The Basic New Keynesian Model, the Labor Market and Sticky Wages

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- Baseline NK model with no capital and with a competitive labor market.
    - private sector equilibrium conditions
    - Details: http://faculty.wcas.northwestern.edu/~lchrist/course/Korea_2012/intro_NK.pdf
- Standard Labor Market Friction: Erceg-Henderson-Levin sticky wages.

New Keynesian Model with Competitive Labor Market: Households

- Problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}$$

$$\text{s.t. } P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + \text{Profits net of taxes}_t$$

- First order conditions:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5)$$

$$\exp(\tau_t) C_t N_t^{\varphi} = \frac{W_t}{P_t}.$$

New Keynesian Model with Competitive Labor Market: Goods

- Final good firms:
  - maximize profits:

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,$$

    subject to:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

  - Foncs:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon} \rightarrow \overbrace{P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}}^{\text{"cross price restrictions"}}$$

New Keynesian Model with Competitive Labor Market: Goods

- Demand curve for i^{th} monopolist:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon}.$$

- Production function:

$$Y_{i,t} = \exp\left(a_t\right) N_{i,t},\ a_t = \rho a_{t-1} + \varepsilon_t^a$$

- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}.$$

- Real marginal cost:

$$s_t = \frac{\frac{d\text{Cost}}{d\text{worker}}}{\frac{d\text{output}}{d\text{worker}}} = \frac{\overbrace{(1-\nu)}^{\text{minimize monopoly distortion by setting } = \frac{\varepsilon-1}{\varepsilon}} \frac{W_t}{P_t}}{\exp\left(a_t\right)}$$

Brief Digression

- With Dixit-Stiglitz final good production function, there is an optimal allocation of resources to all the intermediate activities, $Y_{i,t}$
  - It is optimal to run them all at the same rate, *i.e.*, $Y_{i,t} = Y_{j,t}$ for all $i, j \in [0, 1]$.
- For given N_t, it is optimal to set $N_{i,t} = N_{j,t}$ for all $i, j \in [0, 1]$
- In this case, final output is given by

$$Y_t = e^{a_t} N_t.$$

- Best way to see this is to suppose that labor is *not* allocated equally to all activities.
  - But, this can happen in a million different ways when there is a continuum of inputs!
  - Explore one simple deviation from $N_{i,t} = N_{j,t}$ for all $i, j \in [0, 1]$.

Suppose Labor *Not* Allocated Equally

- Example:

$$N_{it} = \begin{cases} 2\alpha N_t & i \in \left[0, \frac{1}{2}\right] \\ 2(1-\alpha)N_t & i \in \left[\frac{1}{2}, 1\right] \end{cases}, \; 0 \le \alpha \le 1.$$

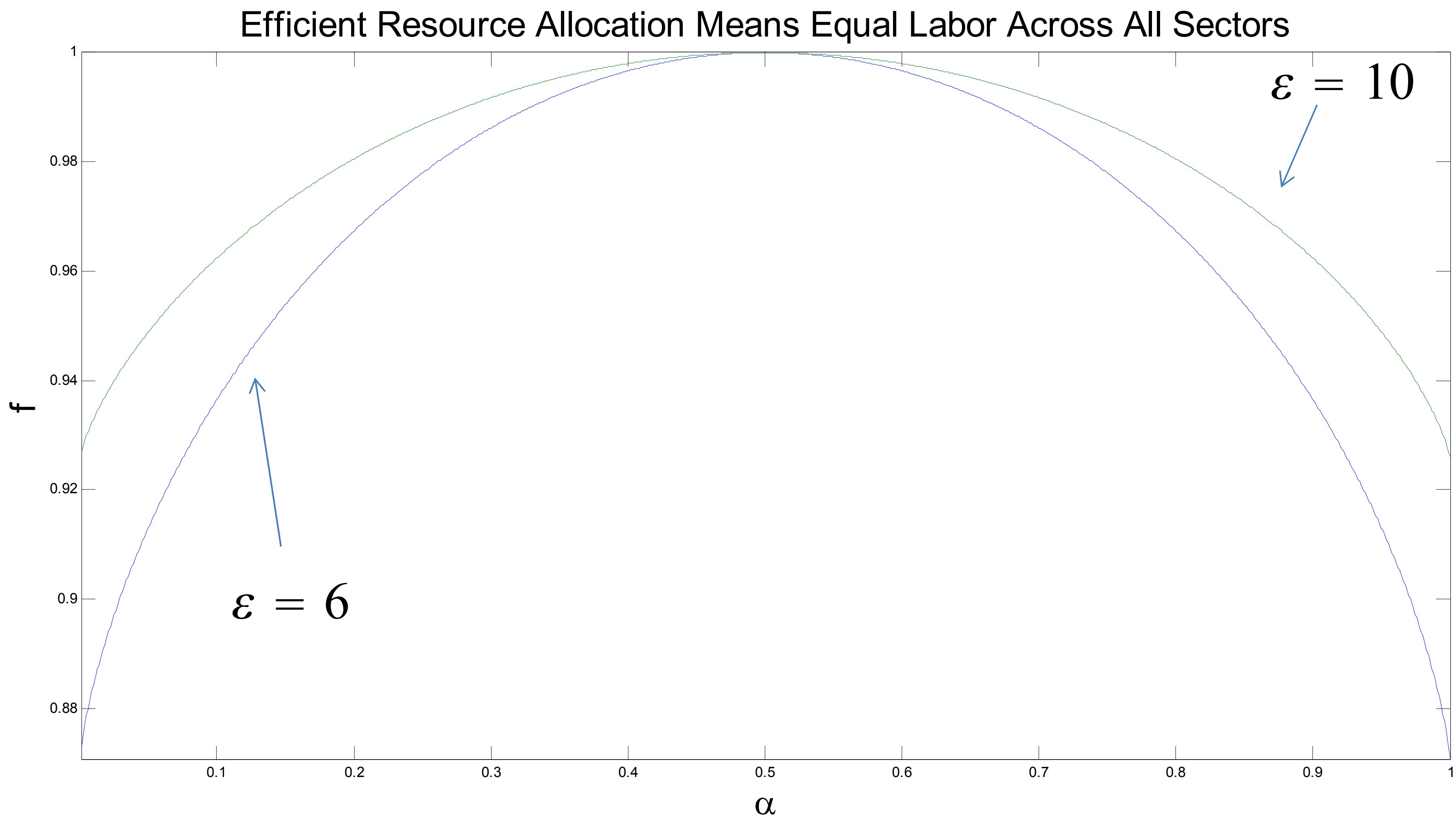
- Note that this is a particular distribution of labor across activities:

$$\int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1-\alpha) N_t = N_t$$

Labor *Not* Allocated Equally, cnt'd

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\int_0^{\frac{1}{2}} Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} (2\alpha N_t)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha)N_t)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\int_0^{\frac{1}{2}} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t f(\alpha)
\end{aligned}
$$

$$f(\alpha) = \left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$



Optimal Price Setting

- Let

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t}, \ \bar{\pi}_t \equiv \frac{P_t}{P_{t-1}}.$$

- Optimal price setting:

$$\tilde{p}_t = \frac{K_t}{F_t},$$

where

$$\begin{aligned} K_t &= \frac{\varepsilon}{\varepsilon - 1} s_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} (1) \\ F_t &= 1 + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} . (2) \end{aligned}$$

- Note:

$$\begin{aligned} K_t &= \frac{\varepsilon}{\varepsilon - 1} s_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+1} \\ &\quad + (\beta\theta)^2 E_t \bar{\pi}_{t+2}^{\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+2} + \dots \end{aligned}$$

Goods and Price Equilibrium Conditions

- Cross-price restrictions imply, given the Calvo price-stickiness:

$$P_t = \left[(1-\theta) \tilde{P}_t^{(1-\varepsilon)} + \theta P_{t-1}^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}.$$

- Dividing latter by P_t and solving:

$$\tilde{p}_t = \left[\frac{1-\theta \bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \rightarrow \frac{K_t}{F_t} = \left[\frac{1-\theta \bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \quad (3)$$

- Relationship between aggregate output and aggregate inputs:

$$\begin{aligned} C_t &= p_t^* A_t N_t, \ (6) \\ \text{where } p_t^* &= \left[(1-\theta) \left(\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{1-\varepsilon}} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \quad (4) \end{aligned}$$

Linearizing around Efficient Steady State

- In steady state (assuming $\bar{\pi} = 1$, $1 - \nu = \frac{\varepsilon - 1}{\varepsilon}$)

$$p^* = 1,\ K = F = \frac{1}{1 - \beta\theta},\ s = \frac{\varepsilon - 1}{\varepsilon},\ \Delta a = \tau = 0,\ N = 1$$

- Linearizing the Tack Yun distortion, (4), about steady state: $\hat{p}_t^* = \theta \hat{p}_{t-1}^* \rightarrow$

$$\boxed{\hat{p}_t^* = 0,\ t \text{ large}}$$

- Denote the output gap in ratio form by X_t:

$$X_t \equiv \frac{C_t}{A_t \exp\left(-\frac{\tau_t}{1+\varphi}\right)} = p_t^* N_t \exp\left(\frac{\tau_t}{1+\varphi}\right),$$

so that (using $x_t \equiv \hat{X}_t$):

$$\boxed{x_t = \hat{N}_t + \frac{d\tau_t}{1+\varphi}}$$

NK IS Curve, Baseline Model

- The intertemporal Euler equation, (5), after substituting for C_t in terms of X_t:

$$\frac{1}{X_t A_t \exp\left(-\frac{\tau_t}{1+\varphi}\right)} = \beta E_t \frac{1}{X_{t+1} A_{t+1} \exp\left(-\frac{\tau_{t+1}}{1+\varphi}\right)} \frac{R_t}{\bar{\pi}_{t+1}}$$

$$\frac{1}{X_t} = E_t \frac{1}{X_{t+1} R^*_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}},$$

where

$$R^*_{t+1} \equiv \frac{1}{\beta} \exp\left(a_{t+1} - a_t - \frac{\tau_{t+1} - \tau_t}{1+\varphi}\right)$$

then, use

$$\widehat{z_t u_t} = \hat{z}_t + \hat{u}_t, \; \widehat{\left(\frac{u_t}{z_t}\right)} = \hat{u}_t - \hat{z}_t$$

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$$\frac{1}{X_t} = E_t \frac{1}{X_{t+1} R^*_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}},$$

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then, use

$$\widehat{z_t u_t} = \hat{z}_t + \hat{u}_t, \ \widehat{\left(\frac{u_t}{z_t}\right)} = \hat{u}_t - \hat{z}_t$$

to obtain:

$$\hat{X}_t = E_t\left[\hat{X}_{t+1} - \left(\hat{R}_t - \hat{\bar{\pi}}_{t+1} - \hat{R}^*_{t+1}\right)\right]$$

NK IS Curve, Baseline Model

- Let:

$$\begin{aligned} R_t &\equiv \exp(r_t) \\ &\rightarrow \hat{R}_t = \frac{d\exp(r_t)}{\exp(r)} = \frac{\exp(r)\,dr_t}{\exp(r)} = dr_t \equiv r_t - r. \end{aligned}$$

- Also,

$$\begin{aligned} R^*_{t+1} &= \exp\left(\log R^*_{t+1}\right) \\ &\rightarrow \hat{R}^*_{t+1} = \frac{d\exp\left(\log R^*_{t+1}\right)}{\exp\left(\log R^*\right)} = d\log R^*_{t+1} \\ &= \log R^*_{t+1} - \overbrace{\log R^*}^{=r \text{ in efficient steady state, with } \bar{\pi}=1} \end{aligned}$$

- So, (letting $r^*_t \equiv E_t \log R^*_{t+1}$)

$$E_t\left(\hat{R}_t - \hat{R}^*_{t+1}\right) = dr_t - E_t d\log R^*_{t+1} = r_t - r^*_t.$$

NK IS Curve, Baseline Model

- Substituting

$$E_t\left(\hat{R}_t - \hat{R}^*_{t+1}\right) = r_t - r^*_t$$

into

$$\hat{X}_t = E_t\left[\hat{X}_{t+1} - \left(\hat{R}_t - \widehat{\bar{\pi}}_{t+1} - \hat{R}^*_{t+1}\right)\right],\ x_t \equiv \hat{X}_t,$$

and using

$$\widehat{\bar{\pi}}_{t+1} = \pi_{t+1},\ \text{when } \bar{\pi} = 1,$$

we obtain NK IS curve:

$$\boxed{x_t = E_t x_{t+1} - E_t\left[r_t - \pi_{t+1} - r^*_t\right]}$$

- Also,

$$r^*_t = -\log(\beta) + E_t\left[a_{t+1} - a_t - \frac{\tau_{t+1} - \tau_t}{1+\varphi}\right].$$

Linearized Marginal Cost in Baseline Model

- Marginal cost (using $da_t = a_t$, $d\tau_t = \tau_t$ because $a = \tau = 0$):

$$\begin{aligned} s_t &= (1-\nu)\frac{\bar{w}_t}{A_t},\ \bar{w}_t = \exp(\tau_t) N_t^{\varphi} C_t \\ &\rightarrow \widehat{\bar{w}}_t = \tau_t + a_t + (1+\varphi)\hat{N}_t \end{aligned}$$

- Then,

$$\boxed{\hat{s}_t = \widehat{\bar{w}}_t - a_t = (\varphi+1)\left[\tfrac{\tau_t}{\varphi+1} + \hat{N}_t\right] = (\varphi+1)\, x_t}$$

Linearized Phillips Curve in Baseline Model

- Log-linearize equilibrium conditions, (1)-(3), around steady state:

$$\boxed{\hat{K}_t = (1-\beta\theta)\,\hat{s}_t + \beta\theta\left(\varepsilon\hat{\bar{\pi}}_{t+1} + \hat{K}_{t+1}\right)\ (1)}$$
$$\boxed{\hat{F}_t = \beta\theta\,(\varepsilon - 1)\,\hat{\bar{\pi}}_{t+1} + \beta\theta\hat{F}_{t+1}\ (2)}$$
$$\boxed{\hat{K}_t = \hat{F}_t + \tfrac{\theta}{1-\theta}\hat{\bar{\pi}}_t\ (3)}$$

- Substitute (3) into (1)

$$\hat{F}_t + \frac{\theta}{1-\theta}\hat{\pi}_t = (1-\beta\theta)\,\hat{s}_t + \beta\theta\left(\varepsilon\hat{\bar{\pi}}_{t+1} + \hat{F}_{t+1} + \frac{\theta}{1-\theta}\hat{\pi}_{t+1}\right)$$

- Simplify the latter using (2), to obtain the NK Phillips curve:

$$\boxed{\pi_t = \tfrac{(1-\theta)(1-\beta\theta)}{\theta}\hat{s}_t + \beta\pi_{t+1}}$$

The Linearized Private Sector Equilibrium Conditions of the Competitive Labor Market Model

$x_t = x_{t+1} - \left[r_t - \pi_{t+1} - r_t^*\right]$
$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{s}_t + \beta\pi_{t+1}$
$\hat{s}_t = (\varphi + 1)\, x_t$
$r_t^* = -\log(\beta) + E_t\left[a_{t+1} - a_t - \frac{\tau_{t+1}-\tau_t}{1+\varphi}\right]$

Equations of Actual Equilibrium Closed by Adding Policy Rule

$$\beta E_t \pi_{t+1} + \kappa x_t - \pi_t \;=\; 0 \text{ (Phillips curve)}$$

$$-\left[r_t - E_t \pi_{t+1} - r_t^*\right] + E_t x_{t+1} - x_t \;=\; 0 \text{ (IS equation)}$$

$$\alpha r_{t-1} + (1-\alpha)\phi_\pi \pi_t + (1-\alpha)\phi_x x_t - r_t \;=\; 0 \text{ (policy rule)}$$

$$r_t^* - \rho \Delta a_t - \frac{1}{1+\varphi}(1-\lambda)\tau_t \;=\; 0 \text{ (definition of natural rate)}$$

Solving the Model

$$s_t = \begin{pmatrix} \Delta a_t \\ \tau_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \lambda \end{bmatrix} \begin{pmatrix} \Delta a_{t-1} \\ \tau_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^{\tau} \end{pmatrix}$$

$$s_t = P s_{t-1} + \epsilon_t$$

$$\begin{bmatrix} \beta & 0 & 0 & 0 \\ \frac{1}{\sigma} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \\ r_{t+1} \\ r_{t+1}^{*} \end{pmatrix} + \begin{bmatrix} -1 & \kappa & 0 & 0 \\ 0 & -1 & -\frac{1}{\sigma} & \frac{1}{\sigma} \\ (1-\alpha)\phi_{\pi} & (1-\alpha)\phi_{x} & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ rr_t^{*} \end{pmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \\ r_{t-1} \\ r_{t-1}^{*} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} s_{t+1} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\sigma\psi\rho & -\frac{1}{\sigma+\varphi}(1-\lambda) \end{pmatrix} s_t$$

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

Solving the Model

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

$$s_t - P s_{t-1} - \epsilon_t = 0.$$

- Solution:

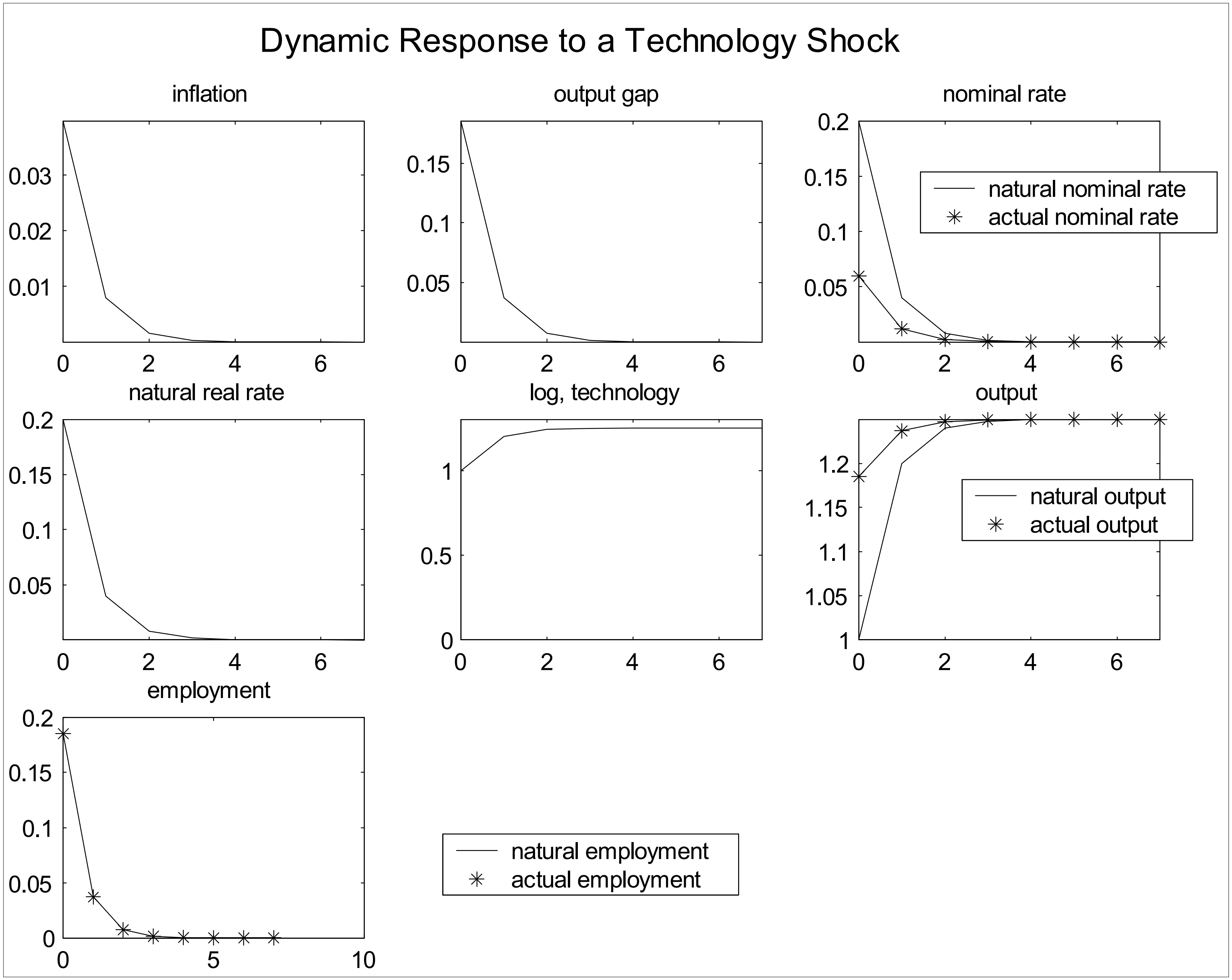
$$z_t = A z_{t-1} + B s_t$$

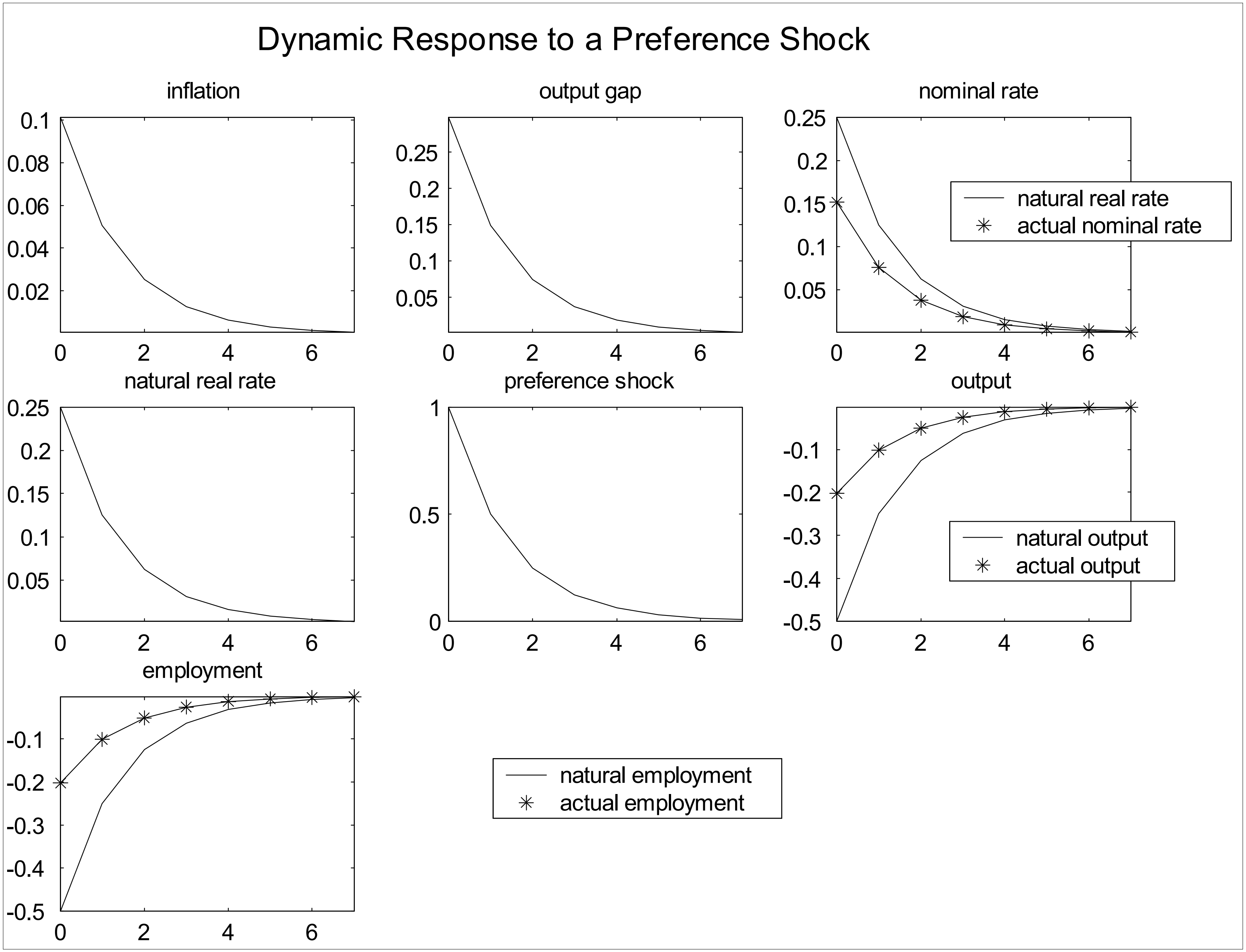
- As before:

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0,$$

$$F = (\beta_0 + \alpha_0 B)P + [\beta_1 + (\alpha_0 A + \alpha_1)B] = 0$$

$$\phi_x = 0,\ \phi_\pi = 1.5,\ \beta = 0.99,\ \varphi = 1,\ \rho = 0.2,\ \theta = 0.75,\ \alpha = 0,\ \delta = 0.2,\ \lambda = 0.5.$$




Dynamic Response to a Preference Shock
inflation
output gap
nominal rate
natural real rate
actual nominal rate
natural real rate
preference shock
output
natural output
actual output
employment
natural employment
actual employment

Reasons to consider frictions in the labor market:

- Play an essential role in accounting for response to a monetary policy shock.
  - With flexible wages, wage costs rise too fast in the wake of expansionary monetary policy shock.
  - High costs limit firms' incentive to expand employment.
  - High costs imply sharp rise in inflation.
  - But, the data suggest that after an expansionary monetary policy shock inflation hardly rises and output rises a lot!
  - Wage frictions play essential role in making possible an account of monetary non-neutrality (see CEE, 2005JPE).
- Important for understanding employment response to other shocks too.
- Introducing sticky wages and monopoly power in labor market provides a theory of unemployment.

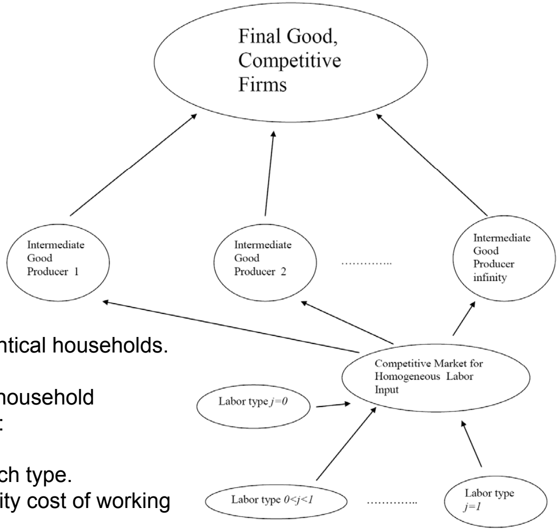
Sticky Wages

- Basic model is due to Erceg-Henderson-Levin.
  - We will follow the interpretation of EHL suggested by Gali, so that we have a theory of unemployment (see also Gali-Smets-Wouters).
- We will not go into the details here
  - see Christiano-Trabant-Walentin Handbook chapter.
  - also, detailed online lecture notes (links in course syllabus).

Outline

- Provide a broad sketch of the model.
- Discuss some linearized equations of the model.
- Provide a critical assessment of the model.

Model

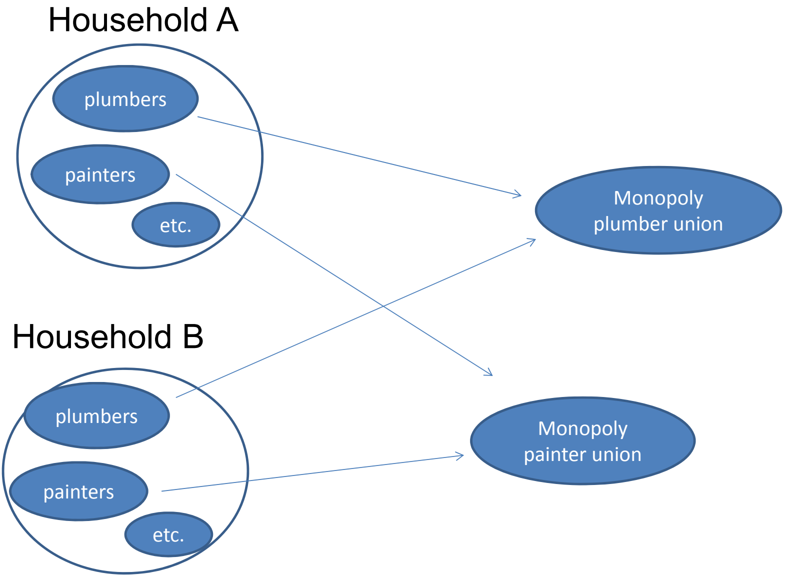


There are many identical households.

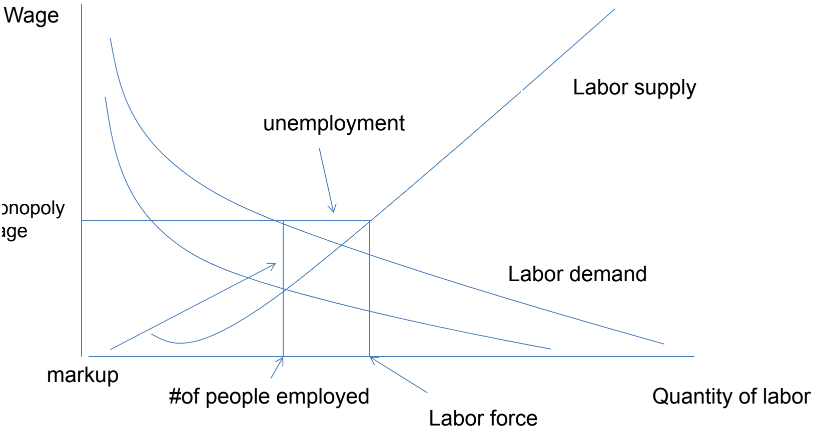
The representative household
contains all workers:

Many workers of each type.
Differentiated by utility cost of working

Every worker enjoys the same level of
consumption.


Household A
plumbers
painters
etc.
Household B
plumbers
painters
etc.
Monopoly plumber union
Monopoly painter union

Type *j* Monopoly Union



Does the Degree of Union Power Affect the Unemployment Rate?

OECD Employment Outlook (2006, chap 7)

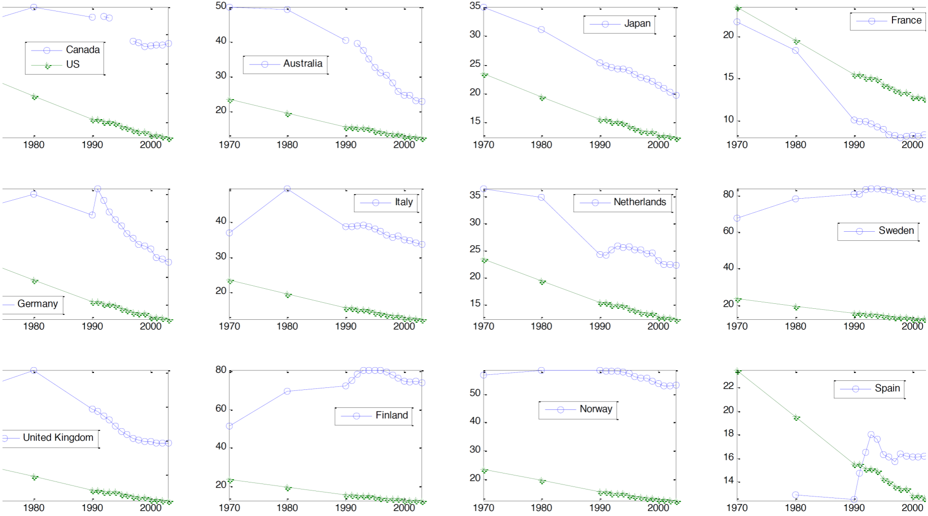
Norway and Denmark have unionization rates near 80 percent. Before the current crisis their unemployment rate was under 3.0 percent.

Union Density Rates

Jelle Visser, 2006 Monthly Labor Review

- Union density rates, 1970, 1980 and 1990–2003, adjusted for comparability.
- Definition: union membership as a proportion of wage and salary earners in employment.

Union Density Rates

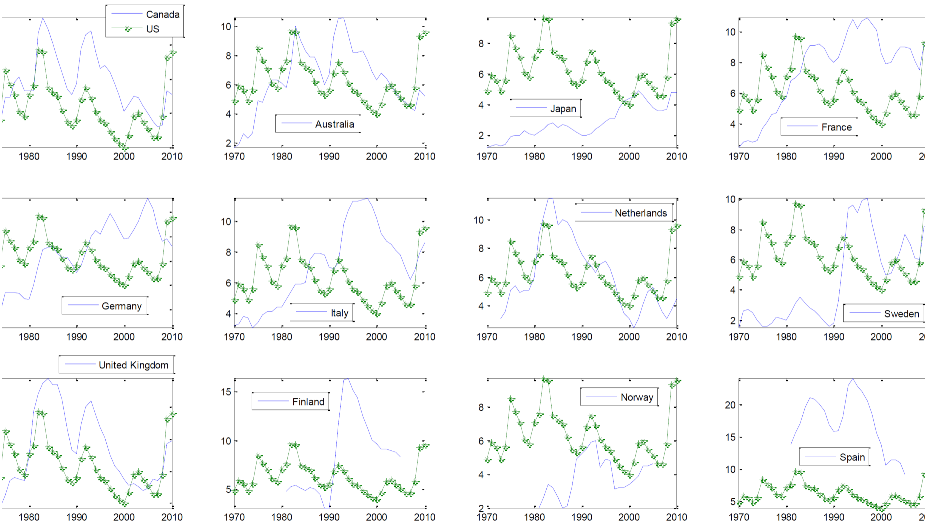


Unemployment Rates: Sources

BLS, "International Comparisons of Annual Labor Force Statistics," Adjusted to U.S. Concepts, 10 Countries, 1970-2010, Table 1-2.

Finland, Norway and Spain taken from ILO, "Comparable annual employment and unemployment estimates, adjusted averages"

Unemployment Rates



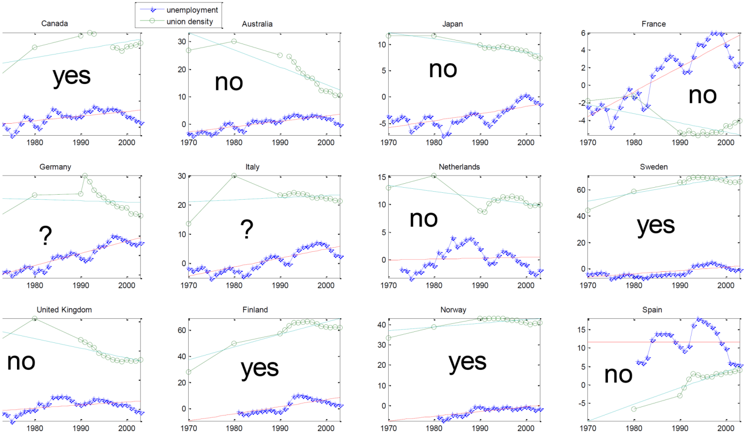
Monopoly Power Hypothesis

- If union density in country A grows faster than union density in US, then
  - Expect unemployment in country A to rise more than unemployment in US.

- Test is based on low frequency part of the data, not on the levels.

Data Consistent With Monopoly Power Hypothesis?

monopoly power ↑ → unemployment ↑



Wage Setting with Frictions

- Suppose, for simplicity, that there are no shocks to labor preferences.
- The solution to union problem gives rise to a 'wage Phillips curve':

$$
\begin{aligned}
\hat{\pi}_{w,t} &= \frac{\kappa_w}{1+\varphi\varepsilon_w}\left(\overbrace{\hat{C}+\varphi\hat{N}_t}^{=\widehat{C_t N_t^{\varphi}}}-\hat{\bar{w}}_t\right)+\beta\hat{\pi}_{w,t+1} \\
\kappa_w &= \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w},\ \pi_{w,t}\equiv\frac{W_t}{W_{t-1}},\ \bar{w}_t\equiv\frac{W_t}{P_t}.
\end{aligned}
$$

Wage Setting with Frictions

- Suppose, for simplicity, that there are no shocks to labor preferences.
- The solution to union problem gives rise to a 'wage Phillips curve':

$$\hat{\pi}_{w,t} = \frac{\kappa_w}{1+\varphi\varepsilon_w}\left(\overbrace{\hat{C}+\varphi\hat{N}_t}^{=\widehat{C_t N_t^{\varphi}}} - \hat{\bar{w}}_t\right) + \beta\hat{\pi}_{w,t+1}$$

$$\kappa_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w},\ \pi_{w,t} \equiv \frac{W_t}{W_{t-1}},\ \bar{w}_t \equiv \frac{W_t}{P_t}.$$

- Wage inflation rises (falls) when cost of working is greater (less) than the real wage.

Collecting the Equations

- Variables to be determined:

$$x_t, r_t, \pi_t, \hat{r}_t^*, \hat{s}_t, \widehat{\bar{w}}_t, \hat{\pi}_{w,t}$$

- Equations:

$x_t = x_{t+1} - \left[r_t - \pi_{t+1} - r_t^*\right]$
$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{s}_t + \beta\pi_{t+1}$
$\hat{s}_t = \widehat{\bar{w}}_t - a_t$
$\hat{\pi}_{w,t} = \frac{\kappa_w}{1+\varphi\varepsilon_w}\left(\hat{C}_t + \varphi\hat{N}_t - \widehat{\bar{w}}_t\right) + \beta\hat{\pi}_{w,t+1}$
$r_t^* = -\log(\beta) + E_t\left[a_{t+1} - a_t\right]$

- Need more equations: relate $\hat{C}_t, \hat{N}_t$ to x_t and shocks, and connect $\widehat{\bar{w}}_t$ to $\hat{\pi}_{w,t}$ and π_t.

- Recall definition of level of output gap: X_t :

$$X_t \equiv \frac{C_t}{A_t} = p_t^* N_t.$$

- Then,

$$\begin{aligned} \hat{C}_t &= x_t + a_t,\ \hat{N}_t \qquad \overbrace{=}^{\text{in the linear approximation about undistored steady state,}} \\ &\rightarrow \hat{C} + \varphi \hat{N}_t = (1+\varphi)\, x_t + a_t. \end{aligned}$$

- Also,

$$\begin{aligned} \pi_{w,t} &\equiv \frac{W_t}{W_{t-1}} = \frac{\frac{W_t}{P_t}}{\frac{W_{t-1}}{P_{t-1}}} \frac{P_t}{P_{t-1}} = \frac{\bar{w}_t}{\bar{w}_{t-1}} \bar{\pi}_t \\ &\rightarrow \hat{\pi}_{w,t} = \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1} + \widehat{\bar{\pi}}_t \end{aligned}$$

Equations of the Sticky Wage Model

- Six private sector equations in seven variables:

$$x_t = x_{t+1} - \left[r_t - \pi_{t+1} - r_t^*\right]$$

$$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{s}_t + \beta\pi_{t+1}$$

$$\hat{s}_t = \widehat{\bar{w}}_t - a_t$$

$$\hat{\pi}_{w,t} = \frac{\kappa_w}{1+\varphi\varepsilon_w}\left[(1+\varphi)\,x_t + a_t - \widehat{\bar{w}}_t\right] + \beta\hat{\pi}_{w,t+1}$$

$$\hat{\pi}_{w,t} = \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1} + \hat{\pi}_t$$

$$r_t^* = \Delta a_{t+1}$$

where r_t and r_t^* now stand for their deviations from steady state, r.

- Monetary policy rule:

$$r_t = \alpha r_{t-1} + (1-\alpha)\left[\phi_\pi \pi_t + \phi_x x_t\right] + u_t,$$

where u_t denotes a monetary policy shock.

Conclusion

- Sticky wages effective in getting wage not to rise much after a monetary policy shock, limiting the rise in inflation and amplifying the rise in output.
- But,
  - Underlying monopoly power theory of unemployment does not receive strong support in the data.
  - Important recent policy question: what will be the effect of extending the duration of unemployment benefits?
    - some say, more unemployment benefits will make an already weak economy even weaker.
    - others say, the number of available jobs is low because of weakness in aggregate demand.
    - "if workers search less in response to better unemployment benefits, that won't change the (low) number of existing jobs because it won't affect aggregate demand (see Christiano, Eichenbaum and Trabandt (in process))".
- We will discuss alternative approaches to labor markets next (see Christiano, Eichenbaum and Trabandt (2013)).