Foundations for the New Keynesian Model II

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Standard New Keynesian Model (NK model of last time, with Taylor rule)

- Taylor rule: designed, so that in steady state, inflation is zero ($\bar{\pi} = 1$)

- Employment subsidy extinguishes monopoly power in steady state:

$$(1-\nu)\frac{\varepsilon}{\varepsilon-1} = 1$$

Equations of the NK Model

$$\frac{1}{C_t} - E_t \frac{\beta}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} = 0 \qquad 1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta\theta F_{t+1} - F_t = 0$$

$$C_t - p_t^* e^{a_t} N_t = 0 \qquad F_t \left(\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} - K_t = 0$$

$$(1-\nu)\frac{\varepsilon}{\varepsilon-1} \frac{C_t \exp(\tau_t) N_t^{\varphi}}{e^{a_t}} + E_t \beta\theta\bar{\pi}_{t+1}^{\varepsilon} K_{t+1} - K_t = 0$$

$$\frac{1}{p_t^*} - \left((1-\theta)\left(\frac{1-\theta(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right) = 0$$

- In steady state:

$$R = \frac{1}{\beta},\ p^* = 1,\ F = K = \frac{1}{1-\beta\theta},\ N = \exp\left(-\frac{0}{1+\varphi} \right).$$

Natural Rate of Interest

- Intertemporal euler equation in Ramsey equilibrium:

$$\overbrace{a_t - \frac{1}{1+\varphi}\tau_t}^{y_t^*} = -\left[r_t^* - \overbrace{r}^{\equiv -\log\beta} \right] + E_t \overbrace{\left(a_{t+1} - \frac{1}{1+\varphi}\tau_{t+1} \right)}^{y_{t+1}^*}$$

- Back out the natural rate:

$$r_t^* = r + E_t \Delta a_{t+1} - \frac{1}{1+\varphi} E_t(\tau_{t+1} - \tau_t)$$

- Shocks:

$$\tau_t = \lambda\tau_{t-1} + \varepsilon_t^{\tau},\ \Delta a_t = \rho\Delta a_{t-1} + \varepsilon_t$$

NK *IS* Curve

- Euler equation in two equilibria:

$$\text{Taylor rule equilibrium: } y_t = -[r_t - E_t\pi_{t+1} - r] + E_t y_{t+1}$$

$$\text{Natural equilibrium: } y_t^* = -[r_t^* - r] + E_t y_{t+1}^*$$

- Subtract:

Output gap

$$x_t = -[r_t - E_t\pi_{t+1} - r_t^*] + E_t x_{t+1}$$

The Tak Yun distortion:

$$p_t^* = \left((1-\theta)\left(\frac{1-\theta(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*}\right)^{-1}$$
$$= f(\bar{\pi}_t, p_{t-1}^*),$$

say, where

$$p^* = f(\bar{\pi}, p^*).$$

The first order Taylor series expansion of f about $\bar{\pi}_t = \bar{\pi}$ and $p_{t-1}^* = p^*$ is:

$$p_t^* - f(\bar{\pi}, p^*) = f_1(\bar{\pi}, p^*)(\bar{\pi}_t - \bar{\pi}) + f_2(\bar{\pi}, p^*)(p_{t-1}^* - p^*)$$
$$= [f_1(\bar{\pi}, p^*)\bar{\pi}]\hat{\bar{\pi}}_t + [f_2(\bar{\pi}, p^*)p^*]\hat{p}_{t-1}^*,$$

where

$$\hat{\bar{\pi}}_t \equiv \frac{\bar{\pi}_t - \bar{\pi}}{\bar{\pi}} = \frac{d\bar{\pi}_t}{\bar{\pi}},\ \hat{p}_{t-1}^* = \frac{p_{t-1}^* - p^*}{p^*} = \frac{dp_{t-1}^*}{p^*}.$$

Then,

$$f_1(\bar{\pi}, p^*) = -(p^*)^2\left(\frac{\varepsilon}{\varepsilon-1}(1-\theta)\left(\frac{1-\theta(\bar{\pi})^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}-1}\left(\frac{-\theta(\varepsilon-1)(\bar{\pi})^{\varepsilon-2}}{1-\theta}\right) + \frac{\varepsilon\theta\bar{\pi}^{\varepsilon-1}}{p^*}\right)$$

$$f_1(1,1) = 0$$

$$f_2(\bar{\pi}, p^*) = -(p^*)^2\left(-\frac{\theta\bar{\pi}^{\varepsilon}}{(p^*)^2}\right)$$

$$f_2(1,1) = \theta$$

so,

$$\hat{p}_t^* = \overbrace{\left[f_1(\bar{\pi}, p^*)\frac{\bar{\pi}}{p^*}\right]}^{=0}\hat{\bar{\pi}}_t + \overbrace{[f_2(\bar{\pi}, p^*)]}^{=\theta}\hat{p}_{t-1}^*$$

Output in NK Equilibrium

- Agg output relation:

$$y_t = \log p_t^* + n_t + a_t, \ \log p_t^* = \begin{cases} = 0 & \text{if } P_{i,t} = P_{j,t} \text{ for all } i,j \\ \leq 0 & \text{otherwise} \end{cases} .$$

- To first order approximation,

$$\hat{p}_t^* \approx \theta \hat{p}_{t-1}^* + 0 \times \bar{\pi}_t, \ (\rightarrow p_t^* \approx 1)$$

Price Setting Equations

- Log-linearly expand the price setting equations about steady state.

$$1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta\theta F_{t+1} - F_t = 0 \qquad F_t \left(\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} - K_t = 0$$

$$(1-\nu)\frac{\varepsilon}{\varepsilon-1}\frac{C_t \exp(\tau_t) N_t^{\varphi}}{e^{a_t}} + E_t \beta\theta\bar{\pi}_{t+1}^{\varepsilon} K_{t+1} - K_t = 0$$

- Log-linearly expand about steady state:

$$\hat{\bar{\pi}}_t = \frac{(1-\beta\theta)(1-\theta)}{\theta}(1+\varphi)x_t + \beta\hat{\bar{\pi}}_{t+1}$$

- See http://faculty.wcas.northwestern.edu/~lchrist/course/solving_handout.pdf

Taylor Rule

- Policy rule

$$r_t = \alpha r_{t-1} + (1-\alpha)[r + \phi_\pi \pi_t + \phi_x x_t]$$

Equations of Actual Equilibrium Closed by Adding Policy Rule

- Here, r_t* denotes r_t*-r and r_t denotes r_t-r.

$$\beta E_t \pi_{t+1} + \kappa x_t - \pi_t = 0 \text{ (Calvo pricing equation)}$$

$$-\left[r_t - E_t \pi_{t+1} - r_t^*\right] + E_t x_{t+1} - x_t = 0 \text{ (intertemporal equation)}$$

$$\alpha r_{t-1} + (1-\alpha)\phi_\pi \pi_t + (1-\alpha)\phi_x x_t - r_t = 0 \text{ (policy rule)}$$

$$r_t^* - E_t \Delta a_{t+1} + \frac{1}{1+\varphi} E_t(\tau_{t+1} - \tau_t) = 0 \text{ (definition of natural rate)}$$

$$\Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t^a,\ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^\lambda$$

Solving the Model

$$s_t = \begin{pmatrix} \Delta a_t \\ \tau_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \lambda \end{bmatrix} \begin{pmatrix} \Delta a_{t-1} \\ \tau_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^\tau \end{pmatrix}$$

$$s_t = P s_{t-1} + \epsilon_t$$

$$\begin{bmatrix} \beta & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \\ r_{t+1} \\ r^*_{t+1} \end{pmatrix} + \begin{bmatrix} -1 & \kappa & 0 & 0 \\ 0 & -1 & -1 & 1 \\ (1-\alpha)\phi_\pi & (1-\alpha)\phi_x & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ r^*_t \end{pmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \\ r_{t-1} \\ r^*_{t-1} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & \frac{1}{1+\varphi} \end{pmatrix} s_{t+1} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{1+\varphi} \end{pmatrix} s_t$$

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

Solving the Model

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

$$s_t - P s_{t-1} - \epsilon_t = 0.$$

- Solution:

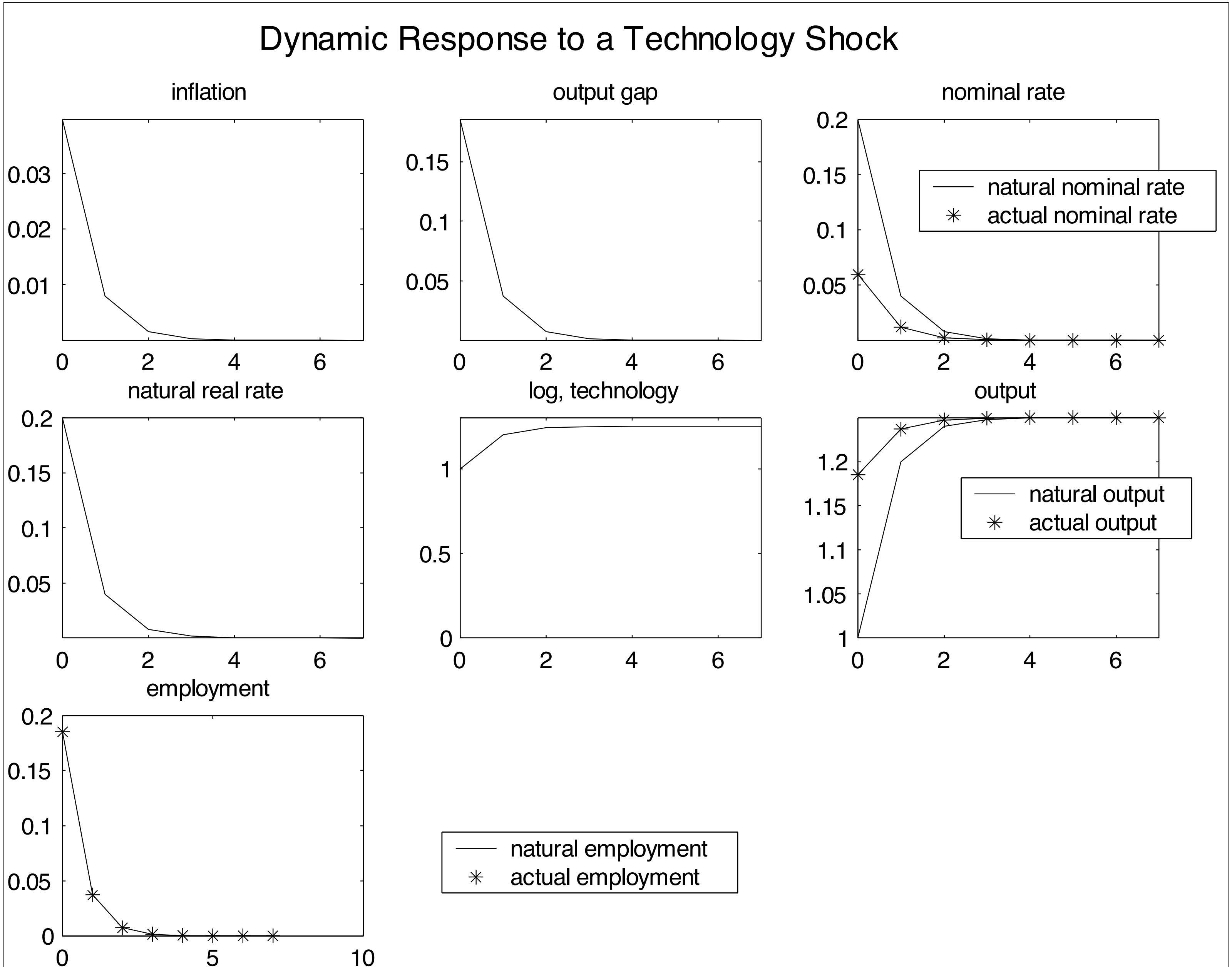
$$z_t = A z_{t-1} + B s_t$$

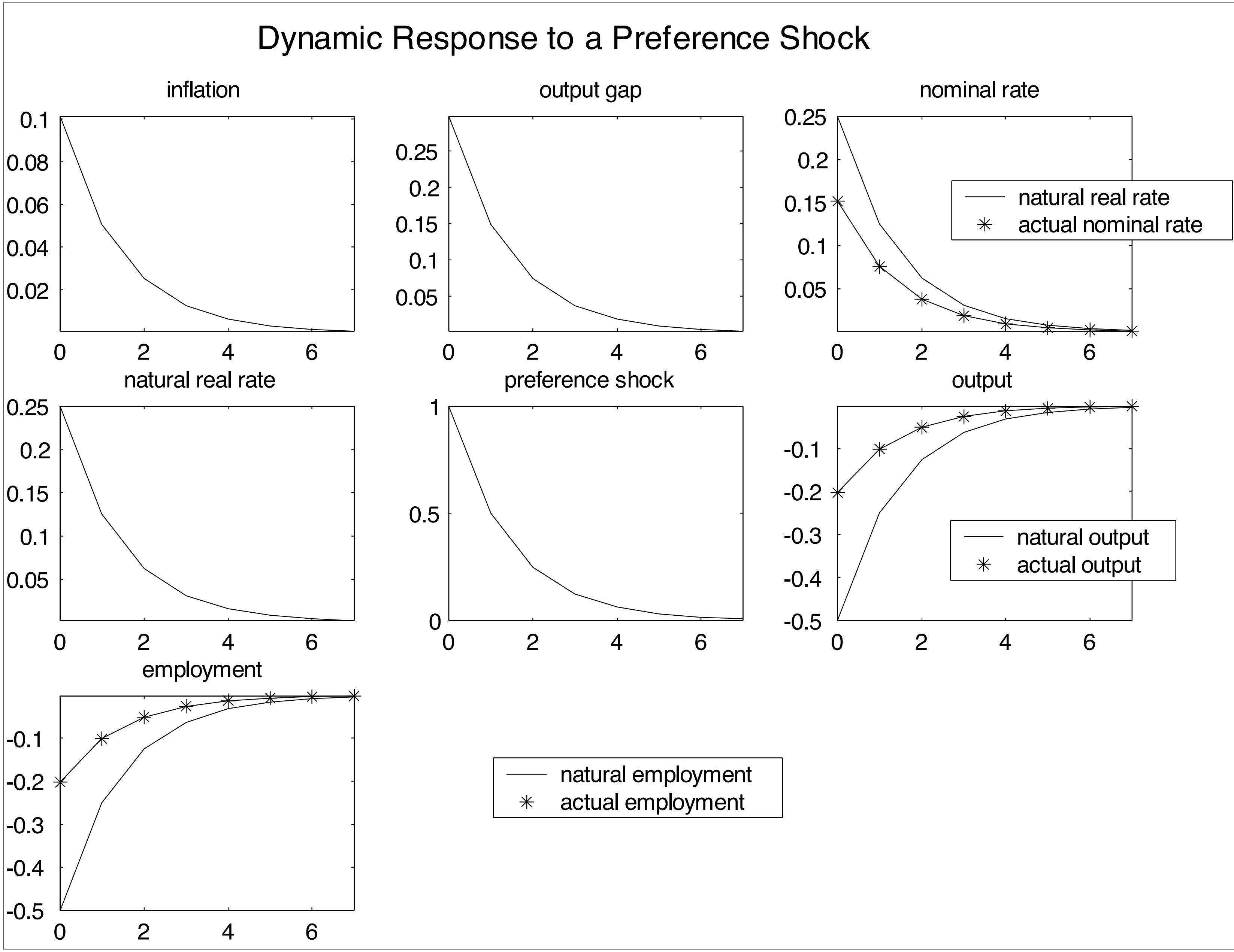
- To ensure that simulations from the previous equation satisfy the equilibrium conditions, require that *A* and *B* satisfy:

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0,$$

$$(\beta_0 + \alpha_0 B)P + [\beta_1 + (\alpha_0 A + \alpha_1)B] = 0$$

$$\phi_x = 0,\ \phi_\pi = 1.5,\ \beta = 0.99,\ \varphi = 1,\ \rho = 0.2,\ \theta = 0.75,\ \alpha = 0,\ \delta = 0.2,\ \lambda = 0.5.$$




Dynamic Response to a Preference Shock
inflation
output gap
nominal rate
natural real rate
actual nominal rate
natural real rate
preference shock
output
natural output
actual output
employment
natural employment
actual employment