Simple New Keynesian Model without Capital: Implications of Networks

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Background

- The simple ('three equation') New Keynesian model assumes:
  - firms have no need to borrow to finance variable inputs (e.g., no working capital requirement)
  - *all* production is sold to final consumers, none to other firms.
    - Basu (AER, 1996) argues that about 1/2 of gross production is sold to other firms.
    - Acemoglu, Akcigit and Kerr (forthcoming, Macro Annual) also draw attention to the 'network' nature of production.
    - See Christiano, Trabandt and Walentin (Handbook of Monetary Economics, 2011) for an extended discussion of the approach to networks developed here.
- Some possible implications of thinking about networks:
  - may make working capital more important and convert the 'Taylor principle' into the 'Taylor curse'.
  - may make the allocative inefficiencies associated with price-setting frictions more severe (see Chad Jones, 'Misallocation, Economic Growth, and Input-Output Economics', 2011).

What We Do Here

- Derive the equilibrium conditions for a version of the simple New Keynesian model with networks.
  - Will use a huge short cut (following Basu (1996, AER)).
- This will put us in a position to evaluate the implications of networking for monetary DSGE models.
- On a technical level, we extend a variant of the simple NK model developed by Prof. Walsh last week.
  - Will be useful for doing exercises with perturbation method in Dynare.
  - Will exploit Prof. Walsh's presentation to streamline the discussion here.

Substantive Part of the Analysis

- The New Keynesian literature ignores implications of the NK model for resource misallocation arising from frictions in the setting of prices.
- In these notes, we suggest that the distortions may in fact be non-negligible based on:
  - results from a relatively 'model-free' estimate, based on US data, of the consequence of distortions induced by price-setting frictions.
  - steady state distortions in a parameterized NK model.
- In the computer lab, we can evaluate inefficiencies implied for economic dynamics using Dynare.
  - We will also examine the implications for the Taylor principle there.

Households

- Problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \; \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}$$

$$\text{s.t. } P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + \text{Profits net of taxes}_t$$

- First order conditions:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5)$$

$$\exp(\tau_t) C_t N_t^{\varphi} = \frac{W_t}{P_t}.$$

Homogeneous Goods Production

- Competitive firms:
  - maximize profits:

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,$$

subject to:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon - 1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon - 1}}.$$

  - Foncs:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon} \to \overbrace{P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}}^{\text{"cross price restrictions"}}$$

Intermediate Goods Production

- Demand curve for i^{th} monopolist:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon}.$$

- Production function:

$$\begin{aligned} Y_{i,t} &= \exp(a_t) N_{i,t}^{\gamma} I_{i,t}^{1-\gamma},\ a_t \text{ ~exogenous shock to technology,} \\ 0 &< \gamma \leq 1. \end{aligned}$$

- $I_{i,t}$ ~'materials' these are purchases of the homogeneous output good (Basu's simplified way of capturing that firms buy goods from other firms).
- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}.$$

Cost Minimization Problem

- Price setting by intermediate good firms is discussed later.
  - The intermediate good firm must produce the quantity demanded, $Y_{i,t}$, at the price that it sets.
  - Right now we take $Y_{i,t}$ as given and we investigate the cost minimization problem that determines the firm's choice of inputs.
- Cost minimization problem:

$$\min_{N_{i,t},I_{i,t}} \bar{W}_t N_{i,t} + \bar{P}_t I_{i,t} + \overbrace{\lambda_{i,t}}^{\text{marginal cost (money terms)}} \left[Y_{i,t} - A_t N_{i,t}^{\gamma} I_{i,t}^{1-\gamma} \right]$$

with resource costs:

$$\bar{W}_t = \overbrace{(1-\nu)}^{\text{subsidy, if } \nu>0} \times \overbrace{(1-\psi_H+\psi_H R_t) W_t}^{\text{cost, including finance, of a unit of labor}}$$

$$\bar{P}_t = (1-\nu) \times \overbrace{(1-\psi_I+\psi_I R_t) P_t}^{\text{cost, including finance, of a unit of materials}}.$$

Cost Minimization Problem

- Problem:

$$\min_{N_{i,t}, I_{i,t}} \bar{W}_t N_{i,t} + \bar{P}_t I_{i,t} + \lambda_{i,t} \left[Y_{i,t} - A_t N_{i,t}^{\gamma} I_{i,t}^{1-\gamma} \right]$$

- First order conditions:

$$\bar{P}_t I_{i,t} = (1-\gamma)\, \lambda_{i,t} Y_{i,t},\ \bar{W}_t N_{i,t} = \gamma \lambda_{i,t} Y_{i,t},$$

so that,

$$\begin{aligned} \frac{I_{it}}{N_{it}} &= \frac{1-\gamma}{\gamma} \frac{\bar{W}_t}{\bar{P}_t} = \frac{1-\gamma}{\gamma} \frac{(1-\psi_N+\psi_N R_t)}{(1-\psi_I+\psi_I R_t)} \exp(\tau_t)\, C_t N_t^{\varphi} \\ &\rightarrow \frac{I_{it}}{N_{it}} = \frac{I_t}{N_t},\ \text{for all } i. \end{aligned}$$

Cost Minimization Problem

- Firm first order conditions imply

$$\lambda_{i,t} = \left(\frac{\bar{P}_t}{1-\gamma}\right)^{1-\gamma} \left(\frac{\bar{W}_t}{\gamma}\right)^{\gamma} \frac{1}{A_t}.$$

- Divide marginal cost by P_t :

$$\begin{aligned} s_t \quad &\equiv \quad \frac{\lambda_{i,t}}{P_t} = (1-\nu) \left(\frac{1-\psi_I+\psi_I R_t}{1-\gamma}\right)^{1-\gamma} \\ &\times \left(\frac{1-\psi_N+\psi_N R_t}{\gamma} \exp\left(\tau_t\right) C_t N_t^{\varphi}\right)^{\gamma} \frac{1}{A_t} \text{ (9)}, \end{aligned}$$

  after substituting out for $\bar{P}_t$ and $\bar{W}_t$ and using the household's labor first order condition.

- Note from (9) that i^{th} firm's marginal cost, s_t, is independent of i and $Y_{it,}$.

Share of Materials in Intermediate Good Output

- Firm i materials proportional to $Y_{i,t}$:

$$I_{i,t} = \frac{(1-\gamma)\,\lambda_{i_it} Y_{i,t}}{\bar{P}_t} = \mu_t Y_{i,t},$$

where

$$\mu_t \frac{(1-\gamma)\, s_t}{(1-\nu)\,(1-\psi_I+\psi_I R_t)} \text{ (10).}$$

- "Share of materials in gross output", μ_t.

Decision By Firm that Can Change Its Price

- i^{th} intermediate good firm's objective:

$$E_t^i \sum_{j=0}^{\infty} \beta^j \ v_{t+j} \overbrace{\left[\overbrace{P_{i,t+j} Y_{i,t+j}}^{\text{revenues}} - \overbrace{P_{t+j} s_{t+j} Y_{i,t+j}}^{\text{total cost}} \right]}^{\text{period } t+j \text{ profits sent to household}}$$

v_{t+j} - Lagrange multiplier on household budget constraint

- Firm that gets to reoptimize its price is concerned only with future states in which it does not change its price:

$$\begin{aligned} & E_t^i \sum_{j=0}^{\infty} \beta^j v_{t+j} \left[P_{i,t+j} Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right] \\ = \ & E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right] + X_t, . \end{aligned}$$

where $\tilde{P}_t$ denotes a firm's price-setting choice at time t and X_t not a function of $\tilde{P}_t$.

Decision By Firm that Can Change Its Price

- Substitute out demand curve:

$$
\begin{aligned}
& E_t \sum_{j=0}^{\infty} (\beta\theta)^j v_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j}\right] \\
= \; & E_t \sum_{j=0}^{\infty} (\beta\theta)^j v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[\tilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon}\right].
\end{aligned}
$$

- Differentiate with respect to $\tilde{P}_t$:

$$
E_t \sum_{j=0}^{\infty} (\beta\theta)^j v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[(1-\varepsilon)\left(\tilde{P}_t\right)^{-\varepsilon} + \varepsilon P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon-1}\right] = 0,
$$

or,

$$
E_t \sum_{j=0}^{\infty} (\beta\theta)^j v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon-1} s_{t+j}\right] = 0.
$$

- When $\theta = 0$, get standard result - price is fixed markup over marginal cost.

Decision By Firm that Can Change Its Price

- Substitute out the multiplier:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \overbrace{\frac{u'\left(C_{t+j}\right)}{P_{t+j}}}^{= v_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0.$$

- Using assumed log-form of utility,

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0,$$

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t},\ \bar{\pi}_t \equiv \frac{P_t}{P_{t-1}},\ X_{t,j} = \left\{ \begin{array}{c} \frac{1}{\bar{\pi}_{t+j} \bar{\pi}_{t+j-1} \cdots \bar{\pi}_{t+1}},\ j \geq 1 \\ 1,\ j = 0. \end{array} \right. ,$$

$$X_{t,j} = X_{t+1,j-1} \frac{1}{\bar{\pi}_{t+1}},\ j > 0$$

Decision By Firm that Can Change Its Price

- Want $\tilde{p}_t$ in:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j}\right] = 0$$

- Solving for $\tilde{p}_t$, we conclude that prices are set as follows:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+1}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_t}{F_t}.$$

- Need convenient expressions for K_t, F_t.

Decision By Firm that Can Change Its Price

$$
\begin{aligned}
K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j} \\
&= \frac{\varepsilon}{\varepsilon-1} \frac{Y_t}{C_t} s_t \\
&\quad + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} \overbrace{E_{t+1} \sum_{j=0}^{\infty} (\beta\theta)^j X_{t+1,j}^{-\varepsilon} \frac{Y_{t+j+1}}{C_{t+j+1}} \frac{\varepsilon}{\varepsilon-1} s_{t+1+j}}^{\text{exactly } K_{t+1}!} \\
&= \frac{\varepsilon}{\varepsilon-1} \frac{Y_t}{C_t} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1}
\end{aligned}
$$

For a detailed derivation, see, e.g., http://faculty.wcas.northwestern.edu/~lchrist/course/IMF2015/intro_NK_handout.pdf.

Decision By Firm that Can Change Its Price

- Conclude:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{-\varepsilon} \frac{Y_{t+j}}{C_{t+j}} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}} = \frac{K_t}{F_t},$$

where

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{Y_t}{C_t} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1} \text{ (1)}$$

- Similarly,

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{1-\varepsilon} F_{t+1} \text{ (2)}$$

Interpretation of Price Formula

- Note,

$$\frac{1}{P_{t+j}} = \frac{1}{P_t} X_{t,j},\ s_{t+j} = \frac{\lambda_{t+j}}{P_{t+j}} = \frac{\lambda_{t+j}}{P_t} X_{t,j},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t}.$$

  Multiply both sides of the expression for $\tilde{p}_t$ by P_t :

$$\tilde{P}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}} \frac{\varepsilon}{\varepsilon-1} \lambda_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}} = \frac{\varepsilon}{\varepsilon-1} \sum_{j=0}^{\infty} E_t \omega_{t+j} \lambda_{t+j}$$

  where

$$\omega_{t+j} = \frac{(\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}},\ \sum_{j=0}^{\infty} E_t \omega_{t+j} = 1.$$

  Evidently, price is set as a markup over a weighted average of future marginal cost, where the weights are shifted into the future depending on how big θ is.

Restriction Between Aggregate and Intermediate Good Prices

- 'Calvo result':

$$P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}} = \left[(1-\theta)\, \tilde{P}_t^{(1-\varepsilon)} + \theta P_{t-1}^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}.$$

- Divide by P_t :

$$1 = \left[(1-\theta)\, \tilde{p}_t^{(1-\varepsilon)} + \theta \left(\frac{1}{\bar{\pi}_t} \right)^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}.$$

- Rearrange:

$$\tilde{p}_t = \left[\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}}$$

Aggregate inputs and outputs

- Technically, there is no 'aggregate production function':
  - there is no exact relationship between output, Y_t, and aggregate inputs, N_t, I_t, A_t.
  - must also know the *distribution* of resources across intermediate good firms.

- Tack Yun (JME) developed a simple approach that can be used to determine the connection between N, A, I, Y and the distribution of resources.

Gross Output and Aggregate Inputs

- Define Y_t^*:

$$
\begin{aligned}
Y_t^* &\equiv \int_0^1 Y_{i,t} di \\
&\overbrace{=}^{\text{demand curve}} Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} di = Y_t P_t^{\varepsilon} \int_0^1 \left(P_{i,t}\right)^{-\varepsilon} di \\
&= Y_t P_t^{\varepsilon} \left(P_t^*\right)^{-\varepsilon}
\end{aligned}
$$

where, using 'Calvo result':

$$
P_t^* \equiv \left[\int_0^1 P_{i,t}^{-\varepsilon} di\right]^{\frac{-1}{\varepsilon}} = \left[(1-\theta)\,\tilde{P}_t^{-\varepsilon} + \theta \left(P_{t-1}^*\right)^{-\varepsilon}\right]^{\frac{-1}{\varepsilon}}
$$

- Then

$$
Y_t = p_t^* Y_t^*,\ p_t^* = \left(\frac{P_t^*}{P_t}\right)^{\varepsilon}.
$$

Law of Motion of Tack Yun Distortion

- We have

$$P_t^* = \left[(1-\theta)\, \tilde{P}_t^{-\varepsilon} + \theta \left(P_{t-1}^* \right)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}$$

- Then,

$$\begin{aligned} p_t^* &\equiv \left(\frac{P_t^*}{P_t} \right)^{\varepsilon} = \left[(1-\theta)\, \tilde{p}_t^{-\varepsilon} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \\ &= \left[(1-\theta) \left(\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \qquad (4) \end{aligned}$$

using the restriction between $\tilde{p}_t$ and aggregate inflation.

Gross Output and Aggregate Input

- Relationship between aggregate inputs and outputs:

$$
\begin{aligned}
Y_t &= p_t^* Y_t^* = p_t^* \int_0^1 Y_{i,t} di \\
&= p_t^* A_t \int_0^1 N_{i,t}^{\gamma} I_{i,t}^{1-\gamma} di = p_t^* A_t \int_0^1 \left(\frac{N_{i,t}}{I_{i,t}} \right)^{\gamma} I_{i,t} di, \\
&= p_t^* A_t \left(\frac{N_t}{I_t} \right)^{\gamma} I_t,
\end{aligned}
$$

or,

$$
Y_t = p_t^* A_t N_t^{\gamma} I_t^{1-\gamma} \text{ (6)}
$$

- Tack Yun distortion p_t^*:

$$
p_t^* : \begin{cases} \leq 1 & \\ = 1 & P_{i,t} = P_{j,t}, \text{ all } i, j \end{cases} .
$$

Working Towards an Expression for Gross Domestic Product

- Recall

$$I_{i,t} = \mu_t Y_{i,t},$$

so,

$$I_t \equiv \int_0^1 I_{i,t} di = \mu_t \int_0^1 Y_{i,t} d = \mu_t Y_t^* = \frac{\mu_t}{p_t^*} Y_t.$$

- Then,

$$\begin{aligned} Y_t &= p_t^* A_t N_t^\gamma I_t^{1-\gamma} \\ &= p_t^* A_t N_t^\gamma \left(\frac{\mu_t}{p_t^*} Y_t \right)^{1-\gamma} \\ \longrightarrow & \quad Y_t = \left(p_t^* A_t \left(\frac{\mu_t}{p_t^*} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} N_t \end{aligned}$$

Gross Domestic Product (GDP)

- We have

$$
\begin{aligned}
GDP_t &= Y_t - I_t = \left(1 - \frac{\mu_t}{p_t^*}\right) Y_t \\
&= \left(1 - \frac{\mu_t}{p_t^*}\right) \left(p_t^* A_t \left(\frac{\mu_t}{p_t^*}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}} N_t \\
&= \overbrace{\left(p_t^* A_t \left(1 - \frac{\mu_t}{p_t^*}\right)^{\gamma} \left(\frac{\mu_t}{p_t^*}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}^{=\text{Total Factor Productivity}} N_t
\end{aligned}
$$

- Note how an increase in technology at the firm level, by A_t, gives rise to a bigger increase in TFP by $A_t^{1/\gamma}$.
  - In the literature on networks, $1/\gamma$ is referred to as a 'multiplier effect' (see Jones, 2011).
- The Tack Yun distortion, p_t^*, seems to have the same multiplier phenomenon.

Decomposition for Total Factor Productivity

- To maximize GDP for given aggregate N_t and A_t :

$$\max_{0<p_t^*\leq 1,\ 0\leq\lambda_t\leq 1}\left(p_t^* A_t\left(1-\lambda_t\right)^{\gamma}\left(\lambda_t\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}$$

$$\rightarrow\quad \lambda_t = 1-\gamma,\ p_t^* = 1.$$

- So,

$$TFP_t \;=\; \overbrace{\left(p_t^*\left(\frac{1-\frac{\mu_t}{p_t^*}}{\gamma}\right)^{\gamma}\left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}^{\text{Component due to market distortions}\equiv\chi_t}$$

$$\times\overbrace{\left(A_t\left(\gamma\right)^{\gamma}\left(1-\gamma\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}^{\text{Technology component}}$$

Evaluating the Distortions

- The equations characterizing the TFP distortion, χ_t :

$$\chi_t = \left(p_t^* \left(\frac{1 - \frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}}$$

$$p_t^* = \left[(1-\theta) \left(\frac{1 - \theta \bar{\pi}_t^{(\varepsilon - 1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon - 1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1}.$$

- Potentially, NK model provides an 'endogenous theory of TFP'.
- Standard practice in NK literature is to set $\chi_t = 1$ for all t.
  - Set $\gamma = 1$ and linearize around $\bar{\pi}_t = p_t^* = 1$.
  - With $\gamma = 1$, $\chi_t = p_t^*$, and first order expansion of p_t^* around $\bar{\pi}_t = p_t^* = 1$ is:

$$p_t^* = p^* + 0 \times \bar{\pi}_t + \theta \left(p_{t-1}^* - p^* \right), \text{ with } p^* = 1,$$

so $p_t^* \to 1$ and is invariant to shocks.

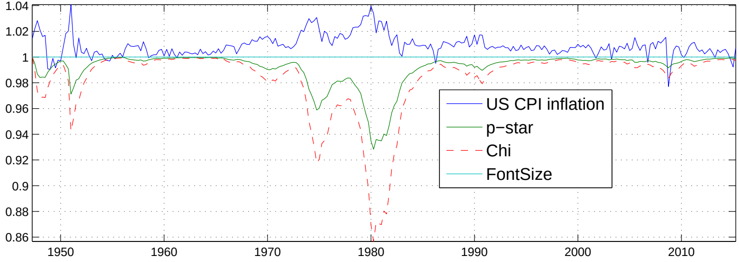
Empirical Assessment of the Distortions

- The TFP distortion, χ_t :

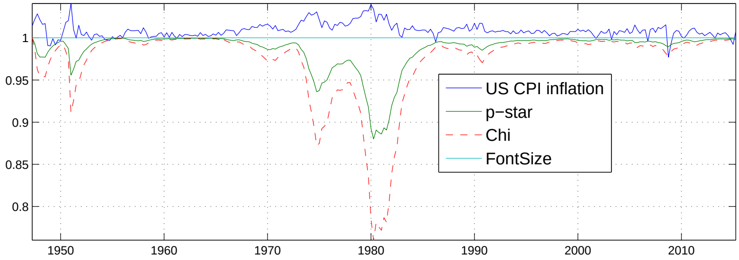
$$\chi_t = \left(p_t^* \left(\frac{1 - \frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}}$$

- Problem: the objects, χ_t and p_t^*, are not quite observable.
  - Still, if we assume μ_t is constant, at $1-\gamma$, we can get a feel about the magnitudes using US inflation data.
- Will consider $\gamma = 1/2$ (Basu's empirical estimate) and $\gamma = 1$ (standard assumption in NK literature).
- Will consider two values for the markup:
  - $\varepsilon / (\varepsilon - 1) = 1.20$, the baseline estimate in CEE (JPE, 2005), which corresponds to $\varepsilon = 6$,
  - $\varepsilon / (\varepsilon - 1) = 1.15$, more competition, i.e., $\varepsilon = 7.7$.

Graph of Quarterly, Gross US CPI inflation, p−star and chi, assumed markup is 1.2



Graph of Quarterly, Gross US CPI inflation, p−star and chi, assumed markup is 1.15



The Distortions Displayed in the Figure are Not Negligible

- With $\varepsilon = 6$,
  - mean$(\chi_t) = 0.98$, a 2% loss of GDP.
  - frequency, $\chi_t < 0.955$, is 10% (i.e., 10% of the time, the output loss is greater than 4.5 percent).
- With more competition (i.e., ε higher), the losses are greater.
  - with higher elasticity of demand, given movements in inflation imply much greater substitution away from high priced items, thus greater misallocation (caveat: this intuition is incomplete since with greater ε the consequences of a given amount of misallocation are smaller).
- Distortions with $\gamma = 1/2$ are roughly twice the size of distortions in standard case, $\gamma = 1$.
  - To see this, let $p^* = 1 - r$. Then,

$$\chi_t \simeq (p^*)^{\frac{1}{\gamma}} \simeq 1 - \frac{1}{\gamma} r.$$

Next

- Summarize the equilibrium conditions.
- Compare flexible price and sticky price equilibria
  - sticky price equilibrium incomplete.
    - One equation short because real allocations in private economy co-determined along with the nominal quantities.
  - flexible price equilibrium (at least, the one without working capital) dichotomizes.
    - real allocations in flexible price model are determined and monetary policy only delivers inflation and the nominal interest, things that do not affect utility.
- Evaluate distortions in steady state.

Summarizing the Equilibrium Conditions

- Break up the equilibrium conditions into three sets:
  - Conditions (1)-(4) for prices: $K_t, F_t, \bar{\pi}_t, p_t^*, s_t$
  - Conditions (6)-(10) for: $C_t, Y_t, N_t, I_t, \mu_t$
  - Conditions (5) and (11) for R_t and χ_t.
- Consider
  - conditions for the model as is.
  - conditions pertaining to the case of flexible prices, no working capital and efficient subsidy for monopoly power:

$$\theta = 0,\ \psi_I = \psi_N = 0,\ \frac{\varepsilon}{\varepsilon - 1}(1 - \nu) = 1.$$

    - equilibrium supports 'first best' allocations: those that would occur if a benevolent planner chose the allocations rather than the market.

Equilibrium Conditions for Prices

$$K_t = \frac{\varepsilon}{\varepsilon - 1}\frac{Y_t}{C_t}s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1} \text{ (1)}$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{1-\varepsilon} F_{t+1} \text{ (2)}$$

$$\frac{K_t}{F_t} = \left[\frac{1 - \theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \quad \text{(3)}$$

$$p_t^* = \left[(1-\theta)\left(\frac{1 - \theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1} \quad \text{(4)}$$

- When $\theta = 0$, these boil down to (i) zero price dispersion and (ii) everyone sets price as markup, $\varepsilon/(\varepsilon - 1)$, over marginal cost:

$$p_t^* = 1,\ \frac{\varepsilon}{\varepsilon - 1}s_t = 1,\ K_t = F_t = C_t/Y_t, \text{ no restriction on } \bar{\pi}_t$$

Other, Static, Equilibrium Conditions

- Variables:

$$C_t, Y_t, N_t, I_t, \mu_t$$

- Equations:

$$\begin{aligned}
Y_t &= p_t^* A_t N_t^{\gamma} I_t^{1-\gamma} \text{ (6)},\ C_t + I_t = Y_t \text{ (7)},\ I_t = \mu_t \frac{Y_t}{p_t^*} \text{ (8)} \\
s_t &= (1-\nu)\left(\frac{1-\psi_I+\psi_I R_t}{1-\gamma}\right)^{1-\gamma} \\
&\quad \times \left(\frac{1-\psi_N+\psi_N R_t}{\gamma}\exp(\tau_t)\, C_t N_t^{\varphi}\right)^{\gamma} \frac{1}{A_t} \text{ (9)} \\
\mu_t &= \frac{(1-\gamma)\, s_t}{(1-\nu)(1-\psi_I+\psi_I R_t)} \text{ (10)},
\end{aligned}$$

Other Variables in Flexible Price, no Working Capital Case

- Suppose $\varepsilon(1-\nu)/(\varepsilon-1)=1$, $\theta=\psi_I=\psi_N=0$,

$$
\begin{aligned}
Y_t &= \left[A_t\mu_t^{1-\gamma}\right]^{\frac{1}{\gamma}} N_t \text{ (6)}, \; C_t = \left[A_t\left(1-\mu_t\right)^{\gamma}\mu_t^{1-\gamma}\right]^{\frac{1}{\gamma}} N_t \text{ (6,7,8)} \\
1 &= \frac{\varepsilon}{\varepsilon-1}\left(1-\nu\right)\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{1}{\gamma}\exp\left(\tau_t\right)C_tN_t^{\varphi}\right)^{\gamma}\frac{1}{A_t} \text{ (9)} \\
\mu_t &= \frac{\varepsilon-1}{\varepsilon}\frac{\left(1-\gamma\right)}{\left(1-\nu\right)} = 1-\gamma \text{ (10)},
\end{aligned}
$$

- Substituting (6,7,8) and (10) into (9)

$$
\begin{aligned}
N_t &= \exp\left(-\frac{\tau_t}{1+\varphi}\right) \\
C_t(&= GDP_t) = \left[\left(\gamma\right)^{\gamma}\left(1-\gamma\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}\exp\left(\frac{1}{\gamma}a_t-\frac{\tau_t}{1+\varphi}\right)
\end{aligned}
$$

- Note that in this case all the ‘real variables’ are determined,

Last Equilibrium Conditions

- Distortion:

$$\chi_t = \left(p_t^* \left(\frac{1 - \frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} \quad (11)$$

in $\varepsilon(1-\nu) / (\varepsilon - 1) = 1$, $\theta = \psi_I = \psi_N = 0$ case,

$$\chi_t = 1, \text{ for all } t.$$

- Intertemporal equation

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5)$$

Real Interest Rate in Flex P Equilibrium

- The real interest rate, $R_t/\bar{\pi}_{t+1}$.
  - Absent uncertainty, $R_t/\bar{\pi}_{t+1}$ determined uniquely:
$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}}.$$
  - With uncertainty, household intertemporal condition simply places a single linear restriction across all the period $t+1$ values for $R_t/\bar{\pi}_{t+1}$ that are possible given period t.
- The real interest rate, $\tilde{r}_t$, on a risk free one-period bond that pays in $t+1$ *is* uniquely determined:
$$\frac{1}{C_t} = \tilde{r}_t \beta E_t \frac{1}{C_{t+1}}.$$
- By no-arbitrage, only the following weighted average of $R_t/\bar{\pi}_{t+1}$ across period $t+1$ states of nature is determined:
$$\tilde{r}_t = \frac{E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}}}{E_t \frac{1}{C_{t+1}}}.$$

Steady State

The steady state may found by implementing the following calculations in sequence, for given $\bar{\pi}$:

$$
\begin{aligned}
R &= \frac{\bar{\pi}}{\beta},\ K_f \equiv \frac{K}{F} = \left[\frac{1-\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}},\ s = K_f \frac{\varepsilon-1}{\varepsilon}\frac{1-\beta\theta\bar{\pi}^{\varepsilon}}{1-\beta\theta\bar{\pi}^{\varepsilon-1}} \\
p^* &= \frac{1-\theta\bar{\pi}^{\varepsilon}}{(1-\theta)\left(\frac{1-\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}},\ \mu = \frac{(1-\gamma)\, s}{(1-\nu)\,(1-\psi_I+\psi_I R)}, \\
C_Y &\equiv \frac{C}{Y} = 1-\frac{\mu}{p^*},\ Y = \left[p^*\left(\frac{\mu}{p^*}\right)^{1-\gamma}\right]^{\frac{1}{\gamma}} N, \\
C &= \overbrace{\left[p^*\left(1-\frac{\mu}{p^*}\right)^{\gamma}\left(\frac{\mu}{p^*}\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}}^{Q} N,
\end{aligned}
$$

Steady State, Continued

$$
\begin{aligned}
N &= \left[\frac{s}{(1-\nu)\left(\frac{1-\psi_I+\psi_I R}{1-\gamma}\right)^{1-\gamma}\left(\frac{1-\psi_N+\psi_N R}{\gamma}Q\right)^{\gamma}}\right]^{\frac{1}{(1+\varphi)\gamma}} \\
C &= QN,\ Y = \frac{C}{1-\frac{\mu}{p^*}},\ I = \mu\frac{Y}{p^*},\ F = \frac{1/C_Y}{1-\beta\theta\bar{\pi}^{1-\varepsilon}},\ K = K_f \times F
\end{aligned}
$$

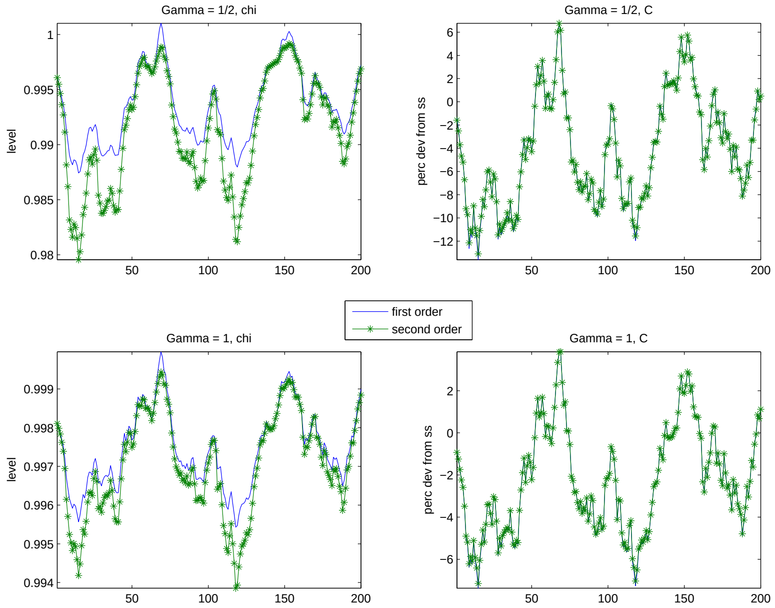
Magnitude of TFP Distortion Stochastic Simulations

- Parameter values

$$\begin{aligned}
\bar{\pi} &= 1.025^{\frac{1}{4}},\ \psi_I = \psi_N = 1,\ \gamma = \frac{1}{2},\ \beta = 1.03^{-0.25}, \\
\theta &= 0.75,\ \varepsilon = 6\ \left(\frac{\varepsilon}{\varepsilon - 1} = 1.2\right),\ \varphi = 1,\ \nu = \frac{1}{\varepsilon}, \\
\sigma_a &= 0.01,\ \sigma_\tau = 0.01,\ \rho_a = 0.95,\ \rho_\tau = 0.90.
\end{aligned}$$

- Monetary policy rule:

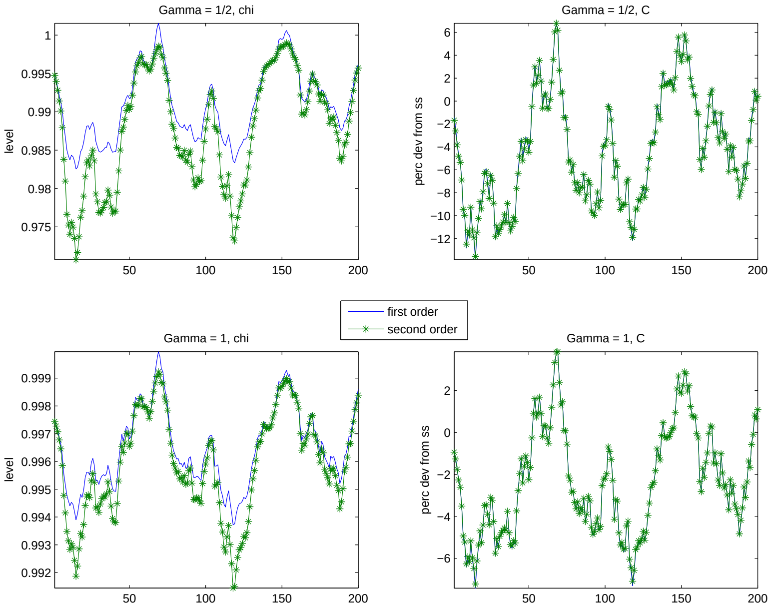
$$R_t / R = (R_{t-1}/R)^{0.8} \exp\left[(1 - 0.8)\, 1.5(\bar{\pi}_t - 1.0062)\right]$$



Gamma = 1/2, chi
level
Gamma = 1/2, C
perc dev from ss
Gamma = 1, chi
level
Gamma = 1, C
perc dev from ss
first order
second order

Results in Previous Graph

- Differences between first and second order perturbations
  - Negligible for consumption, but non-neglible for distortion, χ.
- Effect of reducing γ to $1/2$.
  - Volatility of consumption rises noticeably, consistent with the 'multiplier' discussed in the input-output literature.
  - Distortion, χ_t, not as great as the emprical estimate.
    - In a sense, this is a failure of the model, since the data distortion is a statistic of the data that it does not match.
- The overall volatility of GDP in the example is somewhat higher than in the data. Prescott (1986) reports the standard deviation of log, HP filtered GDP to be around 2 percent. For the model, the standard deviation of log consumption is around 2.5 percent $(\gamma = 1)$ and around 4.7 percent $(\gamma = 1/2)$.
- The US data calculations suggest that the distortions are increased when the degree of competition is increased, as one can see in the next figure where ε was increased from 6 to 7.7.



Gamma = 1/2, chi
level
0.975
0.98
0.985
0.99
0.995
1
50
100
150
200
Gamma = 1/2, C
perc dev from ss
−12
−10
−8
−6
−4
−2
0
2
4
6
50
100
150
200
first order
second order
Gamma = 1, chi
level
0.992
0.993
0.994
0.995
0.996
0.997
0.998
0.999
50
100
150
200
Gamma = 1, C
perc dev from ss
−6
−4
−2
0
2
50
100
150
200

Conclusion

- Some evidence of misallocation distortions from price setting frictions when production done in networks.
  - The evidence is very substantial when measured from the data using minimal restrictions from the model.
  - The evidence is less dramatic (though still non-negligible) when based on all the restrictions of the model using stochastic simulation.
- An extensive discussion of the implications for the Taylor principle appears in my 2011 handbook chapter.
  - When the smoothing parameter is set to zero and $\psi_I = \psi_N = 1$, then the model has indeterminacy, even when the coefficient on inflation is 1.5. So, the likelihood of the Taylor principle breaking down goes up when γ is reduced, consistent with intuition.
  - When the smoothing parameter is at its empirically plausible value of 0.8, then the solution of the model does not display indeterminacy.