Simplest New Keynesian Model

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Outline

- Basic components of a model analysis:
  - What is the best possible allocation of labor and consumption, independent of how these is brought about.
  - How might markets cause this to happen (the ‘invisible hand’).
  - What happens with markets that resemble actual real world markets where prices don’t adjust instantly.

- Do these things in the basic New Keynesian model without capital

- Implications of model for monetary policy:
  - Clarifying the concepts of ‘excess and inadequate aggregate demand’.
  - The Taylor principle and inflation targeting.
  - Cases where ‘overzealous inflation targeting’ can go awry:
    - News shocks and the relationship between monetary policy and stock market volatility
    - The working capital channel and the Taylor principle.

Model

- Household preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right\},$$

$$\tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau},\ \varepsilon_t^{\tau} \sim iidN(0, \sigma_{\varepsilon}^2)$$

Production

- Final output requires lots of intermediate inputs:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 1$$

- Production of intermediate inputs:

$$Y_{i,t} = e^{a_t} N_{i,t}, \ \Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t^a, \ \varepsilon_t^a \sim iidN(0, \sigma_a^2)$$

- Constraint on allocation of labor:

$$\int_0^1 N_{it} di = N_t$$

Efficient Allocation of Total Labor

- Suppose total labor, N_t , is fixed.

- What is the best way to allocate N_t among the various activities, $0 \leq i \leq 1$?

- Answer:
  - allocate labor equally across all the activities

$$N_{it} = N_t, \text{ all } i$$

Suppose Labor *Not* Allocated Equally

- Example:

$$N_{it} = \begin{cases} 2\alpha N_t & i \in \left[0, \frac{1}{2}\right] \\ 2(1-\alpha)N_t & i \in \left[\frac{1}{2}, 1\right] \end{cases}, \; 0 \leq \alpha \leq 1.$$

- Note that this is a particular distribution of labor across activities:

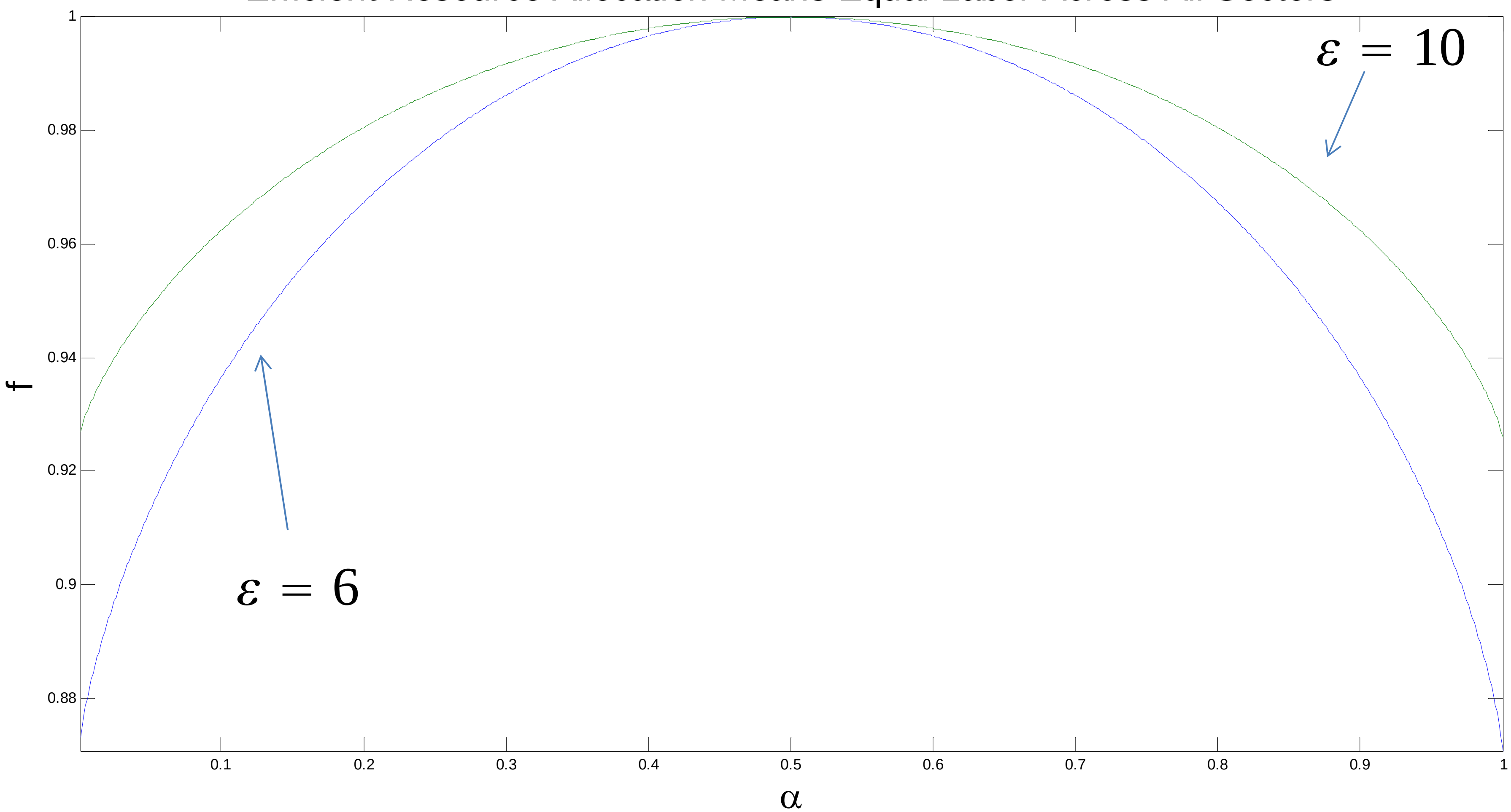
$$\int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1-\alpha) N_t = N_t$$

Labor *Not* Allocated Equally, cnt'd

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}}\,di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\int_0^{\frac{1}{2}} Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}}\,di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}}\,di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t}\left[\int_0^{\frac{1}{2}} N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}}\,di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}}\,di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t}\left[\int_0^{\frac{1}{2}} (2\alpha N_t)^{\frac{\varepsilon-1}{\varepsilon}}\,di + \int_{\frac{1}{2}}^1 (2(1-\alpha)N_t)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t}N_t\left[\int_0^{\frac{1}{2}} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}}\,di + \int_{\frac{1}{2}}^1 (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}}\,di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t}N_t\left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t}N_t f(\alpha)
\end{aligned}
$$

$$f(\alpha) = \left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Efficient Resource Allocation Means Equal Labor Across All Sectors



Economy with Efficient *N* Allocation

- Efficiency dictates

$$N_{it} = N_t \text{ all } i$$

- So, with efficient production:

$$Y_t = e^{a_t} N_t$$

- Resource constraint:

$$C_t \le Y_t$$

- Preferences:

$$E_0 \sum_{t=0}^{\infty} \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right),\ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau},\ \varepsilon_t^{\tau} \sim iid,$$

Efficient Determination of Labor

- Lagrangian:

$$\max_{C_t, N_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \overbrace{u(C_t, N_t, \tau_t)}^{=\log C_t - \exp(\tau_t)\frac{N_t^{1+\varphi}}{1+\varphi}} + \lambda_t [e^{a_t} N_t - C_t] \right\}$$

- First order conditions:

$$u_c(C_t, N_t, \tau_t) = \lambda_t, \; u_n(C_t, N_t, \tau_t) + \lambda_t e^{a_t} = 0$$

- or:

$$u_{n,t} + u_{c,t} e^{a_t} = 0$$

$$\overbrace{\frac{-u_{n,t}}{u_{c,t}}}^{\text{marginal cost of labor in consumption units} = \frac{-\frac{du}{dN_t}}{\frac{du}{dC_t}} = \frac{dC_t}{dN_t}} = \overbrace{e^{a_t}}^{\text{marginal product of labor}}$$

Efficient Determination of Labor, cont'd

- Solving the fonc's:

$$\frac{-u_{n,t}}{u_{c,t}} = e^{a_t}$$

$$C_t \exp(\tau_t) N_t^{\varphi} = e^{a_t}$$

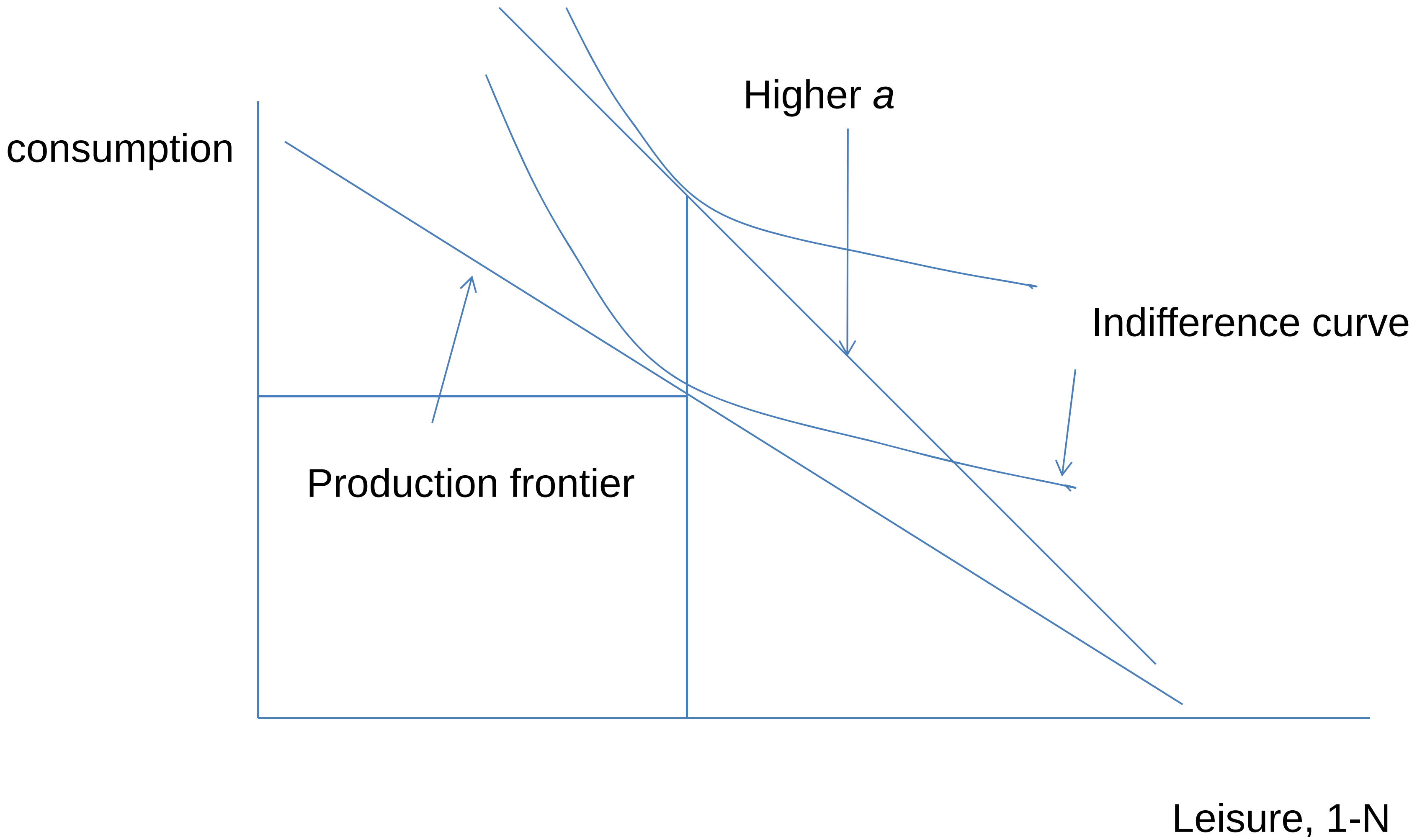
$$e^{a_t} N_t \exp(\tau_t) N_t^{\varphi} = e^{a_t}$$

$$\rightarrow N_t = \exp\left(\frac{-\tau_t}{1+\varphi}\right)$$

$$\rightarrow C_t = \exp\left(a_t - \frac{\tau_t}{1+\varphi}\right)$$

- Note:
  - Labor responds to preference shock, *not* to tech shock

Response to a Jump in *a*



Case Where Markets Work Beautifully (triumph of the 'invisible hand')

- Give households budget constraints, put them in markets and let them pursue their individual interests.
- Give the production functions to firms and suppose that they seek to maximize profits.
- There is monopoly power….extinguish the effects of that by a subsidy.
- Let markets work perfectly: prices and wages adjust instantly all the time, to clear markets.

Households

- Solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right\},$$

- Subject to:

Profits, *net of government taxes*

$$C_t + \overbrace{B_{t+1}}^{\text{bonds purchases in } t} \leq \overbrace{w_t}^{\text{wage rate}} N_t + \overbrace{\pi_t}^{\text{profits}} + \overbrace{r_{t-1}}^{\text{(real) interest on bonds}} B_t$$

- First order conditions:

$$\frac{-u_{n,t}}{u_{c,t}} = C_t \exp(\tau_t) N_t^{\varphi} = w_t$$ ‘marginal cost of working equals marginal benefit’

$$u_{c,t} = \beta E_t u_{c,t+1} r_t$$ ‘marginal cost of saving equals marginal benefit’

Final Good Firms

- Final good firms buy $Y_{i,t}, i \in (0,1)$, at given prices, $P_{i,t}$, to maximize profits:

$$Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$

- Subject to

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- Fonc's:

$$P_{i,t} = \left(\frac{Y_t}{Y_{i,t}} \right)^{\frac{1}{\varepsilon}}$$

$$\rightarrow Y_{i,t} = P_{i,t}^{-\varepsilon} Y_t, \; 1 = \int_0^1 P_{i,t}^{1-\varepsilon} di$$

Intermediate Good Firms

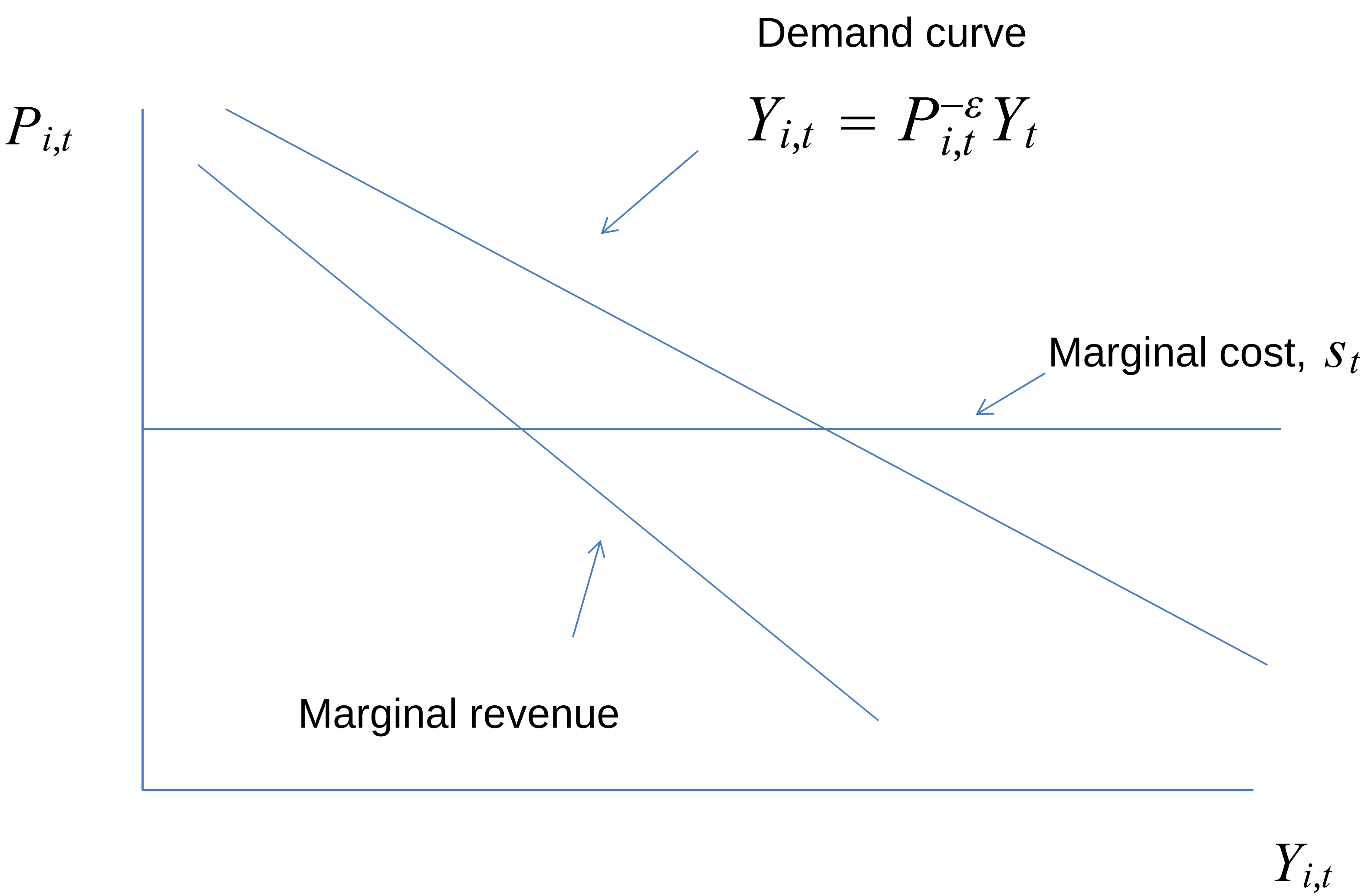
- For each $Y_{i,t}$ there is a single producer who is a monopolist in the product market and hires labor, $N_{i,t}$ in competitive labor markets.

- Marginal cost of production:

$$\text{(real) marginal cost} = s_t = \frac{\frac{dCost}{dworker}}{\frac{doutput}{dworker}} = \frac{\left(1 - \overbrace{\nu}^{\text{subsidy payment to firm}}\right) w_t}{\exp(a_t)}$$

- Subsidy will be required to ensure markets work efficiently.

Intermediate Good Firms



i*th* Intermediate Good Firm

- Problem: $\max_{N_{it}} P_{it} Y_{it} - s_t Y_{it}$
- Subject to demand for $Y_{i,t}$: $Y_{i,t} = P_{i,t}^{-\varepsilon} Y_t$
- Problem:

$$\max_{N_{it}} P_{it} P_{i,t}^{-\varepsilon} Y_t - s_t P_{i,t}^{-\varepsilon} Y_t$$

fonc : $(1-\varepsilon) P_{it}^{-\varepsilon} Y_t + \varepsilon s_t P_{i,t}^{-\varepsilon-1} Y_t = 0$

$P_{it} = \frac{\varepsilon}{\varepsilon - 1} s_t$ ‘price is markup over marginal cost’

- Note: all prices are the same, so resources allocated efficiently across intermediate good firms.

$P_{i,t} = P_{j,t} = 1,$ because $1 = \int_0^1 P_{i,t}^{1-\varepsilon} di$

Equilibrium

- Pulling things together:

$$1 = \frac{\varepsilon}{\varepsilon - 1} s_t = \frac{\varepsilon}{\varepsilon - 1} \frac{(1-\nu) w_t}{\exp(a_t)}$$

$$\overbrace{=}^{\text{household fonc}} \quad \frac{\varepsilon(1-\nu)}{\varepsilon - 1} \frac{\frac{-u_{n,t}}{u_{c,t}}}{\exp(a_t)}$$

$$\overbrace{=}^{\text{if } \frac{\varepsilon(1-\nu)}{\varepsilon-1} = 1} \quad \frac{\frac{-u_{n,t}}{u_{c,t}}}{\exp(a_t)}.$$

If proper subsidy is provided to monopolists, employment is efficient:

if $1 - \nu = \frac{\varepsilon - 1}{\varepsilon}$, then $\frac{-u_{n,t}}{u_{c,t}} = \exp(a_t)$

Equilibrium Allocations

- With efficient subsidy,

$$\frac{-u_{n,t}}{u_{c,t}} \overbrace{=}^{\text{functional form}} C_t \exp(\tau_t) N_t^{\varphi} \overbrace{=}^{\text{resource constraint}} \exp(a_t)\exp(\tau_t) N_t^{1+\varphi} = \exp(a_t)$$

$$\rightarrow N_t = \exp\left(\frac{-\tau_t}{1+\varphi}\right)$$

$$C_t = e^{a_t} N_t = \exp\left(a_t - \frac{\tau_t}{1+\varphi}\right)$$

- Bond market clearing implies:

$$B_t = 0 \text{ always}$$

Interest Rate in Equilibrium

- Interest rate backed out of household intertemporal Euler equation:

$$u_{c,t} = \beta E_t u_{c,t+1} r_t \rightarrow \frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} r_t$$

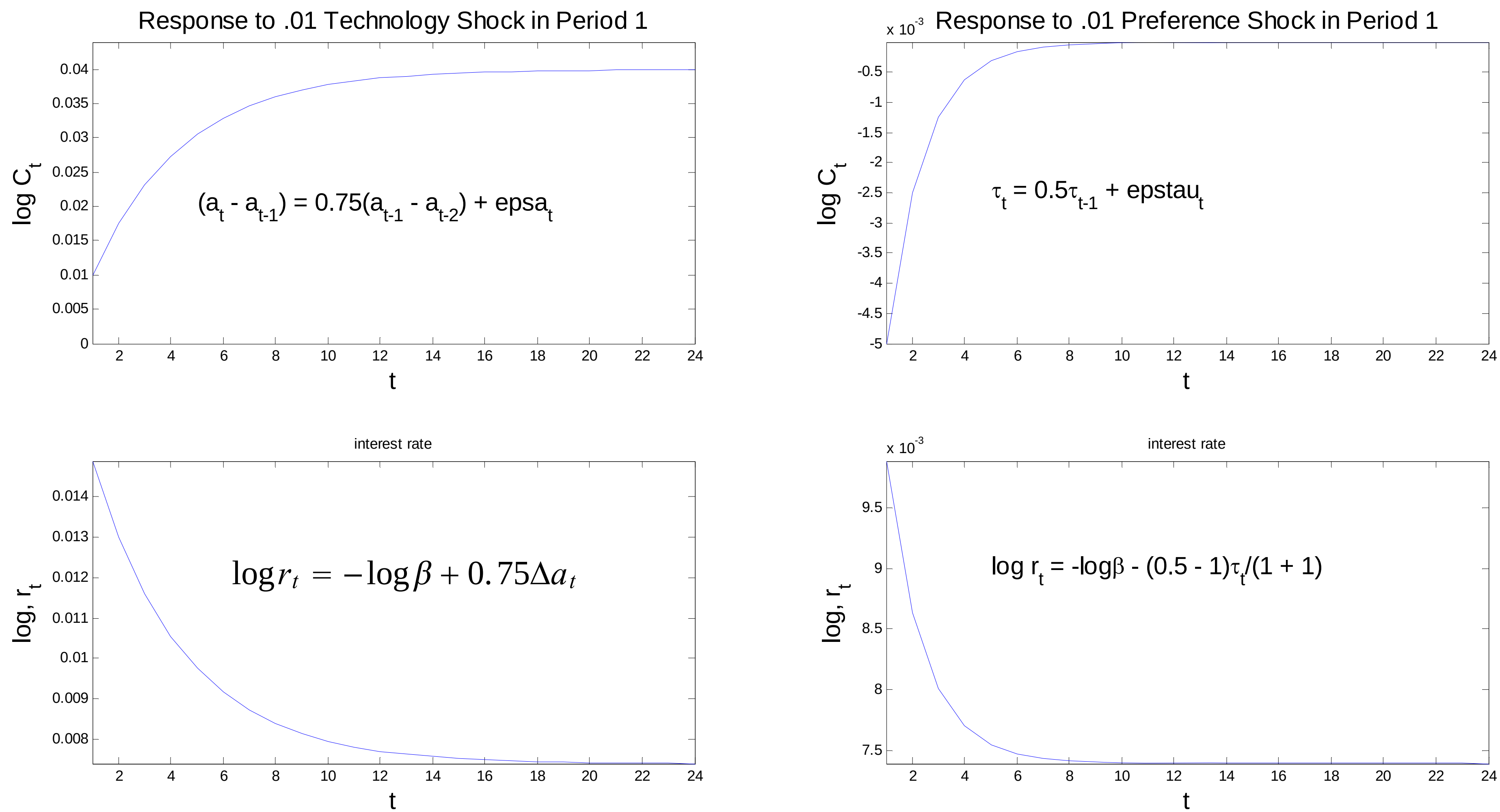
$$\rightarrow r_t = \frac{1}{\beta E_t \frac{C_t}{C_{t+1}}} = \frac{1}{\beta E_t \exp[c_t - c_{t+1}]} = \frac{1}{\beta E_t \exp\left[a_t - a_{t+1} - \frac{\tau_t - \tau_{t+1}}{1+\varphi}\right]}$$

$$= \frac{1}{\beta \exp\left[E_t\left(-\Delta a_{t+1} - \frac{\tau_t - \tau_{t+1}}{1+\varphi}\right) + \frac{1}{2}V\right]},\ V = \sigma_a^2 + \left(\frac{1}{1+\varphi}\right)^2 \sigma_\lambda^2$$

$$\log r_t = -\log\beta + E_t\left(\overbrace{\Delta a_{t+1} - \frac{\tau_{t+1} - \tau_t}{1+\varphi}}^{c_{t+1}-c_t}\right) + \frac{1}{2}V$$

$$\underbrace{=}_{\text{using assumptions about } \Delta a_t \text{ and } \tau_t} -\log\beta + \rho\Delta a_t - \frac{(\lambda-1)\tau_t}{1+\varphi} + \frac{1}{2}V$$

Dynamic Properties of the Model



Key Features of Equilibrium Allocations

- Allocations *efficient* (as long as monopoly power neutralized)
- Employment does not respond to technology
  - Improvement in technology raises marginal product of labor and marginal cost of labor by same amount.
- First best consumption not a function of intertemporal considerations
  - Discount rate irrelevant.
  - Anticipated future values of shocks irrelevant.
- Natural rate of interest steers consumption and employment towards their natural levels, ‘as if guided by an invisible hand’.

Introducing Price Setting Frictions (Clarida-Gali-Gertler Model)

- Households maximize:

$$E_0 \sum_{t=0}^{\infty} \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}, \ \varepsilon_t^{\tau} \sim iid,$$

- Subject to:

Profits, net of taxes raised by Government to finance subsidies.

$$P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + T_t$$

- Intratemporal first order condition:

$$C_t \exp(\tau_t) N_t^{\varphi} = \frac{W_t}{P_t}$$

Household Intertemporal FONC

- Condition:

$$1 = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} \frac{R_t}{1+\pi_{t+1}}$$

  – or

$$\begin{aligned} 1 &= \beta E_t \frac{C_t}{C_{t+1}} \frac{R_t}{1+\pi_{t+1}} \\ &= \beta E_t \exp[\log(R_t) - \log(1+\pi_{t+1}) - \Delta c_{t+1}] \\ &\simeq \beta \exp[\log(R_t) - E_t \pi_{t+1} - E_t \Delta c_{t+1}],\ c_t \equiv \log(C_t) \end{aligned}$$

  – take log of both sides:

$$0 = \log(\beta) + r_t - E_t \pi_{t+1} - E_t \Delta c_{t+1},\ r_t = \log(R_t)$$

  – or

$$c_t = -\log(\beta) - [r_t - E_t \pi_{t+1}] + c_{t+1}$$

NK *IS* Curve

- Euler equation in two equilibria:

Output equals consumption

Actual equilibrium: $y_t = -[r_t - E_t\pi_{t+1} - rr] + E_t y_{t+1}$

Natural equilibrium: $y_t^* = -[r_t^* - rr] + E_t y_{t+1}^*$

- Subtract:

Output gap

$$x_t = -[r_t - E_t\pi_{t+1} - r_t^*] + E_t x_{t+1}$$

Final Good Firms

- Buy $Y_{i,t},\ i \in [0,1]$ at prices $P_{i,t}$ and sell Y_t for P_t
- Take all prices as given (competitive)
- Profits:

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$

- Production function:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} di,\ \varepsilon > 1,$$

- First order condition:

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \qquad \rightarrow P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}$$

Intermediate Good Firms

- Each ith good produced by a single monopoly producer.
- Demand curve:

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon}$$

- Technology:

$$Y_{i,t} = \exp(a_t) N_{i,t}, \; \Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t^a,$$

- Calvo Price-setting Friction

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}$$

Marginal Cost

real marginal cost = $s_t = \frac{\frac{dCost}{dworker}}{\frac{dOutput}{dworker}} = \frac{(1-\nu)W_t/P_t}{\exp(a_t)}$

$$= \frac{\overbrace{(1-\nu)}^{=\frac{\varepsilon-1}{\varepsilon} \text{ in efficient setting}} \quad C_t \exp(\tau_t) N_t^{\varphi}}{\exp(a_t)}$$

The Intermediate Firm's Decisions

- i*th* firm is required to satisfy whatever demand shows up at its posted price.

- Its only real decision is to adjust price whenever the opportunity arises.

- It sets its price as a function of current and expected future marginal cost.
  - Inflation evolves (to first order approximation) as follows:

$$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\left[\hat{s}_t + \beta E_t \hat{s}_{t+1} + \beta^2 E_t \hat{s}_{t+2} + \ldots\right]$$

$$\hat{s}_t \equiv \frac{s_t - s}{s},\ \pi_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}}$$

NK Phillips Curve

$$\beta E_t \pi_{t+1} = \frac{(1-\theta)(1-\beta\theta)}{\theta}[\beta E_t \hat{s}_{t+1} + \beta^2 E_t \hat{s}_{t+2} + \beta^3 E_t \hat{s}_{t+3} + \dots]$$

$$\pi_t - \beta E_t \pi_{t+1} = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{s}_t$$

$$\hat{s}_t = (1+\varphi)x_t$$

$$\rightarrow \pi_t = \beta E_t \pi_{t+1} + \kappa x_t,\ \kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}(1+\varphi)$$

Taylor Rule

- Policy rule

$$r_t = \alpha r_{t-1} + (1-\alpha)[rr + \phi_\pi \pi_t + \phi_x x_t] \qquad , x_t \equiv y_t - y_t^*.$$

Equations of Actual Equilibrium Closed by Adding Policy Rule

$$\beta E_t \pi_{t+1} + \kappa x_t - \pi_t \; = \; 0 \text{ (Phillips curve)}$$

$$-\left[r_t - E_t \pi_{t+1} - r_t^*\right] + E_t x_{t+1} - x_t \; = \; 0 \text{ (IS equation)}$$

$$\alpha r_{t-1} + (1-\alpha)\phi_\pi \pi_t + (1-\alpha)\phi_x x_t - r_t \; = \; 0 \text{ (policy rule)}$$

$$r_t^* - \rho \Delta a_t - \frac{1}{1+\varphi}(1-\lambda)\tau_t \; = \; 0 \text{ (definition of natural rate)}$$

Solving the Model

$$s_t = \begin{pmatrix} \Delta a_t \\ \tau_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \lambda \end{bmatrix} \begin{pmatrix} \Delta a_{t-1} \\ \tau_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^{\tau} \end{pmatrix}$$

$$s_t = P s_{t-1} + \epsilon_t$$

$$\begin{bmatrix} \beta & 0 & 0 & 0 \\ \frac{1}{\sigma} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \\ r_{t+1} \\ r^*_{t+1} \end{pmatrix} + \begin{bmatrix} -1 & \kappa & 0 & 0 \\ 0 & -1 & -\frac{1}{\sigma} & \frac{1}{\sigma} \\ (1-\alpha)\phi_\pi & (1-\alpha)\phi_x & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ rr^*_t \end{pmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \\ r_{t-1} \\ r^*_{t-1} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} s_{t+1} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\sigma\psi\rho & -\frac{1}{\sigma+\varphi}(1-\lambda) \end{pmatrix} s_t$$

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

Solving the Model

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

$$s_t - P s_{t-1} - \epsilon_t = 0.$$

- Solution:

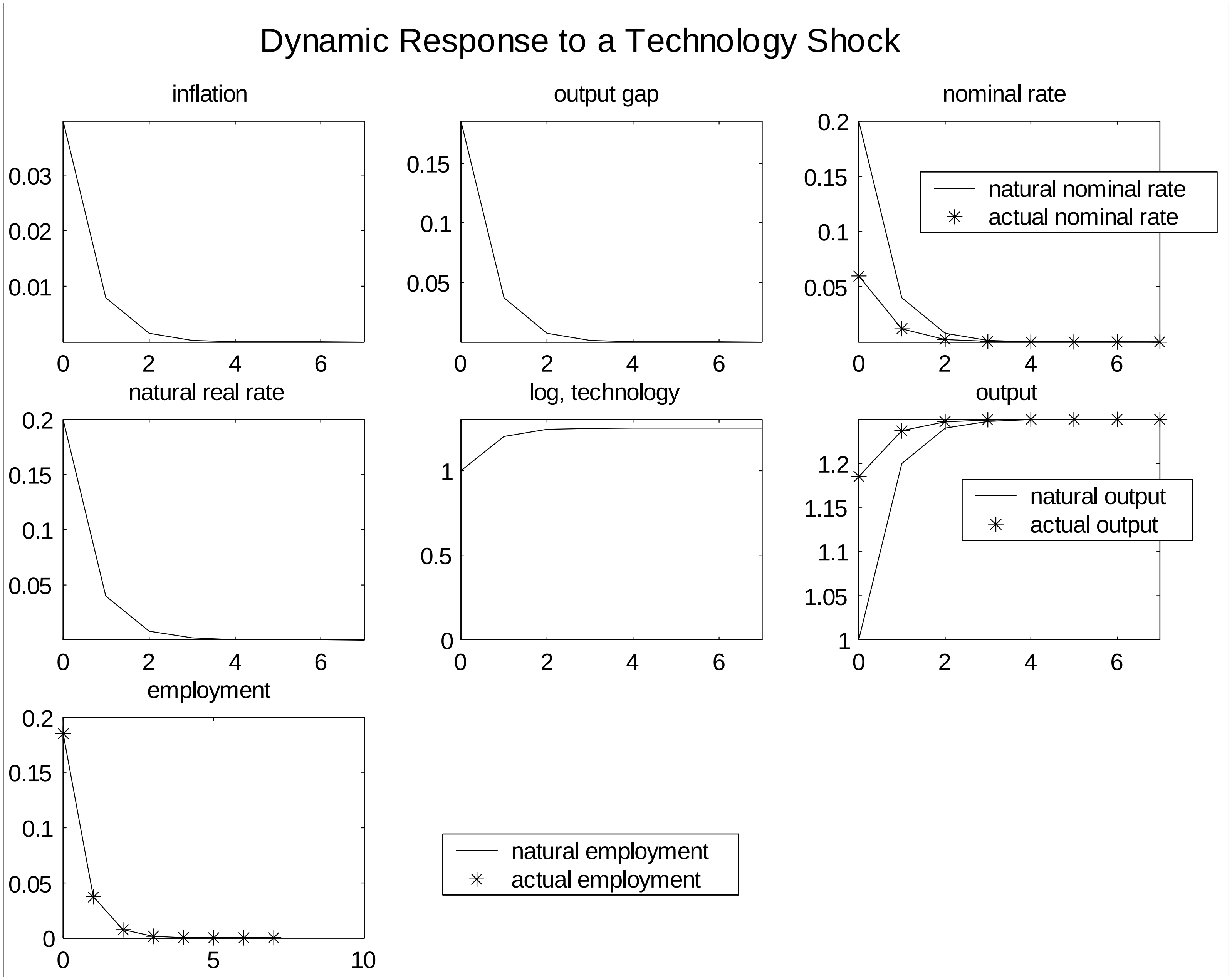
$$z_t = A z_{t-1} + B s_t$$

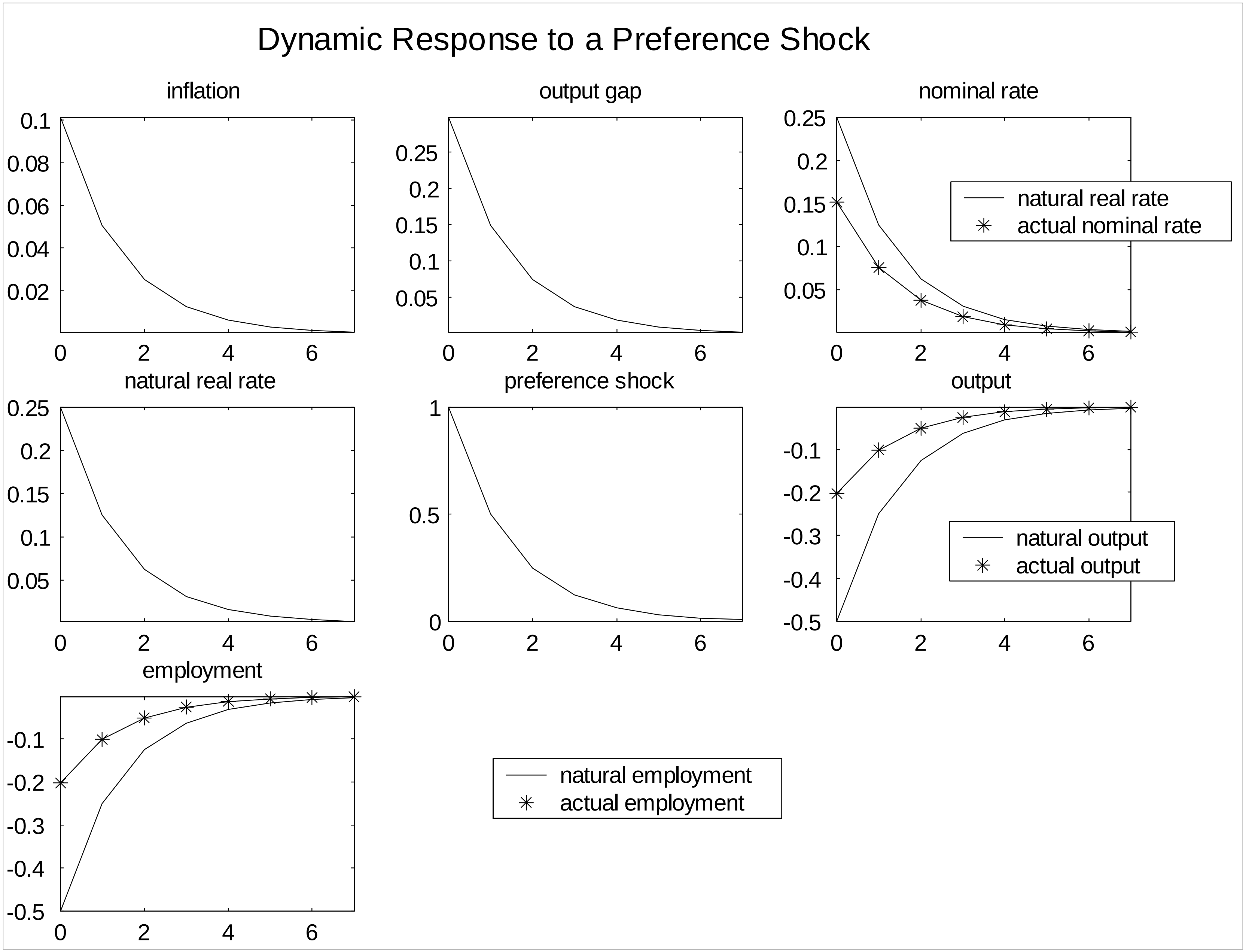
- As before:

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0,$$

$$F = (\beta_0 + \alpha_0 B)P + [\beta_1 + (\alpha_0 A + \alpha_1)B] = 0$$

$\phi_x = 0,\ \phi_\pi = 1.5,\ \beta = 0.99,\ \varphi = 1,\ \rho = 0.2,\ \theta = 0.75,\ \alpha = 0,\ \delta = 0.2,\ \lambda = 0.5.$





Dynamic Response to a Preference Shock
inflation
output gap
nominal rate
natural real rate
actual nominal rate
natural real rate
preference shock
output
natural output
actual output
employment
natural employment
actual employment
0.1
0.08
0.06
0.04
0.02
0.25
0.2
0.15
0.1
0.05
1
0.5
0
-0.1
-0.2
-0.3
-0.4
-0.5
2
4
6