Networks in a Simple New Keynesian Model without Capital

Lawrence J. Christiano

October 25, 2015

What we do

- Introduce *networks*: fact that for the typical firm, 50% of output is sold to other firms and 50% of costs is for materials purchased from other firms.
- In absence of monopoly power and price-setting frictions (e.g., real business cycle (RBC) model), fact that materials are typically ignored involves no loss of generality.
  - RBC strategy of adopting representative firm, backing out technology from production function that relates GDP to capital and labor (ignoring materials) is fine.
- When there are frictions, networks
  - render RBC strategy invalid, because TFP is endogenous and no longer structural.
  - blow up the costs of inflation and monopoly power.
  - create 'strategic complementarity' in price-setting: flatten the Phillips curve.
  - raise concerns about the effectiveness of the Taylor Principle.

Background Readings on Networks

- Basu, Susanto, 1995, 'Intermediate goods and business cycles: Implications for productivity and welfare,' *American Economic Review*, 85 (3), 512–531.
- Rotemberg, J., and M. Woodford, 1995, 'Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets,' in, T. Cooley, ed., *Frontiers of Business Cycle Research,* Princeton University Press (also, NBER wp 4502).
- Nakamura, Emi and Jon Steinsson, 2010, 'Monetary Non-Neutrality in a Multisector Menu Cost Model,' *The Quarterly Journal of Economics*, August.
- Jones, Chad, 2013, 'Misallocation, Economic Growth, and Input-Output Economics,' in D. Acemoglu, M. Arellano, and E. Dekel, *Advances in Economics and Econometrics*, Tenth World Congress, Volume II, Cambridge University Press.
- Daron Acemoglu, Ufuk Akcigit, William Kerr, 'Networks and the Macroeconomy: An Empirical Exploration,' forthcoming, NBER Macroeconomics Annual 2015.

Networks and the Consequences of Inflation

- Christiano, 2015, 'Networks and the Macroeconomic Consequences of Inflation'.
  - Following notes based on latter.
- Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin, 2011, 'DSGE Models for Monetary Policy Analysis,' In Benjamin M. Friedman, and Michael Woodford, editors: *Handbook of Monetary Economics*, Vol. 3A, The Netherlands: North-Holland, 2011, pp. 285-367.
- Ascari, Guido, Louis Phaneuf and Eric Sims, 2015, 'On the Welfare and Cyclical Implications of Moderate Trend Inflation,' National Bureau of Economic Research Working Paper 21392.

Outline

- Decisions by individual agents
  - Households, homogeneous good producers, intermediate good producers.
  - Efficient allocation of resources in production.
- Equilibrium for Aggregate Variables
  - Comparing RBC and Sticky Price version of the model.
  - Gross output versus value-added.
  - Tack Yun distortion, allocative cost of monopoly power and inflation.
  - Classical Dichotomy.
- The costs of inflation through the lense of the model.
- Steady state: useful for model intuition, and needed for solution methods.
- Phillips curve - linearized equilibrium condition for inflation.
  - Networks flatten Phillips curve by magnifying strategic complementarities in price-setting.

Households

- Problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}$$

$$\text{s.t. } P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + \text{Profits net of taxes}_t$$

- First order conditions:

$$\begin{aligned} \frac{1}{C_t} &= \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5) \\ \exp(\tau_t) C_t N_t^{\varphi} &= \frac{W_t}{P_t}. \end{aligned}$$

Goods Production

- A homogeneous final good is produced using the following (Dixit-Stiglitz) production function:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

- Each intermediate good, $Y_{i,t}$, is produced as follows:

$$\begin{aligned} Y_{i,t} &= \exp(a_t) N_{i,t}^{\gamma} I_{i,t}^{1-\gamma}, \ a_t \sim \text{exogenous shock to technology,} \\ 0 &< \gamma \leq 1. \end{aligned}$$

- $I_{i,t}$ ~'materials' these are purchases of the homogeneous output good (Basu's simplified way of capturing that firms buy goods from other firms).
- Before discussing the firms that operate these production functions, we briefly investigate the socially efficient ('First Best') allocation of resources across i.
  - simplify the discussion with $\gamma = 1$ (no materials).

Efficient Sectoral Allocation of Resources Across Sectors

- With Dixit-Stiglitz final good production function, there is a socially optimal allocation of resources to all the intermediate activities, $Y_{i,t}$
  - It is optimal to run them all at the same rate, *i.e.*, $Y_{i,t} = Y_{j,t}$ for all $i, j \in [0, 1]$.
- For given N_t, it is optimal to set $N_{i,t} = N_{j,t}$, for all $i, j \in [0, 1]$
- In this case, final output is given by

$$Y_t = e^{a_t} N_t.$$

- Best way to see this is to suppose that labor is *not* allocated equally to all activities.
  - Explore one simple deviation from $N_{i,t} = N_{j,t}$ for all $i, j \in [0, 1]$.

Suppose Labor *Not* Allocated Equally

- Example:

$$N_{it} = \begin{cases} 2\alpha N_t & i \in \left[0, \frac{1}{2}\right] \\ 2(1-\alpha)N_t & i \in \left[\frac{1}{2}, 1\right] \end{cases}, \; 0 \le \alpha \le 1.$$

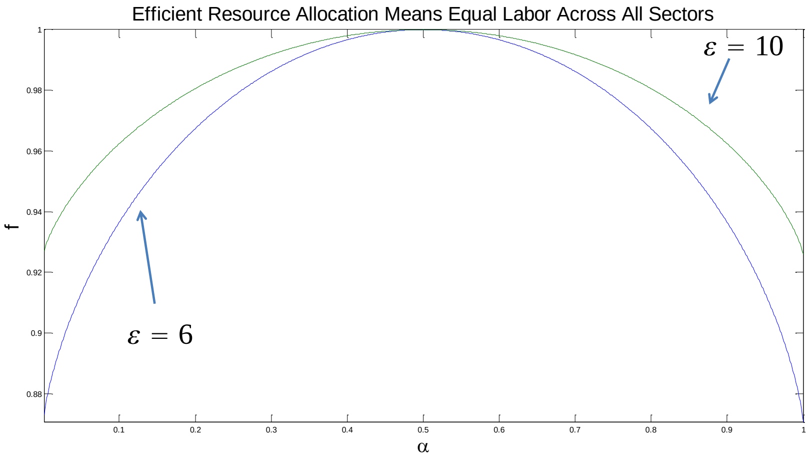
- Note that this is a particular distribution of labor across activities:

$$\int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1-\alpha) N_t = N_t$$

Labor *Not* Allocated Equally, cnt'd

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\int_0^{\frac{1}{2}} Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} (2\alpha N_t)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha)N_t)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\int_0^{\frac{1}{2}} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\frac{1}{2} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t f(\alpha)
\end{aligned}
$$

$$f(\alpha) = \left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$



Homogeneous Good Production

- Competitive firms:
  - maximize profits:

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,$$

subject to:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

  - Foncs:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon} \rightarrow \overbrace{P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}}^{\text{"cross price restrictions"}}$$

Intermediate Goods Production

- Demand curve for i^{th} monopolist:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}}\right)^{\varepsilon}.$$

- Production function, $0 < \gamma \leq 1$:

$$Y_{i,t} = \exp\left(a_t\right) N_{i,t}^{\gamma} I_{i,t}^{1-\gamma},\ a_t \text{ ~exogenous shock to technology.}$$

- $I_{i,t}$ ~'materials' these are purchases of the homogeneous output good (Basu's simplified way of capturing that firms buy goods from other firms).

- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}.$$

Cost Minimization Problem

- Price setting by intermediate good firms is discussed later.
  - The intermediate good firm must produce the quantity demanded, $Y_{i,t}$, at the price that it sets.
  - Right now we take $Y_{i,t}$ as given and we investigate the cost minimization problem that determines the firm's choice of inputs.

Cost minimization problem:

$$\min_{N_{i,t},I_{i,t}} \bar{W}_t N_{i,t} + \bar{P}_t I_{i,t} + \overbrace{\lambda_{i,t}}^{\text{marginal cost (money terms)}} \left[Y_{i,t} - A_t N_{i,t}^{\gamma} I_{i,t}^{1-\gamma} \right]$$

with resource costs:

$$\bar{W}_t = \overbrace{(1-\nu)}^{\text{subsidy, if } \nu>0} \times \overbrace{(1-\psi+\psi R_t)\, W_t}^{\text{cost, including finance, of a unit of labor}}$$

$$\bar{P}_t = (1-\nu) \times \overbrace{(1-\psi+\psi R_t)\, P_t}^{\text{cost, including finance, of a unit of materials}} .$$

Cost Minimization Problem

- Problem:

$$\min_{N_{i,t}, I_{i,t}} \bar{W}_t N_{i,t} + \bar{P}_t I_{i,t} + \lambda_{i,t} \left[Y_{i,t} - A_t N_{i,t}^{\gamma} I_{i,t}^{1-\gamma} \right]$$

- First order conditions:

$$\bar{P}_t I_{i,t} = (1-\gamma)\, \lambda_{i,t} Y_{i,t},\ \bar{W}_t N_{i,t} = \gamma \lambda_{i,t} Y_{i,t},$$

so that,

$$\begin{aligned} \frac{I_{it}}{N_{it}} &= \frac{1-\gamma}{\gamma} \frac{\bar{W}_t}{\bar{P}_t} = \frac{1-\gamma}{\gamma} \exp(\tau_t)\, C_t N_t^{\varphi} \\ &\rightarrow \frac{I_{it}}{N_{it}} = \frac{I_t}{N_t},\ \text{for all } i. \end{aligned}$$

Cost Minimization Problem

- Firm first order conditions imply

$$\lambda_{i,t} = \left(\frac{\bar{P}_t}{1-\gamma}\right)^{1-\gamma} \left(\frac{\bar{W}_t}{\gamma}\right)^{\gamma} \frac{1}{A_t}.$$

- Divide marginal cost by P_t :

$$\begin{aligned} s_t \quad &\equiv \quad \frac{\lambda_{i,t}}{P_t} = (1-\nu)\,(1-\psi+\psi R_t) \left(\frac{1}{1-\gamma}\right)^{1-\gamma} \\ &\times \left(\frac{1}{\gamma}\exp\left(\tau_t\right) C_t N_t^{\varphi}\right)^{\gamma} \frac{1}{A_t} \text{ (9)}, \end{aligned}$$

after substituting out for $\bar{P}_t$ and $\bar{W}_t$ and using the household's labor first order condition.

- Note from (9) that i^{th} firm's marginal cost, s_t, is independent of i and $Y_{it,}$.

Share of Materials in Intermediate Good Output

- Firm i materials proportional to $Y_{i,t}$:

$$I_{i,t} = \frac{(1-\gamma)\,\lambda_{i_it} Y_{i,t}}{\bar{P}_t} = \mu_t Y_{i,t},$$

where

$$\mu_t = \frac{(1-\gamma)\,s_t}{(1-\nu)\,(1-\psi+\psi R_t)} \text{ (10).}$$

- "Share of materials in firm-level gross output", μ_t.

Decision By Firm that Can Change Its Price

- i^{th} intermediate good firm's objective:

$$E_t^i \sum_{j=0}^{\infty} \beta^j \; v_{t+j} \overbrace{\left[\overbrace{P_{i,t+j} Y_{i,t+j}}^{\text{revenues}} - \overbrace{P_{t+j} s_{t+j} Y_{i,t+j}}^{\text{total cost}} \right]}^{\text{period } t+j \text{ profits sent to household}}$$

v_{t+j} - Lagrange multiplier on household budget constraint

- Firm that gets to reoptimize its price is concerned only with future states in which it does not change its price:

$$\begin{aligned} & E_t^i \sum_{j=0}^{\infty} \beta^j v_{t+j} \left[P_{i,t+j} Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right] \\ = \; & E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right] + X_t, . \end{aligned}$$

where $\tilde{P}_t$ denotes a firm's price-setting choice at time t and X_t not a function of $\tilde{P}_t$.

Decision By Firm that Can Change Its Price

- Substitute out demand curve:

$$
\begin{aligned}
& E_t \sum_{j=0}^{\infty} (\beta\theta)^j \upsilon_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j}\right] \\
= \; & E_t \sum_{j=0}^{\infty} (\beta\theta)^j \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[\tilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon}\right].
\end{aligned}
$$

- Differentiate with respect to $\tilde{P}_t$:

$$
E_t \sum_{j=0}^{\infty} (\beta\theta)^j \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[(1-\varepsilon)\left(\tilde{P}_t\right)^{-\varepsilon} + \varepsilon P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon-1}\right] = 0,
$$

or,

$$
E_t \sum_{j=0}^{\infty} (\beta\theta)^j \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon-1} s_{t+j}\right] = 0.
$$

- When $\theta = 0$, get standard result - price is fixed markup over marginal cost.

Decision By Firm that Can Change Its Price

- Substitute out the multiplier:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \overbrace{\frac{u'\left(C_{t+j}\right)}{P_{t+j}}}^{=\, v_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon-1} s_{t+j} \right] = 0.$$

- Using assumed log-form of utility,

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon-1} s_{t+j} \right] = 0,$$

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t},\ \bar{\pi}_t \equiv \frac{P_t}{P_{t-1}},\ X_{t,j} = \begin{cases} \frac{1}{\bar{\pi}_{t+j}\bar{\pi}_{t+j-1}\cdots\bar{\pi}_{t+1}},\ j \geq 1 \\ 1,\ j = 0. \end{cases},$$

$$X_{t,j} = X_{t+1,j-1} \frac{1}{\bar{\pi}_{t+1}},\ j > 0$$

Decision By Firm that Can Change Its Price

- Want $\tilde{p}_t$ in:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j}\right] = 0$$

- Solving for $\tilde{p}_t$, we conclude that prices are set as follows:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+1}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_t}{F_t}.$$

- Need convenient expressions for K_t, F_t.

Decision By Firm that Can Change Its Price

$$
\begin{aligned}
K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j} \\
&= \frac{\varepsilon}{\varepsilon-1}\frac{Y_t}{C_t} s_t \\
&\quad + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} \overbrace{E_{t+1} \sum_{j=0}^{\infty} (\beta\theta)^j X_{t+1,j}^{-\varepsilon} \frac{Y_{t+j+1}}{C_{t+j+1}} \frac{\varepsilon}{\varepsilon-1} s_{t+1+j}}^{\text{exactly } K_{t+1}!} \\
&= \frac{\varepsilon}{\varepsilon-1}\frac{Y_t}{C_t} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1}
\end{aligned}
$$

For a detailed derivation, see, e.g., http://faculty.wcas.northwestern.edu/~lchrist/course/IMF2015/intro_NK_handout.pdf.

Decision By Firm that Can Change Its Price

- Conclude:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{-\varepsilon} \frac{Y_{t+j}}{C_{t+j}} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}} = \frac{K_t}{F_t},$$

where

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{Y_t}{C_t} s_t + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}} \right)^{-\varepsilon} K_{t+1} \text{ (1)}$$

- Similarly,

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}} \right)^{1-\varepsilon} F_{t+1} \text{ (2)}$$

Interpretation of Price Formula

- Note,

$$\frac{1}{P_{t+j}} = \frac{1}{P_t} X_{t,j},\ s_{t+j} = \frac{\lambda_{t+j}}{P_{t+j}} = \frac{\lambda_{t+j}}{P_t} X_{t,j},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t}.$$

Multiply both sides of the expression for $\tilde{p}_t$ by P_t :

$$\tilde{P}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}} \frac{\varepsilon}{\varepsilon-1} \lambda_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}} = \frac{\varepsilon}{\varepsilon - 1} \sum_{j=0}^{\infty} E_t \omega_{t+j} \lambda_{t+j}$$

where

$$\omega_{t+j} = \frac{(\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}},\ \sum_{j=0}^{\infty} E_t \omega_{t+j} = 1.$$

Evidently, price is set as a markup over a weighted average of future marginal cost, where the weights are shifted into the future depending on how big θ is.

Moving On to Aggregates

- Aggregate price level.
- Aggregate measures of production.
  - Value added.
  - Gross output.

Aggregate Price Index

- Rewrite the aggregate price index.
  - let $p \in (0, \infty)$ the set of logically possible prices for intermediate good producers.
  - let $g_t(p) \geq 0$ denote the measure (e.g., 'number') of producers that have price, p, in t
  - let $g_{t-1,t}(p) \geq 0$, denote the measure of producers that had price, p, in $t-1$ and could not reoptimize in t
- Then,

$$P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}} = \left(\int_0^\infty g_t(p) \, p^{(1-\varepsilon)} dp \right)^{\frac{1}{1-\varepsilon}}.$$

- Note:

$$P_t = \left(\theta \tilde{P}_t^{1-\varepsilon} + \int_0^\infty g_{t-1,t}(p) \, p^{(1-\varepsilon)} dp \right)^{\frac{1}{1-\varepsilon}}.$$

Aggregate Price Index

- Calvo randomization assumption:

$$
\begin{aligned}
\overbrace{g_{t-1,t}\left(p\right)}^{\text{measure of firms that had price, } p\text{, in } t-1 \text{ and could not change}} \\
= \quad \theta \times \overbrace{g_{t-1}\left(p\right)}^{\text{measure of firms that had price } p \text{ in } t-1}
\end{aligned}
$$

- Then,

$$
\begin{aligned}
P_t &= \left(\left(1-\theta\right)\tilde{P}_t^{1-\varepsilon} + \int_0^\infty g_{t-1,t}\left(p\right) p^{(1-\varepsilon)} dp \right)^{\frac{1}{1-\varepsilon}} \\
&= \left(\left(1-\theta\right)\tilde{P}_t^{1-\varepsilon} + \theta \overbrace{\int_0^\infty g_{t-1}\left(p\right) p^{(1-\varepsilon)} dp}^{=P_{t-1}^{1-\varepsilon}} \right)^{\frac{1}{1-\varepsilon}}
\end{aligned}
$$

Restriction Between Aggregate and Intermediate Good Prices

- 'Calvo result':

$$P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}} = \left[(1-\theta)\, \tilde{P}_t^{(1-\varepsilon)} + \theta P_{t-1}^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}.$$

- Divide by P_t :

$$1 = \left[(1-\theta)\, \tilde{p}_t^{(1-\varepsilon)} + \theta \left(\frac{1}{\bar{\pi}_t} \right)^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}.$$

- Rearrange:

$$\tilde{p}_t = \left[\frac{1-\theta}{1-\theta \bar{\pi}_t^{(\varepsilon-1)}} \right]^{\frac{1}{\varepsilon-1}}$$

Aggregate inputs and outputs

- *Gross output* of firm i :

$$Y_{i,t} = \exp\left(a_t\right) N_{i,t}^{\gamma} I_{i,t}^{1-\gamma}.$$

  – Net output or *value-added* would subtract out the materials that were bought from other firms.

- Economy-wide *gross output*: sum of value of $Y_{i,t}$ across all firms:

$$\begin{aligned}\int_0^1 P_{i,t} Y_{i,t} di &= \int_0^1 P_t \left(\frac{Y_t}{Y_{i,t}}\right)^{\frac{1}{\varepsilon}} Y_{i,t} di \\ &= P_t Y_t^{\frac{1}{\varepsilon}} \overbrace{\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di}^{=Y_t^{\frac{\varepsilon-1}{\varepsilon}}} = P_t Y_t.\end{aligned}$$

- *Gross output production function*: relation between Y_t and non-produced inputs, N_t.

Aggregate inputs and outputs, cnt'd

- Gross output, Y_t, is not a good measure of economic output, because it double counts.
  - Some of the output that firm i 'produced' is materials produced by another firm, which is counted in that firm's output.
  - If wheat is used to make bread, wrong to measure production by adding all wheat and all bread. That double counts the wheat.
- Want aggregate *value-added*: sum of firm-level gross output, minus purchases of materials from other firms.
- *Value-added production function*: expression relating aggregate value-added in period t to inputs not produced in period t.
  - capital and labor.

Gross Output Production Function

- Approach developed by Tack Yun (JME, 1996).
- Define Y_t^*:

$$
\begin{aligned}
Y_t^* &\equiv \int_0^1 Y_{i,t} di \\
&\overbrace{=}^{\text{demand curve}} Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} di = Y_t P_t^{\varepsilon} \int_0^1 \left(P_{i,t}\right)^{-\varepsilon} di \\
&= Y_t P_t^{\varepsilon} \left(P_t^*\right)^{-\varepsilon}
\end{aligned}
$$

where, using 'Calvo result':

$$
P_t^* \equiv \left[\int_0^1 P_{i,t}^{-\varepsilon} di\right]^{\frac{-1}{\varepsilon}} = \left[\left(1-\theta\right)\tilde{P}_t^{-\varepsilon} + \theta\left(P_{t-1}^*\right)^{-\varepsilon}\right]^{\frac{-1}{\varepsilon}}
$$

- Then

$$
Y_t = p_t^* Y_t^*,\ p_t^* = \left(\frac{P_t^*}{P_t}\right)^{\varepsilon}.
$$

Tack Yun Distortion

- Consider the object,

$$p_t^* = \left(\frac{P_t^*}{P_t}\right)^{\varepsilon},$$

where

$$P_t^* = \left(\int_0^1 P_{i,t}^{-\varepsilon} di\right)^{\frac{-1}{\varepsilon}},\ P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{1}{1-\varepsilon}}$$

- In following slide, use Jensen's inequality to show:

$$p_t^* \leq 1.$$

Tack Yun Distortion

- Note

$$\overbrace{\left(\int_0^1 P_{i,t}^{-\varepsilon} di\right)^{\frac{-1}{\varepsilon}}}^{P_t^*} \leq \overbrace{\left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{1}{1-\varepsilon}}}^{P_t}$$

$$\iff \left(\int_0^1 P_{i,t}^{-\varepsilon} di\right) \geq \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\iff \int_0^1 \left(P_{i,t}^{(1-\varepsilon)}\right)^{\frac{\varepsilon}{\varepsilon-1}} di \geq \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

  by convexity.

- Example:
  - let $f(x) = x^4$. Then,

$$\alpha x_1^4 + (1-\alpha) x_2^4 > (\alpha x_1 + (1-\alpha) x_2)^4$$

    for $x_1 \neq x_2$, $0 < \alpha < 1$.

Gross Output Production Function

- Relationship between aggregate inputs and outputs:

$$\begin{aligned} Y_t &= p_t^* Y_t^* = p_t^* \int_0^1 Y_{i,t} di \\ &= p_t^* A_t \int_0^1 N_{i,t}^{\gamma} I_{i,t}^{1-\gamma} di = p_t^* A_t \int_0^1 \left(\frac{N_{i,t}}{I_{i,t}} \right)^{\gamma} I_{i,t} di, \\ &= p_t^* A_t \left(\frac{N_t}{I_t} \right)^{\gamma} I_t, \end{aligned}$$

or,

$$Y_t = p_t^* A_t N_t^{\gamma} I_t^{1-\gamma} \text{ (6)},$$

where

$$p_t^* : \left\{ \begin{array}{ll} \leq 1 & \\ = 1 & P_{i,t} = P_{j,t}, \text{ all } i, j \end{array} \right. .$$

Law of Motion of Tack Yun Distortion

- We have

$$P_t^* = \left[(1-\theta)\, \tilde{P}_t^{-\varepsilon} + \theta \left(P_{t-1}^* \right)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}$$

- Then,

$$\begin{aligned} p_t^* &\equiv \left(\frac{P_t^*}{P_t} \right)^{\varepsilon} = \left[(1-\theta)\, \tilde{p}_t^{-\varepsilon} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \\ &= \left[(1-\theta) \left(\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \qquad (4) \end{aligned}$$

using the restriction between $\tilde{p}_t$ and aggregate inflation developed earlier.

Gross Output Production Function

- Recall

$$I_{i,t} = \mu_t Y_{i,t},$$

so,

$$I_t \equiv \int_0^1 I_{i,t} di = \mu_t \int_0^1 Y_{i,t} d = \mu_t Y_t^* = \frac{\mu_t}{p_t^*} Y_t.$$

- Then, the gross output production function is:

$$\begin{aligned} Y_t &= p_t^* A_t N_t^{\gamma} I_t^{1-\gamma} \\ &= p_t^* A_t N_t^{\gamma} \left(\frac{\mu_t}{p_t^*} Y_t \right)^{1-\gamma} \\ &\longrightarrow \; Y_t = \left(p_t^* A_t \left(\frac{\mu_t}{p_t^*} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} N_t \end{aligned}$$

Value Added (GDP) Production Function

- We have

$$
\begin{aligned}
GDP_t &= Y_t - I_t = \left(1 - \frac{\mu_t}{p_t^*}\right) Y_t \\
&= \left(1 - \frac{\mu_t}{p_t^*}\right) \left(p_t^* A_t \left(\frac{\mu_t}{p_t^*}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}} N_t \\
&= \overbrace{\left(p_t^* A_t \left(1 - \frac{\mu_t}{p_t^*}\right)^{\gamma} \left(\frac{\mu_t}{p_t^*}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}^{=\text{Total Factor Productivity (TFP)}} N_t
\end{aligned}
$$

- Note how an increase in technology at the firm level, by A_t, gives rise to a bigger increase in TFP by $A_t^{1/\gamma}$.
  - In the literature on networks, $1/\gamma$ is referred to as a 'multiplier effect' (see Jones, 2011).
- The Tack Yun distortion, p_t^*, is associated with the same multiplier phenomenon.

Decomposition for TFP

- To maximize GDP for given aggregate N_t and A_t :

$$\max_{0<p_t^*\leq 1,\ 0\leq\lambda_t\leq 1} \left(p_t^* A_t \left(1-\lambda_t\right)^{\gamma} \left(\lambda_t\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}$$

$$\rightarrow \quad \lambda_t = 1-\gamma,\ p_t^* = 1.$$

- So,

$$TFP_t = \overbrace{\left(p_t^* \left(\frac{1-\frac{\mu_t}{p_t^*}}{\gamma}\right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}^{\text{Component due to market distortions}\equiv\chi_t}$$

$$\times \quad \overbrace{\left(A_t \left(\gamma\right)^{\gamma} \left(1-\gamma\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}^{\text{Exogenous, technology component}\equiv\tilde{A}_t}$$

Allocative Cost of Inflation

- Could decompose TFP even more:

$$TFP_t = \tilde{A}_t \times \chi_t,\ \chi_t = \chi_t^{\pi} \times \chi_t^{\text{monopoly}},$$

where

χ_t^{monopoly} ~ what χ_t would be if prices were flexible, $\theta = 0$.

$$\chi_t^{\pi} \sim \frac{\chi_t}{\chi_t^{\text{monopoly}}}.$$

- Allocative cost of inflation:

$$100\left(1 - \chi_t^{\pi}\right).$$

- Total allocative costs (1^{st} order Taylor expansion about $\chi_t^{\pi} = \chi_t^{\text{monopoly}} = 1$) :

$$100\left(1 - \chi_t\right) \simeq 100\left(1 - \chi_t^{\pi}\right) + 100\left(1 - \chi_t^{\text{monopoly}}\right)$$

Evaluating the Distortions

- The equations characterizing the TFP distortion, χ_t :

$$\chi_t = \left(p_t^* \left(\frac{1-\frac{\mu_t}{p_t^*}}{\gamma}\right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}$$

$$p_t^* = \left[(1-\theta)\left(\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1}.$$

- Potentially, NK model provides an 'endogenous theory of TFP'.
- Standard practice in NK literature is to set $\chi_t = 1$ for all t.
  - Set $\gamma = 1$ and linearize around $\bar{\pi}_t = p_t^* = 1$.
  - With $\gamma = 1$, $\chi_t = p_t^*$, and first order expansion of p_t^* around $\bar{\pi}_t = p_t^* = 1$ is:

$$p_t^* = p^* + 0 \times (\bar{\pi}_t - 1) + \theta\left(p_{t-1}^* - p^*\right), \text{ with } p^* = 1,$$

so $p_t^* \to 1$ and is invariant to shocks.

Empirical Assessment of the Distortions

- The TFP distortion, χ_t :

$$\chi_t = \left(p_t^* \left(\frac{1 - \frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}}$$

- Problem: the objects, χ_t and p_t^*, are not quite observable.
  - Still, if we assume μ_t is constant, at $1-\gamma$, we can get a feel about the magnitudes using US inflation data.
- Will consider $\gamma = 1/2$ (Basu's empirical estimate) and $\gamma = 1$ (standard assumption in NK literature).
- Will consider two values for the markup:
  - $\varepsilon / (\varepsilon - 1) = 1.20$, the baseline estimate in CEE (JPE, 2005), which corresponds to $\varepsilon = 6$,
  - $\varepsilon / (\varepsilon - 1) = 1.15$, more competition, i.e., $\varepsilon = 7.7$.

Empirical Assessment of the Distortions

- First, do 'back of the envelope' calculations in a steady state when inflation is constant and p^* is constant.

$$p^* = \left[(1-\theta) \left(\frac{1-\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}^{\varepsilon}}{p^*} \right]^{-1}$$

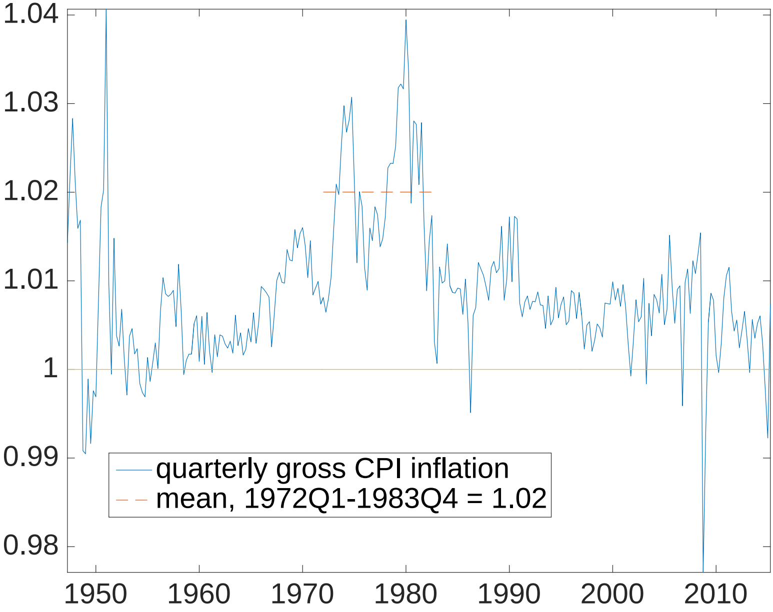
$$\rightarrow p^* = \frac{1-\theta\bar{\pi}^{\varepsilon}}{1-\theta} \left(\frac{1-\theta}{1-\theta\bar{\pi}^{(\varepsilon-1)}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

- Approximate TFP distortion, χ :

$$\chi_t = \left(p_t^* \left(\frac{1-\frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} \overbrace{\simeq}^{\text{more on this later}} (p^*)^{1/\gamma}$$

Three Inflation Rates:

- Average inflation in the 1970s, 8 percent APR.
- Several people have suggested that the US raise its inflation target to 4 percent to raise the nominal rate of interest and thereby reduce the likelihood of the zero lower bound on the interest rate becoming binding again.
  - http://www.voxeu.org/article/case-4-inflation
- Two percent inflation is the average in the recent (pre-2008) low inflation environment.



1.04
1.03
1.02
1.01
1
0.99
0.98
1950
1960
1970
1980
1990
2000
2010
quarterly gross CPI inflation
mean, 1972Q1-1983Q4 = 1.02

Cost of Three Alternative Permanent Levels of Inflation

$$p^* = \frac{1-\theta\bar{\pi}^{\varepsilon}}{1-\theta}\left(\frac{1-\theta}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\right)^{\frac{\varepsilon}{\varepsilon-1}},\ \chi = (p^*)^{1/\gamma}$$

Table 1: Percent of GDP Lost[1] Due to Inflation, 100$(1-\chi_t)$	
Without networks $(\gamma = 1)$	With networks $(\gamma = 1/2)$
a: Steady state inflation: 8 percent per year	
2.41[2] (3.92) [10.85]	4.76 (7.68) [20.53]
b: Steady state inflation: 4 percent per year	
0.46 (0.64) [1.13]	0.91 (1.27) [2.25]
c: Steady state inflation: 2 percent per year	
0.10 (0.13) [0.21]	0.20 (0.27) [0.42]

Note: number not in parentheses assumes a markup of 20 percent; number in parentheses: 15 percent; number in square brackets: 10 percent

Next: Assess Costs of Inflation Using Non-Steady State Formulas

Figure 1a: Percent loss of GDP due to Inflation, assumed markup is 1.2

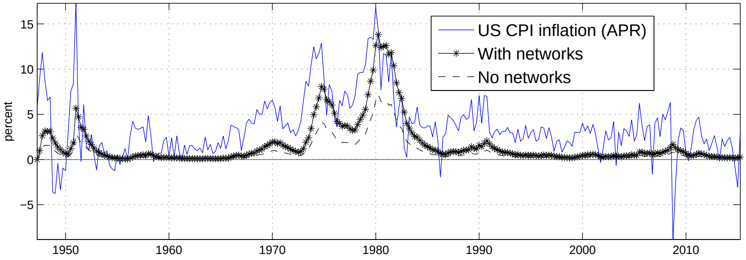
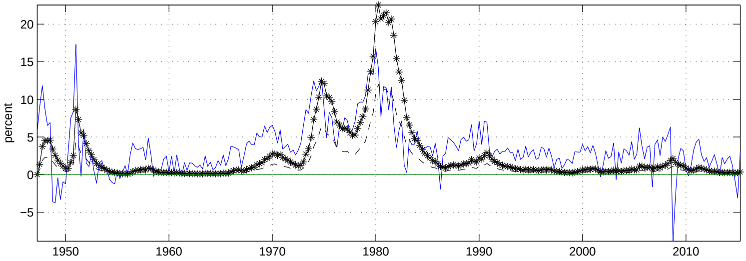


Figure 1b: Percent loss of GDP due to Inflation, assumed markup is 1.15



Inflation Distortions Displayed are Big

- With $\varepsilon = 6$,
  - mean$(\chi_t) = 0.98$, a 2% loss of GDP.
  - frequency, $\chi_t < 0.955$, is 10% (i.e., 10% of the time, the output loss is greater than 4.5 percent).
- With more competition (i.e., ε higher), the losses are greater.
  - with higher elasticity of demand, given movements in inflation imply much greater substitution away from high priced items, thus greater misallocation (caveat: this intuition is incomplete since with greater ε the consequences of a given amount of misallocation are smaller).
- Distortions with $\gamma = 1/2$ are roughly twice the size of distortions in standard case, $\gamma = 1$.
  - To see this, note

$$1 - \chi_t \simeq 1 - (p^*)^{\frac{1}{\gamma}} \overbrace{\simeq}^{\text{Taylor series expansion about } p^*=1} \frac{1}{\gamma}(1 - p^*).$$

Comparison of Steady State and Dynamic Costs of Inflation in 1970s

- Results

Table 1: Fraction of GDP Lost, $100(1-\chi)$, During High Inflation		
	No networks, $\gamma = 1$	Networks, $\gamma = 2$
Steady state lost output	2.41 (3.92)*	4.76 (7.68)
Mean, 1972Q1-1982Q4	3.13 (5.22)	6.26 (10.44)
Note * number not in parentheses - markup of 20 percent (i.e., $\varepsilon = 6$)		
number in parentheses - markup of 15 percent. (i.e., $\varepsilon = 7.7$)		

- Evidently, distortions increase rapidly in inflation,

$$E\left[distortion\left(\text{inflation}\right)\right] > distortion\left(E\text{inflation}\right)$$

Next: RBC versus Sticky Price Equilibrium Conditions

- Two versions of the model:
  - *sticky price version of the model* : $\theta, \psi > 0$, free to choose ν somehow.
  - *RBC version of the model:* flexible prices, $\theta = 0$; no working capital, $\psi = 0$; no monopoly power, $\varepsilon = +\infty$; no subsidy to intermediate good firms, $\nu = 0$.
- Sticky price equilibrium incomplete.
  - One equation short because real allocations in private economy co-determined along with the nominal quantities.
  - Impossible to think about equilibrium allocations without thinking about monetary policy.
- RBC version of model exhibits *classical dichotomy*.
  - real allocations in flexible price model are determined and monetary policy only delivers inflation and the nominal interest rate, things that have no impact on welfare.
- Evaluate distortions in steady state.

Summarizing the Equilibrium Conditions

- Break up the equilibrium conditions into three sets:
  1. Conditions (1)-(4) for prices: $K_t, F_t, \bar{\pi}_t, p_t^*, s_t$
  2. Conditions (6)-(10) for: $C_t, Y_t, N_t, I_t, \mu_t$
  3. Conditions (5) and (11) for R_t and χ_t.
- Consider
  - conditions for the sticky price case.
  - conditions for RBC case: equilibrium allocations are *first best*, they are what a benevolent planner would choose.

First set of Equilibrium Conditions

$$K_t = \frac{\varepsilon}{\varepsilon - 1}\frac{Y_t}{C_t}s_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \text{ (1)}$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \text{ (2)}$$

$$\frac{K_t}{F_t} = \left[\frac{1 - \theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \quad \text{(3)}$$

$$p_t^* = \left[(1-\theta)\left(\frac{1 - \theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1} \quad \text{(4)}$$

- RBC case $(\varepsilon = +\infty, \nu = \theta = 0)$: (i) zero price dispersion and (ii) everyone sets price equal to marginal cost $(\varepsilon / (\varepsilon - 1) = 1)$:

$$p_t^* = 1,\ s_t = 1,\ K_t = F_t = C_t / Y_t, \text{ no restriction on } \bar{\pi}_t$$

Second Set of Equilibrium Conditions

- Equations:

$$
\begin{aligned}
Y_t &= p_t^* A_t N_t^{\gamma} I_t^{1-\gamma} \text{ (6), } C_t + I_t = Y_t \text{ (7), } I_t = \mu_t \frac{Y_t}{p_t^*} \text{ (8)} \\
s_t &= (1-\nu)(1-\psi+\psi R_t)\left(\frac{1}{1-\gamma}\right)^{1-\gamma} \\
&\quad \times \left(\frac{1}{\gamma} \overbrace{\exp(\tau_t) C_t N_t^{\varphi}}^{\text{used household Euler equation to substitute out } W_t/P_t}\right)^{\gamma} \frac{1}{A_t} \\
\mu_t &= \frac{(1-\gamma) s_t}{(1-\nu)(1-\psi+\psi R_t)} \text{ (10),}
\end{aligned}
$$

Second Set of Equilibrium Conditions, RBC Case

- Suppose $\nu = \theta = \psi = 0$, $\varepsilon = +\infty$:

$$
\begin{aligned}
1 &= \left(\frac{1}{1-\gamma}\right)^{1-\gamma} \left(\frac{1}{\gamma} \exp\left(\tau_t\right) C_t N_t^{\varphi}\right)^{\gamma} \frac{1}{A_t} \text{ (9)} \\
\mu_t &= 1-\gamma \text{ (10)}, \\
Y_t &= \left[A_t \left(1-\gamma\right)^{1-\gamma}\right]^{\frac{1}{\gamma}} N_t \text{ (6)}, \\
C_t &= \overbrace{\left[A_t \gamma^{\gamma} \left(1-\gamma\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}}^{\tilde{A}_t} N_t \text{ (6,7,8)}
\end{aligned}
$$

- RBC practice of setting $\gamma = 1$ and backing out technology from aggregate production function involves no error if true $\gamma = 1/2$.

Second Set of Equilibrium Conditions, RBC Case, cnt'd

- Suppose $\nu = \theta = \psi = 0$, $\varepsilon = +\infty$.
- Solve equation (9) for cost of working, $\exp(\tau_t) C_t N_t^{\varphi}$,

$$\overbrace{\exp(\tau_t) C_t N_t^{\varphi}}^{\text{cost of working}} = \overbrace{\left[A_t (\gamma)^{\gamma} (1-\gamma)^{1-\gamma}\right]^{\frac{1}{\gamma}}}^{\text{benefit of working}} \quad (9)$$

- Conditions (6,7,8,10) and (9) imply that first-best levels of consumption and employment occur:

$$\begin{aligned} N_t &= \exp\left(-\frac{\tau_t}{1+\varphi}\right) \\ C_t(&= GDP_t) = \left[A_t (\gamma)^{\gamma} (1-\gamma)^{1-\gamma}\right]^{\frac{1}{\gamma}} \exp\left(-\frac{\tau_t}{1+\varphi}\right) \end{aligned}$$

Third Set of Equilibrium Conditions

- Allocative distortion:

$$\chi_t = \left(p_t^* \left(\frac{1 - \frac{\mu_t}{p_t^*}}{\gamma} \right)^{\gamma} \left(\frac{\frac{\mu_t}{p_t^*}}{1-\gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} \quad (11)$$

in RBC case, i.e., $\nu = \theta = \psi = 0$, $\varepsilon = +\infty$,

$$\chi_t = 1, \text{ for all } t.$$

- Intertemporal equation

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5)$$

Third Set of Equil. Cond., RBC Case

- Absent uncertainty, $R_t/\bar{\pi}_{t+1}$ determined uniquely from C_t :

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}}.$$

  – With uncertainty, household intertemporal condition simply places a single linear restriction across all the period $t+1$ values for $R_t/\bar{\pi}_{t+1}$ that are possible given period t.

- The real interest rate, $\tilde{r}_t$, on a risk free one-period bond that pays in $t+1$ *is* uniquely determined:

$$\frac{1}{C_t} = \tilde{r}_t \beta E_t \frac{1}{C_{t+1}}.$$

- By no-arbitrage, only the following weighted average of $R_t/\bar{\pi}_{t+1}$ across period $t+1$ states of nature is determined:

$$\tilde{r}_t = \frac{E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}}}{E_t \frac{1}{C_{t+1}}} = E_t \frac{\frac{1}{C_{t+1}}}{E_t \frac{1}{C_{t+1}}} \frac{R_t}{\bar{\pi}_{t+1}} = E_t \nu_{t+1} \frac{R_t}{\bar{\pi}_{t+1}}.$$

Classical Dichotomy

- Exhibited by RBC version of model ($\nu = \theta = \psi = 0$, $\varepsilon = +\infty$.)
  - Real variables determined independent of monetary policy.
  - The things that matter - consumption, employment - are first best and there is no constructive role for monetary policy.
  - Monetary policy irrelevant. Money is a veil, is neutral.
- Sticky price version of model.
  - Now, all aspects of the system are interrelated and jointly determined.
  - Whole system depends on the nature of monetary policy.
  - Within the context of a market system, monetary policy has an essential role as a potential 'lubricant', to help the economy to get as close as possible to the first best.
  - Monetary policy:
    - has the potential to do a good job.
    - or, if mismanaged, could get very bad outcomes.

- Monetary Policy Rule

$$R_t/R = (R_{t-1}/R)^{\rho} \exp\left[(1-\rho)\,\phi_{\pi}(\bar{\pi}_t - \bar{\pi}) + u_t\right]$$

- Smoothing parameter: ρ.
  - Bigger is ρ the more persistent are policy-induced changes in the interest rate.
- Monetary policy shock: u_t.

Next: Steady State

- Need steady state for model solution methods.
- We have:

$$
\begin{aligned}
L &= \frac{\text{marginal utility cost of working}}{\text{marginal product of working}} = \frac{CN^{\varphi}}{\chi \tilde{A}} \\
TFP &= \chi \tilde{A}.
\end{aligned}
$$

- Chari-Kehoe-McGrattan (Econometrica, 'Business Cycle Accounting'):
  - $1-\chi$ is the 'efficiency wedge', $1-L$ is the 'labor wedge'.
  - First best: wedges are zero, $L=1$, $\chi=1$.
- First best in steady state can be accomplished by suitable choice of $\bar{\pi}$ and ν.

Steady State

- Equilibrium conditions (1), (2), (3), (4), (5) imply:

$$R = \frac{\bar{\pi}}{\beta},\ K_f \equiv \frac{K}{F} = \left[\frac{1-\theta}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\right]^{\frac{1}{\varepsilon-1}},$$

$$s = K_f \frac{\varepsilon-1}{\varepsilon}\frac{1-\beta\theta\bar{\pi}^{\varepsilon}}{1-\beta\theta\bar{\pi}^{\varepsilon-1}},\ p^* = \frac{1-\theta\bar{\pi}^{\varepsilon}}{1-\theta}\left(\frac{1-\theta}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Equilibrium condition (10) implies steady state materials to gross output ratio:

$$\frac{\mu}{p^*} = \frac{(1-\gamma)\, s/p^*}{(1-\nu)\,(1-\psi+\psi R)},\ (+)$$

Steady State

- Let ν^* be defined by,

$$\frac{\mu}{p^*} = (1-\gamma)\,\frac{1-\nu^*}{1-\nu},\ (++)$$

  so ν^* is the value of the subsidy that puts steady state materials-to-cost ratio to first-best level.

- Solving for ν^* :

$$1-\nu^* = \frac{\varepsilon-1}{(1-\psi+\psi R)\,\varepsilon}\,\frac{1-\beta\theta\bar{\pi}^{\varepsilon}}{1-\theta\bar{\pi}^{\varepsilon}}\,\frac{1-\theta\bar{\pi}^{(\varepsilon-1)}}{1-\beta\theta\bar{\pi}^{(\varepsilon-1)}}.$$

Steady State

- From (11),

$$TFP = \overbrace{\left(p^* \left(\frac{1-(1-\gamma)\frac{1-\nu^*}{1-\nu}}{\gamma}\right)^{\gamma} \left(\frac{1-\nu^*}{1-\nu}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}}^{\chi}$$
$$\times \overbrace{\left(\gamma^{\gamma}(1-\gamma)^{1-\gamma}\right)^{\frac{1}{\gamma}}}^{\tilde{A}}$$

- Thus,
  - when $\nu = \nu^*$, $\chi = \chi^{\pi} = (p^*)^{1/\gamma}$, $\chi^{\text{monopoly}} = 1$.
  - if also, $\bar{\pi} = 1$, then $\chi = 1$ and TFP at its first best level.

Steady State

- Combining (+) and (++),

$$s = (1 - \psi + \psi R)(1 - \nu^*) p^*.$$

- Use this to substitute out for s in steady state version of (9),

$$\frac{1-\nu^*}{1-\nu} p^* (1-\gamma)^{1-\gamma} (\gamma)^\gamma = (CN^\varphi)^\gamma,$$

or, after rearranging:

$$L = \frac{\gamma}{\gamma + \frac{\nu^* - \nu}{1-\nu^*}},$$

- So, labor wedge set to zero (first-best) when $\nu = \nu^*$.

Steady State

- Solve for N using expression for L and $C = \chi\tilde{A}N$:

$$
\begin{aligned}
N &= \left[\frac{\gamma}{\gamma + \frac{\nu^* - \nu}{1-\nu^*}}\right]^{\frac{1}{1+\varphi}}, \; C = \chi\tilde{A}N, \; Y = \frac{C}{\gamma} \\
F &= \frac{1/\gamma}{1 - \beta\theta\bar{\pi}^{\varepsilon-1}}, \; K = K_f \times F.
\end{aligned}
$$

Networks Cut the Slope of the Phillips Curve in Half

- Networks promote strategic complementarity in price setting.
- Phillips curve requires concept of *output gap*.
  - the log deviation of equilibrium output from a benchmark level of output.
  - three possible benchmarks include: (i) output in the Ramsey equilibrium, (ii) the equilibrium when prices are flexible and (iii) the first best equilibrium, when output is chosen by a benevolent planner.
  - When $\psi = 0$ and $\nu = \nu^*$ then (i)-(iii) identical.
  - When $\psi > 0$ (i) and (ii) complicated and so I just go with (iii).
- Derive Phillips Curve
  - Classic Phillips curve depends on absence of price distortions in steady state.

First Best Output

- First best equilibrium solves

$$\max_{C_t,N_t} u\left(C_t\right) - \exp\left(\tau_t\right)\frac{N_t^{1+\varphi}}{1+\varphi},$$

subject to the maximal consumption that can be produced by allocating resources efficiently across sectors and between materials and value-added:

$$C_t = \left(A_t \gamma^{\gamma}\left(1-\gamma\right)^{1-\gamma}\right)^{\frac{1}{\gamma}} N_t.$$

- Solution:

$$\begin{aligned} C_t^* &= \left(A_t \gamma^{\gamma}\left(1-\gamma\right)^{1-\gamma}\right)^{\frac{1}{\gamma}} \exp\left(-\frac{\tau_t}{1+\varphi}\right), \\ N_t^* &= \exp\left(-\frac{\tau_t}{1+\varphi}\right). \end{aligned}$$

Output Gap

$$X_t = \frac{C_t}{C_t^*}.$$

The log deviation of output gap from steady state:

$$\begin{aligned} x_t &\equiv \hat{X}_t = \hat{C}_t - \hat{C}_t^* \\ &= \hat{C}_t - \left(\frac{1}{\gamma}\hat{A}_t - \frac{\tau_t}{1+\varphi}\right), \end{aligned}$$

where

$$\hat{x}_t = \frac{X_t - X}{X} = \log\left(\frac{X_t}{X}\right),$$

for X_t sufficiently close to X.

Phillips Curve

- Linearizing (1), (2) and (3), about steady state,

$$\hat{K}_t = \left(1-\beta\theta\bar{\pi}^{\varepsilon}\right)\left[\hat{Y}_t + \hat{s}_t - \hat{C}_t\right] + \beta\theta\bar{\pi}^{\varepsilon}E_t\left(\varepsilon\hat{\bar{\pi}}_{t+1} + \hat{K}_{t+1}\right) \text{ (a)}$$

$$\begin{aligned}\hat{F}_t &= \left(1-\beta\theta\bar{\pi}^{\varepsilon-1}\right)\left(\hat{Y}_t - \hat{C}_t\right) \\ &\quad +\beta\theta\bar{\pi}^{\varepsilon-1}E_t\left(\left(\varepsilon-1\right)\hat{\bar{\pi}}_{t+1} + \hat{F}_{t+1}\right) \text{ (b)}\end{aligned}$$

$$\hat{K}_t = \hat{F}_t + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\hat{\bar{\pi}}_t. \text{ (c)}$$

- Substitute out for $\hat{K}_t$ in (a) using (c) and then substitute out for $\hat{F}_t$ from (b) to obtain the equation on the next slide.

Phillips Curve

- Performing the substitutions described on the previous slide:

$$\left(1-\beta\theta\bar{\pi}^{\varepsilon-1}\right)\left(\hat{Y}_t-\hat{C}_t\right)+\beta\theta\bar{\pi}^{\varepsilon-1}E_t\left(\left(\varepsilon-1\right)\widehat{\bar{\pi}}_{t+1}+\hat{F}_{t+1}\right)$$
$$+\frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\widehat{\bar{\pi}}_t=\left(1-\beta\theta\bar{\pi}^{\varepsilon}\right)\left[\hat{Y}_t+\hat{s}_t-\hat{C}_t\right]$$
$$+\beta\theta\bar{\pi}^{\varepsilon}E_t\left(\varepsilon\widehat{\bar{\pi}}_{t+1}+\hat{F}_{t+1}+\frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\widehat{\bar{\pi}}_{t+1}\right).$$

Phillips Curve

- Collecting terms,

$$\overbrace{\widehat{\bar{\pi}}_t = \frac{\left(1-\theta\bar{\pi}^{(\varepsilon-1)}\right)\left(1-\beta\theta\bar{\pi}^{\varepsilon}\right)}{\theta\bar{\pi}^{(\varepsilon-1)}}\hat{s}_t + \beta E_t \widehat{\bar{\pi}}_{t+1}}^{\text{familiar Phillips curve}}$$

$$+\left(1-\bar{\pi}\right)\left(1-\theta\bar{\pi}^{(\varepsilon-1)}\right)\beta$$

$$\times\left[\hat{Y}_t - \hat{C}_t + E_t\left(\hat{F}_{t+1} + \left(\varepsilon + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\right)\widehat{\bar{\pi}}_{t+1}\right)\right].$$

- Don't actually get standard Phillips curve unless $\bar{\pi}=1$.
  - More generally, get standard Phillips curve as long as there are no price distortions in steady state.
- Going for the Phillips curve in terms of the output gap.

Linearized Marginal Cost

- Equation (9):

$$
\begin{aligned}
s_t &= (1-\nu)\left(1-\psi+\psi R_t\right)\left(\frac{1}{1-\gamma}\right)^{1-\gamma} \\
&\quad \times \left(\frac{1}{\gamma}\exp\left(\tau_t\right)C_t N_t^{\varphi}\right)^{\gamma}\frac{1}{A_t}
\end{aligned}
$$

- Using $C_t = \tilde{A}_t \chi_t N_t$,

$$
\begin{aligned}
s_t &= (1-\nu)\left(1-\psi+\psi R_t\right)\left(\frac{1}{1-\gamma}\right)^{1-\gamma} \\
&\quad \times \left(\frac{1}{\gamma}\exp\left(\tau_t\right)C_t^{1+\varphi}\right)^{\gamma}\frac{\left(\tilde{A}_t \chi_t\right)^{-\gamma\varphi}}{A_t}.
\end{aligned}
$$

- Linearizing:

$$
\hat{s}_t = \frac{\psi R}{\left(1-\psi+\psi R\right)}\hat{R}_t + \left(1+\varphi\right)\gamma\hat{C}_t + \gamma\tau_t - \varphi\gamma\widehat{\left(\tilde{A}_t\chi_t\right)} - \hat{A}_t
$$

Linearized Marginal Cost

$$
\begin{aligned}
-\varphi\gamma\widehat{\left(\tilde{A}_t\chi_t\right)} - \hat{A}_t &= -\varphi\gamma \overbrace{\widehat{\tilde{A}}_t}^{=\frac{1}{\gamma}\hat{A}_t} - \varphi\gamma\hat{\chi}_t - \hat{A}_t \\
&= -\left(1+\varphi\right)\hat{A}_t - \varphi\gamma\hat{\chi}_t
\end{aligned}
$$

- Adopt the standard New Keynesian assumptions: $\nu = \nu^*$, $\psi = 0$, $\bar{\pi} = 1$, so that $\hat{\chi}_t = 0$ and

$$
\hat{s}_t = \left(1+\varphi\right)\gamma\left[\overbrace{\hat{C}_t - \left(\frac{1}{\gamma}\hat{A}_t - \frac{\tau_t}{1+\varphi}\right)}^{x_t}\right]
$$

- Conclude that the Phillips curve is:

$$
\hat{\bar{\pi}}_t = \frac{\left(1-\theta\right)\left(1-\beta\theta\right)}{\theta}\left(1+\varphi\right)\gamma x_t + \beta E_t\hat{\bar{\pi}}_{t+1}
$$

with slope cut in half by networks with $\gamma = 1/2$.

Conclusion

- Networks alter the New Keynesian model's implications for inflation.
  - Doubles the cost of inflation.
  - Phillips curve is flatter because of strategic complementarities (when there are price frictions, this makes materials prices inertial which makes marginal costs inertial, which reduces firms' interest in changing prices).
- For the result on the Taylor principle, see my 2011 handbook chapter and Christiano (2015).
  - When the smoothing parameter in Taylor rule is set to zero and $\psi = 1$, then the model has indeterminacy, even when the coefficient on inflation is 1.5.
  - So, the likelihood of the Taylor principle breaking down goes up when γ is reduced, consistent with intuition.
  - When the smoothing parameter is at its empirically plausible value of 0.8, then the solution of the model does not display indeterminacy.

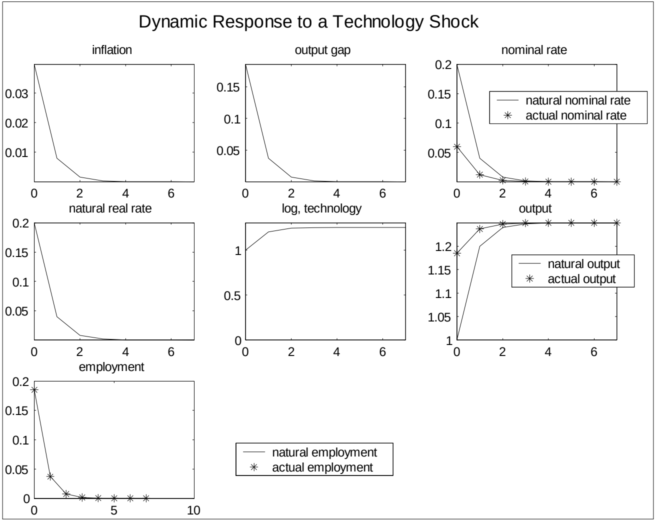
Simulation

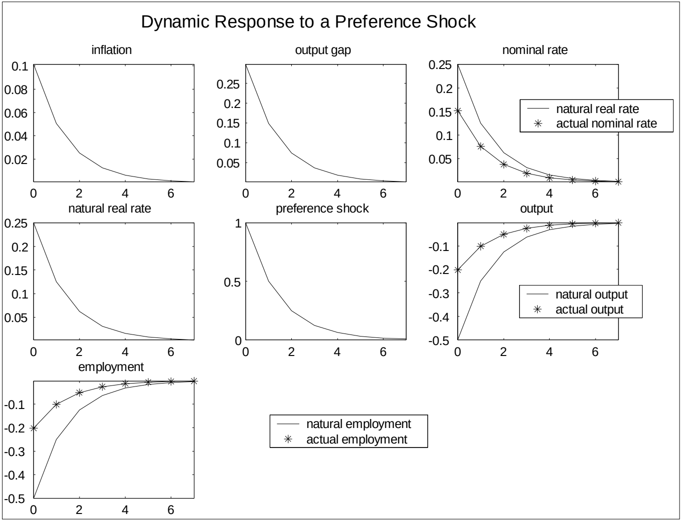
- Set

$$\begin{aligned}
\phi_x &= 0,\ \phi_{\bar{\pi}} = 1.5,\ \beta = 0.99,\ \varphi = 1,\ \rho = 0.2, \\
\theta &= 3/4,\ \alpha = 0,\ \lambda = 0.5,\ \gamma = 1,\ \nu = \nu^*,\ \bar{\pi} = 1, \\
a_t &= \rho a_{t-1} + \varepsilon_t^a,\ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^\tau.
\end{aligned}$$

- Solve the model by first order perturbation about steady state.
- Display the response to ε_t^a and ε_t^τ.

$\phi_x = 0,\ \phi_\pi = 1.5,\ \beta = 0.99,\ \varphi = 1,\ \rho = 0.2,\ \theta = 0.75,\ \alpha = 0,\ \delta = 0.2,\ \lambda = 0.5.$





Dynamic Response to a Preference Shock
inflation
output gap
nominal rate
natural real rate
actual nominal rate
natural real rate
preference shock
output
natural output
actual output
employment
natural employment
actual employment

Why Is Output Inefficiently High or Low Sometimes?

- Brief answer: the Taylor rule sets the wrong interest rate (should be the natural rate).
- Households equate costs with the *private* benefit of working, W_t:

$$\frac{-u_{N,t}}{u_{c,t}} = \exp(\tau_t) C_t N_t^{\varphi} = \frac{W_t}{P_t}.$$

  - So, one reason efficiency may not occur is if W_t / P_t does not correspond to the actual marginal product of labor (the other possibility is that there is an inefficiency wedge).
- The relationship between W_t / P_t and labor productivity may be understood by studying the markup of price over marginal cost.

Price over Marginal Cost (Markup)

- i^{th} firm sets $P_{i,t}$ as a markup, $\mu_{i,t}$, over marginal cost

$$P_{i,t} = \overbrace{\mu_{i,t}}^{\text{markup}} \times \frac{\overbrace{(1-\nu)\, W_t}^{\text{marginal cost}}}{e^{a_t}},$$

  where we have been setting $1 - \nu = \frac{\varepsilon - 1}{\varepsilon}$.

- In the flexible price version of the model the markup is trivial, $\mu_{i,t} = \frac{\varepsilon - 1}{\varepsilon}$ so

$$P_{i,t} = P_t = \frac{W_t}{e^{a_t}},$$

  and W_t / P_t corresponds to the marginal product of labor.

Price over Marginal Cost (Markup)

- In the sticky price version of the model, the markup is more complicated.
  - firms currently setting prices, do so to get the markup to be $\varepsilon/\left(\varepsilon-1\right)$ on average in the current and future periods (see earlier discussion)
  - for firms not able to set prices in the current period, the markup is

$$\mu_{i,t} = \frac{P_{i,t-1}}{\frac{(1-\nu)W_t}{e^{a_t}}}.$$

    - for these firms the markup moves inversely with a shock to marginal cost.
- Need to look at some aggregate of all markups.

Price over Marginal Cost (Markup)

- Weighted average markup, μ_t:

$$\mu_t \equiv \left(\int_0^1 \mu_{i,t}^{(1-\varepsilon)} di\right)^{\frac{1}{1-\varepsilon}} = \frac{P_t}{\frac{(1-\nu)W_t}{e^{a_t}}} = \frac{1}{\frac{1-\nu}{e^{a_t}} e^{\tau_t} C_t N_t^{\varphi}}$$

$$= \frac{1}{(1-\nu)\, e^{\tau_t} p_t^* N_t^{1+\varphi}} = \frac{1}{(1-\nu)\, p_t^*} \left(\frac{e^{-\frac{\tau_t}{1+\varphi}}}{N_t}\right)^{1+\varphi}.$$

- Consider the output gap:

$$X_t = \frac{C_t}{e^{a_t - \tau_t/(1+\varphi)}} = \frac{p_t^* e^{a_t} N_t}{e^{a_t - \tau_t/(1+\varphi)}} = p_t^* \frac{N_t}{e^{-\tau_t/(1+\varphi)}}$$

- Use this to substitute into the markup

$$\mu_t = \frac{(p_t^*)^{\varphi}}{(1-\nu)} X_t^{-(1+\varphi)}.$$

μ_t moves inversely with the output gap.

Price over Marginal Cost (Markup)

- The preceding implies:

$$\frac{-u_{N,t}}{u_{c,t}} \overbrace{=}^{\text{source of efficiency}} \frac{W_t}{P_t} = \frac{e^{a_t}}{(1-\nu)\,\mu_t} = e^{a_t}\,(p_t^*)^{-\varphi}\,X_t^{1+\varphi}.$$

- Suppose $p_t^* = 1$.
- When the output gap is high, $X > 0$, then the markup is low and the real wage *exceeds* e^a.
  - There is too much employment in this case because the private benefit to the workers of working exceeds the actual benefits (the difference comes out of lump sum profits).
- Why are firms willing to produce with a low markup? Because on average it is high $(\mu = \varepsilon / (\varepsilon - 1))$ and so cutting it (not too much!) allows them to still get some profits out of the workers.
- Interpretation: model implies markups are countercyclical.
  - Nekarda and Ramey ('The Cyclical Behavior of the Price-Cost Markup', UCSD 2013) argue that the evidence does not support this.

Wrap Up

- Suppose monetary policy puts the interest rate below the natural rate, driving up the output gap.
- The low interest rate gives people an incentive to spend more.
- Marginal cost rises and since prices sticky, markups

$$\mu_t = \frac{P_t}{MC_t},$$

  go down $\left(MC_t = (1-\nu)\, W_t / e^{a_t}\right)$.
- Low markup means high wage when there is a monetary shock, encouraging more work.
- Profits are reduced, but presumably remain positive:

$$\begin{aligned}
\text{profits} &= P_{i,t} Y_{i,t} - (1-\nu)\, W_t N_{i,t} \\
&= P_{i,t} Y_{i,t} - e^{a_t} MC_t \times N_{i,t} \\
&= \left[P_{i,t} - MC_t\right] Y_{i,t} \\
&= \left[\mu_{i,t} - 1\right] MC_t \times Y_{i,t}
\end{aligned}$$