean time between wage reoptimization = $\frac{1}{1 - \xi_w} = (2.63, 4.55, 16.7)$

Point estimate plus/minus 2 standard deviations



- Parameter estimates

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A big number, implying capital utilization hardly varies

- Parameter estimates

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Habit parameter value similar to others reported in the literature

- Parameter estimates

Estimated Parameter Values, ζ_1

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Benchmark	1.01	0.78 (0.08)	0.014 (0.007)	11.42 (6.86)	0.76 (0.08)	1.50 (0.83)

- Slope of Phillips curve *very* small.

$$\gamma = \frac{(1 - \xi_p)(1 - \beta\xi_p)}{\xi_p} = 0.014 \rightarrow \xi_p = 0.89$$

average amount of time a price remains unchanged = $\frac{1}{1 - \xi_p} = 9$ quarters!

- Apparently, a major failure!

Not a Failure...

- The standard model assumes capital is homogeneous
 - traded freely in homogeneous markets.
 - assumption made for simplicity, not realism.
 - hope: it does not matter.
 - in fact: it matters a lot!
- In reality, much capital is firm-specific
 - once in place, cannot easily be converted to another use.

Homogeneous versus firm-specific capital

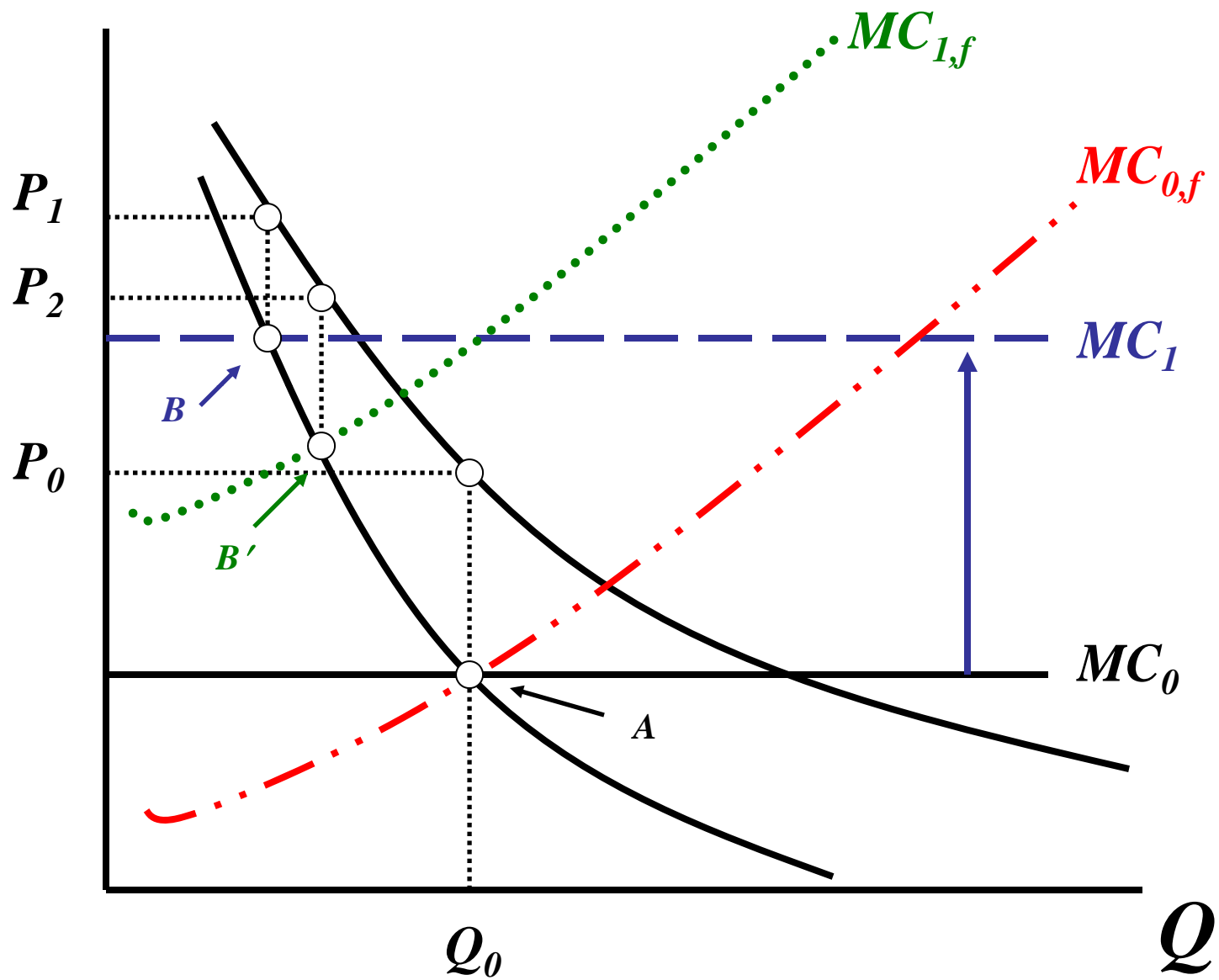
- Homogeneous capital:
 - Marginal cost is independent of firm output.

$$Y_{it} = (u_t \bar{K}_{it})^\alpha (z_t L_{it})^{1-\alpha}$$

- Firm-specific capital:
 - Marginal cost is increasing in firm output.
 - Requires that capital utilization not be variable.
 - As firm expands output, cannot simultaneously increase capital so incur diminishing returns in labor.

Homogeneous versus firm-specific capital, cnt'd...

- When firms have rising marginal cost, a given shock to marginal cost has smaller impact on price.



More Intuition: Rising Marginal Cost and Incentive to Raise Price

- A Firm Contemplates Raising Price
 - This Implies Output Falls
 - Marginal Cost Falls
 - Incentive to Raise Price Falls
- Effect Quantitatively Important When:
 - Marginal Cost Steep (capital firm-specific; no variable utilization, σ_a large)
 - Demand Elastic (elasticity of demand, $\frac{\lambda_f}{\lambda_f - 1}$)

Observational Equivalence Property of Model

- Firm-Specificity of Capital Irrelevant for All Aggregate Equilibrium Conditions, Except One
- Aggregate Inflation Dynamics:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma s_t, \quad s_t = \text{marginal cost}$$

$$\gamma = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p} \chi$$

$$\chi = \begin{cases} 1 & \text{standard, homogeneous capital model} \\ f(\text{slope of marginal cost and demand}) & \text{firm-specific capital model} \end{cases}$$

Degree of Price Stickiness in Model with Firm-specific Capital

IMPLIED AVERAGE TIME (Quarters) BETWEEN REOPTIMIZATION			
	$\frac{1}{1-\xi_P}$		elasticity of demand, $\frac{\lambda_f}{\lambda_f-1}$
Model	Firm-Specific Capital Model	Homogeneous Capital Model	demand: $Y_{it} = P_{it}^{\left(\frac{\lambda_f}{\lambda_f-1}\right)} \times \text{constant}$
Benchmark ($\lambda_f = 1.01$)	1.8	9.4	101
$\lambda_f = 1.05$	3.2	9.1	21
$\lambda_f = 1.10$	4.0	8.8	11
$\lambda_f = 1.20$	4.9	8.3	6

Plausible degree of price stickiness with assumption that capital is firm-specific consistent with the flat slope of the Phillips curve.

Full assessment requires an estimate of firm-level demand elasticity.

But, is the model consistent with evidence that inflation doesn't respond much to a monetary policy shock?

Figure 1: Response to a monetary policy shock (o - Model, - VAR, grey area - 95 % Confidence Interval)

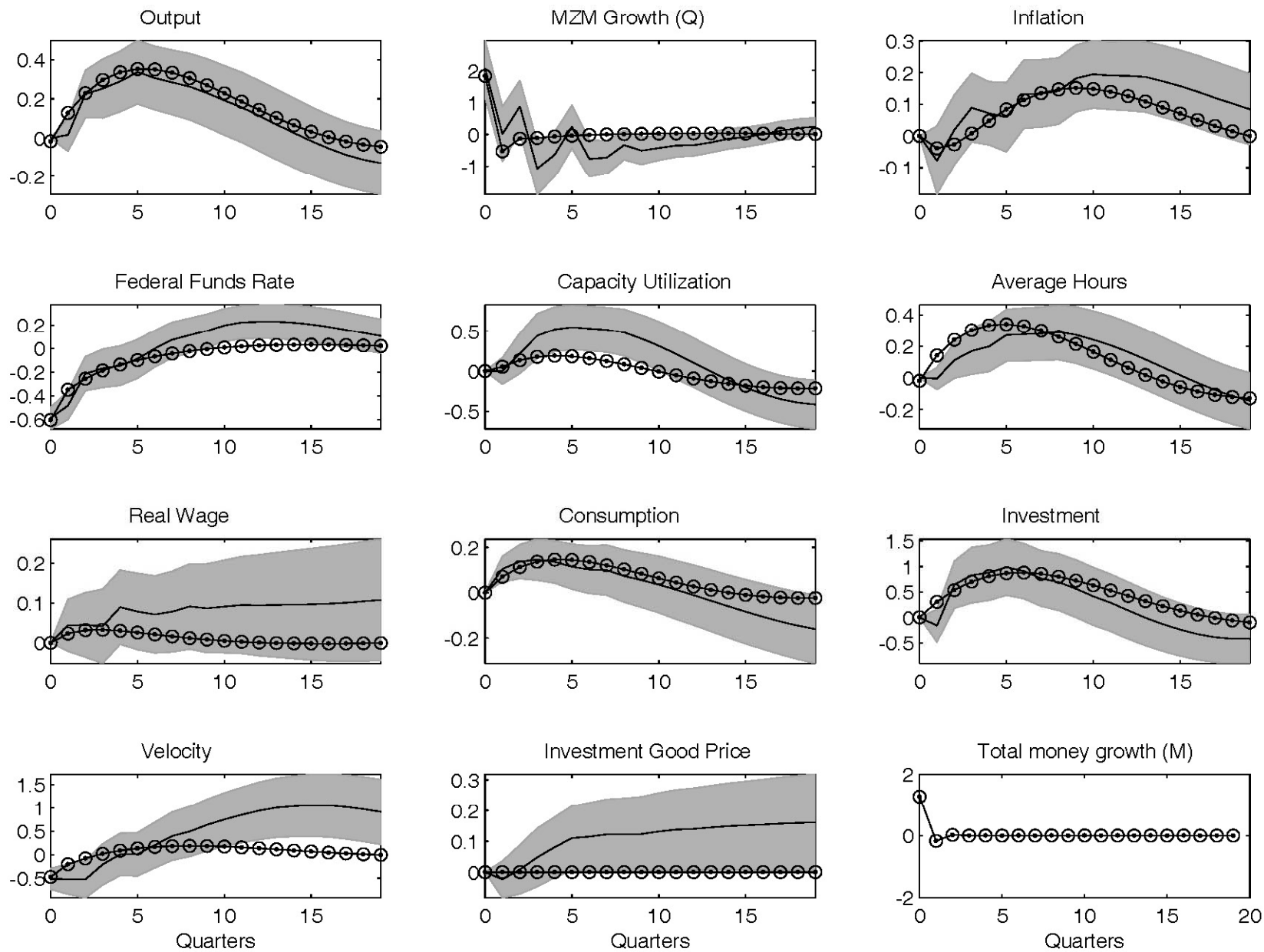


Figure 2: Response to a neutral technology shock (o - Model, - VAR, grey area - 95 % Confidence Interval)

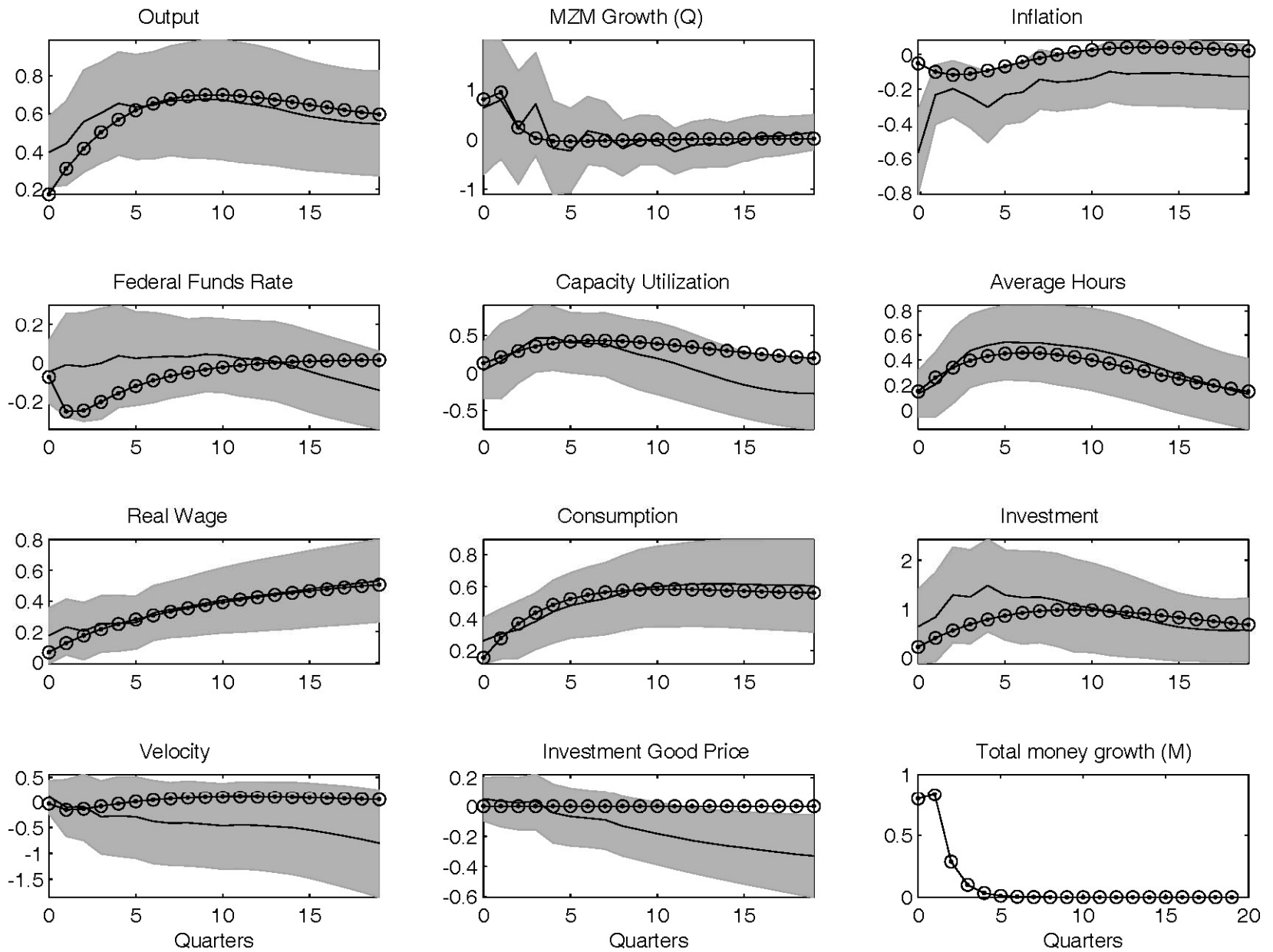
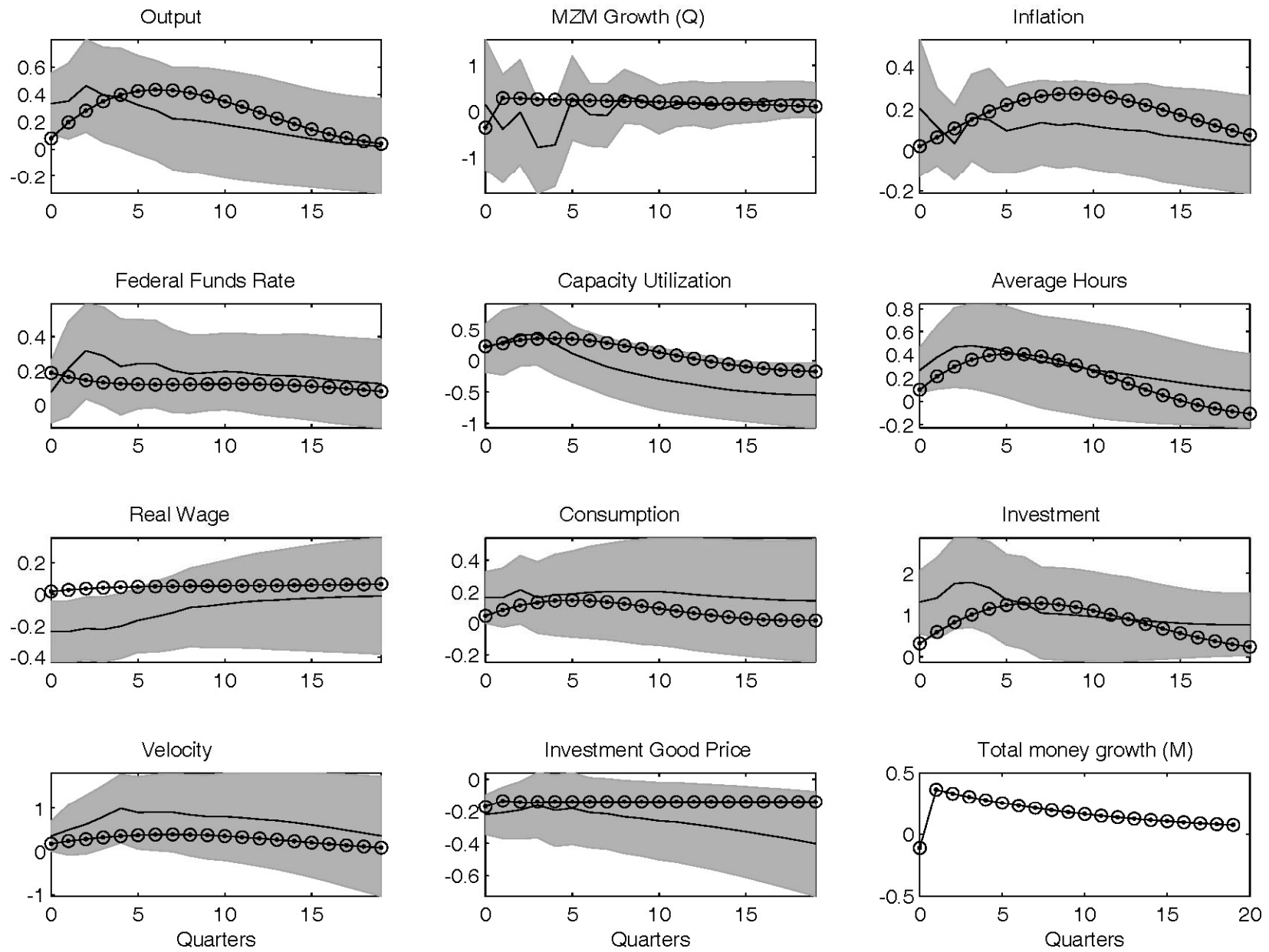


Figure 3: Response to an embodied technology shock (o - Model, - VAR, grey area - 95 % Confidence Interval)



Conclusion of Analysis of Standard Model

- Simple model with various frictions is capable of accounting well for key features of economic responses to monetary and technology shocks.
- But, model is missing financial frictions, and so cannot be used to address many of the policy questions arising from the financial crisis.