Notes on Financial Frictions Under Asymmetric Information and Costly State Verification

by

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Incorporating Financial Frictions into a Business Cycle Model

- General idea:
  - Standard model assumes borrowers and lenders are the same people..no conflict of interest

  - Financial friction models suppose borrowers and lenders are different people, with conflicting interests

  - Financial frictions: features of the relationship between borrowers and lenders adopted to mitigate conflict of interest.

Discussion of Financial Frictions

- Simple model to illustrate the basic costly state verification (csv) model.
  - Original analysis of Townsend (1978), Bernanke-Gertler.

- Integrating the csv model into a full-blown dsge model.
  - Follows the lead of Bernanke, Gertler and Gilchrist (1999).
  - Empirical analysis of Christiano, Motto and Rostagno (JMCB, 2003; AER, 2014).

Simple Model

- There are entrepreneurs with all different levels of wealth, N.
  - Entrepreneurs have different levels of wealth because they experienced different idiosyncratic shocks in the past.

- For each value of N, there are many entrepreneurs.

- In what follows, we will consider the interaction between entrepreneurs with a specific amount of N with competitive banks.

- Later, will consider the whole population of entrepreneurs, with every possible level of N.

Simple Model, cont'd

- Each entrepreneur has access to a project with rate of return,

$$(1 + R^k)\omega$$

- Here, ω is a unit mean, idiosyncratic shock experienced by the individual entrepreneur after the project has been started,

$$\int_0^\infty \omega dF(\omega) = 1$$

- The shock, ω, is privately observed by the entrepreneur.

- *F* is lognormal cumulative distribution function.

Banks, Households, Entrepreneurs

$$\omega \sim F(\omega), \ \int_0^\infty \omega dF(\omega) = 1$$

entrepreneur

entrepreneur

Households

Bank

entrepreneur

entrepreneur
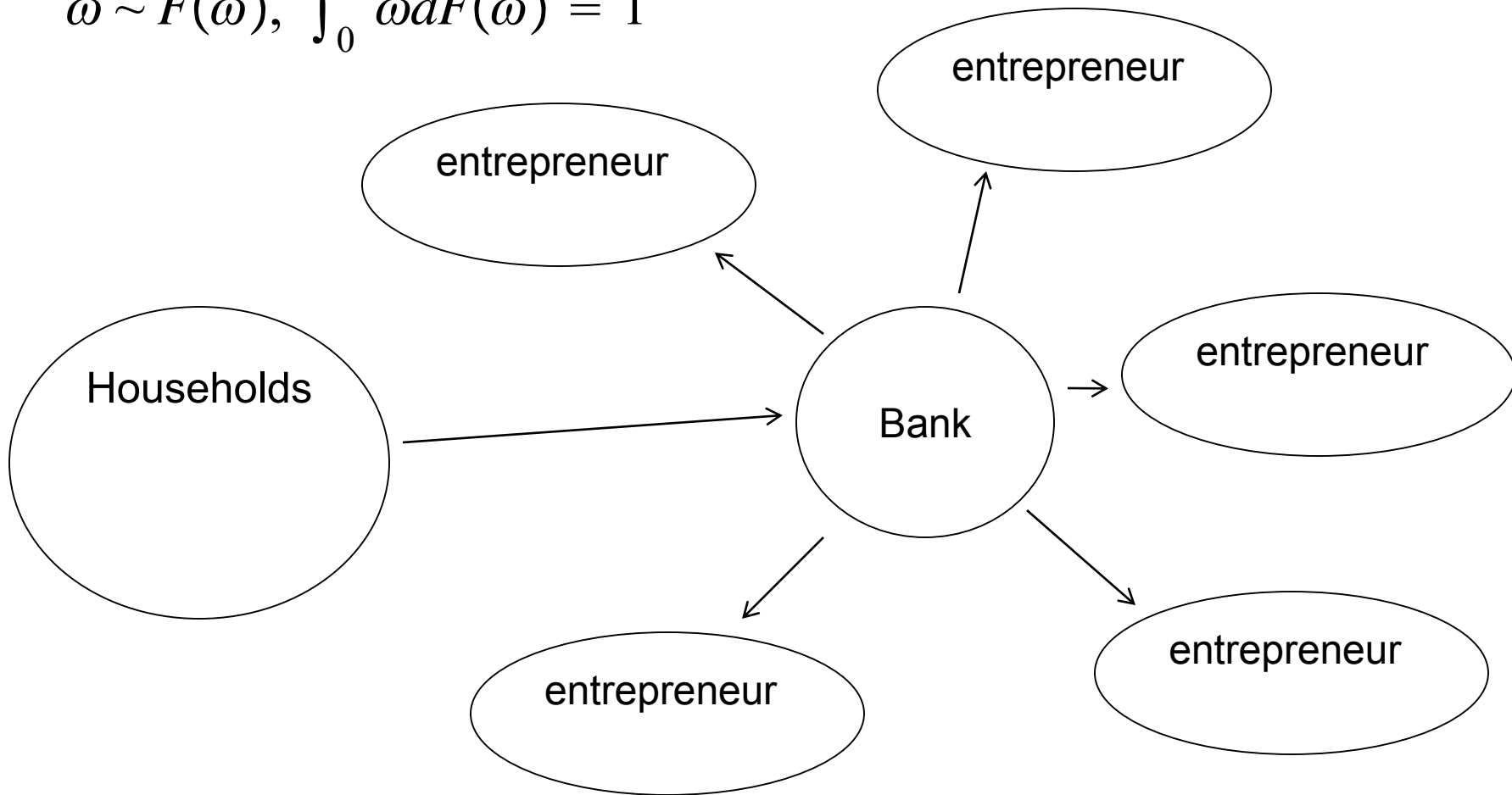
entrepreneur

Standard debt contract

- Entrepreneur receives a contract from a bank, which specifies a rate of interest, *Z*, and a loan amount, *B*.

  - If entrepreneur cannot make the interest payments, the bank pays a monitoring cost and takes everything.

- Total assets acquired by the entrepreneur:

$$\overbrace{A}^{\text{total assets}} = \overbrace{N}^{\text{net worth}} + \overbrace{B}^{\text{loans}}$$

- Entrepreneur who experiences sufficiently bad luck, $\omega \leq \bar{\omega}$, loses everything.

- Cutoff, $\bar{\omega}$

gross rate of return experience by entrepreneur with 'luck', $\bar{\omega}$ total assets

$$\overbrace{(1 + R^k)\bar{\omega}} \qquad\qquad \times \qquad \overbrace{A}$$

interest and principle owed by the entrepreneur

$$= \qquad \overbrace{ZB}$$

$$(1 + R^k)\bar{\omega}A = ZB \;\; \rightarrow$$

leverage $= L$

$$\bar{\omega} = \frac{Z}{(1+R^k)} \frac{\frac{B}{N}}{\frac{A}{N}} = \frac{Z}{(1+R^k)} \frac{\overbrace{\frac{A}{N}} - 1}{\frac{A}{N}} = \frac{Z}{(1+R^k)} \frac{L-1}{L}$$

- Cutoff higher with:
  - higher leverage, L
  - higher $Z/(1 + R^k)$

- Expected return to entrepreneur from operating risky technology, over return from depositing net worth in bank:

Expected payoff for entrepreneur

$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)}$$

For lower values of ω , entrepreneur receives nothing 'limited liability'.

gain from depositing funds in bank ('opportunity cost of funds')

- Rewriting entrepreneur's rate of return:
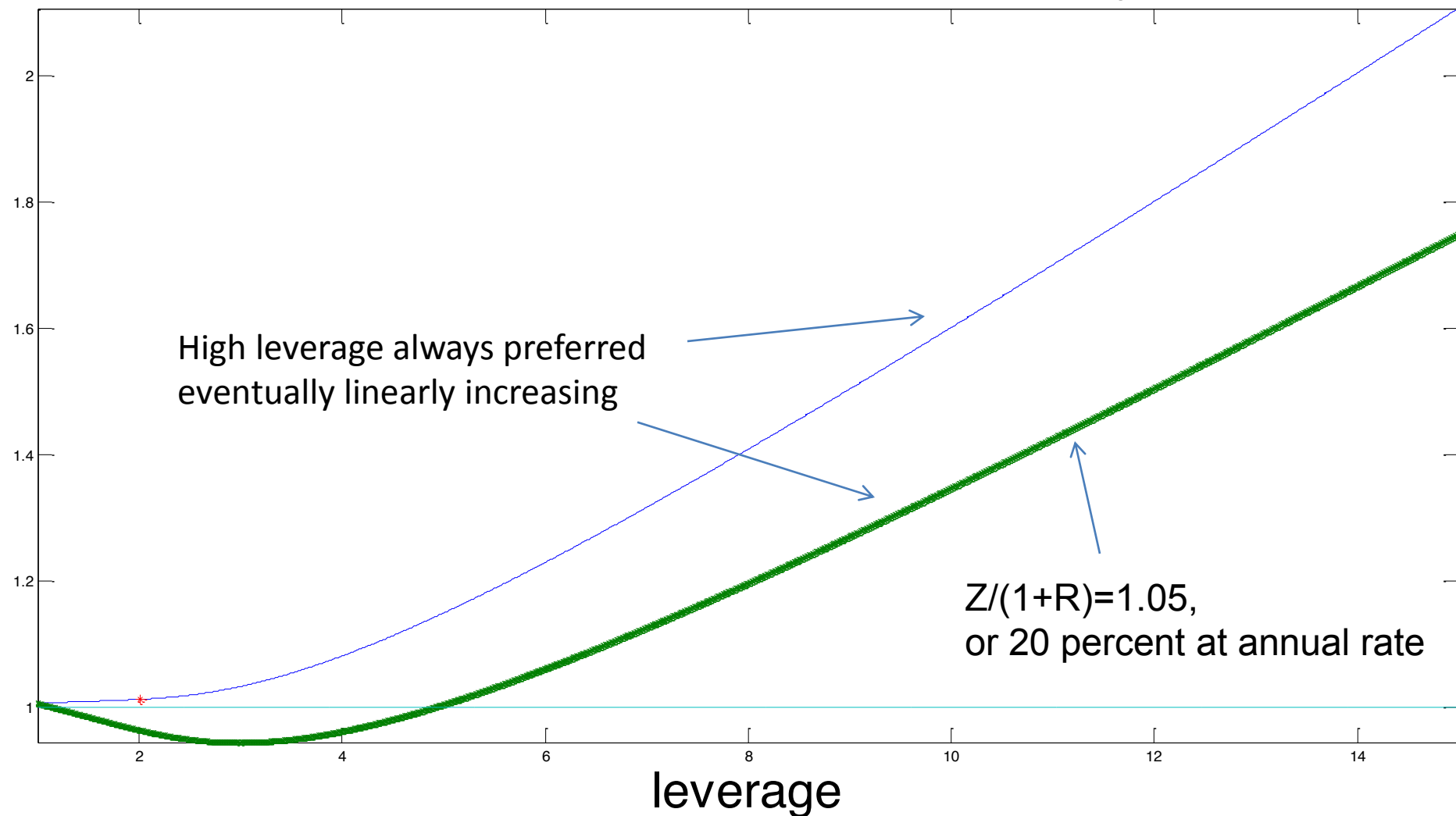
$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - (1+R^k)\bar{\omega}A]dF(\omega)}{N(1+R)}$$

$$= \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$

$$\bar{\omega} = \frac{Z}{(1+R^k)}\frac{L-1}{L}$$

Gets smaller with *L*

Larger with *L*

- Rewriting entrepreneur's rate of return:

$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - (1+R^k)\bar{\omega}A]dF(\omega)}{N(1+R)}$$

$$= \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$

$$\bar{\omega} = \frac{Z}{(1+R^k)}\frac{L-1}{L} \rightarrow_{L\rightarrow\infty} \frac{Z}{(1+R^k)}$$

- Entrepreneur's return unbounded above
  - Risk neutral entrepreneur would always want to borrow an infinite amount (infinite leverage).

Expected entrepreneurial return, over opportunity cost, N(1+R)

High leverage always preferred
eventually linearly increasing

Z/(1+R)=1.05,
or 20 percent at annual rate

leverage

Interest rate spread, Z/(1+R), = 1.0016, or 0.63 percent at annual rate $\qquad \sigma = 0.26$

Return spread, $(1+R^k)/(1+R)$, = 1.0073, or 2.90 percent at annual rate

- If given a fixed interest rate, entrepreneur with risk neutral preferences would borrow an unbounded amount.

- In equilibrium, bank can't lend an infinite amount.

- This is why a loan contract must specify *both* an interest rate, *Z*, and a loan amount, *B*.

Indifference Curves Over *Z* and *B* Problematic

Utility level where entrepreneur is indifferent between depositing in bank and operating risky technology.

Entrepreneurial indifference curves

Utility increasing

Indifference Curves Over *Z* and *B* Problematic
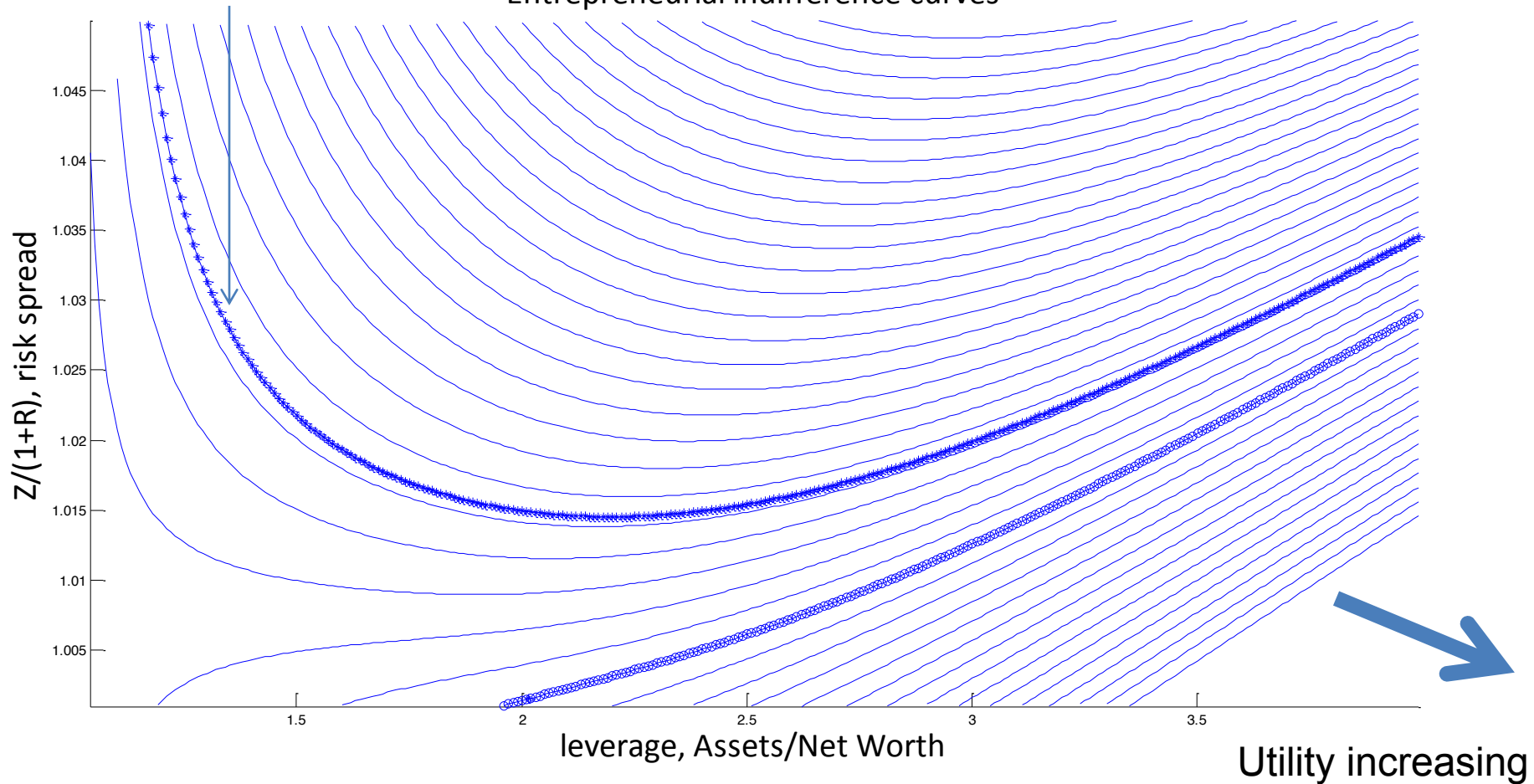
Utility level where entrepreneur is indifferent between depositing in bank and operating risky technology.
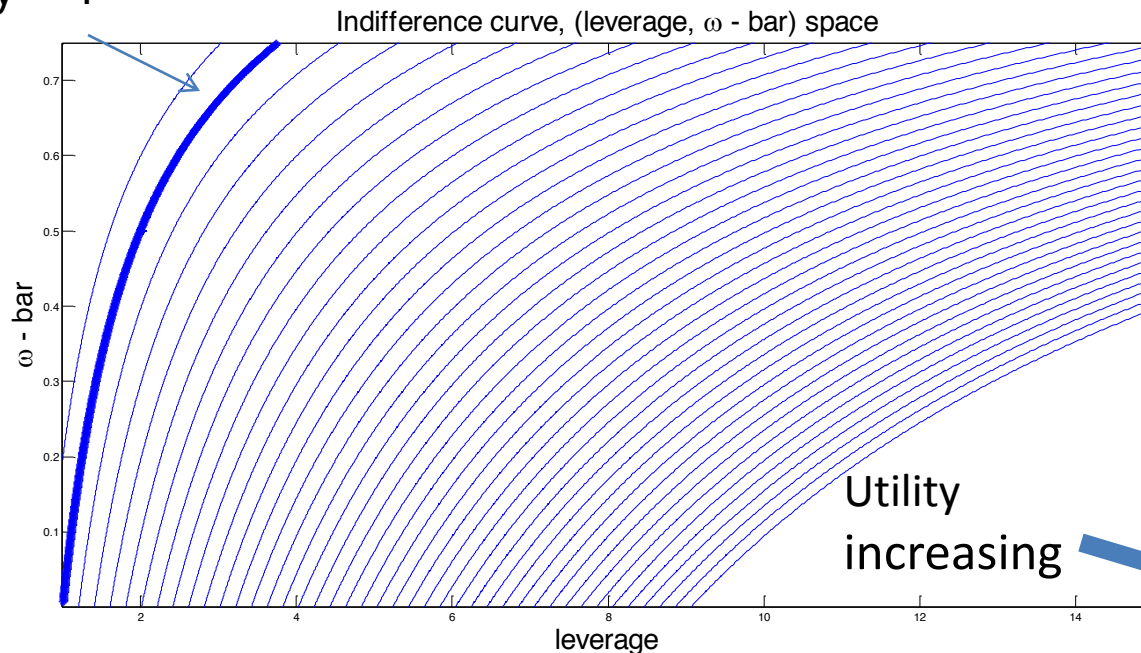
Entrepreneurial indifference curves
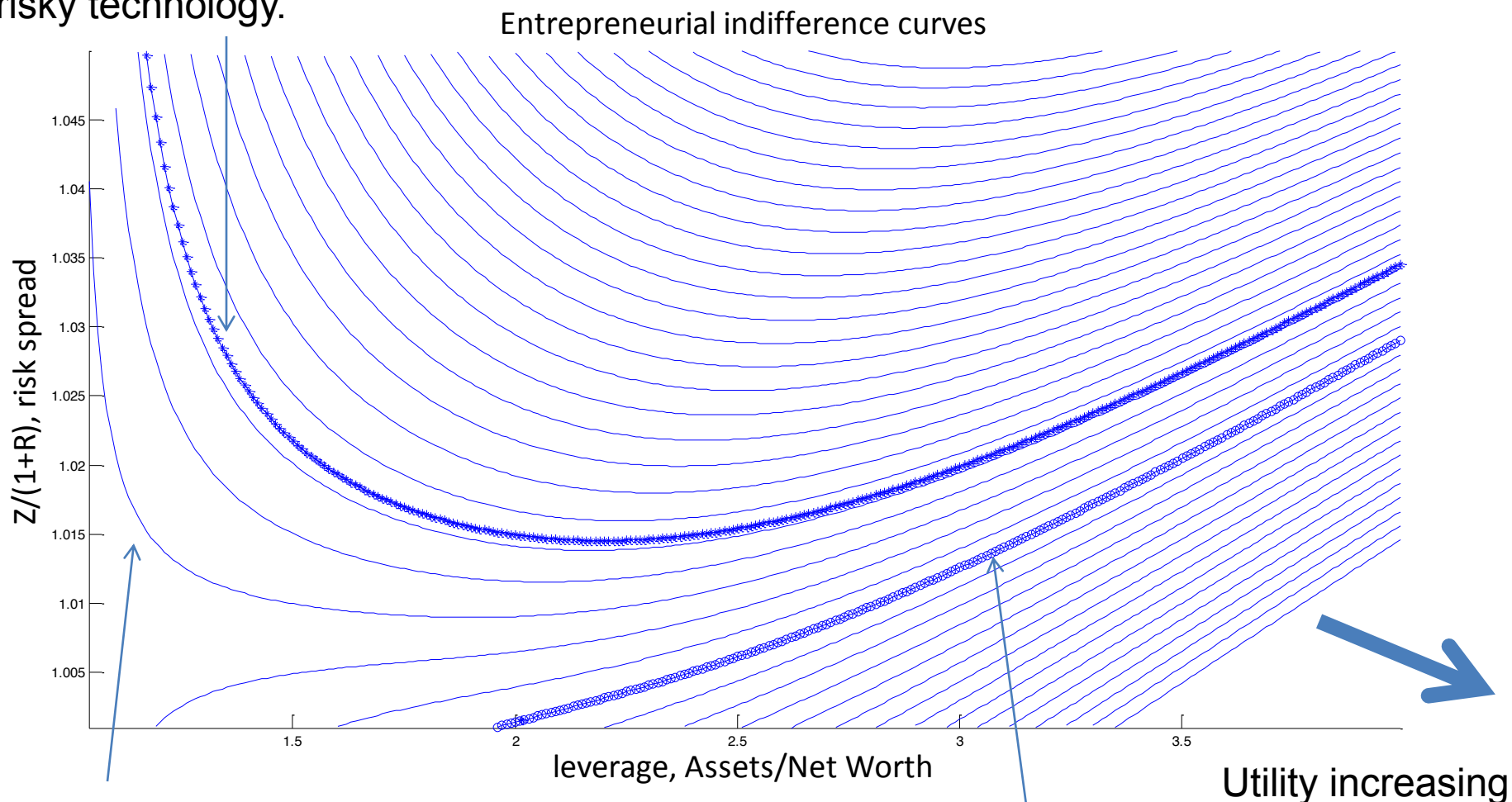
Downward-sloping indifference curves reflect local fall in net worth with rise in leverage when risk premium is high.

Utility increasing

Utility in equilibrium developed In numerical example.

Solution to Technical Problem Posed by Result in Previous Slide

- Think of the loan contract in terms of the loan amount (or, leverage, (N+B)/N) and the cutoff, $\bar{\omega}$

$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$

Utility = 1

$$L = \frac{A}{N} = \frac{N+B}{N}$$

Indifference curve, (leverage, ω - bar) space

ω - bar

leverage

Utility increasing

Simplified Representation of Entrepreneur Utility

- Utility:

$$\int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \frac{1 + R^k}{1 + R} L$$

$$= [1 - \Gamma(\bar{\omega})] \frac{1 + R^k}{1 + R} L$$

- Where

$$\Gamma(\bar{\omega}) \equiv \bar{\omega}(1 - F(\bar{\omega})) + G(\bar{\omega})$$

$$G(\bar{\omega}) \equiv \int_{0}^{\bar{\omega}} \omega dF(\omega)$$

Share of gross entrepreneurial earnings kept by entrepreneur

- Easy to show: $0 \leq \Gamma(\bar{\omega}) \leq 1$

$$\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega}) > 0, \ \Gamma''(\bar{\omega}) < 0$$

$$\lim_{\bar{\omega} \to 0} \Gamma(\bar{\omega}) = 0, \ \lim_{\bar{\omega} \to \infty} \Gamma(\bar{\omega}) = 1$$

$$\lim_{\bar{\omega} \to 0} G(\bar{\omega}) = 0, \ \lim_{\bar{\omega} \to \infty} G(\bar{\omega}) = 1$$

Banks

- Source of funds from households, at fixed rate, *R*

- Bank borrows *B* units of currency, lends proceeds to entrepreneurs.

- Provides entrepreneurs with standard debt contract, *(Z,B)*

Banks, cont'd

- Monitoring cost for bankrupt entrepreneur with $\omega < \bar{\omega}$

Bankruptcy cost parameter

$$\mu(1 + R^k)\omega A$$

- Bank zero profit condition

fraction of entrepreneurs with $\omega > \bar{\omega}$ quantity paid by each entrepreneur with $\omega > \bar{\omega}$

$$[1 - F(\bar{\omega})] \qquad\qquad ZB$$

quantity recovered by bank from each bankrupt entrepreneur

$$+ \quad (1 - \mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A$$

amount owed to households by bank

$$= \quad (1 + R)B$$

Banks, cont'd

- Zero profit condition:

$$[1 - F(\bar{\omega})]ZB + (1 - \mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A = (1 + R)B$$

$$\frac{[1 - F(\bar{\omega})]ZB + (1 - \mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A}{B} = (1 + R)$$

The risk free interest rate here is equated to the 'average return on entrepreneurial projects'.
This is a source of inefficiency in the model. A benevolent planner would prefer that the market price observed by savers correspond to the *marginal* return on projects (Christiano-Ikeda).

Banks, cont'd

- Simplifying zero profit condition:

$$[1 - F(\bar{\omega})]ZB + (1 - \mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A = (1 + R)B$$

$$[1 - F(\bar{\omega})]\bar{\omega}(1 + R^k)A + (1 - \mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A = (1 + R)B$$

share of gross return, $(1+R^k)A$, (net of monitoring costs) given to bank

$$\overbrace{\left([1 - F(\bar{\omega})]\bar{\omega} + (1 - \mu)\int_0^{\bar{\omega}} \omega dF(\omega)\right)}\quad (1 + R^k)A = (1 + R)B$$

$$[1 - F(\bar{\omega})]\bar{\omega} + (1 - \mu)\int_0^{\bar{\omega}} \omega dF(\omega) = \frac{1 + R}{1 + R^k}\frac{B/N}{A/N}$$

$$= \frac{1 + R}{1 + R^k}\frac{L - 1}{L}$$

Expressed naturally in terms of $(\bar{\omega}, L)$

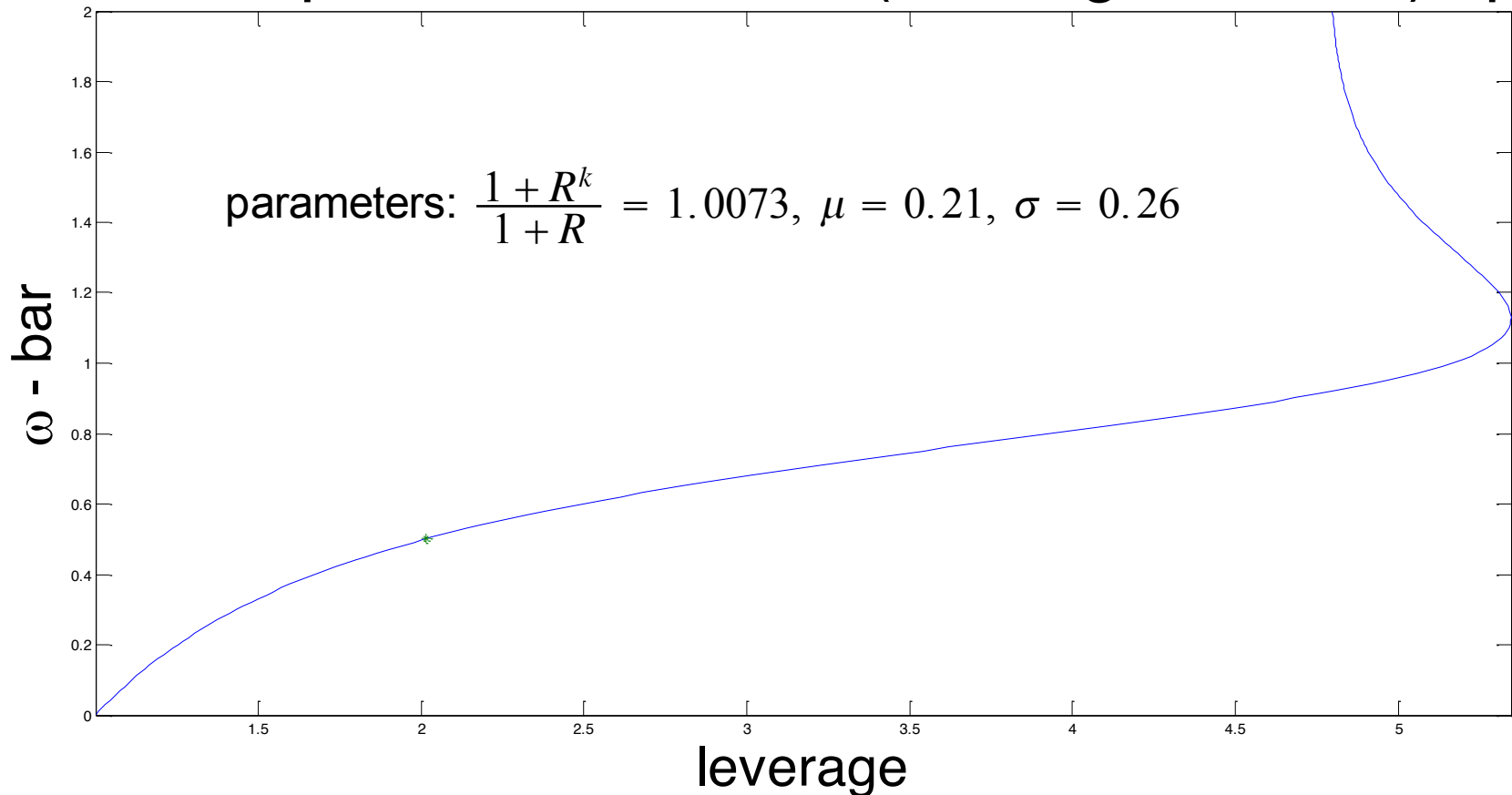# Expressing Zero Profit Condition In Terms of New Notation

share of entrepreneurial profits (net of monitoring costs) given to bank

$$\overbrace{(1 - F(\bar{\omega}))\bar{\omega} + (1 - \mu)\int_0^{\bar{\omega}} \omega dF(\omega)} = \frac{1 + R}{1 + R^k}\frac{L - 1}{L}$$

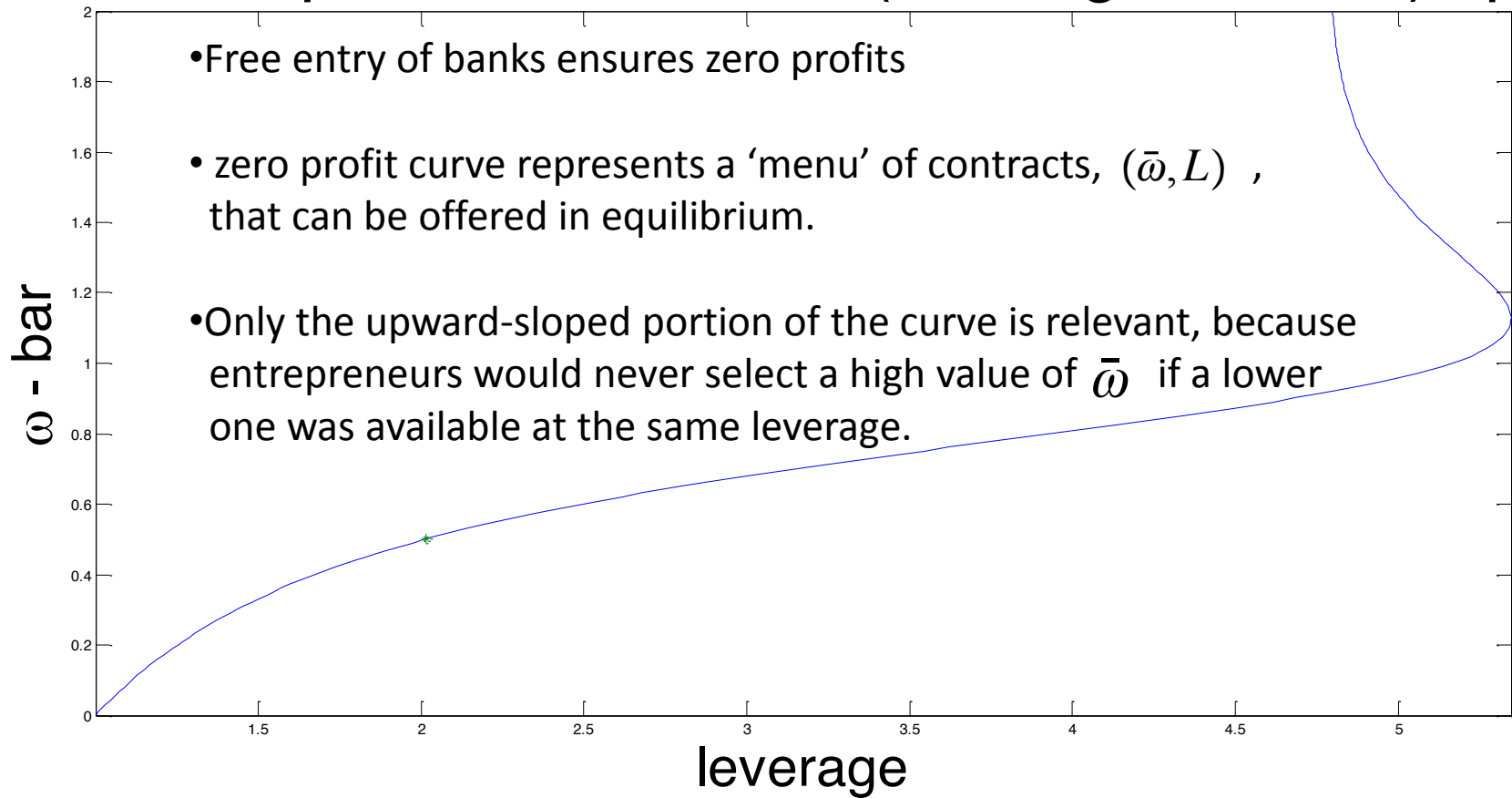$$\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = \frac{1 + R}{1 + R^k}\frac{L - 1}{L}$$

$$L = \frac{1}{1 - \frac{1 + R^k}{1 + R}[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}$$

Bank zero profit condition, in (leverage, ω - bar) space

parameters: $\dfrac{1+R^k}{1+R} = 1.0073,\ \mu = 0.21,\ \sigma = 0.26$

Our value of $\dfrac{1+R^k}{1+R}$, 290 basis points at an annual rate, is a little higher than the 200 basis point value adopted in BGG (1999, p. 1368); the value of μ is higher than the one adopted by BGG, but within the range, 0.20-0.36 defended by Carlstrom and Fuerst (AER, 1997) as empirically relevant; the value of $Var(\log\omega)$ is nearly the same as the 0.28 value assumed by BGG (1999,p.1368).

Bank zero profit condition, in (leverage, ω - bar) space

- Free entry of banks ensures zero profits

- zero profit curve represents a 'menu' of contracts, $(\bar{\omega}, L)$, that can be offered in equilibrium.

- Only the upward-sloped portion of the curve is relevant, because entrepreneurs would never select a high value of $\bar{\omega}$ if a lower one was available at the same leverage.
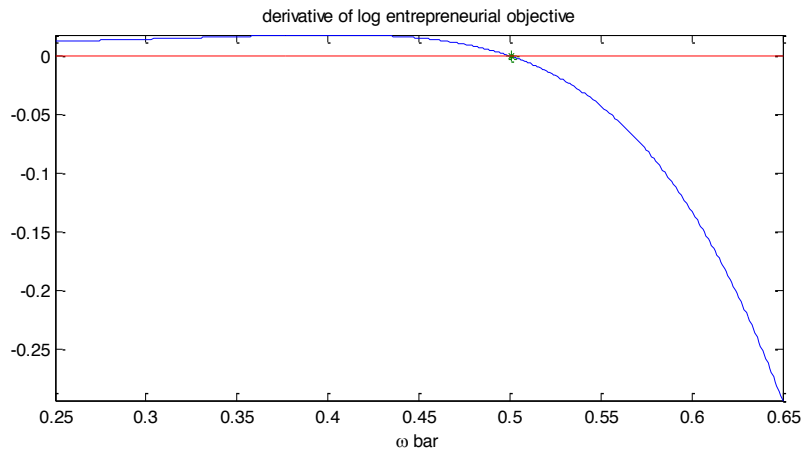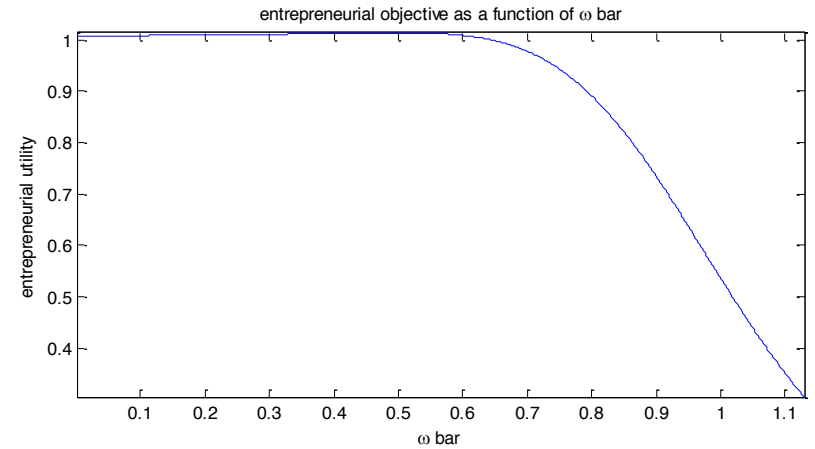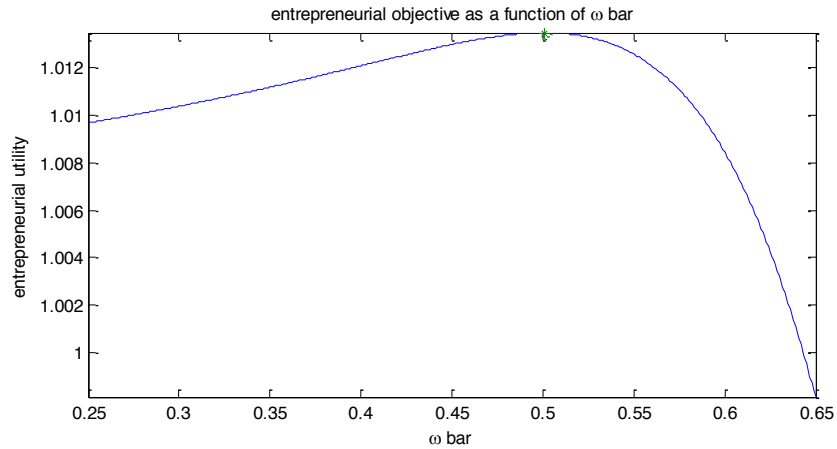
Equilibrium Contract

- Entrepreneur selects the contract is optimal, given the available menu of contracts.

- The solution to the entrepreneur problem is the $\bar{\omega}$ that maximizes, over the relevant domain (i.e., $\bar{\omega} \in [0, 1.13]$ in the example):

$$\log \left\{ \overbrace{\int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \frac{1 + R^k}{1 + R}}^{\text{profits, per unit of leverage, earned by entrepreneur, given } \bar{\omega}} \times \overbrace{\frac{1}{1 - \frac{1+R^k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}}^{\text{leverage offered by bank, conditional on } \bar{\omega}} \right\}$$

$$= \log \overbrace{[1 - \Gamma(\bar{\omega})]}^{\text{higer } \bar{\omega} \text{ drives share of profits to entrepreneur down (bad!)}} + \log \frac{1 + R^k}{1 + R} \overbrace{- \log \left(1 - \frac{1 + R^k}{1 + R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] \right)}^{\text{higher } \bar{\omega} \text{ drives leverage up (good!)}}$$

Entrepreneur Objective
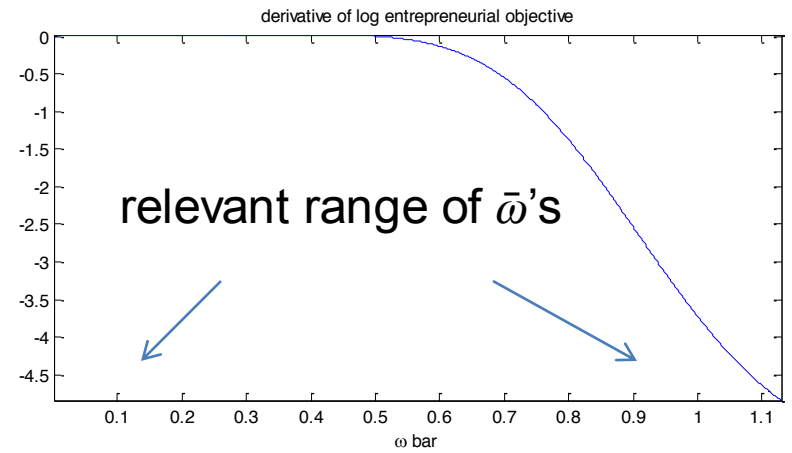
Computing the Equilibrium Contract

- Solve first order optimality condition uniquely for the cutoff, $\bar{\omega}$:

$$\overbrace{\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})}}^{\text{elasticity of entrepreneur's expected return w.r.t. } \bar{\omega}} = \overbrace{\frac{\frac{1+R^k}{1+R}\left[1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})\right]}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}}^{\text{elasticity of leverage w.r.t. } \bar{\omega}}$$

- Given the cutoff, solve for leverage:

$$L = \frac{1}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}$$

- Given leverage and cutoff, solve for risk spread:

$$\text{risk spread} \equiv \frac{Z}{1+R} = \frac{1+R^k}{1+R}\bar{\omega}\frac{L}{L-1}$$

Result

- Leverage, *L*, and entrepreneurial rate of interest, *Z*, **not a function of net worth**, *N*.

- Quantity of loans proportional to net worth:

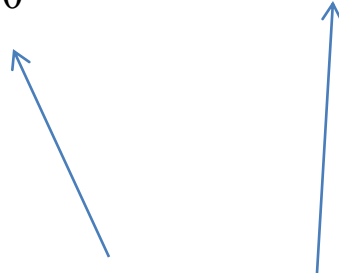$$L = \frac{A}{N} = \frac{N+B}{N} = 1 + \frac{B}{N}$$

$$B = (L-1)N$$

- To compute *L*, *Z/(1+R)*, must make assumptions about *F* and parameters.

$$\frac{1+R^k}{1+R}, \ \mu, \ F$$

Formulas Needed to do the Computations

- Need: $G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega dF(\omega), \ F'(\omega)$

Can get these from the pdf and the cdf of the standard normal distribution.

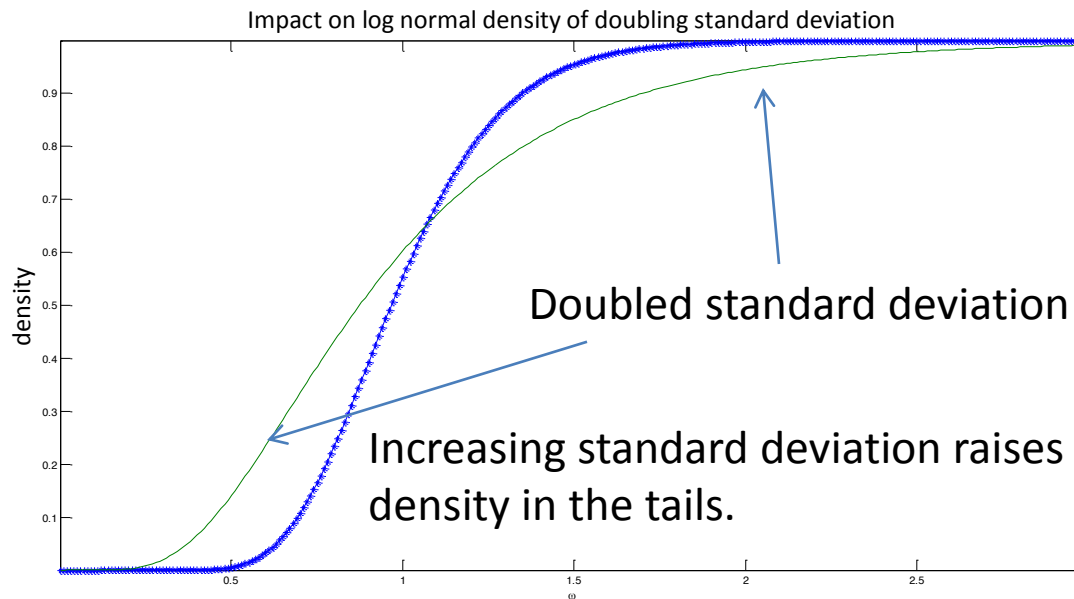These are available in most computational software, like MATLAB.

Also, they have simple analytic representations.

Effect of Increase in Risk, σ

- Keep
$$\int_0^\infty \omega dF(\omega) = 1$$

- But, double standard deviation of Normal underlying *F*.

Impact on log normal density of doubling standard deviation

Doubled standard deviation

Increasing standard deviation raises density in the tails.

Jump in Risk

- σ replaced by $\sigma \times 3$

G/F=0.10

cutoff ω fraction of gross entrepreneurial earnings going to lender   bankruptcy rate: 1.08%   average ω among bankrupt entrepreneurs

$$\bar{\omega} = 0.12, \qquad \Gamma(\bar{\omega}) = 0.12 \qquad , \quad F(\bar{\omega}) = 0.0108 \, , \qquad G(\bar{\omega}) = 0.0011 \qquad ,$$

leverage   interest rate spread   1.66 (APR)   avg earnings of entrepreneur, per unit of net worth

$$L = 1.1418, \qquad \frac{Z}{R} = 1.0041 \, , \qquad [1 - \Gamma(\bar{\omega})]\frac{1+R^k}{1+R}L = 1.0080 \quad > 1$$

- Comparison with benchmark:

G/F = 0.46

cutoff ω fraction of gross entrepreneurial earnings going to lender   bankruptcy rate: 0.56%   average ω among bankrupt entrepreneurs

$$\bar{\omega} = 0.50, \qquad \Gamma(\bar{\omega}) = 0.5008 \qquad , \quad F(\bar{\omega}) = 0.0056 \, , \qquad G(\bar{\omega}) = 0.0026 \qquad ,$$

leverage   interest rate spread   0.62 (APR)   avg earnings of entrepreneur, divided by opportunity cost

$$L = 2.02, \qquad \frac{Z}{R} = 1.0015 \, , \qquad [1 - \Gamma(\bar{\omega})]\frac{1+R^k}{1+R}L = 1.0135 \qquad > 1$$

Impact on standard debt contract of a 5% jump in σ

Risk spread $= 400\left(\dfrac{Z}{1+R}-1\right)$, Leverage $= (B+N)/N$

Zero profit curve

Risk spread=0.616
Leverage = 2.02

Risk spread= 0.635
Leverage = 1.95

Entrepreneur
Indifference curve

risk spread, 400(Z/R-1)

leverage, L = (B+N)/N
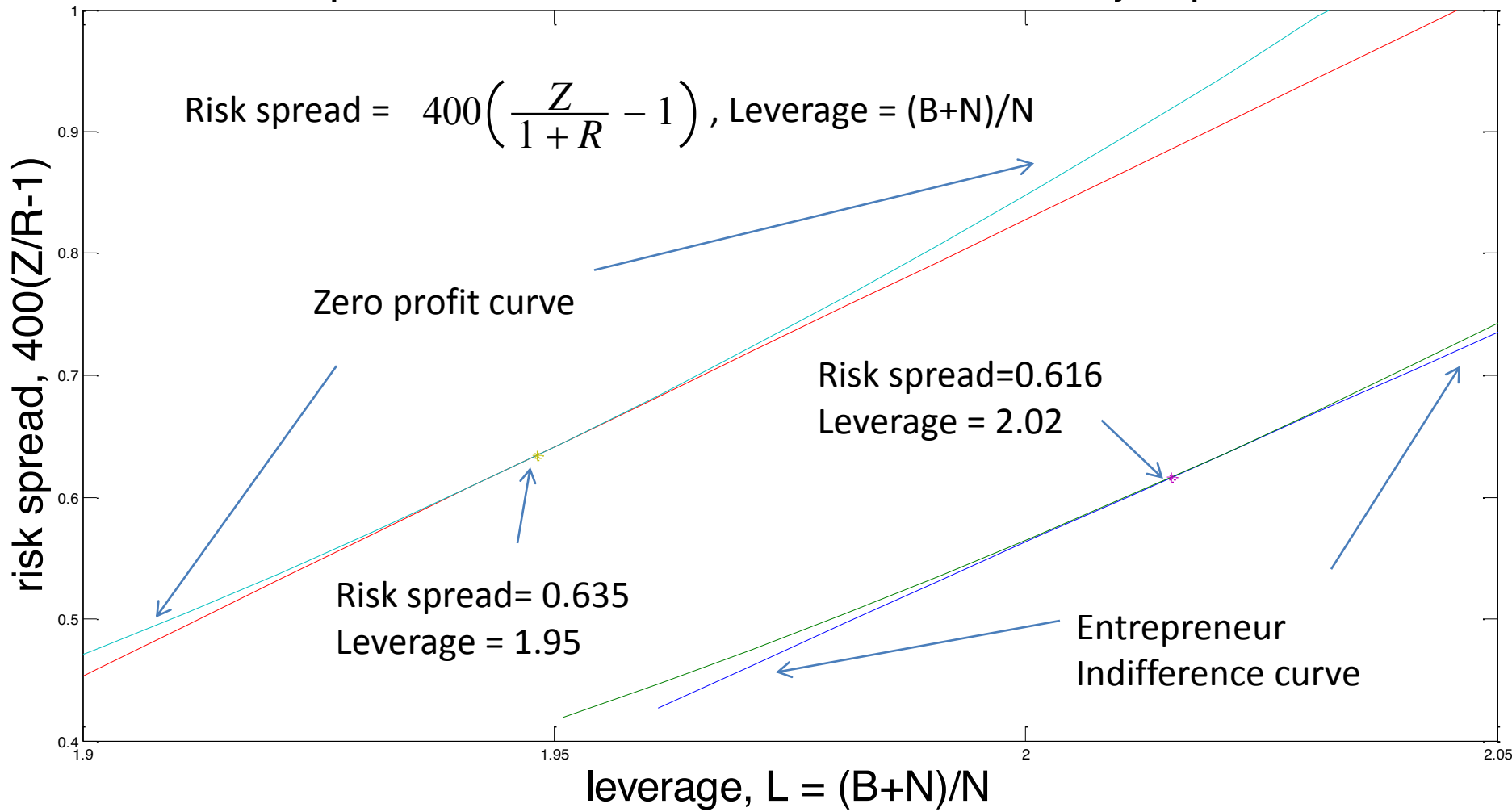
Leverage and interest rate spread, for alternative parameter settings
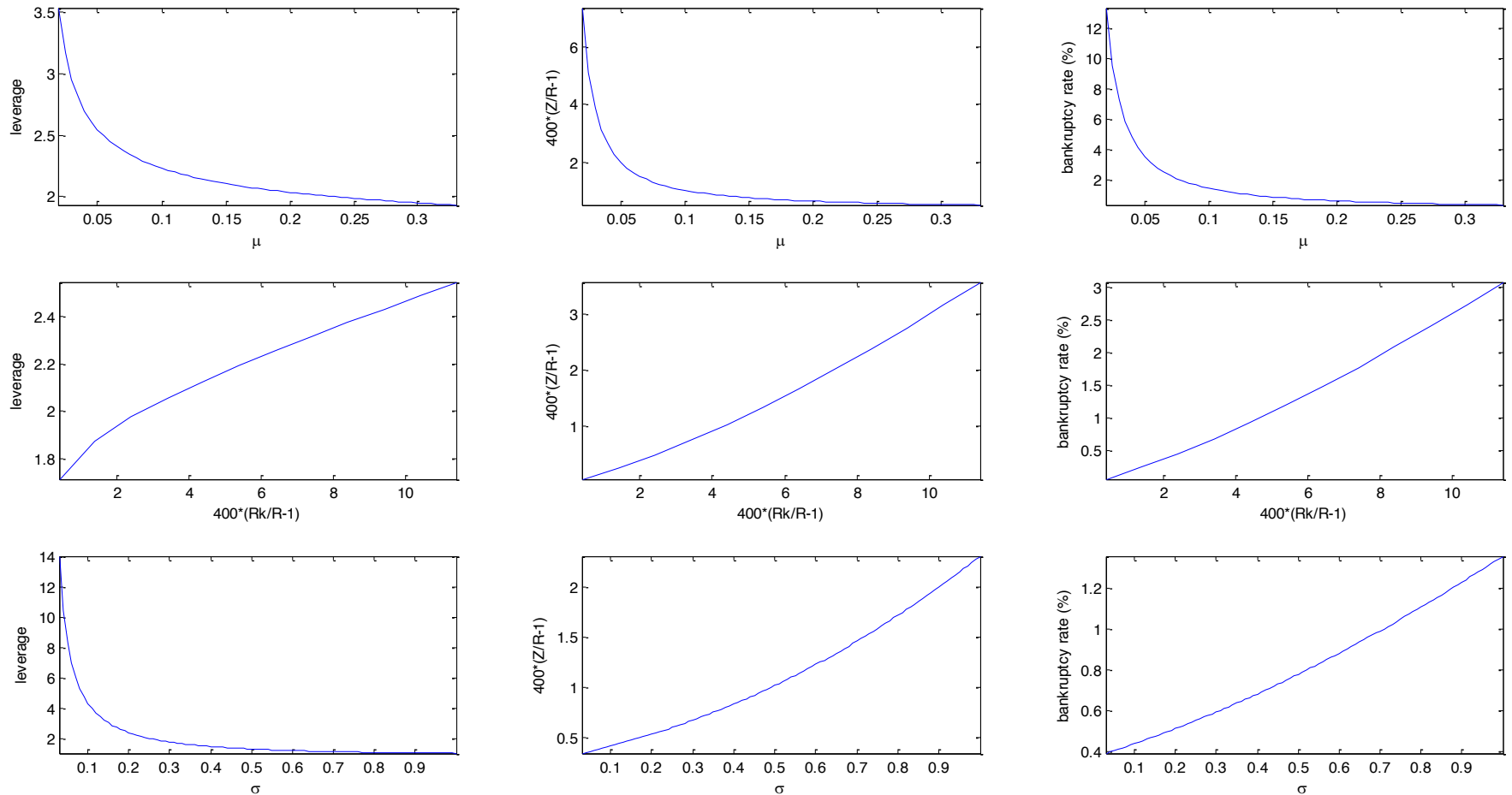
Higher monitoring costs: shifts the menu of contracts up; entrepreneurs choose contracts with lower leverage and lower interest rate; bankruptcy rate falls.

Higher return projects: shifts menu of contracts down; entrepreneurs choose contracts with higher leverage, higher interest rate and higher bankruptcy rates.

Higher risk: shifts menu of contracts up; entrepreneurs choose contracts with lower leverage, higher interest rate, higher bankruptcy rates.

Possible Issues With the Model

- Strictly speaking, applies only to 'mom and pop grocery stores': entities run by entrepreneurs who are bank dependent for outside finance.
  - Not clear how to apply this to actual firms with access to equity markets.

- Assume no long-run connections with banks.

- Entrepreneurial returns independent of scale.

- Overly simple representation of entrepreneurial utility function (assumes entrepreneurs behave as though they are risk neutral)

- Ignores alternative sources of risk spread (risk aversion, liquidity)

- Seems not to allow for bankruptcies in banks.
  - For this, need to study problems on liability side of bank balance sheets (for a review of 5 models, see Christiano-Ikeda, 2013).

Conclusion

- We've reviewed one interesting model of financial frictions.

- Next, see how it works in a NK model with price-setting frictions.