Two-period Version of Gertler-Kiyotaki Model
Risk spreads became extraordinarily high in late 2008, higher than seems easy to explain based on observed default risk.
Puzzle of Interest Rate Spreads

• Very high in late 2008, higher than seems explicable with default risk.

• Two explanations:
  – Liquidity: Kiyotaki-Moore/Moore
    • Banks with cash reluctant to use it to buy firm assets
    • Afraid they’ll need the cash themselves, and the resale market for firm assets would dry up.
    • Classic financial market multiple equilibrium phenomenon (Bagehot)
  – Fear of out-of-equilibrium default (Gertler-Kiradi, Gertler-Kiyotaki).
Two-period Version of GK Model

• Many identical households, each with a unit measure of members:
  – Some members are ‘bankers’
  – Some members are ‘workers’
  – Perfect insurance inside households...workers and bankers consume same amount!

• Period 0
  – Workers endowed with $y$ goods, household makes deposits in a bank
  – Bankers endowed with $N$ goods, take deposits and purchase securities from a firm.
  – Firm issues securities to finance capital used in production in period 1.

• Period 1
  – Household consumes earnings from deposits plus profits from banker.
  – Goods consumed are produced by the firm.
## Problem of the Household

<table>
<thead>
<tr>
<th>period 0</th>
<th>period 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Budget Constraint**

\[ c + d \leq y \quad C \leq R^d d + \pi \]

**Problem**

\[ \max_{d,c^h,c^H} [u(c) + \beta u(C)] \]

### Solution to Household Problem

\[
\frac{u'(c)}{\beta u'(C)} = R^d \quad c + \frac{C}{R^d} \leq y + \frac{\pi}{R^d}
\]

\[
u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad c = \frac{y + \frac{\pi}{R^d}}{1 + \left(\frac{\beta R^d}{R^d}\right)^{\frac{1}{\gamma}}}
\]
## Efficient Benchmark

### Problem of the Bank

<table>
<thead>
<tr>
<th>period 0</th>
<th>period 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>take deposits, $d$</td>
<td>pay $dR^d$ to households</td>
</tr>
<tr>
<td>buy securities, $s = N + d$</td>
<td>receive $sR^k$ from firms</td>
</tr>
</tbody>
</table>

**Problem:** \( \max_d [sR^k - R^d d] \)
Properties of Efficient Benchmark

**Equilibrium:** $R^d, c, C, d, \pi$

(i) household problem solved
(ii) bank problem solved
(iii) market clearing

• Properties:
  – Household faces true social rate of return on saving:
    \[ R^k = R^d \]
  – Equilibrium is ‘first best’, i.e., solves
    \[
    \max_{c, C, k} u(c) + \beta u(C) \\
    c + k \leq y + N, \quad C \leq kR^k
    \]
Friction

- bank combines deposits, $d$, with net worth, $N$, to purchase $N+d$ securities from firms.

- bank has two options:
  - (‘no-default’) wait until next period when $(N + d)R^k$ arrives and pay off depositors, $R^d d$, for profit:
    \[(N + d)R^k - R^d d\]
  - (‘default’) take $\theta(N + d)$ securities, leave banking forever, refuse to pay depositors and wait until next period when securities pay off:
    \[\theta(N + d)R^k\]
Incentive Constraint

• Bank will choose ‘no default’ iff

\[
\begin{align*}
\text{no default} & : (N + d)R^k - R^d \geq \theta(N + d)R^k \\
\text{default} & : (N + d)R^k - R^d \geq \theta(N + d)R^k
\end{align*}
\]

• Rewriting the above expression, the no default condition is equivalent to:

\[
(1 - \theta)(N + d)R^k \geq dR^d
\]

— i.e., banker doesn’t default if defaulting implies the return for depositors goes up.

• Default will never be observed, because depositors would never put their money in a bank that violates the deposit condition.
Collapse in Net Worth

- No default condition:
  \[
  (N + d)R^k - R^d d \geq \theta(N + d)R^k
  \]

- When condition is non-binding, then \( R^k = R^d \) and \( NR^k \geq \theta(N + d)R^k \).

- If \( N \) collapses, then constraint may be violated for \( d \) associated with \( R^d = R^k \)
  
  - Equilibrium requires lower value of \( d \)
  
  - Lower \( d \) requires a spread: \( R^d < R^k \)
  
  - Lower \( d \) is not efficient
Policy Implications

• Inject equity into banks
  – Government raises taxes and uses equity to become part-owner in the bank.
  – This directly increases intermediation, plus may allow additional deposits if households less afraid of default option with government in charge.

• Make direct loans to non-financial firms
  – Hard to interpret in the model, because unique advantage of banks doing the intermediation is not made explicit.

• Make loans to banks.
  – Government may have an advantage here because it’s harder for banks to ‘steal’ from the government
Directions

• Gertler-Kiyotaki place model inside dynamic DSGE model.

• Nominal frictions could be added to the model.