Log Linearized Phillips Curve for Simple New Keynesian Model with No Capital, Networks, or Working Capital

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Equilibrium Conditions

$$K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \ (1), \ F_t = 1 + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \ (2)$$

$$\frac{K_t}{F_t} = \left[\frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} \ (3),$$

$$p_t^* = \left[(1 - \theta) \left(\frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \ (4)$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \ (5), \ Y_t = \exp(a_t) N_t \ (6)$$

Steady State

Conditional on $\bar{\pi}$

$$K = \frac{\frac{\varepsilon}{\varepsilon-1}s}{1-\beta\theta\bar{\pi}^{\varepsilon}} \ (1), \ F = \frac{1}{1-\beta\theta\bar{\pi}^{\varepsilon-1}} \ (2)$$

$$\frac{K}{F} = \left[\frac{1-\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \ (3),$$

$$p^* = \left[(1-\theta)\left(\frac{1-\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}^{\varepsilon}}{p^*}\right]^{-1} \ (4)$$

$$1 = \beta\exp(-\Delta a)\frac{R}{\bar{\pi}} \ (5)$$

Log Linearization

- Hat notation:

$$\hat{x}_t = \frac{dx_t}{x} = \frac{x_t - x}{x} \rightarrow dx_t = \hat{x}_t x.$$

- Log linearize equation (1) about steady state:

$$
\begin{aligned}
K_t &= \frac{\varepsilon}{\varepsilon - 1} s_t + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \\
\hat{K}_t K &= \frac{\varepsilon}{\varepsilon - 1} \hat{s}_t s + \beta \theta \varepsilon \bar{\pi}^{\varepsilon - 1} \widehat{\bar{\pi}}_{t+1} \bar{\pi} K + \beta \theta \bar{\pi}^{\varepsilon} \hat{K}_{t+1} K \\
\hat{K}_t &= \frac{\varepsilon}{\varepsilon - 1} \hat{s}_t \overbrace{\frac{s}{K}}^{= \frac{1 - \beta \theta \bar{\pi}^{\varepsilon}}{\frac{\varepsilon}{\varepsilon - 1}}} + \beta \theta \bar{\pi}^{\varepsilon} \left(\varepsilon \widehat{\bar{\pi}}_{t+1} + \hat{K}_{t+1} \right) \\
\hat{K}_t &= \left(1 - \beta \theta \bar{\pi}^{\varepsilon} \right) \hat{s}_t + \beta \theta \bar{\pi}^{\varepsilon} \left(\varepsilon \widehat{\bar{\pi}}_{t+1} + \hat{K}_{t+1} \right)
\end{aligned}
$$

Phillips Curve

- Linearizing (1), (2) and (3), about steady state,

$$\hat{K}_t = \left(1 - \beta\theta\bar{\pi}^\varepsilon\right)\hat{s}_t + \beta\theta\bar{\pi}^\varepsilon E_t\left(\varepsilon\widehat{\bar{\pi}}_{t+1} + \hat{K}_{t+1}\right) \text{ (a)}$$

$$\hat{F}_t = \beta\theta\bar{\pi}^{\varepsilon-1} E_t\left((\varepsilon - 1)\widehat{\bar{\pi}}_{t+1} + \hat{F}_{t+1}\right) \text{ (b)}$$

$$\hat{K}_t = \hat{F}_t + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta\bar{\pi}^{(\varepsilon-1)}}\widehat{\bar{\pi}}_t. \text{ (c)}$$

- Substitute out for $\hat{K}_t$ in (a) using (c) and then substitute out for $\hat{F}_t$ from (b) to obtain the equation on the next slide.

Phillips Curve

- Performing the substitutions described on the previous slide:

$$\beta\theta\bar{\pi}^{\varepsilon-1}E_t\left((\varepsilon-1)\,\widehat{\bar{\pi}}_{t+1}+\hat{F}_{t+1}\right)$$

$$+\frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\widehat{\bar{\pi}}_t=(1-\beta\theta\bar{\pi}^{\varepsilon})\,\hat{s}_t$$

$$+\beta\theta\bar{\pi}^{\varepsilon}E_t\left(\varepsilon\widehat{\bar{\pi}}_{t+1}+\hat{F}_{t+1}+\frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\widehat{\bar{\pi}}_{t+1}\right).$$

Phillips Curve

- Collecting terms,

$$\overbrace{\widehat{\bar{\pi}}_t = \frac{\left(1 - \theta \bar{\pi}^{(\varepsilon-1)}\right)\left(1 - \beta \theta \bar{\pi}^{\varepsilon}\right)}{\theta \bar{\pi}^{(\varepsilon-1)}} \hat{s}_t + \beta E_t \widehat{\bar{\pi}}_{t+1}}^{\text{familiar Phillips curve}}$$

$$+ (1 - \bar{\pi})\left(1 - \theta \bar{\pi}^{(\varepsilon-1)}\right)\beta$$

$$\times E_t \left(\hat{F}_{t+1} + \left(\varepsilon + \frac{\theta \bar{\pi}^{(\varepsilon-1)}}{1 - \theta \bar{\pi}^{(\varepsilon-1)}}\right) \widehat{\bar{\pi}}_{t+1}\right).$$

- Don't actually get standard Phillips curve unless $\bar{\pi} = 1$.
  - More generally, get standard Phillips curve as long as there are no price distortions in steady state.
- Going for the Phillips curve in terms of the output gap.

Linearized Marginal Cost

- Real Marginal Cost:

$$
\begin{aligned}
s_t &= \frac{(1-\nu)\exp(\tau_t)\,C_t N_t^\varphi}{A_t} \\
&= (1-\nu)\exp(\tau_t)\left(\frac{C_t}{A_t}\right)^{1+\varphi} \\
&= (1-\nu)\exp(\tau_t)\left(\frac{C_t}{A_t\exp\left[-\frac{\tau_t}{1+\varphi}\right]}\right)^{1+\varphi} \\
&= (1-\nu)\,X_t^{1+\varphi},
\end{aligned}
$$

where

$$
X_t = \frac{\text{Actual consumption}}{\text{Ramsey consumption}} = \text{"output gap"}
$$

Phillips Curve in Terms of Output Gap

- Collecting terms,

$$\widehat{\bar{\pi}}_t = \frac{\left(1 - \theta\bar{\pi}^{(\varepsilon-1)}\right)\left(1 - \beta\theta\bar{\pi}^\varepsilon\right)}{\theta\bar{\pi}^{(\varepsilon-1)}} \left(1 + \varphi\right) x_t + \beta E_t \widehat{\bar{\pi}}_{t+1}$$

$$+ \left(1 - \bar{\pi}\right)\left(1 - \theta\bar{\pi}^{(\varepsilon-1)}\right)\beta$$

$$\times E_t \left(\hat{F}_{t+1} + \left(\varepsilon + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta\bar{\pi}^{(\varepsilon-1)}}\right)\widehat{\bar{\pi}}_{t+1}\right),$$

because

$$\hat{s}_t s = \left(1 - \nu\right)\left(1 + \varphi\right)X^{1+\varphi}x_t$$

$$\hat{s}_t = \left(1 + \varphi\right)x_t, \ x_t = \hat{X}_t = \log\left(X_t/X\right).$$