Small Open Economy

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Outline

- Simple Closed Economy Model
- Extend Model to Open Economy
  - Economics
  - Technical issue: scaling the variables, to accommodate (balanced) growth and inflation.
- Analysis:
  - Dynamic responses to shocks.
  - Trend reversion and forecastibility of exchange rates.
- Some shortcomings of the model, and how these are taken care of in more extended versions.
  - Excessive pass-through from exchange rates into domestic prices.
  - Lack of financial frictions, so that currency mismatch between assets and liabilities can be a source of instability could overturn classic channels.
- Simplified version of large scale model used in policy analysis.
  - Christiano-Trabandt-Walentin(CTW) Model, Ramses II model at Riksbank.
    - Based on Christiano-Motto-Rostagno ('Risk Shocks', AER2014)
  - Copaciu-Nalban-Bulete of Romanian Central Bank.

Simple Closed Economy Model

- Results from closed economy model
  - Household preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ \left(u\left(C_t\right) - \exp\left(\tau_t\right) \frac{N_t^{1+\varphi}}{1+\varphi} \right),$$

$$u\left(C_t\right) \equiv \log C_t$$

  - Aggregate resources and household intertemporal optimization:

$$Y_t = p_t^* A_t N_t, \ u_{c,t} = \beta E_t u_{c,t+1} \frac{R_t}{\pi_{t+1}}$$

  - Law of motion of price distortion (see this for details):

$$p_t^* = \left((1-\theta) \left(\frac{1-\theta\left(\pi_t\right)^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \pi_t^{\varepsilon}}{p_{t-1}^*} \right)^{-1} \quad (4)$$

Simple Closed Economy Model

- Equilibrium conditions associated with price setting:

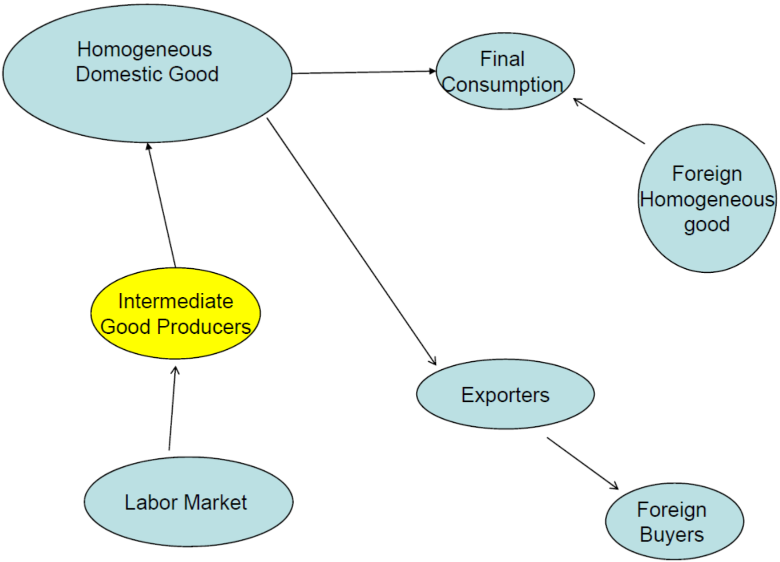
$$\frac{Y_t}{C_t} + E_t \pi_{t+1}^{\varepsilon-1} \beta\theta F_{t+1} = F_t \ (2)$$

$$K_t = \frac{\varepsilon}{\varepsilon-1}(1-\nu) \frac{\overbrace{\exp(\tau_t) N_t^{\varphi}}^{=\frac{W_t}{P_t^c} \text{ by household optimization}}}{u_{c,t}} \times \frac{1}{A_t}\frac{Y_t}{C_t} p_t^c + E_t \beta\theta \pi_{t+1}^{\varepsilon} K_{t+1} \ (1)$$

  - In simple closed economy model, $Y_t = C_t$, not so here.
  - Relative price, p_t^c, is unity in simple closed economy (more below).

- Cross-price restrictions

$$\frac{K_t}{F_t} = \left[\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \ (3)$$


Homogeneous Domestic Good
Final Consumption
Foreign Homogeneous good
Intermediate Good Producers
Exporters
Labor Market
Foreign Buyers

Extensions to Small Open Economy: 17 variables

$$\overbrace{s_t, x_t, a_t^f, p_t^x, q_t}^{\text{rate of depreciation, exports, real (scaled) net foreign assets, terms of trade, real exchange rate}}$$

$$\overbrace{p_t^c}^{\text{relative price of domestic consumption}(c \text{ is composed of domestically produced goods \& imports})}$$

$$\overbrace{p_t^m}^{\text{relative price of imports}} \quad \overbrace{S_t}^{\text{nominal exchange rate}} \quad \overbrace{\pi_t^c}^{\text{consumption price inflation}}$$

$$\overbrace{R_t, \pi_t, y_t, N_t, c_t, K_t, F_t, p_t^*}^{\text{closed economy variables}}$$

Modifications to Simple Model to Create Open Economy

- Unchanged:
  - production of (domestic) homogeneous good, Y_t $(= A_t p_t^* N_t)$
  - Calvo pricing equations (with two adjustments listed above)
- Changes:
  - household budget constraint includes opportunity to acquire foreign assets/liabilities.
  - net foreign assets introduced into household utility for reasons explained below.
  - $Y_t = C_t$ no longer true.
  - introduce exports, imports, balance of payments.
  - exchange rate,

$S_t =$ domestic currency price of one unit of foreign currency
$S_t = \frac{\text{domestic money}}{\text{foreign money}}$

Monetary Policy: two approaches

- Taylor rule

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\, E_t\Big[r_\pi \log\left(\frac{\pi_t^c}{\bar{\pi}^c}\right) + r_y \log\left(\frac{y_t}{y}\right) + r_S log\left(\widetilde{S}_t\right)\Big] + \varepsilon_{R,t} \quad (17)$$

where:

π_t^c consumer price inflation, and target, $\bar{\pi}^c$

$\varepsilon_{R,t}$ iid, mean zero monetary policy shock

$y_t = Y_t / A_t,$ output scaled by technology

R_t nominal rate of interest

$\varepsilon_{R,t}$ mean zero monetary policy shock

$\widetilde{S}_t = S_t / \left(\psi^t \bar{S}\right)$ ˜ nominal exchange rate, S_t,

relative to target, $\psi^t \bar{S},\ \psi > 0.$

Monetary Policy: two approaches

- Second approach (Norges Bank, Riksbank)
  - Solve a type of Ramsey problem in which preferences correspond to preferences of monetary policy committee:

$$E_t \sum_{j=0}^{\infty} \beta^j \Big\{ \left(100 \left[\pi_t^c \pi_{t-1}^c \pi_{t-2}^c \pi_{t-3}^c - (\pi^c)^4\right]\right)^2$$
$$+ \lambda_y \left(100 \log\left(\frac{y_t}{y}\right)\right)^2$$
$$+ \lambda_{\Delta R} \left(400 \left[R_t - R_{t-1}\right]\right)^2 + \lambda_s \left(S_t - \bar{S}\right)^2 \Big\}$$

    straightforward to implement in Dynare.

- We will stress first approach.

Households

- Household preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ \left(u\left(C_t\right) - \exp\left(\tau_t\right) \frac{N_t^{1+\varphi}}{1+\varphi} + \mu_t h_t \left(\frac{S_t A_{t+1}^f}{P_t^c} \right) \right),$$

$$u\left(C_t\right) \equiv \log C_t,$$

where P_t^c denotes the price of the domestic consumption good.

- Note that the real value of net foreign assets are included in the utility function.

Household Budget Constraint

- 'Uses of funds less than or equal to sources of funds'

$$S_t A^f_{t+1} + P^c_t C_t + B_{t+1}$$
$$\leq B_t R_{t-1} + S_t R^f_{t-1} A^f_t + W_t N_t + transfers\ and\ profits_t$$

- Domestic bonds

B_t beginning of period t stock of loans

R_t rate of return on bonds

- Foreign assets

A^f_t beginning-of-period t stock of foreign assets, net of foreign liabilities, held by domestic residents.

Household Intertemporal Conditions: Domestic Assets

- First order condition:

$$\frac{1}{P_t^c C_t} = \beta E_t \frac{R_t}{P_{t+1}^c C_{t+1}}$$

- Scaling:

$$\frac{1}{c_t} = \beta E_t \frac{R_t}{\pi_{t+1}^c c_{t+1} \exp\left(\Delta a_{t+1}\right)} . (5)$$

- Technology:

$$a_t \equiv \log\left(A_t\right), \ \Delta a_t = a_t - a_{t-1}.$$

Household Intertemporal Conditions: Foreign Assets

- Optimality of foreign asset choice (verify this by solving Lagrangian representation of household problem)

$$\frac{\overbrace{u_{c,t}S_t}^{\text{utility cost of 1 unit of foreign currency}=S_t \text{ units of domestic currency, } S_t/P_t^c \text{ units of } C_t}}{P_t^c}$$

$$= \overbrace{\mu_t h_t'\left(\frac{S_t A_{t+1}^f}{P_t^c}\right)\frac{S_t}{P_t^c}}^{\text{marginal utility benefit of extra net foreign assets}} + \beta E_t \overbrace{u_{c,t+1}}^{\text{conversion into utility units}}$$

$$\times \overbrace{\frac{S_{t+1} \overbrace{R_t^f}^{\text{foreign currency payoff next period from one unit of foreign currency today}}}{P_{t+1}^c}}^{\text{quantity of domestic cons. goods purchased from the payoff of 1 unit of foreign currency}}$$

Household Intertemporal Conditions: Foreign Assets

- First order condition:

$$\frac{S_t}{P_t^c C_t} = \mu_t h_t^{'} \left(\frac{A_t S_t A_{t+1}^f}{A_t P_t p_t^c} \right) \frac{S_t}{P_t^c} + \beta E_t \frac{S_{t+1} R_t^f}{P_{t+1}^c C_{t+1}}$$

- Scaling: (we assume $\mu_t = 1/A_t$)

$$\frac{1}{c_t} = h_t^{'} \left(\frac{A_t a_t^f}{p_t^c} \right) + \beta E_t \frac{s_{t+1} R_t^f}{\pi_{t+1}^c c_{t+1} \exp\left(\Delta a_{t+1}\right)},$$

$$c_t \equiv \frac{C_t}{A_t},\ a_t^f \equiv \frac{S_t A_{t+1}^f}{P_t A_t}, s_t \equiv \frac{S_t}{S_{t-1}} = \psi \frac{\tilde{S}_t}{\tilde{S}_{t-1}} (14)$$

Utility Value of Net Foreign Assets

- Suppose that (for reasons not modeled) people have a target level of net foreign assets, summarized in the function, h_t:

$$h_t\left(\frac{S_t A^f_{t+1}}{P^c_t}\right) = -\frac{1}{2}\gamma\left(\frac{\frac{A_t S_t A^f_{t+1}}{A_t P_t p^c_t} - A_t \Upsilon_t}{A_t}\right)^2$$

$$= -\frac{1}{2}\gamma\left(\frac{a^f_t}{p^c_t} - \Upsilon_t\right)^2.$$

- From previous slide:

$$\frac{1}{c_t} = h'_t\left(\frac{A_t a^f_{t+1}}{p^c_t}\right) + \beta E_t \frac{s_{t+1} R^f_t}{\pi^c_{t+1} c_{t+1} \exp\left(\Delta a_{t+1}\right)}$$

- Then,

$$\frac{1}{c_t} = -\gamma\left(\frac{a^f_t}{p^c_t} - \Upsilon_t\right) + \beta E_t \frac{s_{t+1} R^f_t}{\pi^c_{t+1} c_{t+1} \exp\left(\Delta a_{t+1}\right)} \quad (7)$$

Why Put Net Foreign Assets in the Utility Function?

- The primary motivation is technical and is described in, for example, Schmitt-Grohe and Uribe.
- Because the domestic economy is assumed to be small, it has no impact on R_t^f, the return on foreign assets.
  - From the point of view of domestic residents, the foreign asset represents a constant returns investment technology.
  - The consequence is that there does not exist a steady state level of net foreign assets that is independent of the initial net foreign asset position.
  - Why? The answer resembles why there is no steady state capital stock independent of initial capital in the so-called Ak model:
    - That is, if k starts low, there is no incentive to raise investment sharply to raise k to some steady state level. That's because the marginal product of capital, A, is not higher when k is low.
    - Similarly, when k is high, there is no reason to let the stock of capital fall. That is, when k is high, A is not lower.

Why Put Net Foreign Assets in the Utility Function?

- Standard solution methods assume that variables have a steady state that is independent of initial conditions.
- Small open economy models require a small adjustment.
- Consider the first order condition for foreign assets:

$$\frac{1}{c_t} = \overbrace{-\gamma\left(\frac{a_t^f}{p_t^c} - \Upsilon_t\right)}^{\text{second part}} + \overbrace{\beta E_t \frac{s_{t+1} R_t^f}{\pi_{t+1}^c c_{t+1} \exp\left(\Delta a_{t+1}\right)}}^{\text{first part}} \quad (7)$$

- The payoff on the foreign asset corresponds to the two parts of the term on the right of the equality:
  - First part: normal part of the return on the foreign assets, stemming from their cash payoff.
  - Second part: makes overall return higher when households' net foreign asset position is below target, Υ_t, giving households incentive to accumulate more assets. Similarly, the overall return is lower when the net foreign asset position is above target, giving households an incentive to accumulate less.
- So, model has a unique steady state, for $\gamma > 0$.

Final Domestic Consumption Goods

- Produced by representative, competitive firm using:

$$C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}} \left(C_t^d\right)^{\frac{\eta_c-1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(C_t^m\right)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}$$

  where

C_t^d	domestic homogeneous output good, price P_t
C_t^m	imported good, price $P_t^m \left(\equiv S_t P_t^f\right)$
C_t	final consumption good, P_t^c
η_c	elasticity of substitution, domestic and foreign goods.

- The firm takes the prices, P_t, P_t^m, P_t^c, as given and beyond its control.

Final Domestic Consumption Goods

- Profit maximization by representative firm:

$$\max_{C_t, C_t^m, C_t^d} P_t^c C_t - P_t^m C_t^m - P_t C_t^d,$$

subject to production function.

- First order conditions associated with maximization:

$$C_t^m : P_t^c \overbrace{\frac{dC_t}{dC_t^m}}^{=\left(\omega_c \frac{C_t}{C_t^m}\right)^{\frac{1}{\eta_c}}} = P_t^m, \; C_t^d : P_t^c \overbrace{\frac{dC_t}{dC_t^d}}^{=\left((1-\omega_c)\frac{C_t}{C_t^d}\right)^{\frac{1}{\eta_c}}} = P_t$$

so that the demand functions are:

$$C_t^m = \omega_c \left(\frac{P_t^c}{P_t^m}\right)^{\eta_c} C_t, \; C_t^d = (1-\omega_c)\left(\frac{P_t^c}{P_t}\right)^{\eta_c} C_t.$$

Final Good Prices

- Substituting demand functions back into the production function:

$$C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}} \left(C_t \left(\frac{P_t^c}{P_t} \right)^{\eta_c} (1-\omega_c) \right)^{\frac{\eta_c - 1}{\eta_c}} \right.$$
$$\left. + \omega_c^{\frac{1}{\eta_c}} \left(\omega_c \left(\frac{P_t^c}{P_t^m} \right)^{\eta_c} C_t \right)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}},$$

to obtain,

$$p_t^c = \overbrace{\left[(1-\omega_c) + \omega_c \left(p_t^m \right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}}^{\text{marginal cost, in units of the homogeneous good}} \qquad (8)$$

$$p_t^c \equiv \frac{P_t^c}{P_t}, \; p_t^m \equiv \frac{P_t^m}{P_t}.$$

Pass-Through

- Multiplying (8) by P_t 'price $=$ marginal cost':

$$P_t^c = \left[\left(1-\omega_c\right) \left(P_t\right)^{1-\eta_c} + \omega_c \left(P_t^m\right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}},$$

or, using $P_t^m = S_t P_t^f$:

$$P_t^c = \left[\left(1-\omega_c\right) \left(P_t\right)^{1-\eta_c} + \omega_c \left(S_t P_t^f\right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}.$$

- Note that if the exchange rate depreciates, i.e., S_t rises, then marginal cost rises so that the depreciation is 'passed through' marginal cost and into the final good price, P_t^c. This pass-through occurs, no matter how sticky the prices underlying P_t are.
- The high degree of pass through in this model reflects its simplicity. See CTW for a discussion of how this model can be modified to slow down the pass through of exchange rate changes into final good prices.

Consumer Price Inflation

- Consumption good inflation and homogeneous good inflation:

$$\pi_t^c \equiv \frac{P_t^c}{P_{t-1}^c} = \frac{P_t p_t^c}{P_{t-1} p_{t-1}^c} = \pi_t \left[\frac{(1-\omega_c) + \omega_c \left(p_t^m\right)^{1-\eta_c}}{(1-\omega_c) + \omega_c \left(p_{t-1}^m\right)^{1-\eta_c}} \right]^{\frac{1}{1-\eta_c}} \quad (10)$$

Real Exchange Rate

- Real Exchange Rate, q

$$p_t^m = \frac{P_t^c}{P_t^c} \frac{\overbrace{P_t^m}^{\text{zero profits for importers, } P_t^m = S_t P_t^f}}{P_t}$$

$$= p_t^c \times \overbrace{q_t}^{\text{real exchange rate, } q_t \equiv \frac{S_t P_t^f}{P_t^c}} \qquad (9)$$

- Scaling:

$$1 \overbrace{=}^{\text{zero profit condition for exporters}} \frac{S_t P_t^x}{P_t} = \frac{P_t^c S_t P_t^f P_t^x}{P_t P_t^c P_t^f} = q_t p_t^x p_t^c (12)$$

- Also,

$$\frac{q_t}{q_{t-1}} = s_t \frac{\pi_t^f}{\pi_t^c}, (13), \;\; \pi_t^f \equiv \frac{P_t^f}{P_{t-1}^f}$$

Exports

- Foreign demand for domestic goods:

$$X_t = \left(\frac{P_t^x}{P_t^f}\right)^{-\eta_f} Y_t^f = \left(p_t^x\right)^{-\eta_f} Y_t^f, \qquad \overbrace{p_t^x}^{\text{terms of trade}} = \frac{P_t^x}{P_t^f}$$

Y_t^f	foreign output
P_t^f	foreign currency price of foreign good
P_t^x	foreign currency price of export good

- Foreign demand is exogenous to the domestic economy.

Balanced Growth

- The model exhibits growth when A_t grows over time.
- We require the growth to be balanced, so that when growing variables are scaled by A_t the ratios converge in steady state.
- Mathematically, balanced growth requires that after all variables are scaled by A_t, A_t itself disappears from the system.
  - This places certain restrictions on preferences and technology, restrictions which our model satisfies.
- So, in the case of Y_t^f, balanced growth requires that Y_t^f grows at the same rate as A_t.
- An obvious way to proceed is to assume

$$Y_t^f = y_t^f A_t,$$

  where y_t^f is an exogenous shock to Y_t^f. But, this formulation implies that a shock to technology simultaneously expands the demand for exports.
  - Seems implausible.
  - Inconvenient when we compute impulse response functions. Here, we want to study the effects of a disturbance that originates in just one part of the system.

Exports

- Preceding slide suggests we want to express Y_t^f in the following form:

$$Y_t^f = y_t^f Z_t,$$

where Z_t grows with A_t, yet Z_t responds extremely slowly to A_t.

- We apply the approach in Christiano-Trabandt-Walentin (2011, section 2.3):

$$Y_t^f = y_t^f Z_t, Z_t = A_t^{1-\delta} Z_{t-1}^{\delta},\ 0 < \delta < 1,$$
$$z_t \equiv \frac{Z_t}{A_t} = \left(\frac{A_{t-1}}{A_t} \frac{Z_{t-1}}{A_{t-1}}\right)^{\delta} = exp\left(-\delta \Delta a_t\right) z_{t-1}^{\delta} (18)$$

$$x_t = \left(p_t^x\right)^{-\eta_f} y_t^f z_t (11)$$

- Note: with δ close, but less than, unity:
    - Z_t grows at the same as A_t in the sense that Z_t / A_t converges to a constant in steady state.
    - Z_t hardly responds to a shock to a shock in A_t.

Homogeneous Goods Market Clearing

Clearing in domestic homogeneous goods market:

$$\text{output of domestic homogeneous good, } Y_t = \text{uses of domestic homogeneous goods}$$

or,

$$\begin{aligned} Y_t &= \overbrace{C_t^d}^{\text{goods used in production of final consumption, } C_t} + \overbrace{X_t}^{\text{exports}} + \overbrace{G_t}^{\text{govenment}} \\ &= (1-\omega_c)(p_t^c)^{\eta_c} C_t + X_t + G_t. \end{aligned}$$

Aggregate Employment and Uses of Homogeneous Goods

- Substituting out in previous expression for Y_t:

$$A_t p_t^* N_t = (1-\omega_c)\left(p_t^c\right)^{\eta_c} C_t + X_t + G_t,$$

or,

$$p_t^* N_t = (1-\omega_c)\left(p_t^c\right)^{\eta_c} c_t + x_t + g_t z_t, (6)$$

$$c_t \equiv \frac{C_t}{A_t},\ x_t \equiv \frac{X_t}{A_t},\ G_t = g_t Z_t, z_t = \frac{Z_t}{A_t}.$$

- Also,

$$y_t = \frac{Y_t}{A_t} = p_t^* N_t (16)$$

- For an extended discussion of (16), see this.

Balance of Payments

- expenses on imports net of receipts from exports equals flow of financial assets abroad.

$$\overbrace{S_t A^f_{t+1}}^{\text{acquisition of new net foreign assets, in domestic currency units}}$$

$$+ \text{ expenses on imports}_t$$

$$= \text{receipts from exports}_t \; + \overbrace{S_t R^f_{t-1} A^f_t}^{\text{receipts from existing stock of net foreign assets}}$$

Balance of Payments, the Pieces

- Exports and imports:

$$\text{expenses on imports}_t = S_t P_t^f \omega_c \left(\frac{p_t^c}{p_t^m} \right)^{\eta c} C_t$$
$$\text{receipts from exports}_t = S_t P_t^x X_t.$$

- Balance of payments:

$$S_t A_{t+1}^f + S_t P_t^f \omega_c \left(\frac{p_t^c}{p_t^m} \right)^{\eta_c} C_t$$
$$= S_t P_t^x X_t + S_t R_{t-1}^f A_t^f.$$

Balance of Payments, Scaling

- Scaling by $P_t A_t$:

$$\frac{S_t A_{t+1}^f}{P_t A_t} + \frac{S_t P_t^f}{P_t} \omega_c \left(\frac{p_t^c}{p_t^m} \right)^{\eta_c} c_t$$
$$= \frac{S_t P_t^x}{P_t} x_t + \frac{S_t R_{t-1}^f A_t^f}{P_t A_t},$$

or,

$$a_t^f + p_t^m \omega_c \left(\frac{p_t^c}{p_t^m} \right)^{\eta_c} c_t = p_t^c q_t p_t^x x_t + \frac{s_t R_{t-1}^f a_{t-1}^f}{\pi_t \exp\left(\Delta a_t\right)}, (15)$$

where a_t^f is 'scaled, homogeneous goods value of net foreign assets'

Gross Domestic Product

- GDP: 'C + I + G + Net Exports'.
  - Problem: these are different goods, with different prices.
- GDP in domestic consumption units - nominal divided by P_t^c:

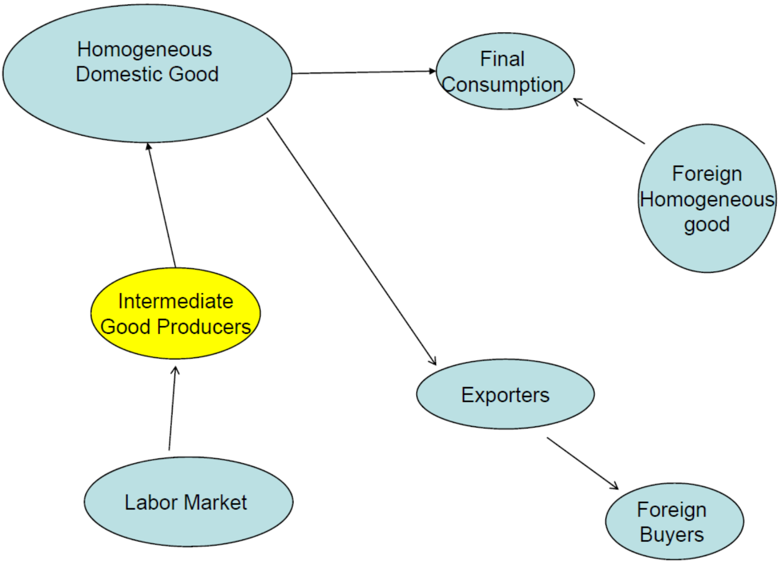
$$GDP_t \equiv \frac{\overbrace{P_t^c A_t c_t}^{\text{nominal expenditures on consumption}} + \overbrace{P_t g_t Z_t}^{\text{nominal government exp}}}{P_t^c}$$

$$+ \frac{\overbrace{-S_t P_t^f \omega_c \left(\frac{p_t^c}{p_t^m}\right)^{\eta c} C_t}^{\text{nominal imports}} + \overbrace{x_t P_t A_t}^{\text{nominal exports}}}{P_t^c}$$

$$= A_t \overbrace{\left[c_t + \frac{g_t z_t}{p_t^c} - \left(\frac{p_t^m}{p_t^c}\right)^{1-\eta c} \omega_c c_t + \frac{x_t}{p_t^c}\right]}^{\equiv gdp_t\text{, scaled by } A_t}$$

- So, GDP (in consumption units) growth is:

$$\log(GDP_t) - \log(GD_{t-1}) = \Delta a_t + \log(gdp_t) - \log(gdp_{t-1}).$$


Homogeneous Domestic Good
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Pulling the Equations Together

- The 18 endogenous variables:

$$K_t, F_t, N_t, y_t, \pi_t, c_t, p_t^*, R_t, a_t^f, p_t^m, p_t^c, q_t, p_t^x, \pi_t^c, x_t, s_t, \widetilde{S}_t, z_t$$

- The 8 exogenous variables: $\Upsilon_t, \tau_t, \Delta a_t, \varepsilon_{R,t}, g_t, \pi_t^f, y_t^f, R_t^f$
- Equilibrium conditions resembling those in closed economy:

$$K_t = \frac{\varepsilon(1-\nu)}{\varepsilon-1} exp(\tau_t) N_t^{\varphi} y_t p_t^c + \beta\theta E_t(\frac{1}{\pi_{t+1}})^{-\varepsilon} K_{t+1} \quad (1)$$

$$F_t = \frac{y_t}{c_t} + E_t \pi_{t+1}^{\varepsilon-1} \beta\theta F_{t+1} \ (2) \ \frac{K_t}{F_t} = \left[\frac{1-\theta\pi_t^{\varepsilon-1}}{1\text{-}\theta}\right]^{\frac{1}{1-\varepsilon}} \quad (3)$$

$$p_t^* = \left((1-\theta)\left(\frac{1-\theta(\pi_t)^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\pi_t^{\varepsilon}}{p_{t-1}^*}\right)^{-1} \quad (4)$$

$$\frac{1}{c_t} = \beta E_t \frac{R_t}{\pi_{t+1}^c c_{t+1} \exp(\Delta a_{t+1})} \ (5)$$

$$y_t = (1-\omega_c)(p_t^c)^{\eta_c} c_t + x_t + g_t z_t \ (6)$$

Pulling the Equations Together

$$\frac{1}{c_t} = -\gamma\left(\frac{a_t^f}{p_t^c} - \Upsilon_t\right) + \beta E_t \frac{s_{t+1} R_t^f}{\pi_{t+1}^c c_{t+1} \exp\left(\Delta a_{t+1}\right)} \quad (7)$$

$$p_t^c = \left[(1-\omega_c) + \omega_c \left(p_t^m\right)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}} \quad (8)$$

$$p_t^m = p_t^c q_t \quad (9)$$

$$\pi_t^c = \pi_t \left[\frac{(1-\omega_c) + \omega_c \left(p_t^m\right)^{1-\eta_c}}{(1-\omega_c) + \omega_c \left(p_{t-1}^m\right)^{1-\eta_c}}\right]^{\frac{1}{1-\eta_c}} \quad (10)$$

$$x_t = \left(p_t^x\right)^{-\eta_f} y_t^f z_t \quad (11)$$

$$1 = q_t p_t^x p_t^c \quad (12)$$

Pulling the Equations Together

- Last equations:

$$\frac{q_t}{q_{t-1}} = s_t \frac{\pi_t^f}{\pi_t^c} \ (13)$$

$$s_t = \psi \frac{\widetilde{S}_t}{\widetilde{S}_{t-1}} \ (14)$$

$$a_t^f + p_t^m \omega_c \left(\frac{p_t^c}{p_t^m} \right)^{\eta_c} c_t = p_t^c q_t p_t^x x_t + \frac{s_t R_{t-1}^f a_{t-1}^f}{\pi_t \exp \left(\Delta a_t \right)} \ (15)$$

$$y_t = p_t^* N_t \ (16)$$

$$\log \left(\frac{R_t}{R} \right) = \rho_R \log \left(\frac{R_{t-1}}{R} \right) + \left(1 - \rho_R \right) E_t [r_\pi \log \left(\frac{\pi_t^c}{\bar{\pi}^c} \right) + r_y \log \left(\frac{y_t}{y} \right) + r_S log \left(\widetilde{S}_t \right)] + \varepsilon_{R,t} \ \ (17)$$

$$z_t = exp \left(-\delta \Delta a_t \right) z_{t-1}^{\delta} \ (18)$$

Summing Up

- We now have 18 equations in 18 unknowns.
- Parameters:

$$\delta, \theta, \omega_c, \beta, \eta_c, \eta_g, \eta_f, \varepsilon, \nu, \psi, \varphi, \gamma, \tau, R^f, y^f, \pi^f, \Delta a, \rho_R, r_\pi, r_y, r_S, \bar{\pi}^c, \Upsilon$$

- Next:
  - compute steady state
  - dynamic model analysis.
    - Predictability of exchange rate (url: Eichenbaum-Johanssen-Rebelo)
    - Uncovered Interest Rate Parity

Steady State

- Computing the steady state of a model can be surprisingly tricky.
- Obtain the steady state equations by deleting time subscripts from the dynamic equations.
- Solve 18 equations for 18 unknown steady state variables:

$$K, F, N, y, \pi, c, p^*, R, a^f, p^m, p^c, q, p^x, \pi^c, x, s, \widetilde{S}, z$$

- In practice, solving a system like this is too big.
  - Must cleverly discover simplifications.
  - Ideally, find a way to solve the equations recursively, one variable at a time.
    - Possible in this model in special case, $\Upsilon = 0$.
- To illustrate what is involved, following slides go into the details.

Steady State: Some Easy Variables

- Foreign variables fixed exogenously: π^f, R^f.
  - π^f and R^f assumed to be linked by foreigners' intertemporal Euler equation, same as domestic:

$$1 = \beta \frac{R^f}{\pi^f \exp(\Delta a)}, \; \pi_t^f \equiv \frac{P_t^f}{P_{t-1}^f} \text{ (exogenous)}$$

- Domestic monetary policy forces $\pi^c = \bar{\pi}^c$, $\bar{\pi}^c$ ~ steady state of inflation target.
- Additional steady state equations:

$$\pi \equiv \frac{P_t}{P_{t-1}} = \bar{\pi}^c \text{ (inflation target)} \qquad (10)$$

$$\pi^f = \pi^c / \psi \; (13), \; s = \psi \; (14)$$

$$R = sR^f, \quad \text{('UIP holds in steady state') (5) and (7)}$$

- Easily verified that domestic intertemporal Euler equation is satisfied:

$$1 = \beta \frac{R}{\pi^c \exp(\Delta a)} \; (5)$$

Steady State: Some Easy Variables

- Tack Yun distortion:

$$p^* = \frac{\frac{1-\theta\pi^{\varepsilon}}{1-\theta}}{\left(\frac{1-\theta\pi^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}} \quad (4)$$

- The growth factor:

$$z = exp\left(-\frac{\delta}{1-\delta}\Delta a\right) \quad (18)$$

- Euler equation for foreign assets, (7), in steady state:

$$1 = -\gamma c\left(\frac{a^f}{p^c} - \Upsilon\right) + \beta\frac{sR^f}{\pi^c \exp\left(\Delta a\right)} \quad (7).$$

- Combine with domestic Euler equation, (5), and $sR^f = R$, conclude:

$$\frac{a^f}{p^c} = \Upsilon.$$

  Not ready for computations, since has two unknowns.

Steady State

- Let η_g denote the share of government consumption in homogeneous output:

$$g = \eta_g y/z.$$

- Using the latter and (16), (6) reduces to:

$$0 = (1 - \omega_c)\,(p^c)^{\eta_c}\, c + x - (1 - \eta_g)\, Np^* \;(6)$$

- Making use of (12), (15) becomes:

$$0 = p^m \omega_c \left(\frac{p^c}{p^m}\right)^{\eta_c} c - x - \left(\frac{1}{\beta} - 1\right) a^f \;(15)$$

- Substituting out for x from (15) into (6), using (8) and rearranging:

$$p^c c = \left(\frac{1}{\beta} - 1\right) a^f + (1 - \eta_g)\, Np^* \;(6),$$

  which says that the value of scaled consumption in steady state equals interest on foreign assets plus homogeneous output, minus government consumption.

Steady State

- Using (2) and (3) to substitute out for K and F in (1):

$$0 = \frac{\varepsilon(1-\nu)}{\varepsilon-1} N^{1+\varphi} p^* p^c + \frac{(\beta\theta\pi^{\varepsilon}-1)\, p^* N}{c\left(1-\pi^{\varepsilon-1}\beta\theta\right)} \left[\frac{1-\theta\pi^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \quad (1)$$

- After rearranging:

$$N^{\varphi} p^c c = \frac{\varepsilon-1}{\varepsilon(1-\nu)} \frac{(1-\beta\theta\pi^{\varepsilon})}{\left(1-\pi^{\varepsilon-1}\beta\theta\right)} \left[\frac{1-\theta\pi^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \quad (1)$$

- Substituting out for $p^c c$ from (6):

$$\begin{aligned}
0 &= N^{1+\varphi} + b_1 N^{\varphi} - b_2, \; (1) \\
b_1 &= \frac{\left(\frac{1}{\beta}-1\right) a^f}{\left(1-\eta_g\right) p^*}, \\
b_2 &= \frac{\varepsilon-1}{\left(1-\eta_g\right) p^* \varepsilon (1-\nu)} \frac{(1-\beta\theta\pi^{\varepsilon})}{\left(1-\pi^{\varepsilon-1}\beta\theta\right)} \left[\frac{1-\theta\pi^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}.
\end{aligned}$$

Steady State: Special Case, $\Upsilon = 0$

- In this case, $a^f = 0$. Then, $b_1 = 0$ in previous slide and

$$N = b_2^{\frac{1}{1+\varphi}}.$$

- Compute $p^c c$ using (6).
- But, not obvious how to compute the other variables, e.g., $p^c, p^x, \ldots$ one at a time, in a nice recursive sequence.
  - We now switch to a nonlinear search.
  - We do so in a way that maximizes the chance we discover multiple steady state if this is the case (it appears not to be in this model).

Steady State: Non-recursive, Iterative Approach

- To illustrate how we proceed, consider the following joint system of two equations in two unknowns, y, x :

$$f(x, y) = 0$$
$$g(x, y) = 0$$

  - We solve this system as a nested set of two one-dimensional problems.
- For each fixed x let $y(x)$ denote the value of y that solves the one-dimensional problem, $g(x, y) = 0$.
  - The problem of finding $y(x)$ is *the inner loop problem*, and y is *the inner loop variable*.
  - For the *outer loop problem*, find a value for *the outer loop variable*, x, such that $f(x, y(x)) = 0$.

Steady State: Non-recursive, Iterative Approach

- It is convenient to define the following variable: $\tilde{\varphi} = p^c q$.
  - $\tilde{\varphi}$ is the outer loop variable in our solution strategy.
- We now define the inner loop problem. Solve:

$$p^m = \tilde{\varphi} \;\; (9) \text{ and } p^x = \frac{1}{\tilde{\varphi}} \;\; (12)$$

$$p^c = \left[(1-\omega_c) + \omega_c \left(p^m\right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \;\; (8)$$

$$q = \frac{\tilde{\varphi}}{p^c}$$

$$a^f = p^c \Upsilon.$$

- Then, solve (1) for N and (6) for $p^c c$.
  - When $a^f \neq 0$ this an inner loop nonlinear search is required to find N.
  - Actually, when $\varphi = 1$ the nonlinear search simplifies to finding the zeros of a second order polynomial.
- With $p^c c$ in hand, c is recovered by dividing with respect to p^c from (8).
- Use (15) to compute x. Also, (2) and (3) can be used to compute K and F.

Model Solution and Parameter Values

- We use standard linearization techniques to solve the model.
- This requires having values for the model parameters. We adopt the following:

$\bar{\pi}^c = 1.005$	$\Upsilon = 0$	$\beta = 1.03^{-1/4}$
$\theta = 3/4$	$\varphi = 1$	$\varepsilon = 6$
$1 - \nu = \frac{\varepsilon - 1}{\varepsilon}$	$\eta_c = 5$	$\omega_c = 0.4$
$\eta_g = 0.3$	$\gamma = 2$	$\eta_f = 1.5$
$\rho_R = 0.9$	$r_\pi = 1.5$	$r_y = 0.15$
r_S=0.02	ψ=1.0118	Δa=0
$\tilde{\varphi}$=1	τ=0	δ=0.999

- We activated three shocks in the system, Δa_t, a shock to Υ, and the monetary policy shock, $\varepsilon_{R,t}$. The monetary policy shock is iid with standard deviation, 0.0025. The other two shocks are scalar AR(1)'s. The autocorrelation of the technology and Υ shocks are 0.90 and 0.95, respectively. Their standard deviations are 0.01 and 0.05.
- In the following, we discuss the values assigned to the non-standard parameters, r_S and γ.

Exchange Rate in Taylor Rule

- We include the interest rate (relative to a deterministic trend) in the Taylor rule:

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[r_\pi \log\left(\frac{\pi_t^c}{\bar{\pi}^c}\right)\right.$$
$$\left. + r_y \log\left(\frac{y_t}{y}\right) + r_S log\left(\widetilde{S}_t\right)\right] + \varepsilon_{R,t} \quad (17)$$

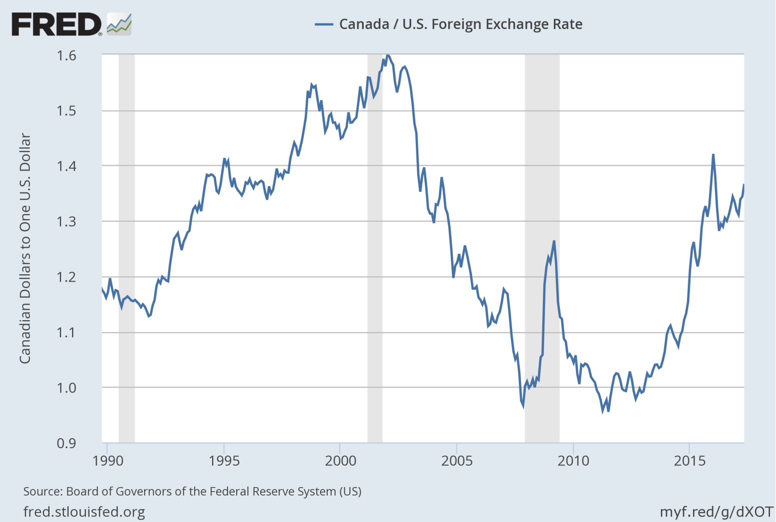
$$\widetilde{S}_t = \frac{S_t}{\psi^t \bar{S}} .$$

- Only $s_t = S_t / S_{t-1}$ (not $\widetilde{S}_t$ or S_t) appears in the other equations, so when $r_S = 0$ only s_t is determined.
  - In this case, the level of the exchange rate has a unit root.
- When $r_S > 0$ then the level of the exchange rate must return to trend, $\psi^t \bar{S}$.
  - This return could take a very long time if r_S is tiny, but positive.
  - The return could be very quick if r_S is very large.

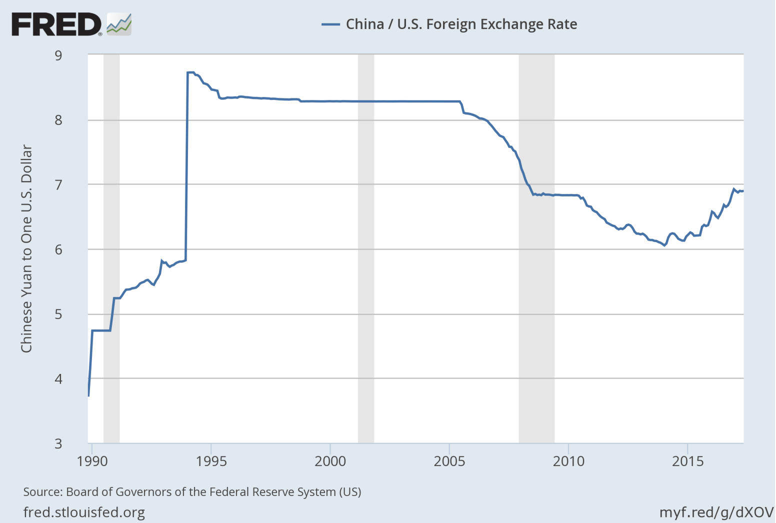
Exchange Rate in Taylor Rule

- Should we put the exchange rate in Taylor rule?
  - Is there evidence that policymakers keep the exchange rate to some form of target?
- Some casual evidence described below suggests that in many cases S_t can be interpreted as exhibiting a return to trend, though this happens at best over the longer run.
  - See the following slides....
- Long-run return to trend in exchange rate may be part of the explanation for the 'uncovered interest parity' puzzle. See below.

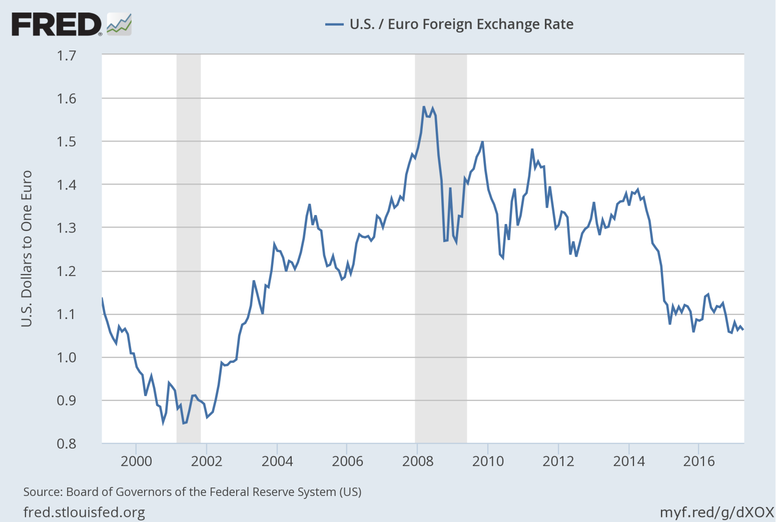
Canada-US: Highly Persistent, But Consistent With (Long-run) Return to Constant ($\psi = 1$)



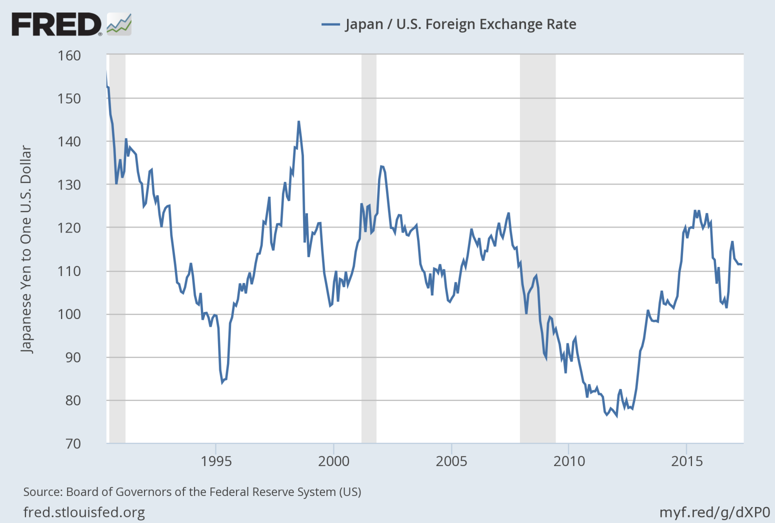
China-US: Persistent, but Change in Regime (i.e., change in the value of ψ) in mid 1990s?



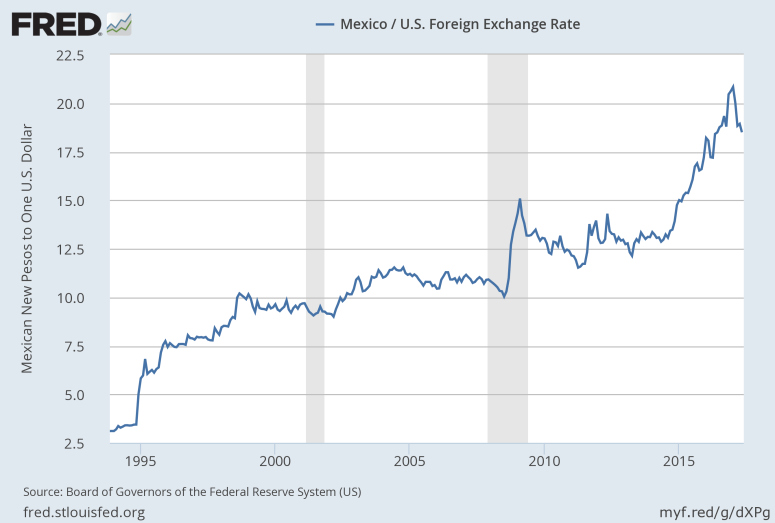
Euro-US: Highly Persistent, but Around Dollar Depreciation Trend ($\psi > 1$)?



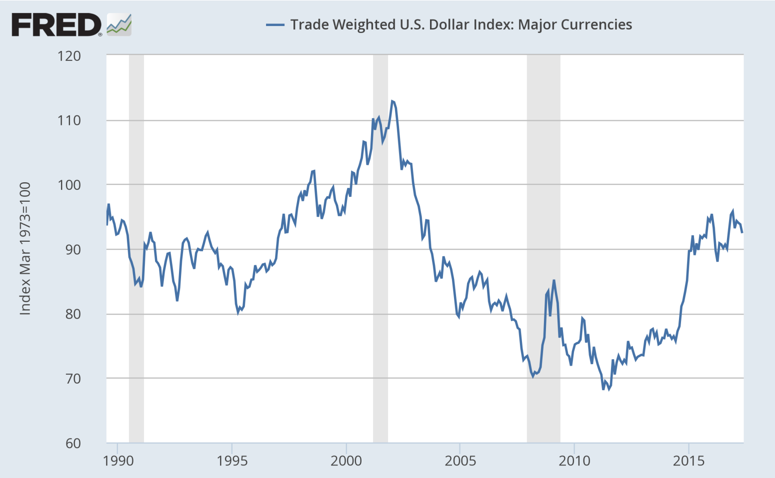
Japan-US: Highly Persistent, but Around Dollar Depreciation Trend ($\psi > 1$)?



Mexico-US: Highly Persistent, but Around Dollar Appreciation Trend ($\psi < 1$)?



US and its Major Trading Partners (Trade-weighted): Highly Persistent, Around (Slight) Dollar Depreciation Trend ($\psi > 1$)?



Empirical Measurement of 'Long-Run Return to Trend'

- Convenient to consider the following reduced form, AR(1) representation of log exchange rate (suppose $\psi = 1$):

$$\log(S_t) = (1-\rho)\log(\bar{S}) + \rho\log(S_{t-1}) + \varepsilon_t,$$

so that

$$\log(S_{t+j}) = \left(1-\rho^j\right)\log(\bar{S}) + \rho^j\log(S_t) + \varepsilon_{t+j} + \rho\varepsilon_{t+j-1} + \ldots + \rho^{j-1}\varepsilon_t$$

or,

$$E_t\log(S_{t+j}) - \log(S_t) = \beta(j)\left(\log(S_t) - \log(\bar{S})\right), \ \beta(j) \equiv \left(\rho^j - 1\right)$$

- Get full return to trend if $\beta(j) \to -1$ as $j \to \infty$.

Empirical Measurement of 'Long-Run Return to Trend'

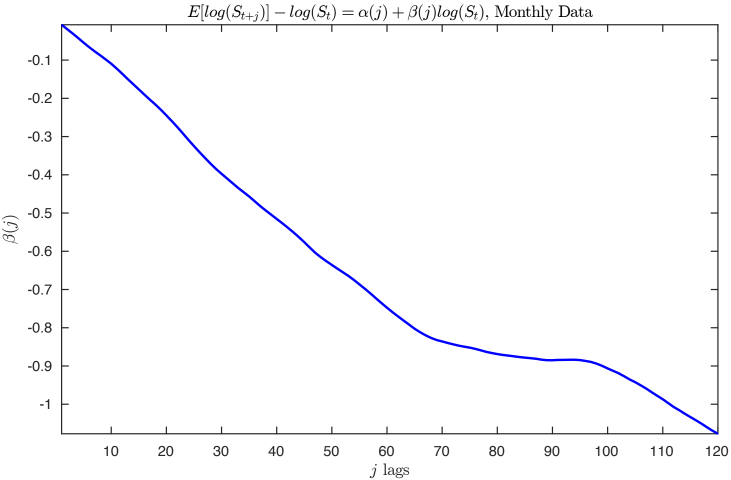
- Expected change in exchange rate and gap:

$$E_t \log\left(S_{t+j}\right) - \log\left(S_t\right) = \beta\left(j\right)\left(\log(S_t) - \log\left(\bar{S}\right)\right),\ \beta\left(j\right) \equiv \left(\rho^j - 1\right)$$

- The function, $\beta\left(j\right)$ determines the trend behavior of S_t.
  - If $\rho = 1$ then $\beta\left(j\right) = 0$ for all j and exchange rate never moves back to any trend.
    - ★ model may capture this case with $r_S = 0$, when only S_t / S_{t-1} (i.e., not S_t itself) appears in the system.
  - If $\rho < 1$, but ρ is close to unity, then $\beta\left(j\right) \simeq 0$ for j small but $\beta\left(j\right) \to -1$ as $j \to \infty$.
    - ★ In this case, get return to trend behavior over the long-term, but little evidence of that in the short term (resembles behavior of actual interest rates).
    - ★ In the model, this is the case, $r_S > 0$ but close to zero.
- Can estimate $\beta\left(j\right)$ by ordinary least squares regression.

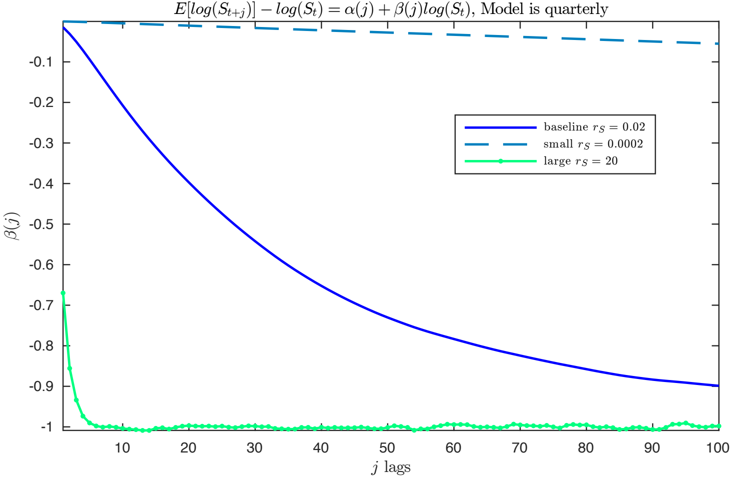
$\beta(j)$ for Monthly Exchange Rate of US and its Major Trading Partners (Trade-weighted)

These calculations confirm visual impression that $\beta(j) \simeq 0$ for low values of j (so, weak return to trend in short run) and $\beta(j) \simeq -1$ for large j.



Model Prediction for $\beta(j)$ as a Function of j

For each j, estimated $\alpha(j), \beta(j)$ by least squares using 100,000 simulated quarterly data with only monetary policy shocks from three different versions of the model, according to value of r_S. Get pattern of empirical data (ignoring mismatch of sampling frequency!) by selecting $r_S \simeq 0.02$.



Interest Parity

- The model implies a relationship between interest rates and the exchange rate.
  - That relationship has been the subject of much research.
- That relationship is best understood by linearizing a portion of the model.

Interest Parity

- Combine the household's intertemporal first order conditions for domestic and foreign bonds, (6) and (7):

$$\gamma\left(\frac{a_t^f}{p_t^c}-\Upsilon_t\right)=E_t\left[\beta\frac{s_{t+1}R_t^f-R_t}{\pi_{t+1}^c c_{t+1}\exp\left(\Delta a_{t+1}\right)}\right]$$

- Linearizing the object in square brackets (using $sR^f=R$):

$$d\beta\frac{s_{t+1}R_t^f-R_t}{\pi_{t+1}^c c_{t+1}\exp\left(\Delta a_{t+1}\right)}=\frac{1}{c}\left(\hat{s}_{t+1}+\hat{R}_t^f-\hat{R}_t\right)$$

where

$$\hat{x}_t\equiv\frac{x_t-x}{x}=\frac{dx_t}{x}.$$

- Also,

$$d\left(\frac{a_t^f}{p_t^c}-\Upsilon_t\right)=\frac{1}{p^c}da_t^f-\Upsilon\hat{p}_t^c-d\Upsilon_t$$

Interest Parity

- From the previous slide:

$$\hat{R}_t = \overbrace{\hat{R}_t^f + E_t\left[\hat{s}_{t+1}\right]}^{\text{Expected return, in domestic currency, to household that invests in } R_t^f} + \hat{\Phi}_t$$

$$\hat{\Phi}_t \equiv c\gamma\left(-\frac{1}{p^c}da_t^f + \Upsilon\hat{p}_t^c + d\Upsilon_t\right)$$

- Uncovered Interest Parity (UIP):

$$\hat{R}_t = \hat{R}_t^f + E_t\left[\hat{s}_{t+1}\right]$$

  - Under UIP, households only consider the expected returns on different assets, so equilibrium requires that the expected returns are equated.
  - Although UIP seems to hold 'in the long run', it does not appear to hold consistently in the short run.
  - Researchers have often used $\hat{\Phi}_t$ as a way to account for short-run failure of UIP.
    - We have introduced it to ensure a steady state for net foreign assets.

Modified Uncovered Interest Parity

- MUIP:

$$\hat{R}_t - \hat{R}_t^f = E_t\left[\hat{s}_{t+1}\right] + \hat{\Phi}_t$$

  - $\hat{\Phi}_t$ often referred to as 'domestic risk premium term'.
  - When $\hat{\Phi}_t > 0$, traders require higher (exchange rate adjusted) domestic rate of interest to be indifferent between domestic and foreign assets.

- Note

$$1 + \hat{x}_t = 1 + \frac{dx_t}{x} = 1 + \frac{x_t - x}{x} = \frac{x_t}{x}$$

$$\text{so,} \widehat{x}_t \simeq \log\left(1 + \widehat{x}_t\right) = \log \frac{x_t}{\mathrm{x}}$$

- Then:

$$log\left(S_{t+1}\right) - log\left(S_t\right) = r_t - r_t^f - \hat{\Phi}_t + u_t,$$

where

$$r_t = log\frac{R_t}{R}, r_t^f = log\frac{R_t^f}{R^f}, \quad u_t \perp \text{variables dated t and earlier}$$

Modified Uncovered Interest Parity

- MIUP:

$$
\begin{aligned}
\log S_{t+1} &= r_t - r_t^f + \log S_t - \hat{\Phi}_t + u_t \\
\log S_{t+2} &= r_{t+1} - r_{t+1}^f + \log S_{t+1} - \hat{\Phi}_{t+1} + u_{t+1} \\
&= r_{t+1} - r_{t+1}^f + r_t - r_t^f + \log S_t - \hat{\Phi}_t - \hat{\Phi}_{t+1} + u_t + u_{t+1}
\end{aligned}
$$

- Then,

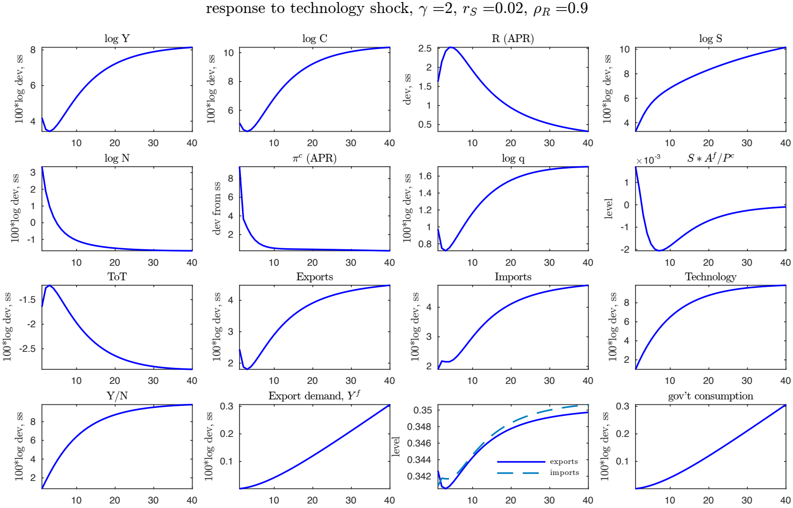
$$
\begin{aligned}
\log S_{t+j} = r_t - r_t^f + r_{t+1} - r_{t+1}^f + .. + r_{t+j-1} - r_{t+j-1}^f + \log S_t \\
- \hat{\Phi}_t - ... - \hat{\Phi}_{t+j-1} + u_t + ... + u_{t+j-1}
\end{aligned}
$$

- A lot of volatility in S_t under UIP and MUIP (i.e., $\hat{\Phi}_t = 0$)
  - Suppose $r_t - r_t^f$ jumps at t and returns to its initial condition from above, then exchange rate depreciates,$\log S_{t+j} - \log S_t \geq 0$, for all $j > 0$.
  - With $r_S > 0$, $E_t \log S_{t+j}$ converges, as $j \to \infty$, to its unshocked path. So, $\log S_t$ drops in period t.

Impulse Responses

- Much of the economics of a model can be found by studying its impulse responses.
- Next, we consider shocks to technology, monetary policy and 'capital flight': $\Delta a_t, \varepsilon_{R,t}, \Upsilon_t$

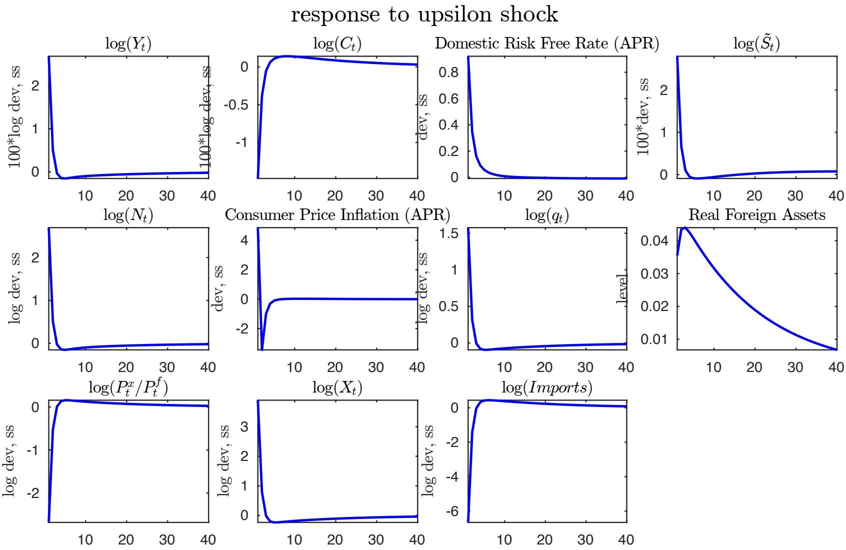
Responses to Technology Shock



Interpreting Responses to Tech Shock

- The 3,4 chart displays the impulse. The state of technology, A_t, jumps 1% in the period of the shock and asymptotically it jumps 10%.
- The strong wealth effect associated with the shock drives up consumption and imports (the latter are an input into the production of consumption goods). See the 1,2 and 3,3 charts.
- Rise in imports drives depreciation (1,4 chart).
- The rise in employment and the exchange rate contribute to the increased production costs that underly the (sharp) rise in inflation in the 2,2 chart. We can see the big role played by exchange rate pass-through here.
- The rise in inflation contributes, via the monetary policy rule, to a rise in the domestic rate of interest. After a brief initial rise in net foreign assets, those assets drop.
- The increase in productivity contributes to a drop in the terms of trade, p_t^x, which stimulates exports. Exports and imports both rise by roughly similar amounts (see 4,3).
- The shift in foreign demand, Y_t^f, for exports and in government spending are small because δ is so close to unity (see 4,2 and 4,4).

Responses to Υ Shock ('Capital Outflow Shock')



Interpreting Responses to Υ Shock

- The capital outflow shock drives households to acquire foreign assets (chart 2,4) and this leads to an immediate nominal depreciation (chart 1,4).
- The depreciation stimulates exports by reducing the terms of trade, p_t^x. To see how, note

$$p_t^x \equiv \frac{P_t^x}{P_t^f} = \frac{S_t P_t^x}{S_t P_t^f} = \frac{P_t}{S_t P_t^f}.$$

  Sticky prices make P_t slow to move and P_t^f is exogenous. So, we expect a jump in S_t to produce a fall in the terms of trade and stimulate exports. See chart 3,1.
- The rise in output and costs (together with the strong pass-through from the exchange rate) lead to a sharp jump in inflation, so that monetary policy generates a rise in the interest rate (charts 1,3 and 2,2).

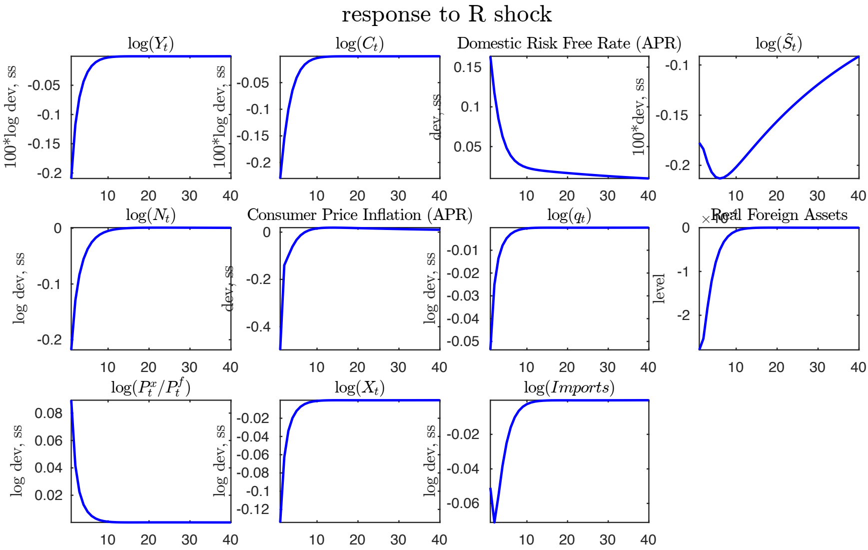
Interpreting Responses to Υ Shock, cnt'd

- The rise in the interest rate drives consumption down (chart 1,2), leading to a fall in imports (chart 3,3). Moreover, the fall in p_t^x leads to a rise in the relative price of imports, p_t^m. To see this, note that

$$p_t^x = \frac{P_t}{S_t P_t^f} = \frac{P_t}{P_t^m} = \frac{1}{p_t^m}.$$

  So, there is substitution away from imports, reinforcing the decline in imports.
- The real exchange rate depreciates, driven by the nominal depreciation (chart 2,3).

Responses to Money Policy Shock



Interpreting Responses to Money Policy Shock

- The monetary policy shock drives up R_t (see chart 1,3), causing consumption to drop (see 1,2).
- This leads to a fall in production and employment, and, by reducing costs, to a drop in inflation (charts 1,1, 2,1 and 2,2).
- The high interest rate causes exchange rate to be depreciated. The effects of pass-through would, other things the same, drive inflation up. In fact, the inflation rate drops. But, the amount of the drop is much smaller than we've seen in the other experiments. So pass-through and the other cost factors roughly cancel.
- The depreciated exchange rate leads to a rise in the terms of trade (see the discussion of previous shocks), and, hence a drop in exports (see charts 3,1 and 3,2).
- The fall in consumption leads to a fall in imports (see chart 3,3).

Interpreting Responses to Money Policy Shock, cnt'd

- The rise in the interest rate leads to a fall in foreign assets. Because the shock hits the economy when it is in steady state, the holdings of foreign assets are now below target. This adds a positive, non-pecuniary, return to those assets because $\gamma > 0$. Thus, traders require a higher return on domestic assets to be willing to hold both. This explains why they are willing to hold both, even though after the shock they receive a higher domestic interest rate and the effective foreign interest rate is lower because the foreign currency is expected to depreciate (i.e., the domestic currency is expected to appreciate). See the downward-sloped portion of the exchange rate trajectory in chart 1,4).
- Because the exchange rate is in the monetary policy rule, it must eventually return to trend. This guarantees that eventually the exchange rate must be higher than it was in the period of the contractionary monetary action, when it exhibits a discrete appreciation. See the upward-sloped portion of the exchange rate trajectory in chart 1,4.

Modified Uncovered Interest Parity

- MIUP:

$$\log S_{t+j} = r_t - r_t^f + r_{t+1} - r_{t+1}^f + .. + r_{t+j-1} - r_{t+j-1}^f + \log S_t$$
$$- \hat{\Phi}_t - ... - \hat{\Phi}_{t+j-1} + u_t + ... + u_{t+j-1}$$

- Long term interest rate in model:

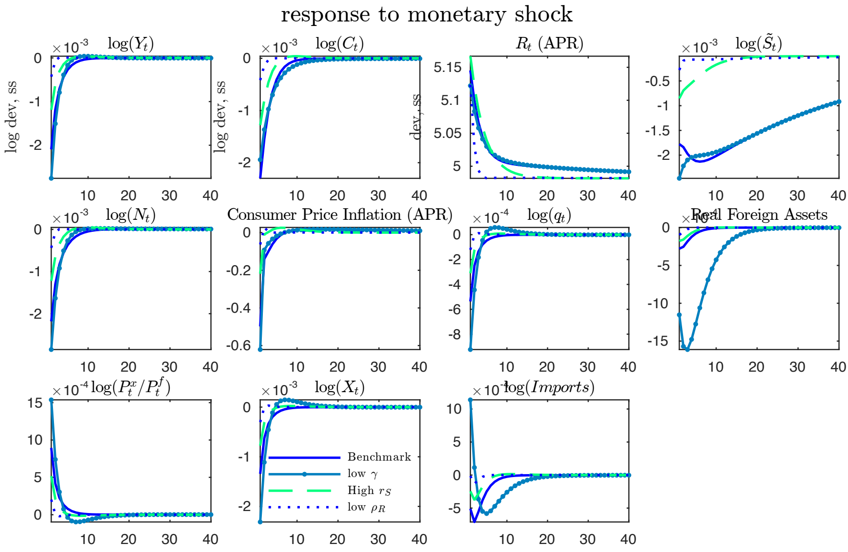
$$r_t^{(j)} = E_t \left[r_t + ... + r_{t+j-1} \right] = r_t + ... + r_{t+j-1} + \eta_{t,t+j},$$
$\eta_{t,t+j} \perp$ information dated t and ealier

- Rewriting MUIP:

$$\log S_{t+j} = r_t^{(j)} - r_t^{f(j)} + \log S_t - \hat{\Phi}_t - ... - \hat{\Phi}_{t+j-1} + \omega_{t+j}$$
$$\omega_{t+j} = u_t + ... + u_{t+j-1} + \eta_{t,t+j} \perp \text{info dated } t \text{ and ealier}$$

- UIP (i.e., $\hat{\Phi}_t = 0$)
  - Under UIP, regression of $logS_{t+j} - logS_t$ on date t, j-period interest rate differential should have unit coefficient, for all $j > 0$.

Impact of γ, r_S, ρ_R on MUIP



MUIP: Summing UP

- The Model can reproduce Chinn and Meredith, 2012 observation on 'sign flip' in UIP regressions at short and long lags.
  - But, success was only relative to monetary policy shocks (it was verified numerically that UIP regressions produce a positive slope on the interest differential for 40-period horizon bonds and a negative slope for one-period horizon bonds).
  - Explaining the Chinn-Meredith sign flip is an important objective.
- Two model features played an important role in the sign flip:
  - The negative slope of $\log S_t$ after an interest rate jump required jump in domestic risk premium term, $\hat{\Phi}_t$.
    - Premium necessary because high domestic interest rate and appreciating currency makes the domestic too attractive for risk neutral traders.
    - The jump in $\hat{\Phi}_t$ reflects the drop in net foreign assets, which makes the domestic currency unattractive because $\gamma > 0$.
- The positive slope in $\log S_t$ at the end occurs because $r_S > 0$ means the exchange rate must eventually return to its unshocked level.

Intuition Behind UIP Puzzle

UIP puzzle: $r_t \uparrow$ *and* expected appreciation of the currency represents a double-boost to the return on domestic assets. On the face of it, it appears that there is an irresistible profit opportunity. Why doesn't the double-boost to domestic returns launch an avalanche of pressure to buy the domestic currency? In standard models, this pressure produces a greater instantaneous appreciation in the exchange rate, until the familiar UIP overshooting result emerges - the pressure to buy the currency leads to such a large appreciation, that expectations of depreciation emerge. In this way, UIP leads to the counterfactual prediction that a higher r_t will be followed (after an instantaneous appreciation) by a period of time during which the currency depreciates.

Intuition Behind 'Resolution' of Puzzle

Model's resolution of the UIP puzzle: when $r_t \uparrow$ the return required for people to hold domestic bonds rises. This is why the double-boost to domestic returns does not create an appetite to buy large amounts of domestic assets. The model's 'explanation' of this lack of appetite is somewhat unconvincing. The idea is that as people respond to the increased return on domestic assets, relative to foreign assets, they start to 'feel bad' because their holdings of net foreign assets are low relative to target, Υ. This may well reflect mechanisms whereby traders who change the composition of their portfolios substantially, draw unwelcome closer attention from the superiors.

Other Questions for Study Using Versions of the Open Economy Model

- Classic analysis of stimulative effects of currency depreciation (caused, for example, by a policy cut in R_t):
  - depreciated currency makes domestic goods cheaper so that foreigners and domestic residents reallocate expenditures towards the domestic economy.
- Two limitations of this analysis.
  - This model does not include financial frictions.
    - With financial frictions, a currency depreciation may hurt the balance sheets of firms/people that have borrowed in foreign currency, forcing them to cut back on spending. This would limit or even reverse the stimulus to spending from the classic channel.
  - Limited pass-through makes domestic prices too sensitive to exchange rate shocks.

Model with Capital, Richer Import Sector and Price Frictions that Slow Down Exchange Rate Pass-Through (CTW)

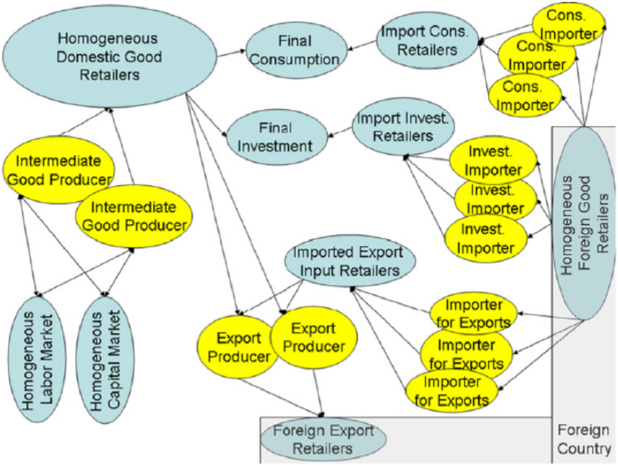
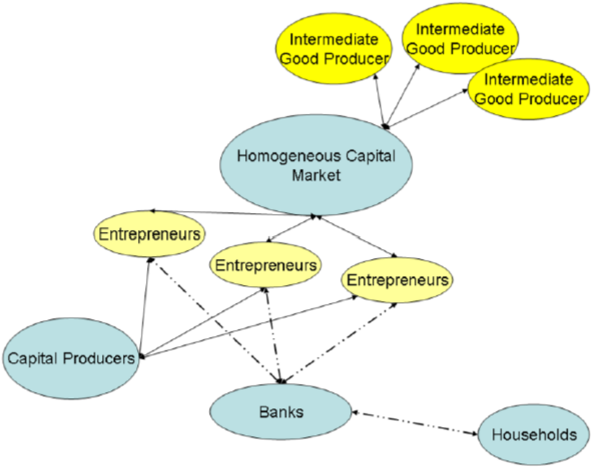


Fig. 1. Graphical illustration of the goods production part of the model.

Financial Frictions as the BGG or CMR in the Open Economy Model



Conclusion

- Open economy model makes best case for why we need DSGE models.
- Economic policies trigger opposing forces in the economy, so that the effects are often ambiguous in theory.
  - Central Bank-induced Exchange rate depreciations: do they expand or contract the economy?
  - Case for expansion: by cheapening cost of domestic goods, more people will buy them.
    - requires good understanding of demand and supply of goods and services.
    - must also understand the extent to which imports are sticky in terms of dollars, or domestic currency.
  - Case for contraction: depreciation will hurt balance sheet of entities that borrow in foreign currency.
    - must understand the workings of the financial sector, where the unhedged currency mismatches are.
- To understand even the simplest economic question: 'will a currency depreciation stimulate or contract the economy?':
  - Requires understanding the structure of very different sectors of the economy.
  - Must understand how those sectors work together.
  - That is more than anyone can work out in their head.

Loose Ends

- Some progress was made understanding departures from UIP (the 'sign switch' observation) and long run return to trend properties of the exchange rate ('regression observation').
  - But, the model was only shown to be able to replicate these two observations when there are only monetary shocks!
- In addition, something we have not explored in these notes is the fact that the model also has difficulty with the following relative volatility observation: in the data the nominal and real exchange rates display roughly the same degree of volatility.
- It would be useful to pursue a systematic search of parameters values to see if the model can explain the three observations above, when there are several shocks beyond just the monetary policy shocks.
  - One possible strategy would be to set up a GMM estimation criterion which includes the standard second moments and also the regression and UIP sign switch observations. A way to do this using Bayesian methods is described and applied in Christiano, et. al. 2011.
  - The Bayesian approach is particularly useful in this context, because some observations (for example, the relative volatility phenomenon) can easily be 'explained' with extremely sticky prices. Such things are ruled out by reasonable priors in a Bayesian analysis.