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Question to assist in studying first model in handout:

http://faculty.wcas.northwestern.edu/~lchrist/course/IMF2012/financial_frictions.pdf

Following is a description of the model economy.

The economy is populated by a large number of identical households. The representative household has a unit measure of agents, and each is either a 'banker' or a 'worker'. Inside the household there is perfect consumption insurance so that each agent consumes the same amount. Periods 1 and 2 consumption by each agent are denoted $c \geq 0$ and $C \geq 0$, respectively. The representative household has preferences,

$$u(c) + \beta u(C), \; 0 < \beta < 1, \; u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \; \gamma > 1.$$

Workers and bankers receive endowments of $y > 0$ and $N > 0$ goods, respectively (all quantities are expressed in household per capita terms). In period 1 each household splits y between c and deposits. Deposits, $d \geq 0$, are placed with a bank run by a banker from another household. Denote the gross return on deposits by R. In period 2, households pay for consumption, C, using the income earned from d and using the profits, π, brought home by their bankers.

In period 1 bankers accept deposits, d, and combine these with their net worth, N, to purchase securities, $s \geq 0$, from firms. Bankers are instructed by their household to maximize profits. A household is able to monitor the actions of its own bankers. We assume that a household has the capacity to enforce its instructions to its own bankers.

Firms possess a linear production technology that can be operated at any scale. For each good put into a firm's technology in period 1, R^k units of goods appear in period 2 (R^k is a technologically given constant). There are no other costs of production and competition implies that a firm which issues s securities to a bank in period 1 returns sR^k goods to the bank in period 2. Participants in all markets are anonymous and competitive.

a) Solve the household problem to obtain an expression relating d to R^d, π and y.

b) Bank profits are:

$$\pi = (N + d) R^k - R^d d.$$

For a given household, the d in the expression for profits represents the deposits of other households. Explain this observation and also explain why it is that in an equilibrium the d's of all households take on the same value.

c) Substitute out for π in the expression relating d to R^d, π and y that you derived above. Solve the resulting expression for d. Now you have an expression relating d to R^d, y, N and R^k. Define this as the *household deposit supply curve.* Note that it is strictly increasing in R^d. Show that if N falls by one unit, then d rises by a fraction of one unit (the fraction is positive and less than unity). Provide intuition into why it is that d rises when N falls. Explain why the supply curve predicts that deposits increase with a rise in y.

d) What is the bank's demand curve for deposits? Draw a diagram indicating the supply and demand curves for deposits, with R^d on the vertical axis and d on the horizontal. The intersection of demand and supply corresponds to the macroeconomic equilibrium.

e) Derive equations that characterize the 'first-best efficient' allocations, in the sense that the allocations solve the planning problem, $\max_{c,C}\left[u(c)+\beta u\left(C\right)\right]$ subject to the period 1 and period 2 resource constraints, $c+k\leq y+N$, $C\leq R^k k$. Here, k denotes goods put into the production technology by the planner in period 1. Show that under our assumptions, the first-best allocations satisfy $c, C>0$. Prove that the allocations in an interior equilibrium (i.e., one in which model parameter values imply $c, C, d>0$) coincide with the 'first-best' allocations.

f) We now introduce a financial friction. Suppose that each banker has two options after it has selected a value for d. Under the first option it can do as assumed above: earn a return on its assets, pay interest to depositors and send the difference home in the form of profits. Under the second option ('default') the bank seizes a fraction, $\theta\in(0,1)$, of the securities, $(N+d)$, and declines to pay anything to its depositors in period 2. A defaulting bank earns profits, $\pi = R^k\theta\left(N+d\right)$, which it sends home in period 2. Depositors in a defaulting bank earn $\left(1-\theta\right)\left(N+d\right)R^k$, on the assets not taken by the banker. Here, θ is an exogenous parameter, not chosen by the banker. At the beginning of period 1, before households have made their deposit decision, banks are required to reveal how many deposits, d, they are willing to accept.

i) Explain why the following condition is necessary and sufficient for a bank to not default (in the case of indifference, assume a bank does not default):

$$R^k \theta (N + d) \leq R^k (N + d) - Rd. \tag{1}$$

ii) Consider an equilibrium in which banks do not default. Explain why it is optimal for an individual bank in such an equilibrium to also not default. Explain why it is that when a bank in that equilibrium selects a value for d, it only considers d's that satisfy (1).

iii) Let an *interior, no-default equilibrium* be a set of numbers, c, C, d, R^d, such that: $c, C, d > 0$; banks do not default; d, c, C solves the household problem given R^d; and the value of d optimizes banks' profits, subject to (1) and given R^d.

   i. Show that $d < \infty$ requires $(1 - \theta) R^k < R^d$ and $d > 0$ requires $R^d \leq R^k$. In light of this, explain why it is that an interior, no-default equilibrium has the property, $(1 - \theta) R^k < R^d \leq R^k$.

   ii. Express the banker problem in Lagrangian form with a multiplier, $\lambda \geq 0$. Exhibit the first order necessary condition for the optimal choice of d as well as the complementary slackness condition.

   iii. Using the first order condition and the complementary slackness condition, derive the bank demand for d for each R^d in $(1 - \theta) R^k < R^d \leq R^k$. (Hint: it is a horizontal line at $R^d = R^k$ up to a specific value of d, after which it is a negatively-sloped curve heading in a south-east direction.)

   iv. Draw an interior, no-default equilibrium as the intersection of demand and supply. Draw an initial intersection where $R^k = R^d$. Then, draw another scenario in which N is lower, causing the equilibrium to occur at a point where $R^d < R^k$. Explain why this scenario illustrates the idea, 'in normal times financial markets do their job efficiently and in crisis times when N is low enough, the banking system is dysfunctional'.

v. Explain how tax-financed direct loans to firms can restore efficiency to financial markets. Define an equity injection to banks as a loan to banks that must be repaid with an interest rate, R^k. Show that a tax financed equity injection into banks can also restore efficiency to financial markets. In both cases, illustrate your argument using the demand-supply diagram for d.