CSV Frictions in A Simple GE Setting

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Two-period Version of CSV Model

- Many identical households, each with a unit measure of members:
  - Members are ‘bankers’, ‘workers’ and ‘entrepreneurs’ Perfect insurance inside households…everyone consumes same amount.
- Period 1
  - Workers endowed with *y* goods, household makes deposits in a bank.
  - Entrepreneurs endowed with *k* goods, go to banks to get *B* (‘standard debt contract’) loans.
- Period 2
  - Household consumes earnings from deposits plus profits from entrepreneur.
  - Goods consumed are produced by the entrepreneur.

Lump sum taxes, to finance τ

Problem of the Household		
	Period 1	Period 2
	If positive, government subsidy	
budget constraint	$c + B \leq y$	$C \leq (1+\tau)(1+R)B + \pi - T$
problem:	$\max_{c,C}[u(c) + \beta u(C)]$	

Return on household deposits received from banks

Profits received from entrepreneurs, net of lump sum taxes paid to government.

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Solution to Household Problem	
$\frac{u'(c)}{\beta u'(C)} = (1+\tau)(1+R)$	$c + \frac{C}{(1+\tau)(1+R)} = y + \frac{\pi - T}{(1+\tau)(1+R)}$
$u(c) = \frac{c^{1-\alpha}-1}{1-\alpha},\ \alpha \geq 0$	$c = \frac{y + \frac{\pi-T}{(1+\tau)(1+R)}}{1+\frac{[\beta(1+R)(1+\tau)]^{\frac{1}{\alpha}}}{(1+\tau)(1+R)}}$

Entrepreneurs

- Endowed with net worth, *N,* in period 1.
- Get loan, *B*, from a bank.
- Use loans and net worth to build capital: *K=B+N*
- Rate of return:

$$(1+R^k)\omega K,\ \omega \sim iid,\ E\omega = 1$$

  – Observed at a cost by bank
  – Observed freely by entrepreneur's household
- Because of asymmetric information and CSV, receive a standard debt contract *(Z,B)* from banks.
- Entrepreneur optimizes expected profits, in exchange for perfect consumption insurance from the household (no problem with enforcement).

Entrepreneurs, cnt'd

- Average entrepreneurial profits:

$$\pi = \overbrace{(N+B)}^{K} (1+R^k)(1-\Gamma(\bar{\omega})),\ \bar{\omega} = \frac{ZB}{K(1+R^k)}$$

- Where: $\Gamma(\bar{\omega}) \equiv G(\bar{\omega}) + \bar{\omega}[1-F(\bar{\omega})],\ G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega dF(\omega)$
- Optimality condition:

$$\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})} = \frac{\frac{1+R^k}{1+R}[1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})]}{1-\frac{1+R^k}{1+R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$$

- Bank zero profit condition:

$$L = \frac{K}{N} = \frac{1}{1-\frac{1+R^k}{1+R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$$

Where did that bank zero profit condition come from??

$$\frac{K}{N} = \frac{1}{1 - \frac{1+R^k}{1+R}[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}$$

$$K - \frac{1+R^k}{1+R} K[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] = N$$

$$K - N = \frac{1+R^k}{1+R} K[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]$$

$$(1+R)\overbrace{(K-N)}^{B} = \overbrace{(1+R^k)K}^{\text{gross return on capital}} \times \overbrace{[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}^{\text{share going to banks, net of monitoring costs}}$$

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Period 2 Resource Constraint and Household Budget Constraint

$$C = (1+\tau)(1+R)B + \pi - T$$

gov't budget constraint: $T = \tau(1+R)B$

$$\overbrace{=}\quad (1+R)B + \pi$$

bank zero profit condition

$$\overbrace{=}\quad (1+R^k)K[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] + \pi$$

entrepreneur profits

$$\overbrace{=}\quad (1+R^k)K[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] + K(1+R^k)(1-\Gamma(\bar{\omega}))$$

$= (1+R^k)K[1 - \mu G(\bar{\omega})]$ ‘resource constraint’ (Walras’ law)

Equilibrium Equations

$C = c[\beta(1+\tau)(1+R)]^{\frac{1}{\alpha}}$	household first order condition
$C = (1+R^k)K[1-\mu G(\bar{\omega})]$	period 2 resource constraint
$c + B = y$	period 1 resource constraint
$\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})} = \frac{\frac{1+R^k}{1+R}[1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})]}{1-\frac{1+R^k}{1+R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$	contract efficiency condition
$\frac{K}{N} = \frac{1}{1-\frac{1+R^k}{1+R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$	bank zero profit condition
$K = N + B$	capital accumulation technology

exogenous variables: β, τ, R^k, N, y

six endogenous variables: $c, C, B, \bar{\omega}, K, R$

Policy Implications

- Recent Fed policy has had the effect of dramatically cutting the costs of funds to banks (looks like an increase in τ)
  - What does this model have to say about this?
- For this, need to compute optimal τ
- Key finding: in general $\tau \neq 0$
- Benevolent planner does not like the fact that in equilibrium households equate intertemporal marginal rate of substitution in consumption with *average* return on investment.

Optimal Interest Rate Subsidy

- Think of the economy as having *seven* endogenous variables (i.e., including τ)and six equations.

- System is underdetermined: many equilibria.

- Pick the best (i.e., Ramsey) equilibrium.

Ramsey Problem

$$\max_{\tau,c,C,B,\bar{\omega},R}\left[\frac{c^{1-\alpha}-1}{1-\alpha}+\beta\frac{C^{1-\alpha}-1}{1-\alpha}\right]$$

subject to:

$C = c[\beta(1+\tau)(1+R)]^{\frac{1}{\alpha}}$ non-binding, use to define τ

$C = (1+R^k)(N+B)[1-\mu G(\bar{\omega})]$ substitute out for C

$c + B = y$ substitute out for c

$\frac{N+B}{N} = \frac{1}{1-\frac{1+R^k}{1+R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$ solve for $\frac{1+R^k}{1+R}$

$\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})} = \frac{\frac{1+R^k}{1+R}[1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})]}{1-\frac{1+R^k}{1+R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$ $\bar{\omega}(B)$ after substituting out for $\frac{1+R^k}{1+R}$

Ramsey problem equivalent to:

$$\max_{B} u\left[\overbrace{y-B}^{c}\right]+\beta u\left[\overbrace{(1+R^k)(N+B)(1-\mu G(\bar{\omega}(B)))}^{C}\right]$$

Ramsey Problem

- First order condition:

$$u'[y-B] = \beta u'[(1+R^k)(N+B)(1-\mu G(\bar{\omega}(B)))] \times \{1-\mu[G(\bar{\omega}(B)) + G'(\bar{\omega}(B))\bar{\omega}'(B)]\},$$

- or

$$\frac{u'(c)}{\beta u'(C)} = \overbrace{(1+R^k)\{1-\mu[G(\bar{\omega}(B)) + G'(\bar{\omega}(B))\bar{\omega}'(B)]\}}^{\text{marginal return on investment, } B}$$

- In equilibrium without gov't intervention

$$\frac{u'(c)}{\beta u'(C)} = 1+R, \qquad 1+R = \frac{\overbrace{(1+R^k)(N+B)[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}^{\text{average return on investment}}}{B}$$

Bank zero profit condition

Average Versus Marginal Return

- Not obvious that they're different, simply by inspection.
- Average=marginal when $\mu = 0$
  - Contract efficiency condition implies $R = R^k$
  - Zero profit condition of banks non-binding and can be used to define $\bar{\omega}$
  - Then problem reduces to 'first best':
  -

$$\max u(c) + \beta u(C)$$

$$\text{subject to: } c + B \leq y,\ C \leq (1 + R^k)(N + B)$$

- In general, average $\neq$ marginal.

Numerical Example

- Parameter values:

$$\alpha = 1 \text{ (log utility)},\ \beta = 0.97,\ R^k = 3.95,$$

$$y = 3.11, \sigma = 0.44,\ k = 1/2,\ \mu = 0.2$$

	$\tau = 0$	Ramsey	First Best
Interest rate subsidy, 100τ	0	1.95	-

The optimal subsidy is positive, indicating that the marginal return on investment exceeds what the private economy equilibrium delivers.

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$100 \times R$	-5.4	-7.1	-

Risk free rate is negative.
The subsidy to household saving drives equilibrium rate down, reducing interest costs of banks.

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$100 \times \left[\frac{Z}{(1+R)} - 1 \right]$	7.7	12.5	-

Risk premium on entrepreneurs rises in Ramsey because loans are larger, so monitoring costs are greater when bankruptcy occurs.

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bankruptcy rate: $100 \times F(\bar{\omega})$	28.95	29.35	0

Bankruptcy rate rises under Ramsey policy because with higher loans, entrepreneurs need more luck to break even.

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bankruptcy rate: $100 \times F(\bar{\omega})$	28.95	29.35	0
Leverage ratio	3.42	3.45	-

Leverage ratio rises under optimal policy, a signal that this is not a good model environment if you think private economy left to its own, generates excessive leverage.

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$k + B$ (investment)	1.71	1.73	1.78

Ramsey tries to move the level of investment towards first best.
First best, however, is not desirable to Ramsey, unless Ramsey could get rid of bankruptcies.

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c	1.91	1.89	1.84
C	1.72	1.74	1.85

Concluding Observations

- Two period model crucial for developing intuition about workings of a model of financial frictions.
  - Model suggests interest rate subsidies a good idea.

- The asymmetric information model raises some questions that can only be addressed in a fully specified DSGE model, brought to data.

- Questions:
  - How should monetary authorities respond to widening interest rate spreads?
  - Are financial markets a source of business cycle shocks?
  - What should the magnitude of interest rate subsidies be?
  - Is there wisdom in injecting funds into banks and/or entrepreneurs in a time of crisis?