Leverage Restrictions in a Business Cycle Model

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Background

- Increasing interest in the following sorts of questions:
  - What restrictions should be placed on bank leverage?
  - How should those restrictions be varied over the business cycle?

What We Do

- Modify a standard medium-sized DSGE model to include a banking sector.

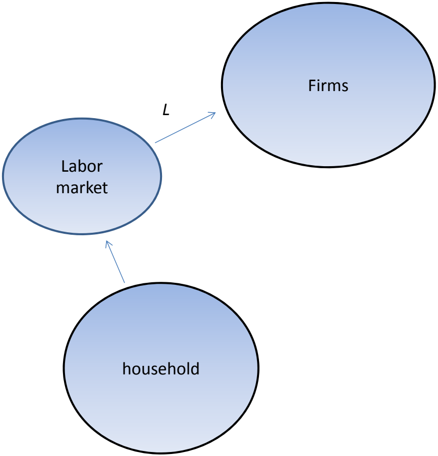
Assets	Liabilities
Loans and other securities	Deposits
	Banker net worth

- Job of bankers is to identify and finance good investment projects.
  - doing this requires exerting costly effort.
- Agency problem between bank and its creditors:
  - banker effort is not observable.
- Consequence: leverage restrictions on banks generate a very substantial welfare gain in steady state.
- Desirable to encourage low leverage in good times, so that banks in better position to absorb bad shocks to net worth.

Outline

- Model
  - first, without leverage restriction
    - observable effort benchmark
    - unobservable case
  - then, with leverage restriction
- Steady state properties of leverage restrictions
- Dynamics

Standard Model



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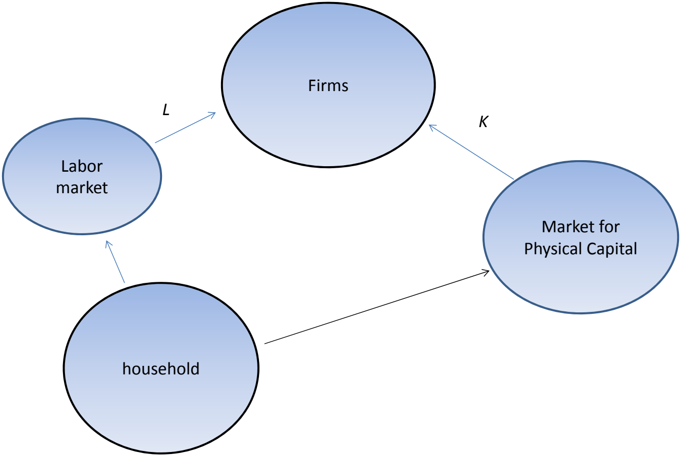

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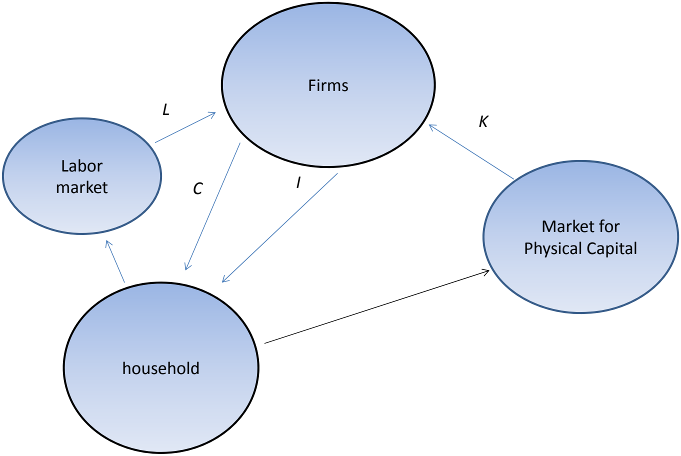
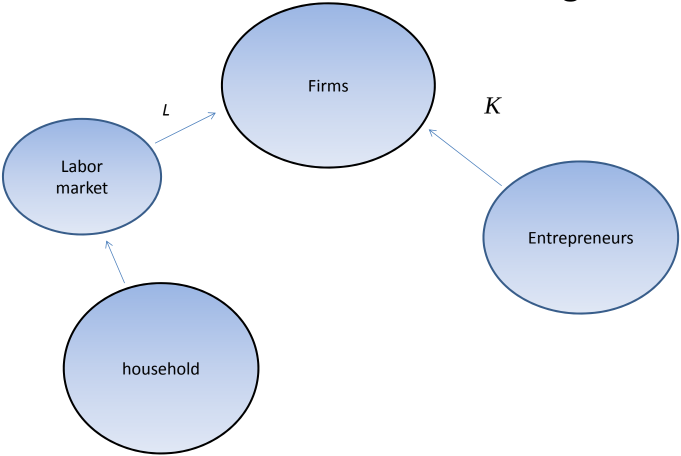


Standard Model with Banking



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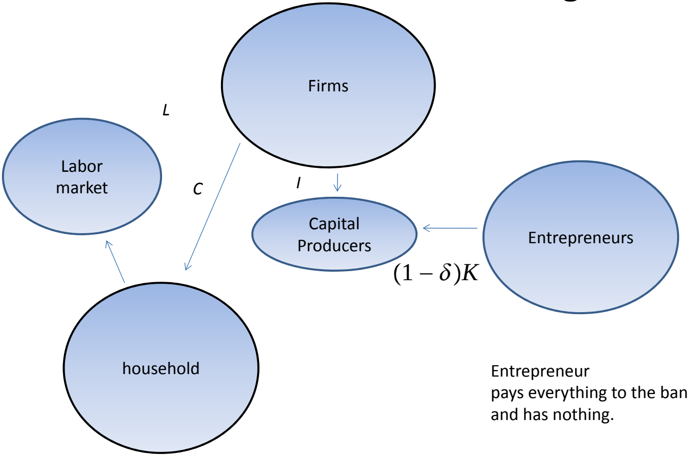
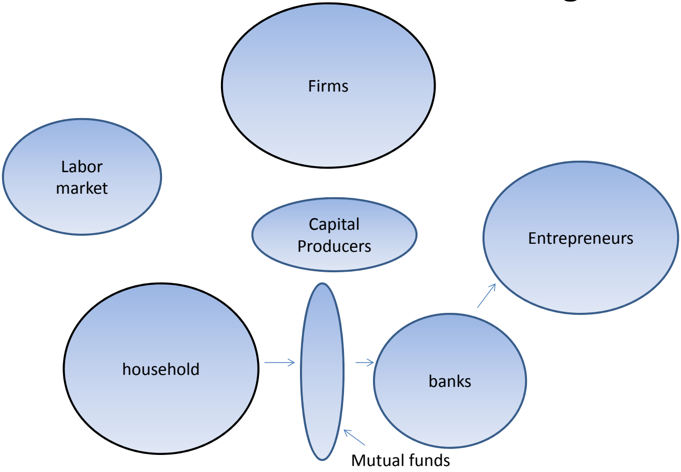


Entrepreneur
pays everything to the bank
and has nothing.

Standard Model with Banking



Entrepreneurs

- After goods production in period t : Purchase raw capital from capital producers, for price $P_{k',t}$.
  - entrepreneurs have no resources of their own and must obtain financing from banks.
- Entrepreneurs convert raw capital into effective capital.
  - Some are good at it and some are bad.
- In period $t+1$:
  - entrepreneurs rent capital to goods-producers in competitive markets, at rental rate, r_{t+1}.
  - after production, sell undepreciated capital back to capital producers at price, $P_{k',t+1}$.
  - entrepreneurs pay all earnings to bank at end of $t+1$, keeping nothing.
  - no agency problems between entrepreneurs and banks.

Earnings of Entrepreneurs

- there are good entrepreneurs and bad entrepreneurs.
- bad: 1 unit, raw capital $\rightarrow e^{b_t}$ units, effective capital
- good: 1 unit, raw capital $\rightarrow e^{g_t} > e^{b_t}$ units, effective capital
- return to capital enjoyed by entrepreneurs:

$$R_{t+1}^{g} = e^{g_t} R_{t+1}^{k},\ R_{t+1}^{b} = e^{b_t} R_{t+1}^{k}$$

$$R_{t+1}^{k} \equiv \frac{r_{t+1}^{k} P_{t+1} + (1-\delta) P_{k,t+1}}{P_{k't}}$$

Bankers

- each has net worth, N_t.
- a banker can only invest in one entrepreneur (asset side of banker balance sheet is risky).
- by exerting effort, e_t, a banker finds a good entrepreneur with probability p :

$$p\left(e_t\right) = \bar{a} + \bar{b}e_t$$

- in t, bankers seek to optimize:

$$E_t\lambda_{t+1}\{p\left(e_t\right)\left[R^g_{t+1}\left(N_t + d_t\right) - R^g_{d,t+1}d_t\right] + \left(1 - p\left(e_t\right)\right)\left[R^b_{t+1}\left(N_t + d_t\right) - R^b_{d,t+1}d_t\right]\} - \frac{1}{2}e_t^2$$

- Bankers have a cash constraint:

$$R^b_{t+1}\left(N_t + d_t\right) \geq R^b_{d,t+1}d_t$$

Bankers and their Creditors

- Bankers and Mutual Funds interact in competitive markets for loan contracts:

$$\left(d_t, e_t, R^g_{d,t+1}, R^b_{d,t+1}\right)$$

- Free entry and competition among mutual funds implies:

$$p\left(e_t\right) R^g_{d,t+1} + \left(1 - p\left(e_t\right)\right) R^b_{d,t+1} = R_t$$

- Two scenarios:
  - banker effort, e_t, is observed by mutual fund
  - banker effort, e_t, is unobserved.

Observed Effort Benchmark

- Set of contracts available to bankers is the $\left(d_t, e_t, R^g_{d,t+1}, R^b_{d,t+1}\right)$'s that satisfy

$$\begin{array}{ll} \text{MF zero profits:} & p\left(e_t\right) R^g_{d,t+1} + \left(1 - p\left(e_t\right)\right) R^b_{d,t+1} = R_t, \\ \text{cash constraint:} & R^b_{t+1}\left(N_t + d_t\right) \geq R^b_{d,t+1} d_t \end{array}$$

- Each banker chooses the most preferred contract from the menu.
- Key feature of observed effort equilibrium:

$$e_t = E_t \lambda_{t+1} p'\left(e_t\right)\left(R^g_{t+1} - R^b_{t+1}\right)\left(N_t + d_t\right)$$

Unobserved Effort

- In this case, banker always sets e_t to its privately optimal level, whatever e_t is specified in the loan contract:

$$\text{incentive:} \qquad e_t = E_t \lambda_{t+1} p'(e_t) \left[\left(R^g_{t+1} - R^b_{t+1} \right) (N_t + d_t) - \left(R^g_{d,t+1} - R^b_{d,t+1} \right) d_t \right].$$

- Set of contracts available to bankers is the $\left(d_t, e_t, R^g_{d,t+1}, R^b_{d,t+1} \right)$'s that satisfy 'incentive' in addition to:

$$\text{MF zero profits:} \qquad p(e_t) R^g_{d,t+1} + (1 - p(e_t)) R^b_{d,t+1} = R_t,$$

$$\text{cash constraint:} \qquad R^b_{t+1}(N_t + d_t) \geq R^b_{d,t+1} d_t$$

- One factor that can make e_t inefficiently low:
  - $R^g_{d,t+1} > R^b_{d,t+1}$.

Law of Motion of Net Worth

- Bankers live in a large representative household, with workers (as in Gertler-Karadi, Gertler-Kiyotaki).
  - Bankers pool their net worth at the end of each period (we avoid worrying about banker heterogeneity)
- Law of motion of banker net worth

$$
\begin{aligned}
N_{t+1} \quad = \quad & \gamma_{t+1}\{p\left(e_t\right) \overbrace{\left[R_{t+1}^g\left(N_t+d_t\right)-R_{d,t+1}^g d_t\right]}^{\text{profits when bank assets good}} \\
& +\left(1-p\left(e_t\right)\right) \overbrace{\left[R_{t+1}^b\left(N_t+d_t\right)-R_{d,t+1}^b d_t\right]}^{\text{profits when bank assets are bad}}\} \\
& + \qquad \overbrace{T_{t+1}}^{\text{lump sum transfer, households to their bankers}}
\end{aligned}
$$

Model Assumption that Banks Don't Systematically Rely on Equity Issues to Finance Assets

- Evidence from two sources provide support for this assumption as a description of the data.
  - Adrian and Shin's examination of the assets and liabilities of two large French financial firms.
  - US flow of funds data on assets and liabilities of financial corporations.
- Adrian and Shin, 'Procyclical Leverage and Value-at-Risk'
  - Changes in financial firm equity not systematically related to their assets.
  - Changes in financial firm debt moves one-for-one with changes in assets.

Material taken from the work of Adrian Shin.
Displays a scatter plot change in equity and debt on the horizontal axis against change in assets on the horizontal axis. Note that the slope of changes in debt against changes in assets is essentially unity, while the slope of changes in equity against changes in assets has a slope of zero.
The results are consistent with the notion that this financial company headquartered in Paris finances changes in assets with changes in debt and not changes in equity.

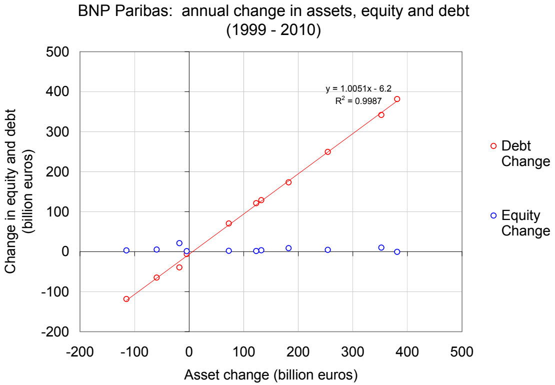


Figure 3. BNP Paribas: annual change in assets, equity and debt (1999-2010) (Source: Bankscope)

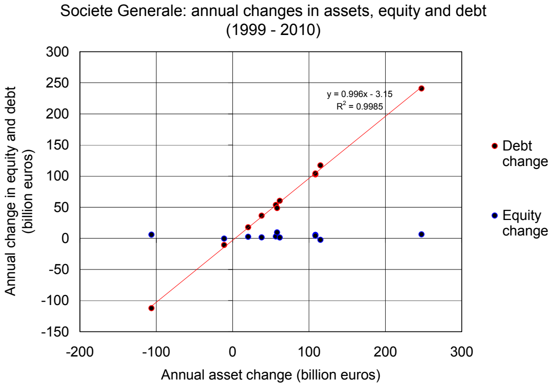
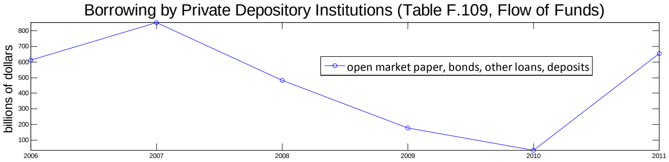


Figure 4. Société Générale: annual change in assets, equity and debt (1999-2010) (Source: Bankscope)

- The model assumes that when bankers want funds, issuing equity is not an option.



This shows how major debt instruments were used at private depository institutions in the wake of the crisis.

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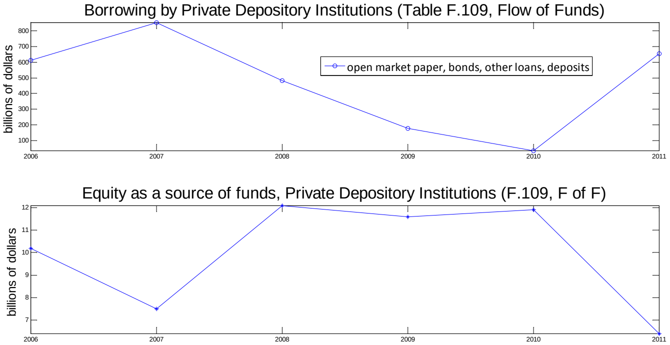


‘Crisis’

- Suppose something makes banker net worth, N_t, drop.
- For given d_t, bank cash constraint gets tighter:

$$R_{t+1}^{b}\left(N_t + d_t\right) \geq R_{d,t+1}^{b} d_t.$$

- So, $R_{d,t+1}^{b}$ has to be low
  - when N_t is low, banks with bad assets cannot cover their own losses and creditors must share in losses.
  - then, creditors require $R_{d,t+1}^{g}$ high
- So, interest rate spread, $R_{d,t+1}^{g} - R_t$, high, banker effort low.
- Banks get riskier (cross sectional mean return down, standard deviation up).

Endogenous Risk

- Rate of return on equity, good banks and bad banks:

$$\begin{aligned} p(e_t) \text{ good banks} &: \frac{R^g_{t+1}(N_t + d_t) - R^g_{d,t+1} d_t}{N_t}, \\ 1 - p(e_t) \text{ bad banks} &: \frac{R^b_{t+1}(N_t + d_t) - R^b_{d,t+1} d_t}{N_t} = 0 \end{aligned}$$

- Mean, E^b_{t+1}, and cross sectional standard deviation, s^b_{t+1}, of return on equity across banks:

$$\begin{aligned} & [p(e_t)(1 - p(e_t))]^{1/2} \frac{R^g_{t+1}(N_t + d_t) - R^g_{d,t+1} d_t}{N_t} \\ E^b_{t+1} &= p(e_t) \frac{R^g_{t+1}(N_t + d_t) - R^g_{d,t+1} d_t}{N_t} \end{aligned}$$

- In a crisis, risk rises and mean return falls.

Macro Model

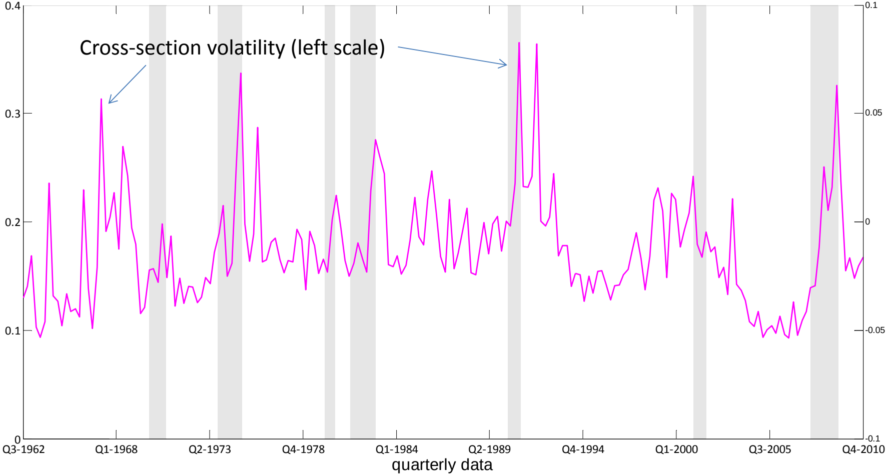
- Sticky wages and prices
- Investment adjustment costs
- Habit persistence in consumption
- Monetary policy rule

Calibration targets

Table 2: Steady state calibration targets for baseline model		
Variable meaning	variable name	magnitude
Cross-sectional standard deviation of quarterly non-financial firm equity returns	s^b	0.20
Fnancial firm interest rate spreads (APR)	$400(R_g^d - R)$	0.60
Financial firm leverage	L	20.00

Data behind calibration targets

Figure 1: Cross-section standard deviation financial firm quarterly return on equity, HP-filtered US real GDP



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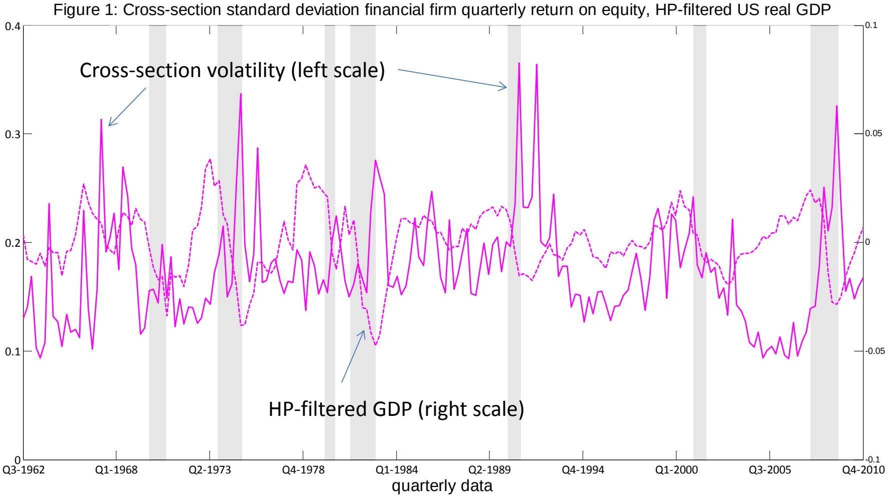
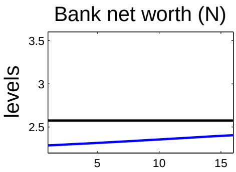
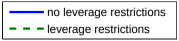


Parameter Values

Table 1: Baseline Model Parameter Values		
Meaning	Name	Value
Panel A: financial parameters		
return parameter, bad entrepreneur	b	-0.09
return parameter, good entrepreneur	g	0.00
constant, effort function	$\bar{a}$	0.83
slope, effort function	$\bar{b}$	0.30
lump-sum transfer from households to bankers	$\tilde{T}$	0.38
fraction of banker net worth that stays with bankers	γ	0.85
Panel B: Parameters that do not affect steady state		
steady state inflation (APR)	$400(\pi - 1)$	2.40
Taylor rule weight on inflation	α_π	1.50
Taylor rule weight on output growth	$\alpha_{\Delta y}$	0.50
smoothing parameter in Taylor rule	ρ_p	0.80
curvature on investment adjustment costs	S''	5.00
Calvo sticky price parameter	ξ_p	0.75
Calvo sticky wage parameter	ξ_w	0.75
Panel C: Nonfinancial parameters		
steady state gdp growth (APR)	μ_{z^*}	1.65
steady state rate of decline in investment good price (APR)	Υ	1.69
capital depreciation rate	δ	0.03
production fixed cost	Φ	0.89
capital share	α	0.40
steady state markup, intermediate good producers	λ_f	1.20
habit parameter	b_u	0.74
household discount rate	$100(\beta^{-4} - 1)$	0.52
steady state markup, workers	λ_w	1.05
Frisch labor supply elasticity	$1/\sigma_L$	1.00
weight on labor disutility	ψ_L	1.00
steady state scaled government spending	$\tilde{g}$	0.89

Impact of Loss of Bank Net Worth





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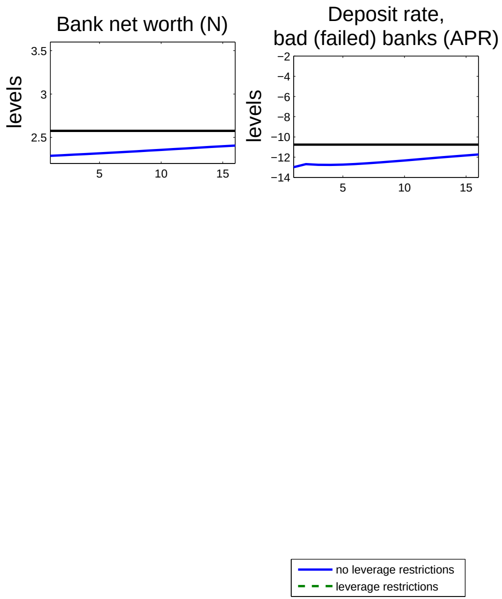

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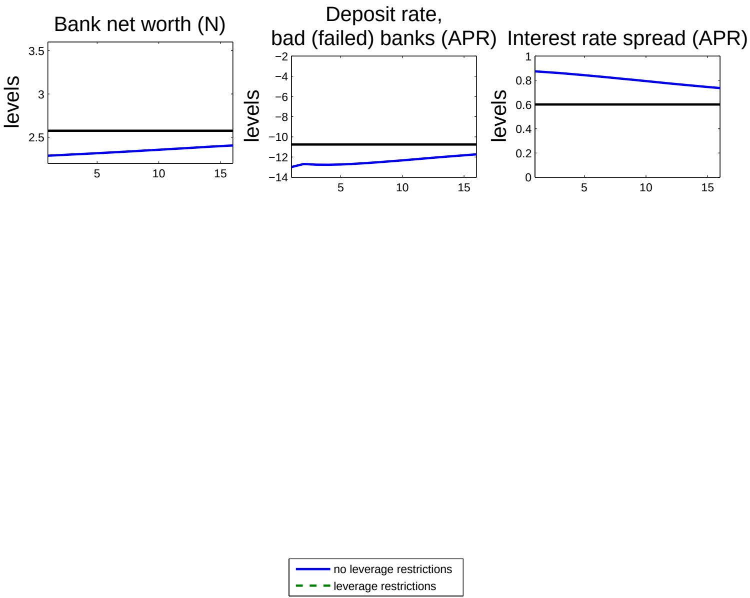

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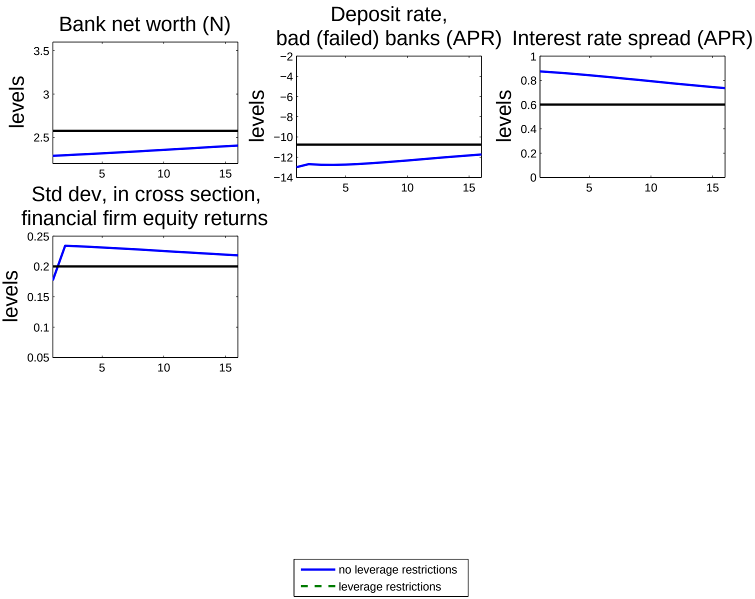

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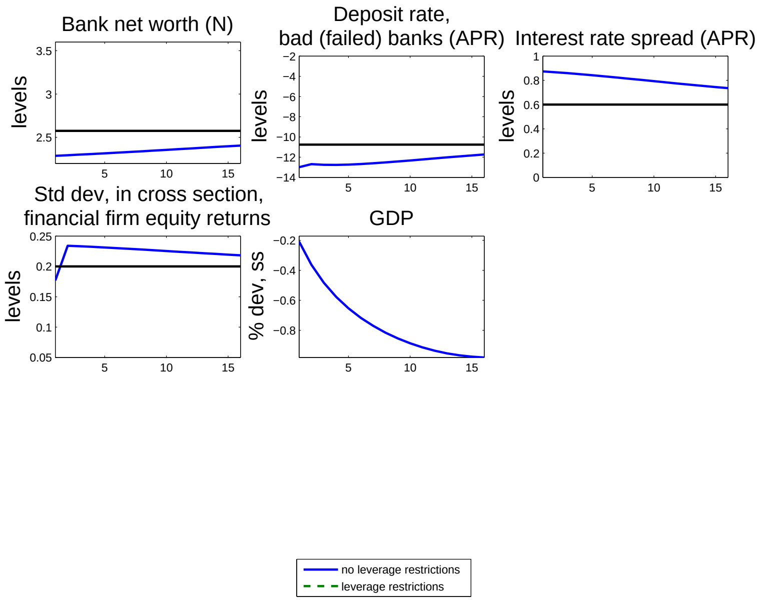
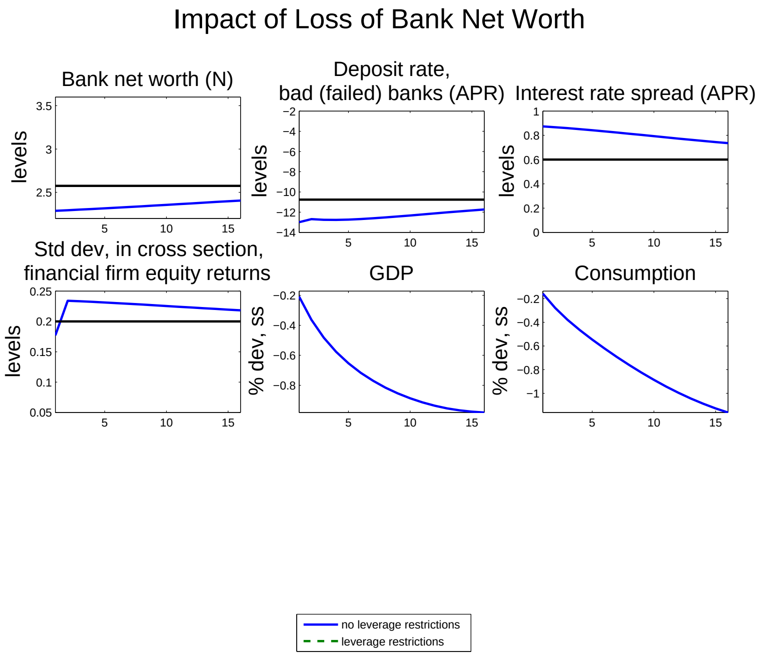
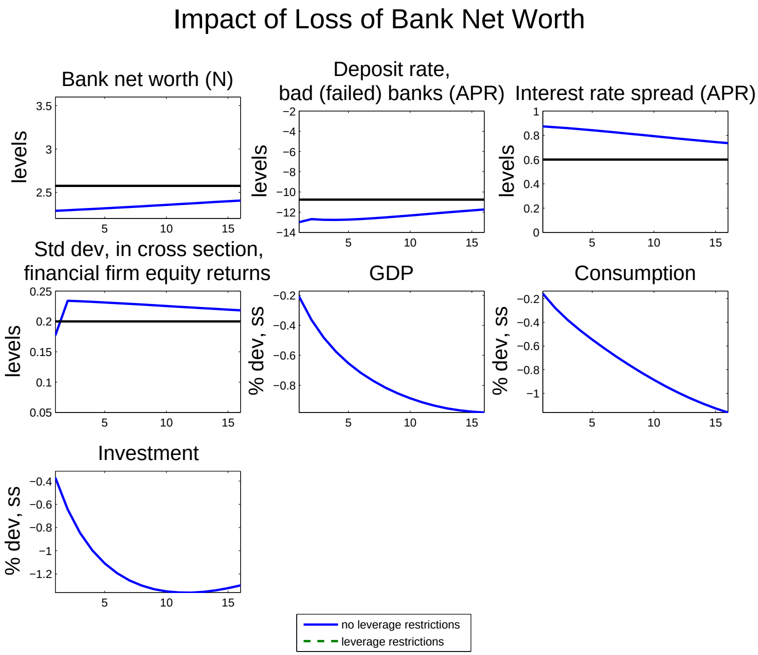




Impact of Loss of Bank Net Worth
Bank net worth (N)
levels
Deposit rate,
bad (failed) banks (APR)
Interest rate spread (APR)
Std dev, in cross section,
financial firm equity returns
GDP
% dev, ss
Consumption
no leverage restrictions
leverage restrictions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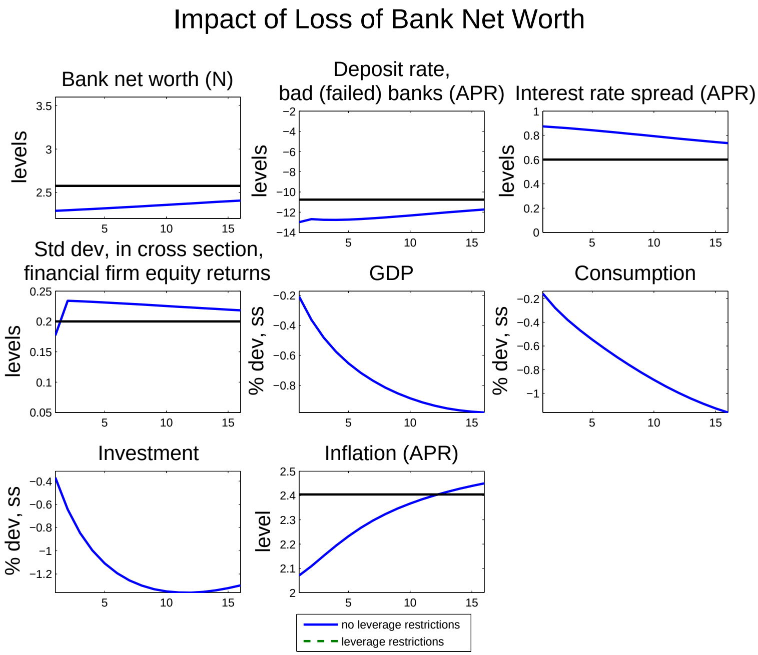
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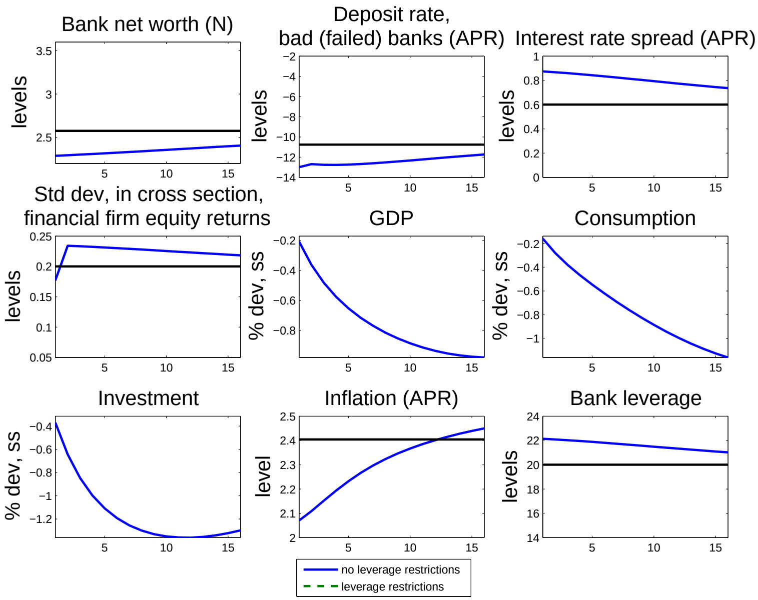


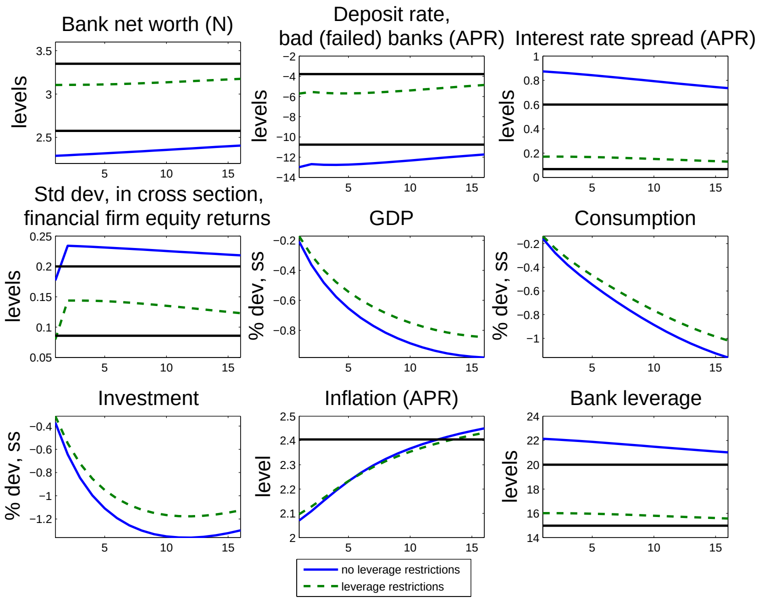
Leverage Restrictions

- Banks taxed for issuing deposits d_t
  - 1.2% AR (versus 3% AR on the risk free nominal rate).
  - revenues redistributed back to banks in lump-sum form.
- What is the consequence of this restriction?
  - With less d_t, banks with bad assets more able to cover losses
    - interest rate spread falls, so banker effort rises.
  - Second effect of leverage restriction,
    - leverage restriction in effect implements collusion among bankers
    - allows them to behave as monopsonists
    - make profits on demand deposits....lots of profits:

$$\left[p\left(e_t\right)\left(R_{t+1}^g - R_{d,t+1}^g\right) + \left(1 - p\left(e_t\right)\right)\left(R_{t+1}^b - R_{d,t+1}^b\right)\right] \overbrace{\frac{d_t}{N_t}}^{\text{big}}$$

    - makes N_t grow, offseting incentive effects of decline in d_t.

Impact of Loss of Bank Net Worth



Conclusion

- Described a model in which there is a problem that is mitigated by the introduction of leverage restrictions.
- Currently exploring what are the optimal dynamic properties of leverage.
  - the cyclical behavior of the tax on leverage depends on which shock drives the cycle.
  - if driven by permanent technology shocks, then act to discourage debt in a boom.

Steady State Calculations

- Next study steady state impact of leverage
  - Quantify role of hidden effort in the analysis (*essential!*)

Table 3: Steady State Properties of the Model					
Variable meaning	Variable name	Unobserved Effort		Observed Effort	
		Leverage Restriction		Leverage Restriction	
		non-binding	binding	non-binding	binding
Spread	$400(R_g^d - R)$	0.600			
scaled consumption	c				
labor	h				
scaled capital stock	k				
bank assets	$N + d$				
bank net worth	N				
bank deposits	d				
bank leverage	$(N + d)/N$	20.00			
bank return on equity (APR)	$400\left(\frac{\left[p(e_t)R_{t+1}^g+(1-p(e_t))R_{t+1}^b\right](N_t+d_t)-R_t d_t}{N_t} - 1\right)$				
fraction of firms with good balance sheets	$p(e)$				
Benefit of leverage (in c units)	100χ				
Benefit of making effort observable (in c units)	100χ				

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Spread	$400(R_g^d - R)$	0.600			
scaled consumption	c	1.84			
labor	h	1.18			
scaled capital stock	k	51.52			
bank assets	$N + d$	51.52			
bank net worth	N	2.58			
bank deposits	d	48.94			
bank leverage	$(N + d)/N$	20.00			
bank return on equity (APR)	$400\left(\frac{\left[p(e_t)R_{t+1}^g+(1-p(e_t))R_{t+1}^b\right](N_t+d_t)-R_t d_t}{N_t} - 1\right)$				
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fraction of firms with good balance sheets	$p(e)$	0.962			
Benefit of leverage (in c units)	100χ	NA			
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labor	h	1.18		1.15	
scaled capital stock	k	51.52		59.75	
bank assets	$N + d$	51.52		59.55	
bank net worth	N	2.58		2.58	
bank deposits	d	48.94		56.98	
bank leverage	$(N + d)/N$	20.00		23.12	
bank return on equity (APR)	$400\left(\frac{\left[p(e_t)R_{t+1}^g+(1-p(e_t))R_{t+1}^b\right](N_t+d_t)-R_t d_t}{N_t} - 1\right)$	4.59		4.59	
fraction of firms with good balance sheets	$p(e)$	0.962		1.000	
Benefit of leverage (in c units)	100χ	NA		NA	
Benefit of making effort observable (in c units)	100χ	NA		6.11	

Making effort observable makes things *a lot* better, equivalent to a 6% permanent jump in consumption!

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bank deposits	d	48.94		56.98	
bank leverage	$(N + d)/N$	20.00		23.12	
bank return on equity (APR)	$400\left(\frac{\left[p(e_t)R_{t+1}^g+(1-p(e_t))R_{t+1}^b\right](N_t+d_t)-R_t d_t}{N_t} - 1\right)$	4.59		4.59	
fraction of firms with good balance sheets	$p(e)$	0.962		1.000	
Benefit of leverage (in c units)	100χ	NA		NA	
Benefit of making effort observable (in c units)	100χ	NA		6.11	

Interestingly, leverage goes up.

Table 3: Steady State Properties of the Model					
Variable meaning	Variable name	Unobserved Effort		Observed Effort	
		Leverage Restriction		Leverage Restriction	
		non-binding	binding	non-binding	binding
Spread	$400(R_g^d - R)$	0.600	0.211	NA	
scaled consumption	c	1.84	1.88	2.01	
labor	h	1.18	1.16	1.15	
scaled capital stock	k	51.52	51.40	59.75	
bank assets	$N+d$	51.52	51.31	59.55	
bank net worth	N	2.58	3.02	2.58	
bank deposits	d	48.94	48.29	56.98	
bank leverage	$(N+d)/N$	20.00	17.00	23.12	
bank return on equity (APR)	$400\left(\frac{\left[p(e_t)R_{t+1}^g+(1-p(e_t))R_{t+1}^b\right](N_t+d_t)-R_t d_t}{N_t}-1\right)$	4.59	14.96	4.59	
fraction of firms with good balance sheets	$p(e)$	0.962	0.982	1.000	
Benefit of leverage (in c units)	100χ	NA	1.19	NA	
Benefit of making effort observable (in c units)	100χ	NA	NA	6.11	

Cut in leverage in the unobserved effort economy moves things towards observed effort.

Table 3: Steady State Properties of the Model					
Variable meaning	Variable name	Unobserved Effort		Observed Effort	
		Leverage Restriction		Leverage Restriction	
		non-binding	binding	non-binding	binding
Spread	$400(R_g^d - R)$			NA	NA
scaled consumption	c			2.01	1.95
labor	h			1.15	1.14
scaled capital stock	k			59.75	53.86
bank assets	$N + d$			59.55	53.68
bank net worth	N			2.58	3.16
bank deposits	d			56.98	50.52
bank leverage	$(N + d)/N$			23.12	17.00
bank return on equity (APR)	$400\left(\frac{\left[p(e_t)R_{t+1}^g+(1-p(e_t))R_{t+1}^b\right](N_t+d_t)-R_t d_t}{N_t} - 1\right)$			4.59	17.63
fraction of firms with good balance sheets	$p(e)$			1.000	1.000
Benefit of leverage (in c units)	100χ			NA	-2.70
Benefit of making effort observable (in c units)	100χ			6.11	2.03

Hidden effort assumption is *essential*. Otherwise, leverage restriction *reduces* utility.

Dynamics

- Here, we consider the dynamic effects of two shocks
  - shock to monetary policy
  - lump sum shock to net worth

$$R_t = 0.80 R_{t-1} + (1 - 0.80)[1.5\pi_{t+1} + 0.5 g_{y,t}] + \varepsilon_t^p$$

$$\varepsilon_0^p = +25 \text{ annual basis points}$$

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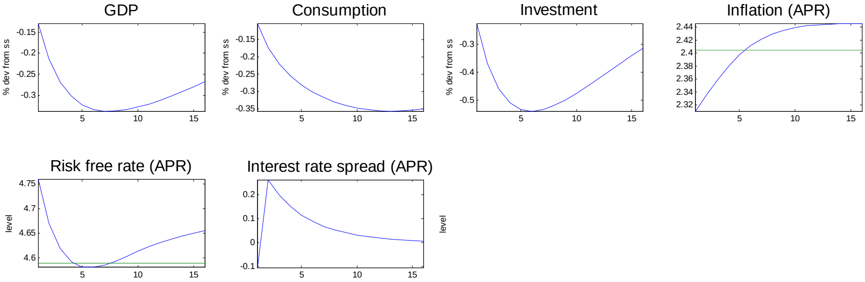
$$\varepsilon_0^p = +25 \text{ annual basis points}$$



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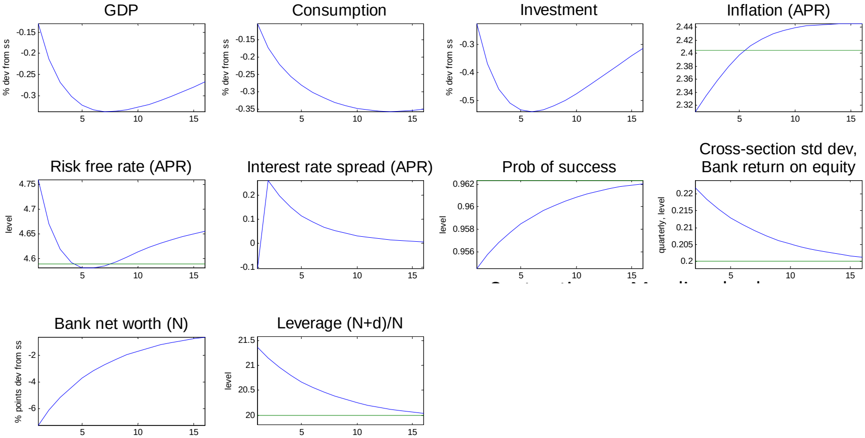

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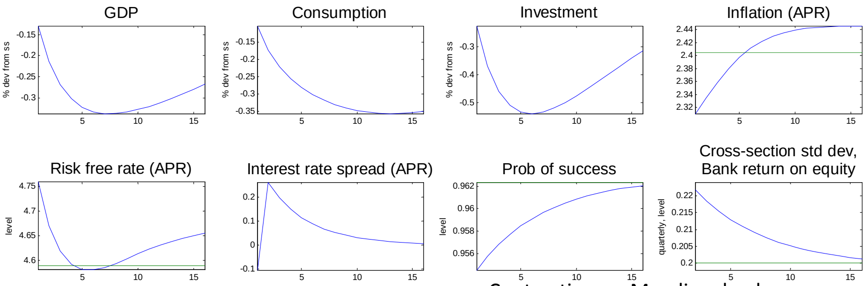
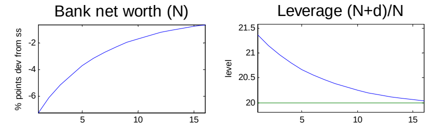




Contractionary M policy shock:

fall in:

- *c, i, y,*
- bank net worth, *N*,
- inflation

Rise in:

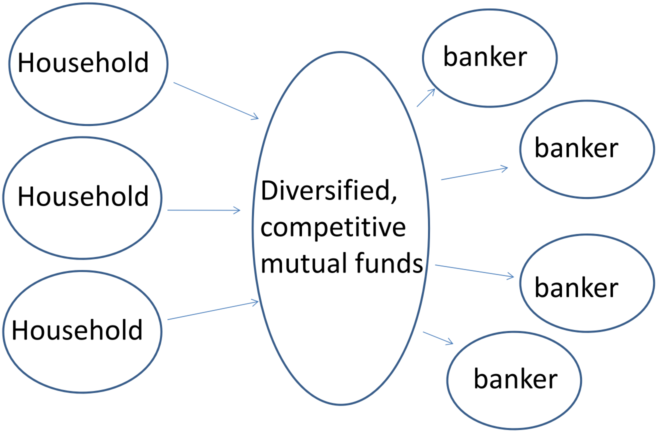
- leverage
- cross-sectional dispersion of bank performance

Bankers and their Creditors

Assets	Liabilities
Loans and other securities	Deposits, d_t
$N_t + d_t$	Banker net worth, N_t

- No agency problems on asset side of bank balance sheet.
- Problems are on liability side.
- Bankers receive credit, d_t, from mutual funds.
  - Mutual funds deal with households.

Risky Bankers Funded By Mutual Funds



$$\log\left(\frac{T_t}{T}\right) = 0.95\log\left(\frac{T_{t-1}}{T}\right) + \varepsilon_t^T$$
$$\varepsilon_0^T = -0.10$$

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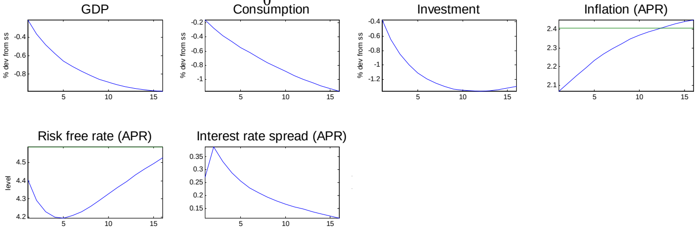

$$\log\left(\frac{T_t}{T}\right) = 0.95\log\left(\frac{T_{t-1}}{T}\right) + \varepsilon_t^T$$

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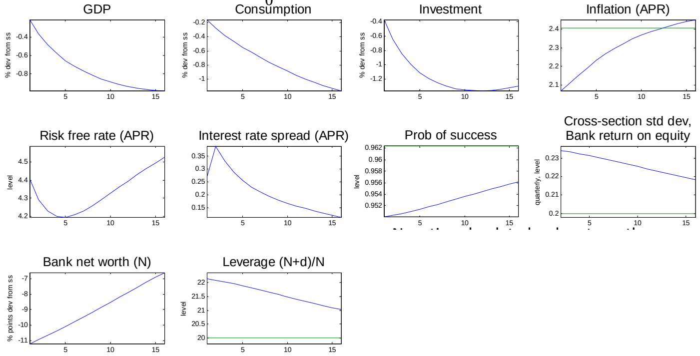

$$\log\left(\frac{T_t}{T}\right) = 0.95\log\left(\frac{T_{t-1}}{T}\right) + \varepsilon_t^T$$

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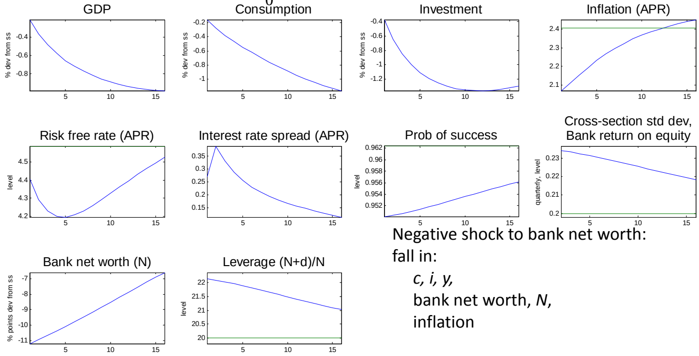


Negative shock to bank net worth:

fall in:

- *c, i, y,*
- bank net worth, *N*,
- inflation

Rise in:

- leverage
- cross-sectional dispersion of bank performance

$$
\begin{aligned}
L^e_t &= \frac{a^f_t}{a^f_t - l^f_t} \\
dL^e_t &= \frac{da^f_t}{a^f - l^f} - \frac{a^f_t}{\left(a^f - l^f\right)^2}\left(da^f_t - dl^f_t\right) \\
&= \frac{a^f}{a^f - l^f}\hat{a}^f_t - \frac{a^f_t}{\left(a^f - l^f\right)^2}\left(a^f\hat{a}^f_t - l^f\hat{l}^f_t\right) \\
\hat{L}^e_t &= \hat{a}^f_t - \frac{1}{a^f - l^f}\left(a^f\hat{a}^f_t - l^f\hat{l}^f_t\right) \\
&= \frac{l^f}{a^f - l^f}\left(\hat{l}^f_t - \hat{a}^f_t\right)
\end{aligned}
$$

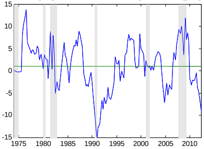
Cyclicality of Leverage

- The model appears to imply countercyclical leverage.
- We took data from the Flow of Funds accounts to measure leverage.
  - Problem: only report *financial* assets (a^f) and liabilities (l^f)

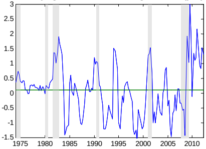
$$L^f = \frac{a^f}{a^f - l^f}$$

  - This measure of leverage can be negative or gigantic.
- We took measures of L^f for three components of financial business, over a period for which L^f does not behave strangely, the 2000s.

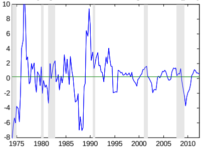
Holding Companies (L.128)
liability growth - asset growth (yoy)



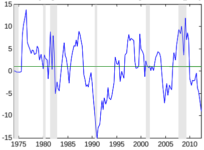
Private Depository Institutions (L.109)
liability growth - asset growth (yoy)



Security Brokers and Dealers (L.127)
liability growth - asset growth (yoy)



Holding Companies (L.128)
liability growth - asset growth (yoy)



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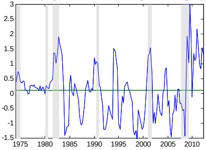
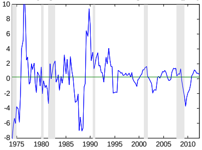
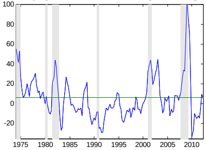
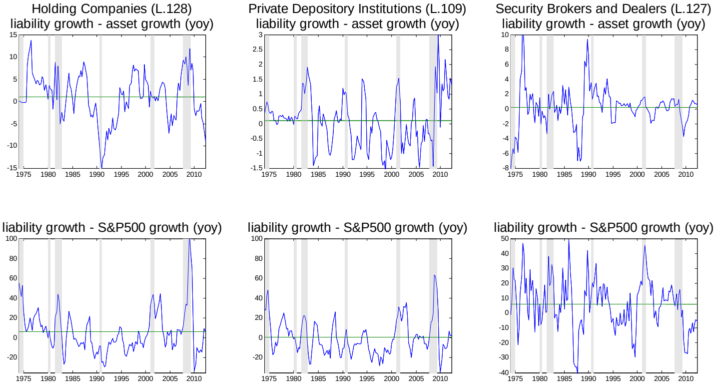


Security Brokers and Dealers (L.127)
liability growth - asset growth (yoy)



liability growth - S&P500 growth (yoy)

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$$
\begin{aligned}
L_t &= \frac{a_t^{nf} + a_t^f}{a_t^{nf} + a_t^f - l_t^f} \\
L\hat{L}_t &= \frac{a^{nf}}{a^{nf} + a^f - l^f}\hat{a}_t^{nf} + \frac{a^f}{a^{nf} + a^f - l^f}\hat{a}_t^f \\
&\quad - \frac{a^{nf} + a^f}{\left(a^{nf} + a^f - l^f\right)^2}\left(a^{nf}\hat{a}_t^{nf} + a^f\hat{a}_t^f - l^f\hat{l}_t^f\right) \\
\hat{L}_t &= \frac{a^{nf}}{a^{nf} + a^f}\hat{a}_t^{nf} + \frac{a^f}{a^{nf} + a^f}\hat{a}_t^f - \frac{1}{a^{nf} + a^f - l^f}\left(a^{nf}\hat{a}_t^{nf} + a^f\hat{a}_t^f - l^f\hat{l}_t^f\right) \\
&= \left[\frac{a^{nf}}{a^{nf} + a^f} - \frac{a^{nf}}{a^{nf} + a^f - l^f}\right]\hat{a}_t^{nf} + \left[\frac{a^f}{a^{nf} + a^f} - \frac{a^f}{a^{nf} + a^f - l^f}\right]\hat{a} \\
&= \left[\frac{a^{nf}}{a^{nf} + a^f} - \frac{a^{nf}}{a^{nf} + a^f - l^f}\right]\hat{a}_t^{nf} - \frac{l^f}{\left(a^{nf} + a^f - l^f\right)\left(a^{nf} + a^f\right)}a^f\hat{a} \\
&= -\frac{l^f a^{nf}}{\left(a^{nf} + a^f\right)\left(a^{nf} + a^f - l^f\right)}\hat{a}_t^{nf} - \frac{l^f a^f}{\left(a^{nf} + a^f\right)\left(a^{nf} + a^f - l^f\right)}\hat{a}_t^f
\end{aligned}
$$