

# Two-Period Version of Gertler-Karadi, Gertler-Kiyotaki Financial Friction Model

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Summary of Christiano-Ikeda, 2012, 'Government Policy, Credit Markets and Economic Activity,' in Federal Reserve Bank of Atlanta conference volume,

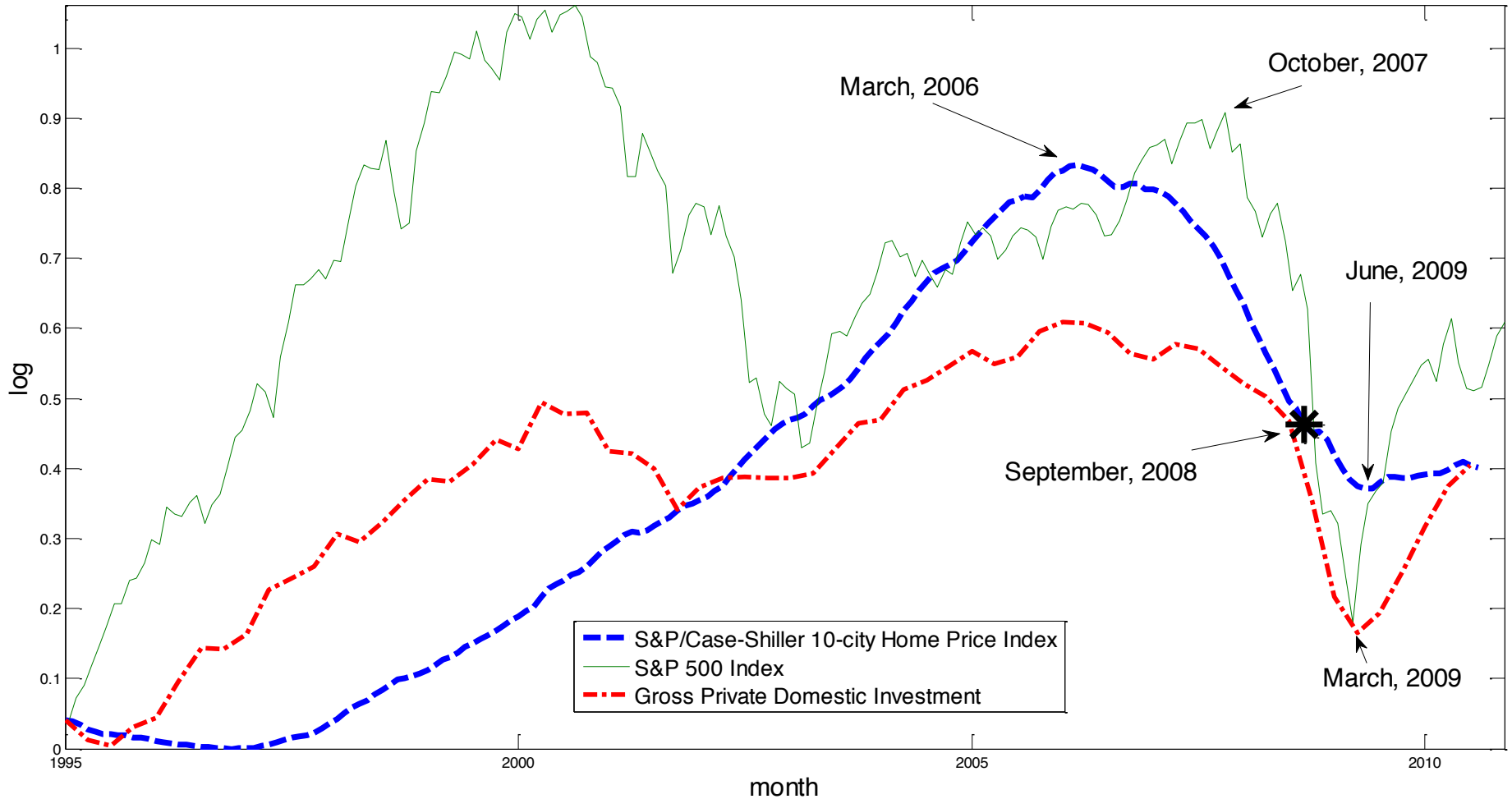
*A Return to Jekyll Island: the Origins, History, and Future of the Federal Reserve*, Cambridge University Press.

# Motivation

- Beginning in 2007 and then accelerating in 2008:
  - Asset values (particularly for banks) collapsed.
  - Intermediation slowed and investment/output fell.
  - Interest rates spreads over what the US Treasury and highly safe private firms had to pay, jumped.
  - US central bank initiated unconventional measures (loans to financial and non-financial firms, very low interest rates for banks, etc.)
- In 2009 – the worst parts of 2007-2008 began to turn around.

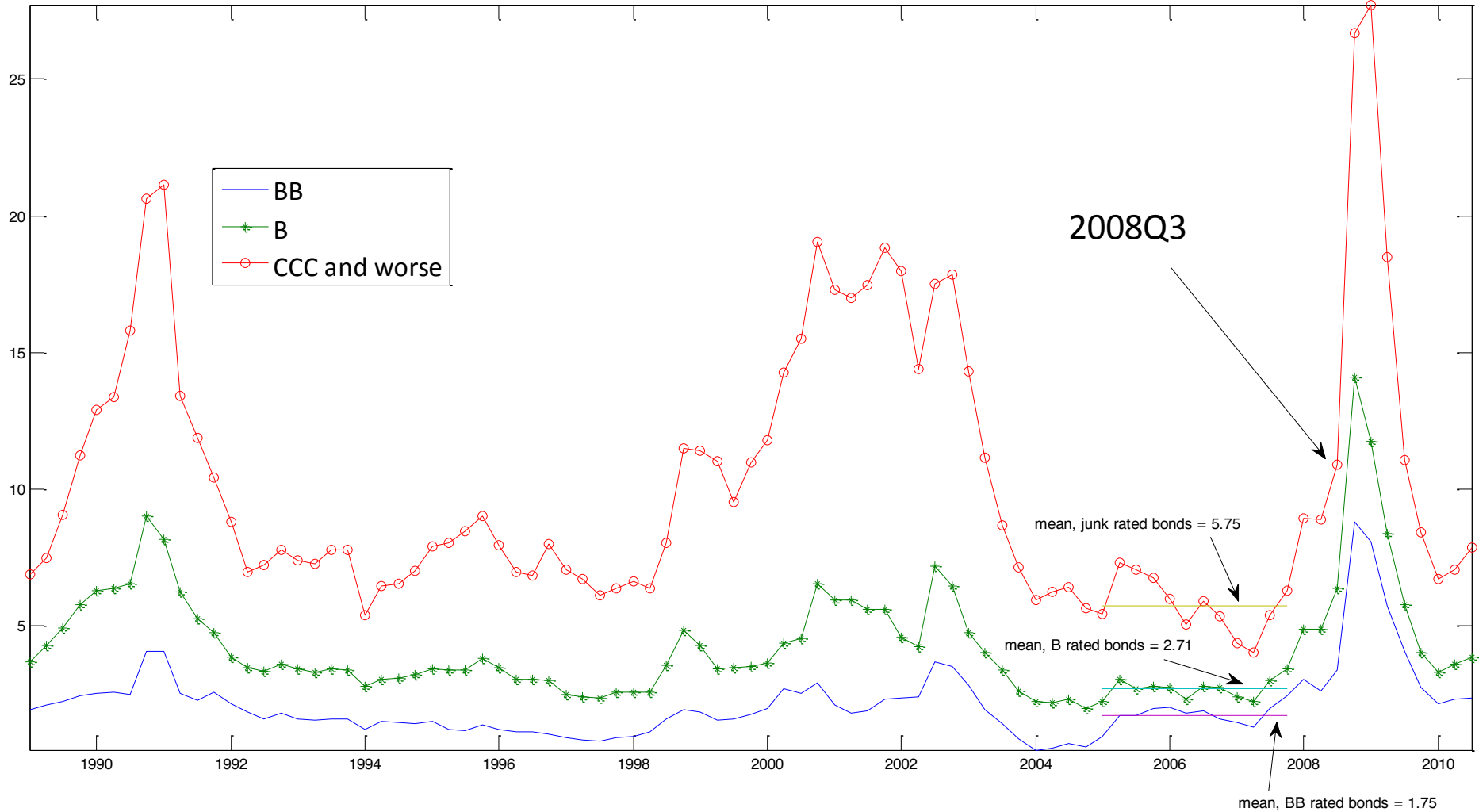
# Collapse in Asset Values and Investment

Log, real Stock Market Index, real Housing Prices and real Investment



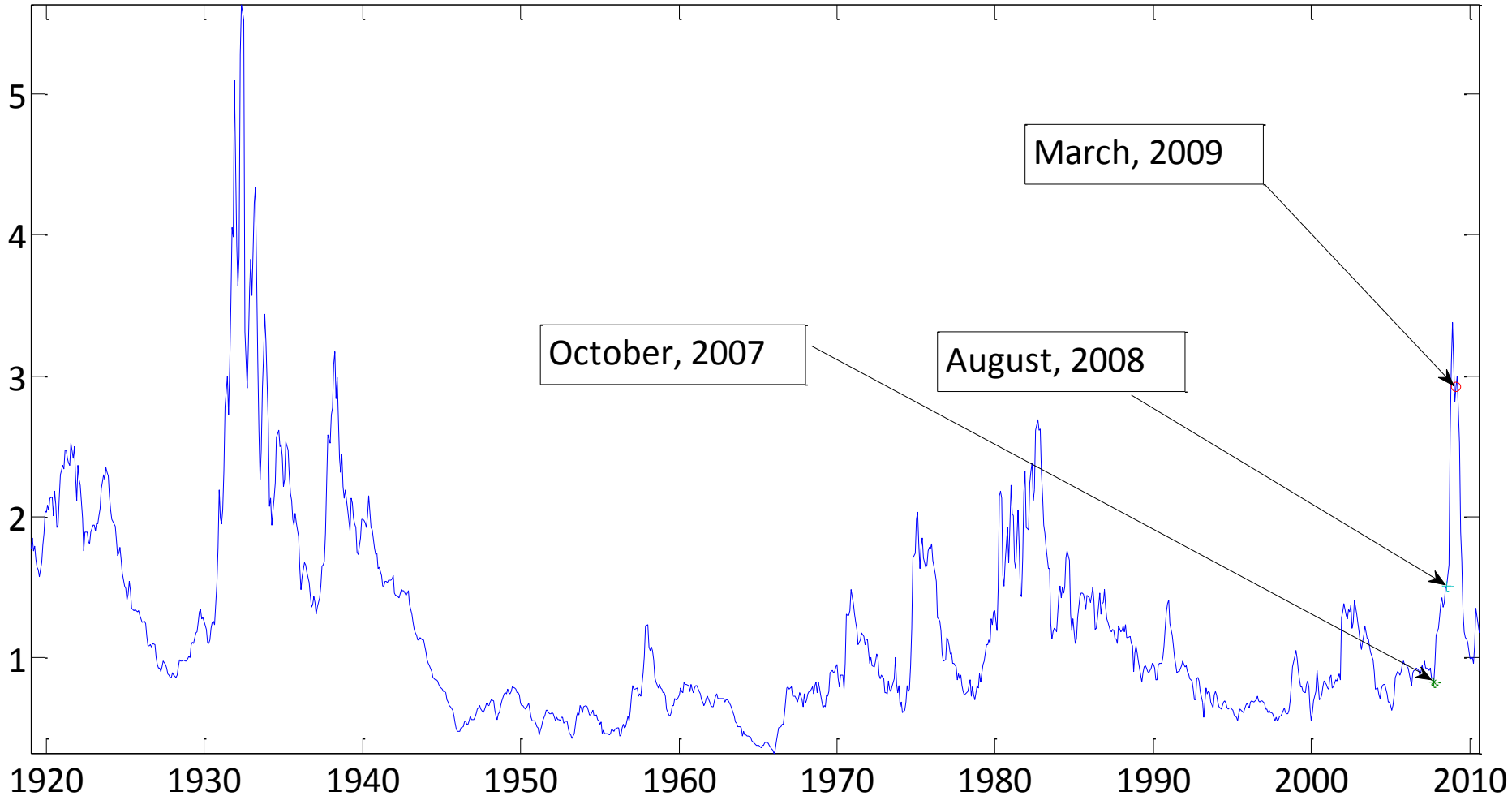
# Spreads for 'Risky' Firms Shot Up in Late 2008

Interest Rate Spread on Corporate Bonds of Various Ratings Over Rate on AAA Corporate Bonds

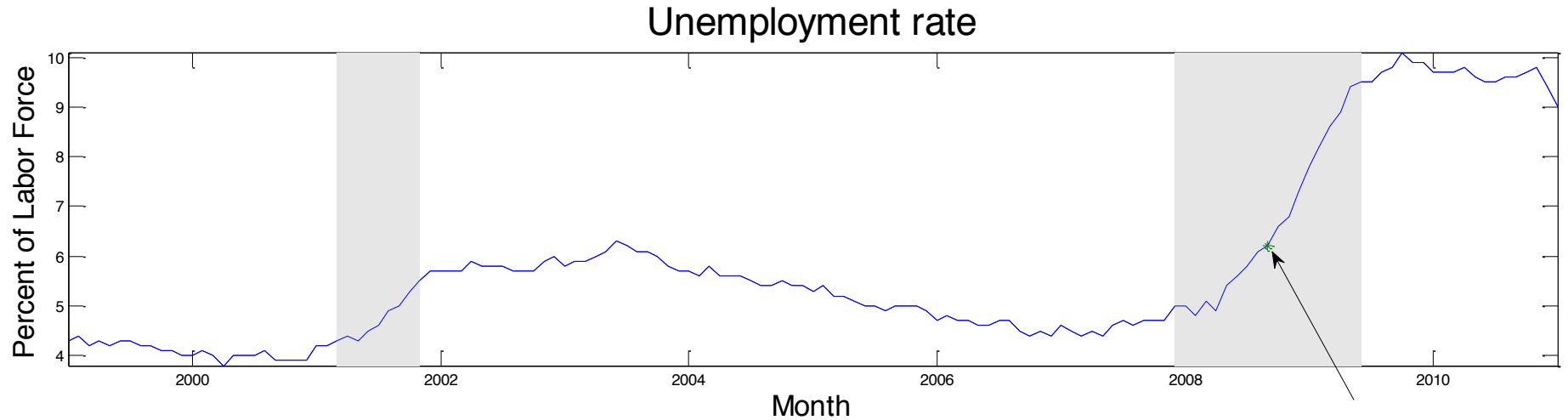


# Must Go Back to Great Depression to See Spreads as Large as the Recent Ones

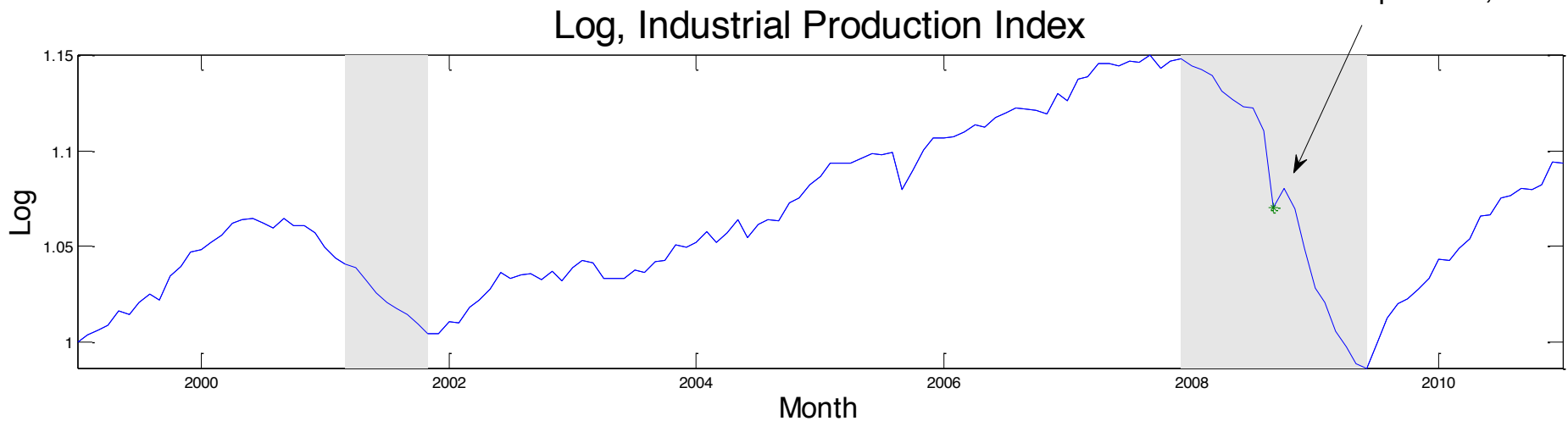
Spread, BAA versus AAA bonds



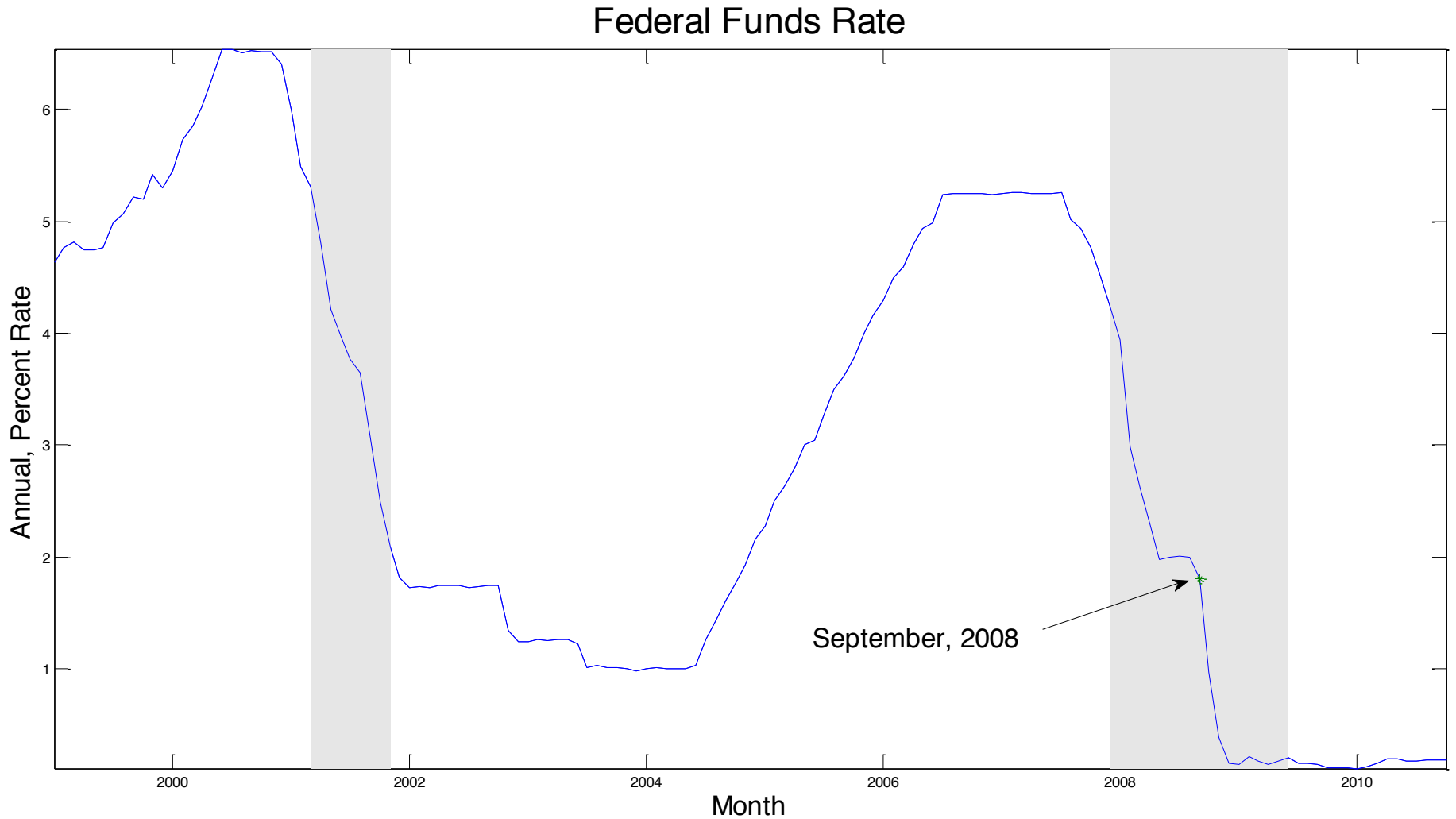
# Economic Activity Shows (anemic!) Signs of Recovery June, 2009



September, 2008



# Banks' Cost of Funds Low



# Characterization of Crisis to be Explored Here

- Bank Asset Values Fell.
- Banking System Became ‘Dysfunctional’
  - Interest rate spreads rose.
  - Intermediation and economy slowed.
- Monetary authority:
  - Transferred funds on various terms to private companies and to banks.
  - Sharply reduced cost of funds to banks.
- Economy in (tentative) recovery.
- Seek to construct models that links these observations together.



# Objective

- Keep analysis simple and on point by:
  - Two periods
  - Minimize complications from agent heterogeneity.
  - Leave out endogeneity of employment.
  - Leave out nominal variables: just look ‘behind the veil of monetary economics’
- Models:
  - Gertler-Kiyotaki/Gertler-Karadi
  - In two-period setting easy to study an interesting nonlinearity that is possible:
    - Participation constraint may be binding in a crisis and not binding in normal times.

# Two-period Version of GK Model

- Many identical households, each with a unit measure of members:
  - Some members are ‘bankers’
  - Some members are ‘workers’
  - Perfect insurance inside households...everyone consumes same amount.
- Period 1
  - Workers endowed with  $y$  goods, household makes deposits,  $d$ , in a bank
  - Bankers endowed with  $N$  goods, take deposits and purchase securities,  $d$ , from a firm.
  - Firm issues securities,  $s$ , to produce  $sR^k$  in period 2.
- Period 2
  - Household consumes earnings from deposits plus profits,  $\pi$ , from banker.
  - Goods consumed are produced by the firm.

| Solution to Household Problem          |  |
|--|--|
| $\frac{u'(c)}{\beta u'(C)} = R^d$      | $c + \frac{C}{R^d} = y + \frac{\pi}{R^d}$  |
| $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ | $c = \frac{y + \frac{\pi}{R^d}}{1 + \frac{(\beta R^d)^{\frac{1}{\gamma}}}{R^d}}$ |

Household budget constraint when gov't buys private assets using tax receipts,  $T$ , and gov't gets the same rate of return,  $R^d$ , as households:

$$c + \frac{C}{R^d} = y - T + \frac{\pi + TR^d}{R^d} = y + \frac{\pi}{R^d}$$

No change!  
(Ricardian-Wallace  
Irrelevance)

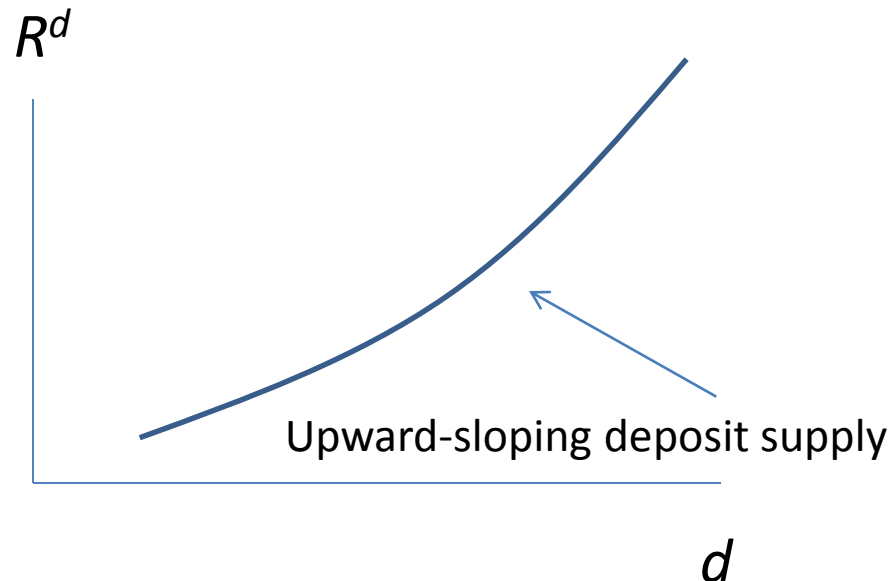
| Problem of the Household |                                    |                      |
|--------------------------|------------------------------------|----------------------|
|                          | period 1                           | period 2             |
|                          |                                    |                      |
| budget constraint        | $c + d \leq y$                     | $C \leq R^d d + \pi$ |
|                          |                                    |                      |
| problem                  | $\max_{c,C,d} [u(c) + \beta u(C)]$ |                      |

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|--|--|
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# Household Supply of Deposits

- For given  $\pi$ ,  $d$  rises or falls with  $R^d$ , depending on parameter values.
- But, in equilibrium  $\pi = R^k(N+d) - R^d d$ .
- Substituting into the expression for  $c$  and solving for  $d$ :

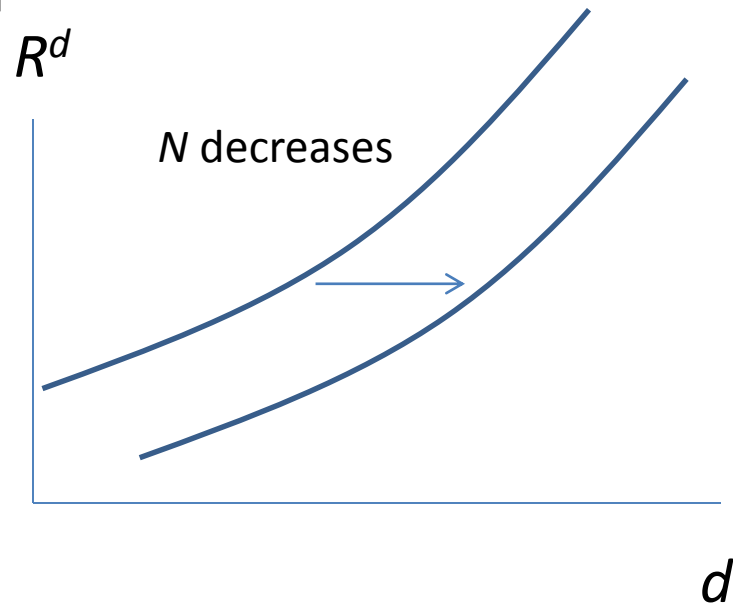
$$d = \frac{(\beta R^d)^{\frac{1}{\gamma}} - \frac{N}{y} R^k}{(\beta R^d)^{\frac{1}{\gamma}} + R^k} y$$



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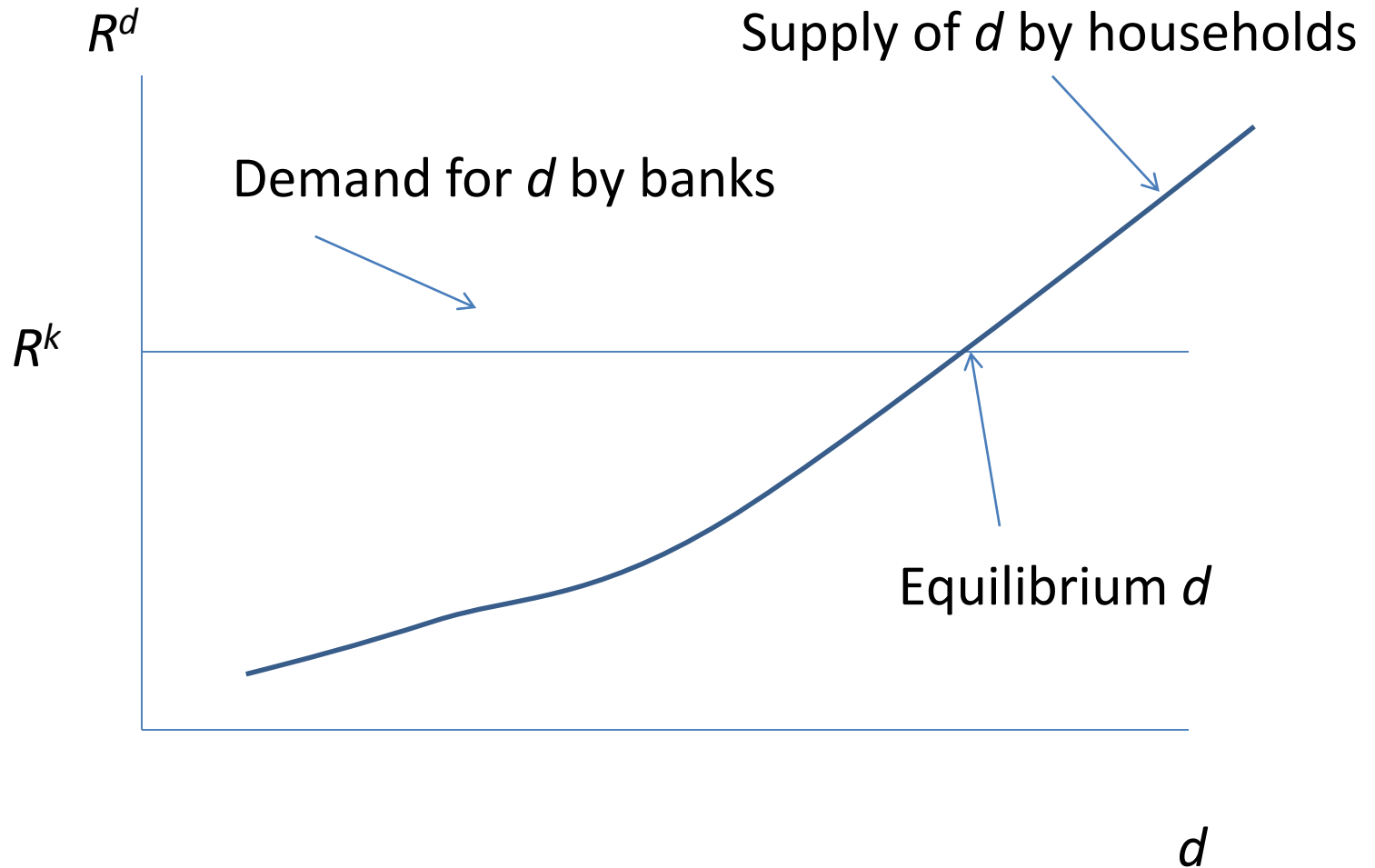
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# Efficient Benchmark

| Problem of the Bank              |                           |
|----------------------------------|---------------------------|
| period 1                         | period 2                  |
|                                  |                           |
| take deposits, $d$               | pay $dR^d$ to households  |
|                                  |                           |
| buy securities, $s = N + d$      | receive $sR^k$ from firms |
|                                  |                           |
| problem: $\max_d [sR^k - R^d d]$ |                           |

# Bank demand for $d$





# Equilibrium in Absence of Frictions

Interior Equilibrium:  $R^d, \pi, d, c, C$

(i)  $c, d, C > 0$

(ii) household problem is solved

(iii) bank problem is solved

(iv) goods and financial markets clear

- Properties:

- Household faces true social rate of return on saving:

$$R^k = R^d$$

- Equilibrium is ‘first best’, i.e., solves

$$\max_{c, C, k} u(c) + \beta u(C)$$

$$c + k \leq y + N, \quad C \leq kR^k$$

# Friction

- bank combines deposits,  $d$ , with net worth,  $N$ , to purchase  $N+d$  securities from firms.
- bank has two options:
  - ('no-default') wait until next period when  $(N+d)R^k$  arrives and pay off depositors,  $R^d d$ , for profit:

$$(N+d)R^k - R^d d$$

- ('default') take  $\theta(N+d)$  securities, refuse to pay depositors and wait until next period when securities pay off:
$$\theta(N+d)R^k$$
- Bank must announce what value of  $d$  it will choose at the beginning of a period.

# Incentive Constraint

- Recall, banks maximize profits
- Choose 'no default' iff

$$\overbrace{(N + d)R^k - R^d d}^{\text{no default}} \geq \overbrace{\theta(N + d)R^k}^{\text{default}}$$

- Next: derive banking system's demand for deposits in presence of financial frictions.

# Result for a no-default equilibrium:

- Consider an individual bank that contemplates defaulting.
- It sets a  $d$  that implies default,

$$R^k(N + d) - R^d d < \theta R^k(d + N) \quad , \text{ or}$$

what the household gets in the other banks

what the household gets in the defaulting bank

$$\overbrace{R^d} > \frac{\overbrace{(1 - \theta)R^k(d + N)}}{d}$$

- A deviating bank will in fact receive no deposits.
- **An optimizing bank would never default**

# Problem of the bank in no-default, interior equilibrium

- Maximize, by choice of  $d$ ,

$$R^k(N + d) - R^d d$$

If interest rate is REALLY low, then bank has no incentive to default because it makes lots of profits not defaulting

subject to:

$$R^k(N + d) - R^d d - R^k \theta(N + d) \geq 0,$$

or,

$$(1 - \theta)R^k N - [R^d - (1 - \theta)R^k]d \geq 0.$$

- Note that  $0 < d < \infty$  requires

if not, then  $d = \infty$

if not, then  $d = 0$

$(1 - \theta)R^k$ 

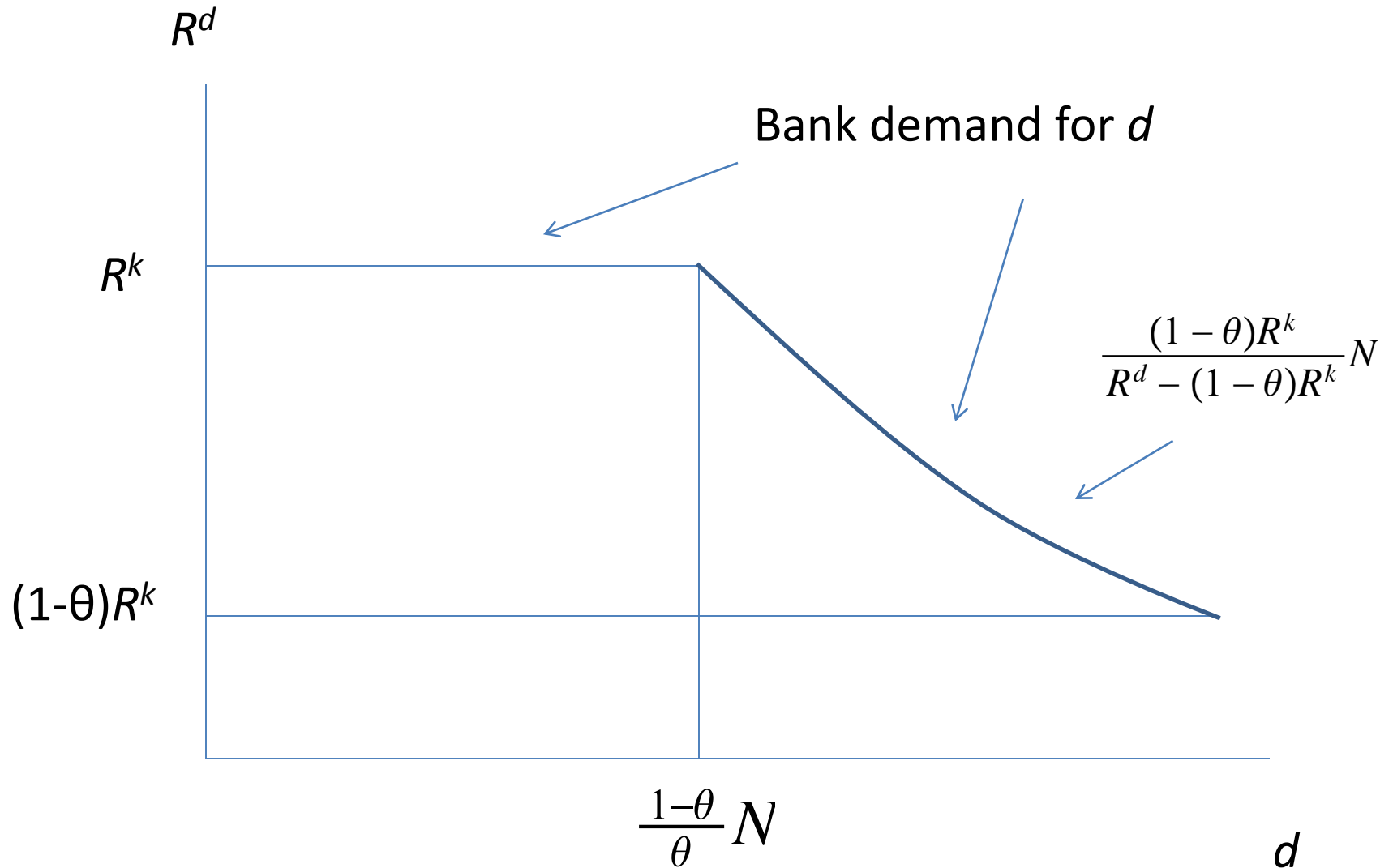
 $\underbrace{\hspace{1em}}_{<}$ 
 $R^d$ 

 $\underbrace{\hspace{1em}}_{\leq}$ 
 $R^k$

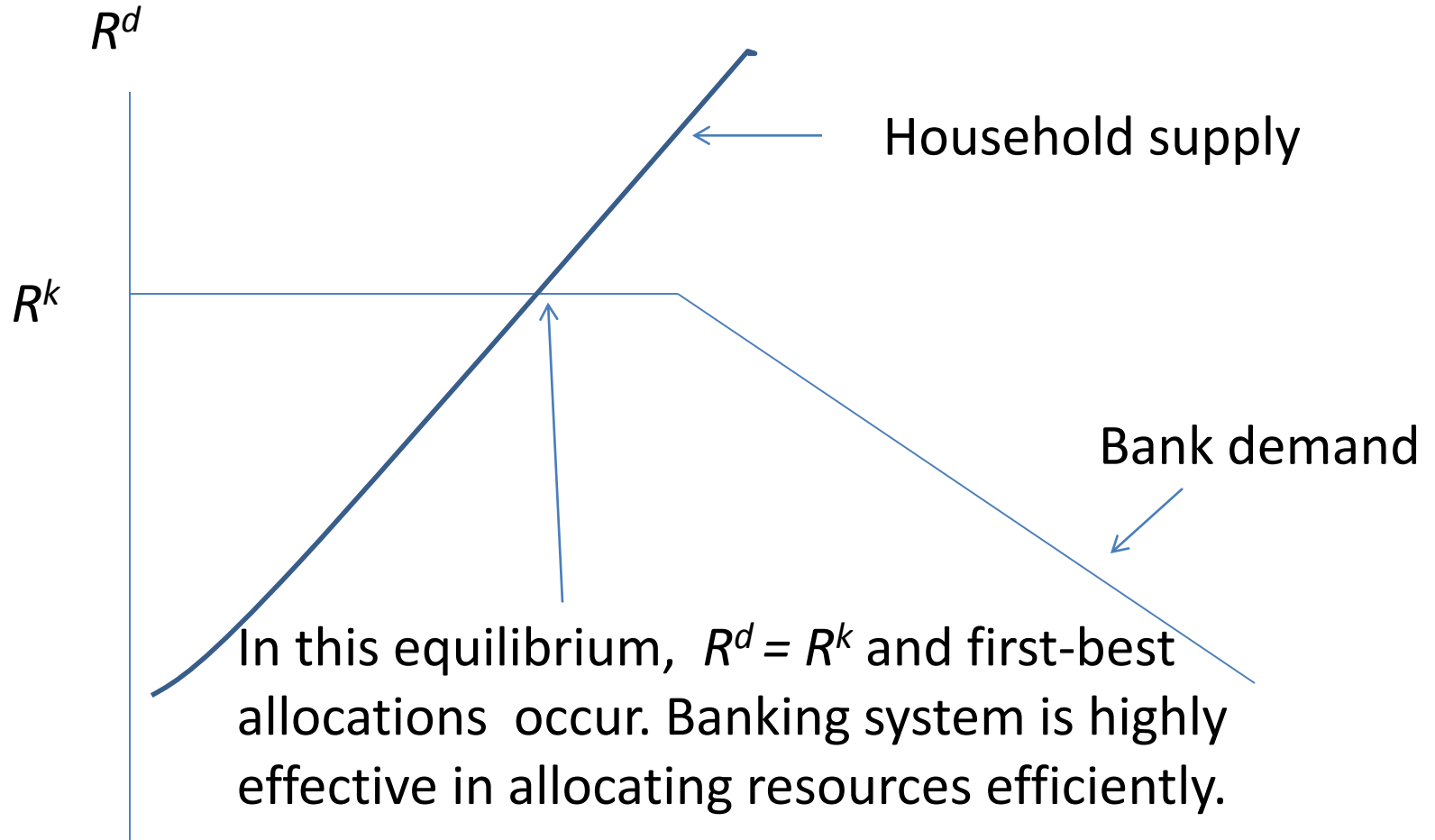
# Problem of the bank in no-default, interior equilibrium, cnt'd

- For  $R^d = R^k$ 
  - a bank makes no profits on  $d$  so – absent default considerations - it is indifferent over all values of  $0 \leq d$
  - Taking into account default, a bank is indifferent over  $0 \leq d \leq N(1-\theta)/\theta$
- For  $(1-\theta)R^k < R^d < R^k$ 
  - Bank wants  $d$  as large as possible, subject to incentive constraint.
  - So,  $d = R^k N(1-\theta) / (R^d - (1-\theta)R^k)$

# Bank demand for $d$



# Interior, no default equilibrium

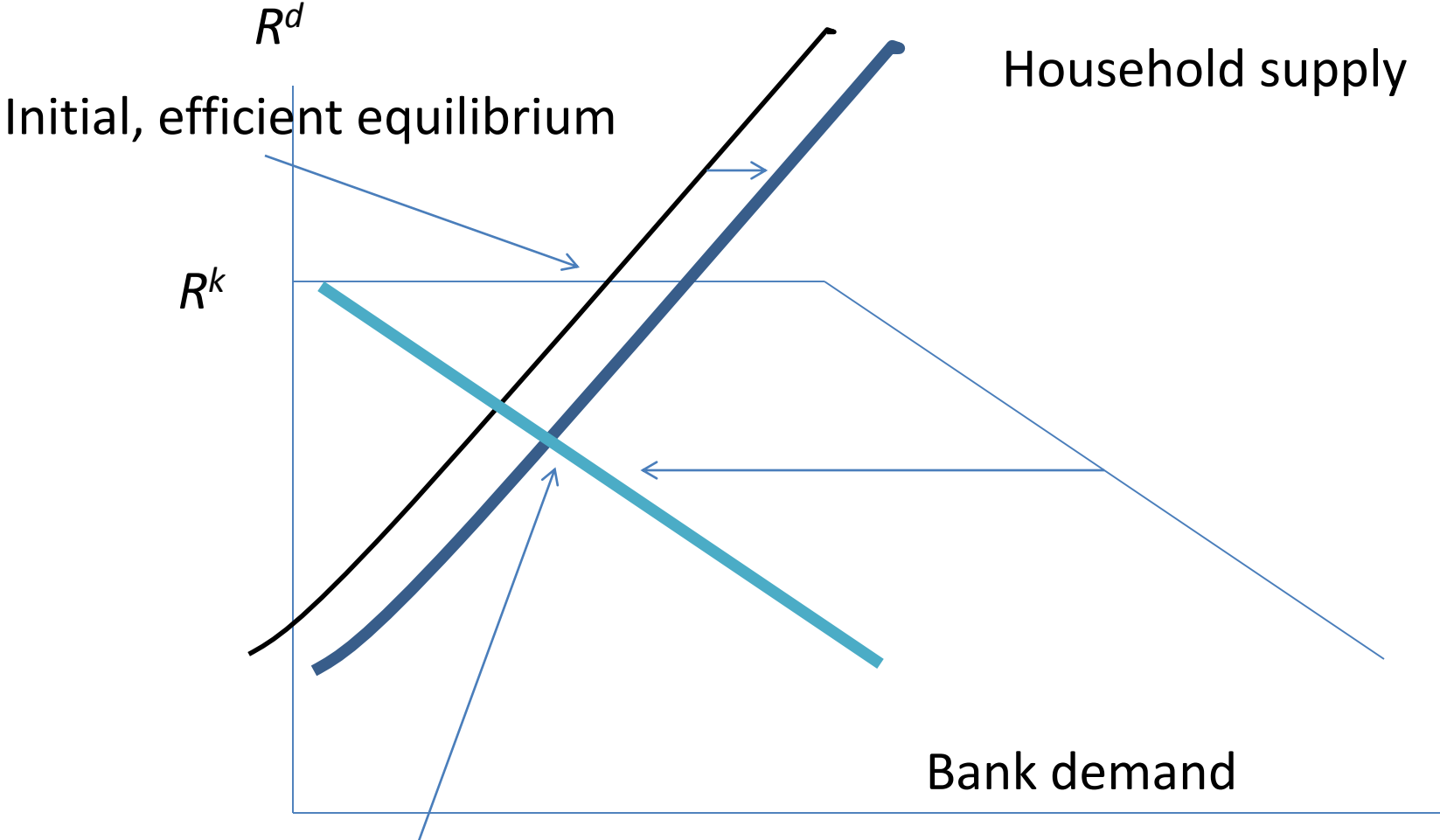




# Collapse in Bank Net Worth

- Suppose that the economy is represented by a sequence of repeated versions of the above model.
- In the periods before the 2007-2008 crisis, net worth was high and the equilibrium was like it is on the previous slide: efficient, with zero interest rate spreads.
  - In practice, spreads are always positive, but that reflects various banking costs that are left out of this model.
- With the crisis,  $N$  dropped a lot, shifting demand to the right and supply to the left.

# Effect of Substantial Drop in Bank Net Worth

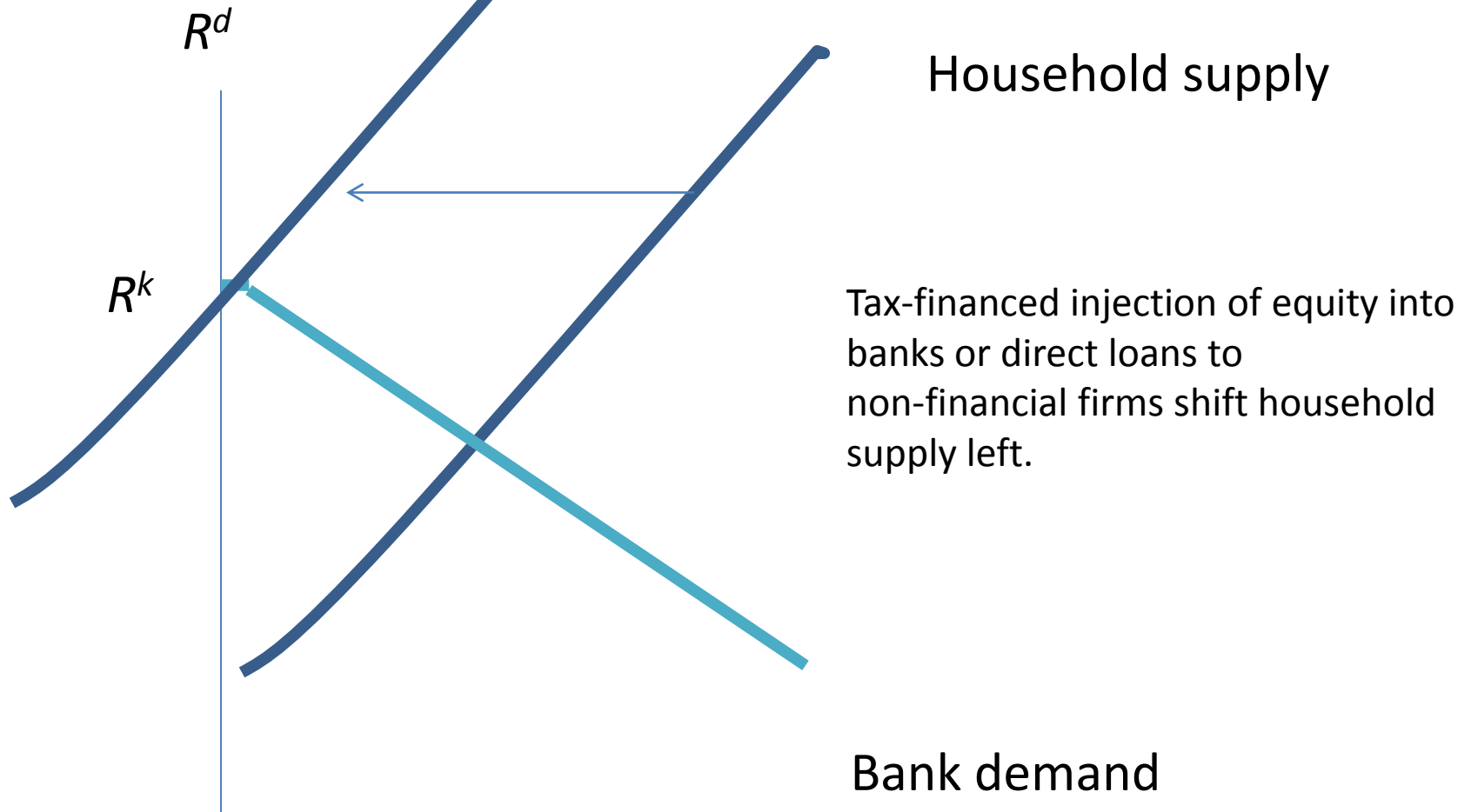


Equilibrium after  $N$  drops is inefficient because  $R^d < R^k$ .

# Government Intervention

- Equity injection.
  - Government raises  $T$  in period 1, provides proceeds to banks and demands  $R^k T$  in return at start of period 2.
  - Rebates earnings to households in 2.
- Has no impact on demand for deposits by banks (no impact on default incentive or profits).
- Reduces supply of deposits by households.
  - $d+T$  rises when  $T$  rises (even though  $d$  falls) because  $R^d$  rises.
- Direct, tax-financed government loans to firms work in the same way.
- An interest rate subsidy to banks will shift their demand for deposits to the right....it will also shift supply to the left.

# Equity Injection and Drop in $N$



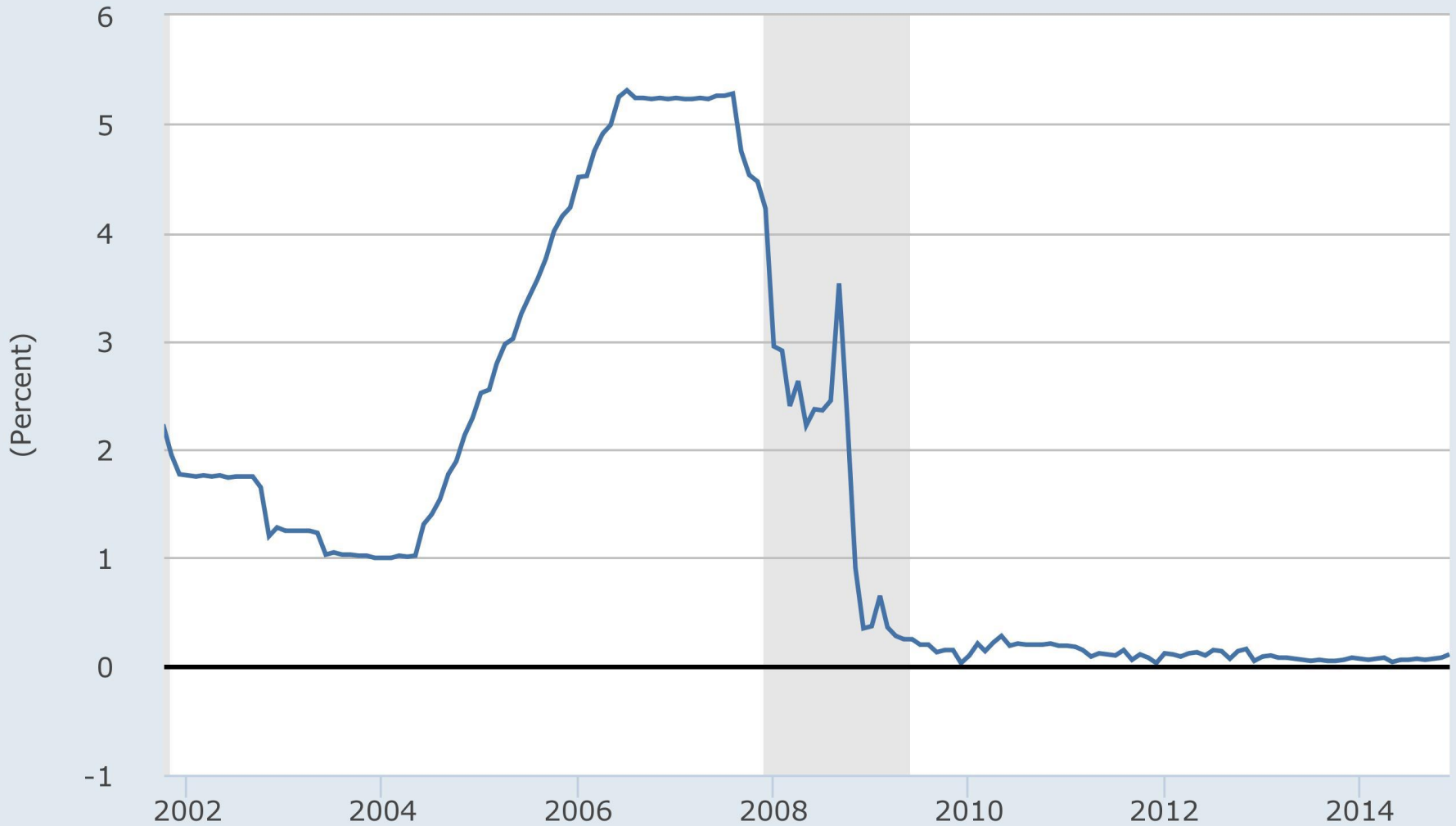
# Recap

- Basic idea:
  - Bankers can run away with a fraction of bank assets.
  - If banker net worth is high relative to deposits, friction not a factor and banking system efficient.
  - If banker net worth falls below a certain cutoff, then banker must restrict the deposits.
    - Bankers fear (correctly) that otherwise depositors would lose confidence and take their business to another bank.
  - Reduction in banker demand for deposits:
    - makes deposit interest rates fall and so spreads rise.
    - Reduced intermediation means investment drops, output drops.
  - Equity injections by the government can revive the banking system.

# Is the Model Narrative Consistent with the Evidence?

- Model says that reduced intermediation of funds through the financial system reflected reduced demand for credit by financial institutions.
- Prediction: interest rate to financial institutions fall.

— 1-Month AA Financial Commercial Paper Rate



Source: Board of Governors of the Federal Reserve System (US)

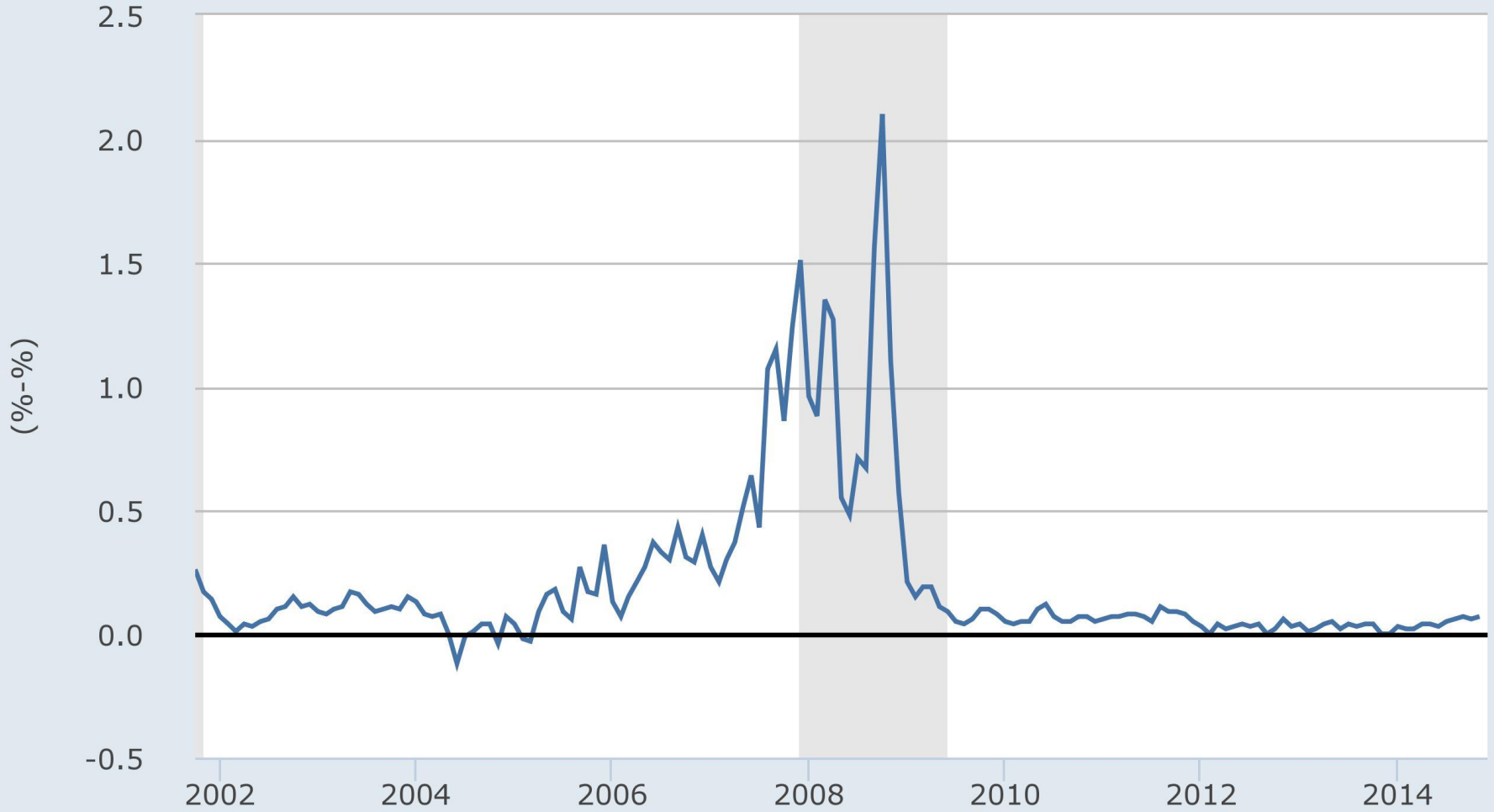
Shaded areas indicate US recessions - 2014 [research.stlouisfed.org](http://research.stlouisfed.org)

- Model prediction for decline in cost of funds to financial institutions seems verified.
- But, other 'risk free' interest rates fell even more.
  - Interest rates on US government debt fell more than interest rate on financial firm commercial paper.





— 1-Month AA Financial Commercial Paper Rate-3-Month Treasury Bill: Secondary Market Rate



Shaded areas indicate US recessions - 2014 research.stlouisfed.org

# Assessment

- Fact that interest rates on US government debt went down more than cost of funds to financial institutions suggests that a complete picture of financial crisis may require two additional features:
  - Risky Banks:
    - Banks in the model are risk free. Default only occurs out of equilibrium.
    - Increased actual riskiness of banks is perhaps also an important part of the picture.
  - Liquidity:
    - Low interest rates on US government debt consistent with idea that high demand for liquidity played an important role in the crisis.

# Macro Prudential Policy

- In recent years there has been increased concern that banks may have a tendency to take on too much debt.
- Has accelerated thinking about debt restrictions on banks.
- There are several models of financial frictions in banks, but they do not necessarily provide a foundation for thinking about debt restrictions on banks.
  - A CSV model of banks implies they issue too *little* debt. (See Christiano-Ikeda).
  - The ‘running away’ model of banks described in these notes does *not* rationalize debt restrictions.
  - Need for the value of assets (held fixed here) that enters participation constraint to be endogenous (see Gertler-Kiyotaki and related literature).