Two Financial Friction Models with Nonlinearities

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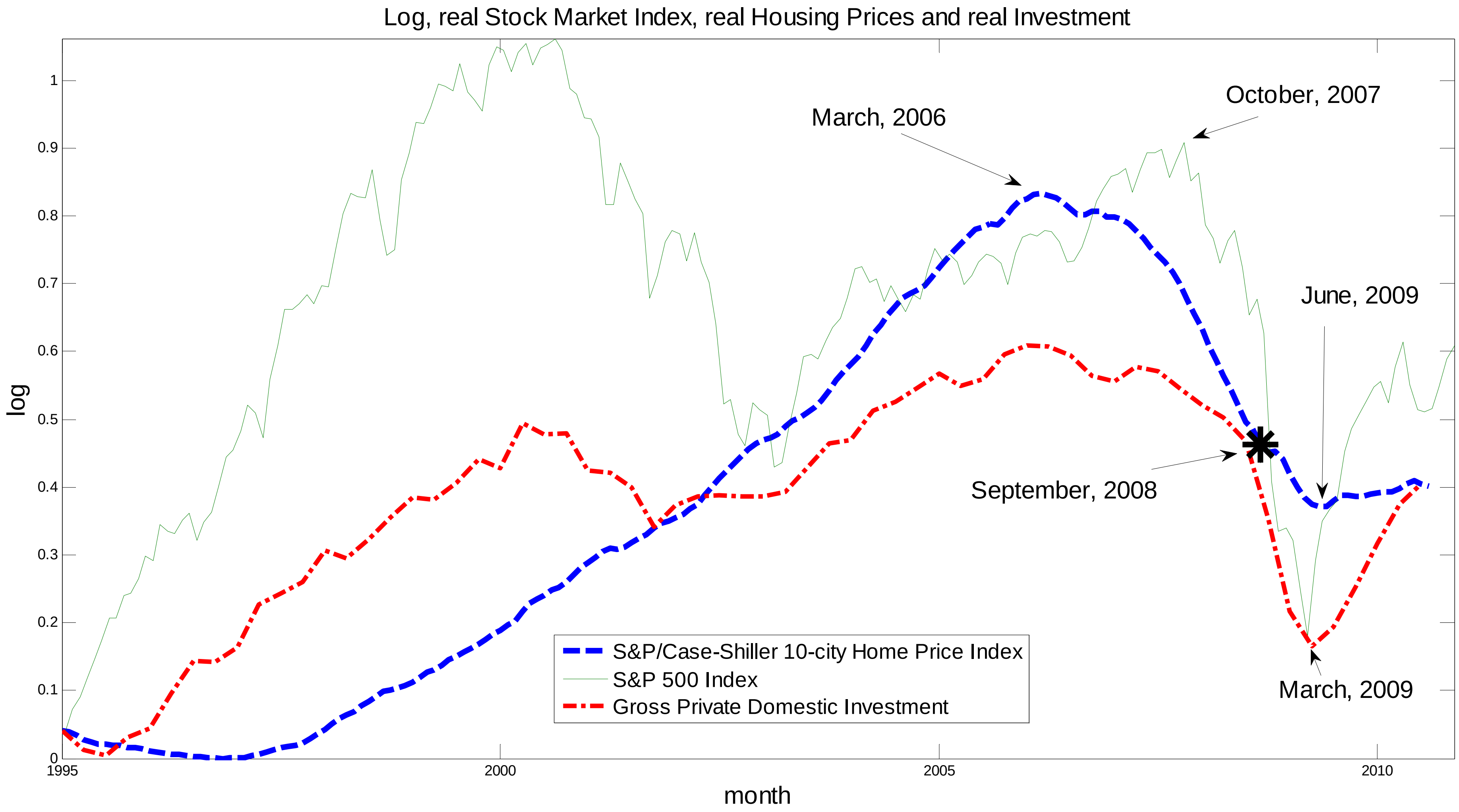
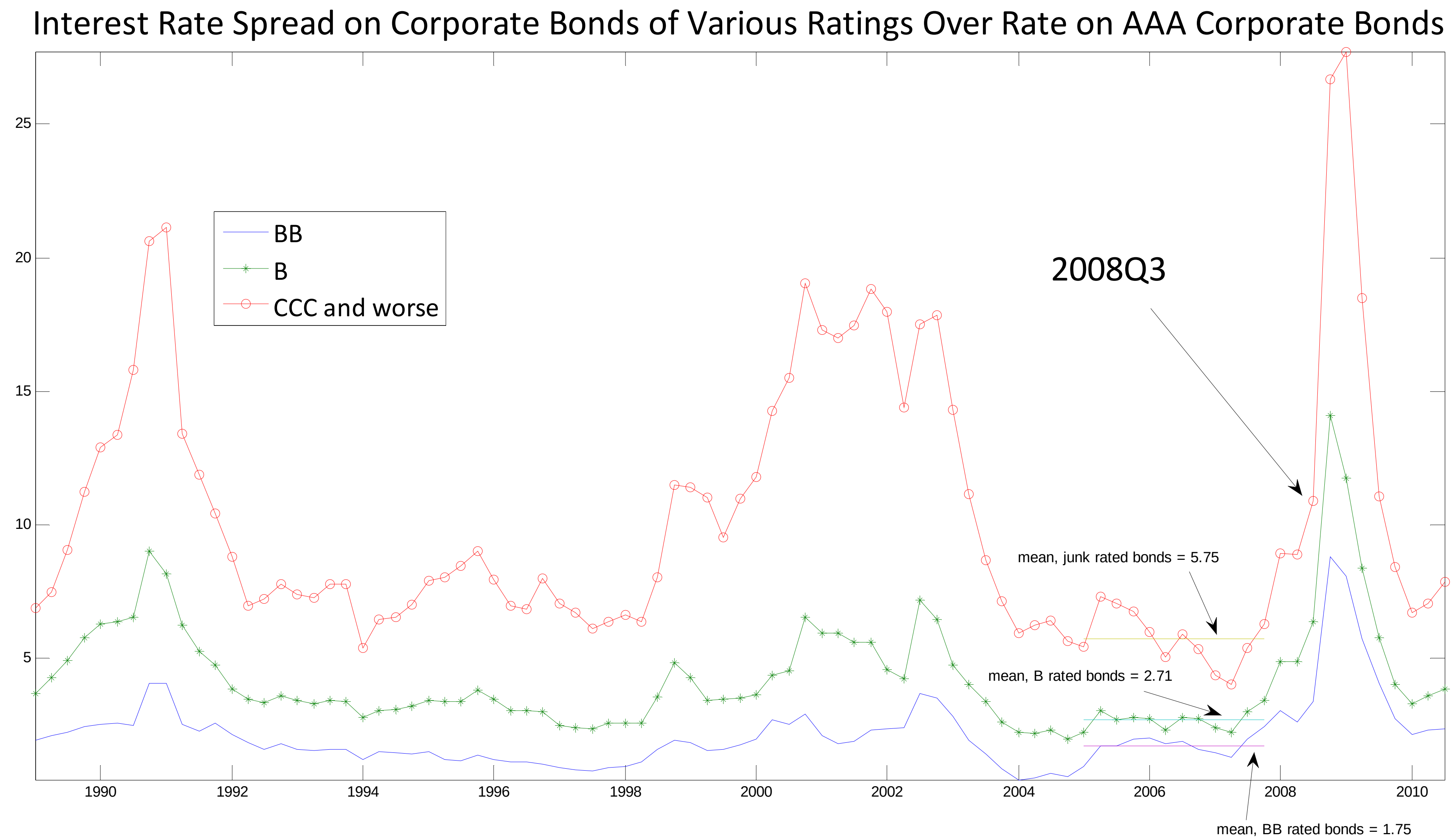
Motivation

- Beginning in 2007 and then accelerating in 2008:
  - Asset values collapsed.
  - Intermediation slowed and investment/output fell.
  - Interest rates spreads over what the US Treasury and highly safe private firms had to pay, jumped.
  - US central bank initiated unconventional measures (loans to financial and non-financial firms, very low interest rates for banks, etc.)
- In 2009 – the worst parts of 2007-2008 began to turn around.

Collapse in Asset Values and Investment

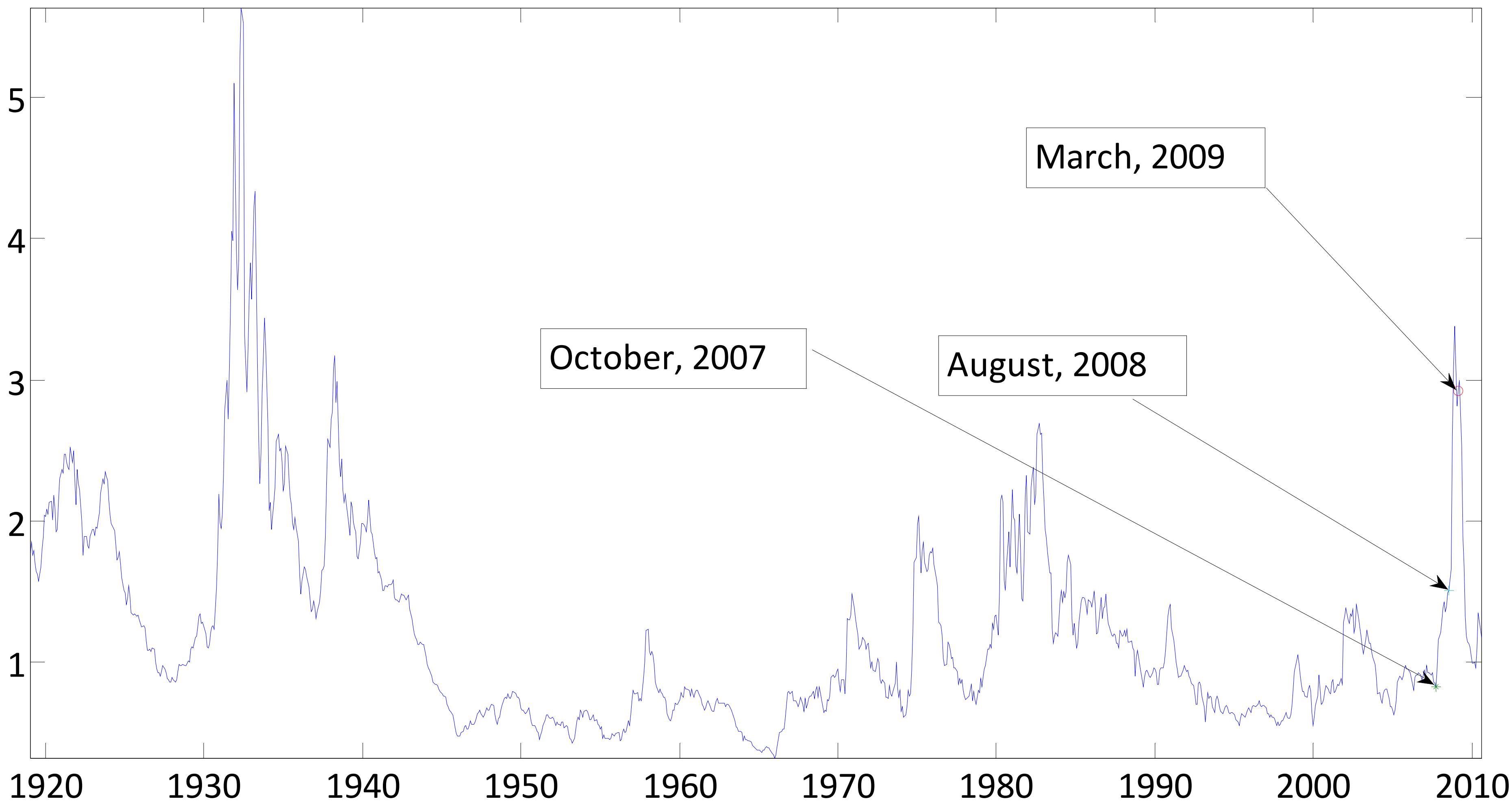


Spreads for ‘Risky’ Firms Shot Up in Late 2008

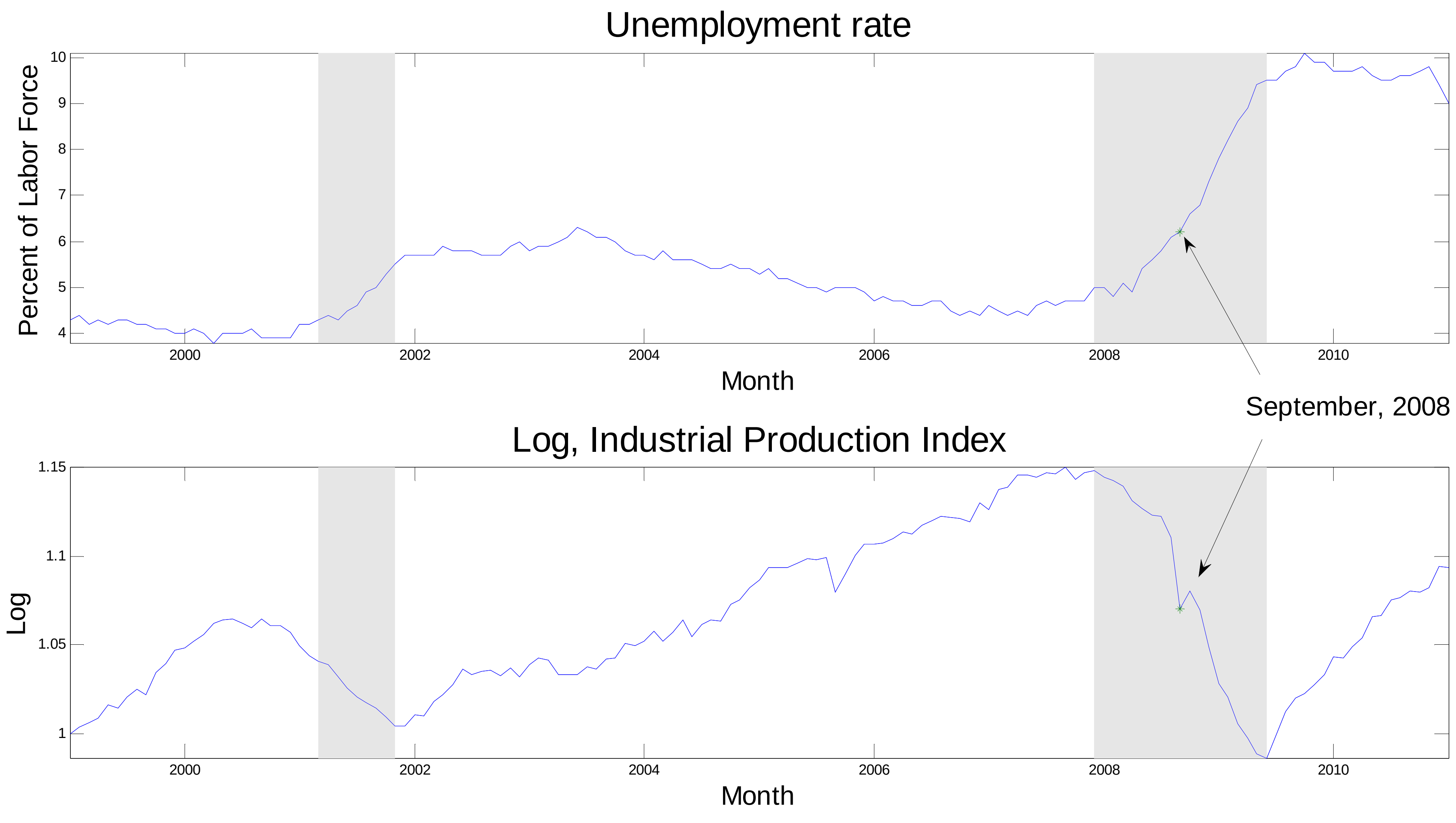


Must Go Back to Great Depression to See Spreads as Large as the Recent Ones

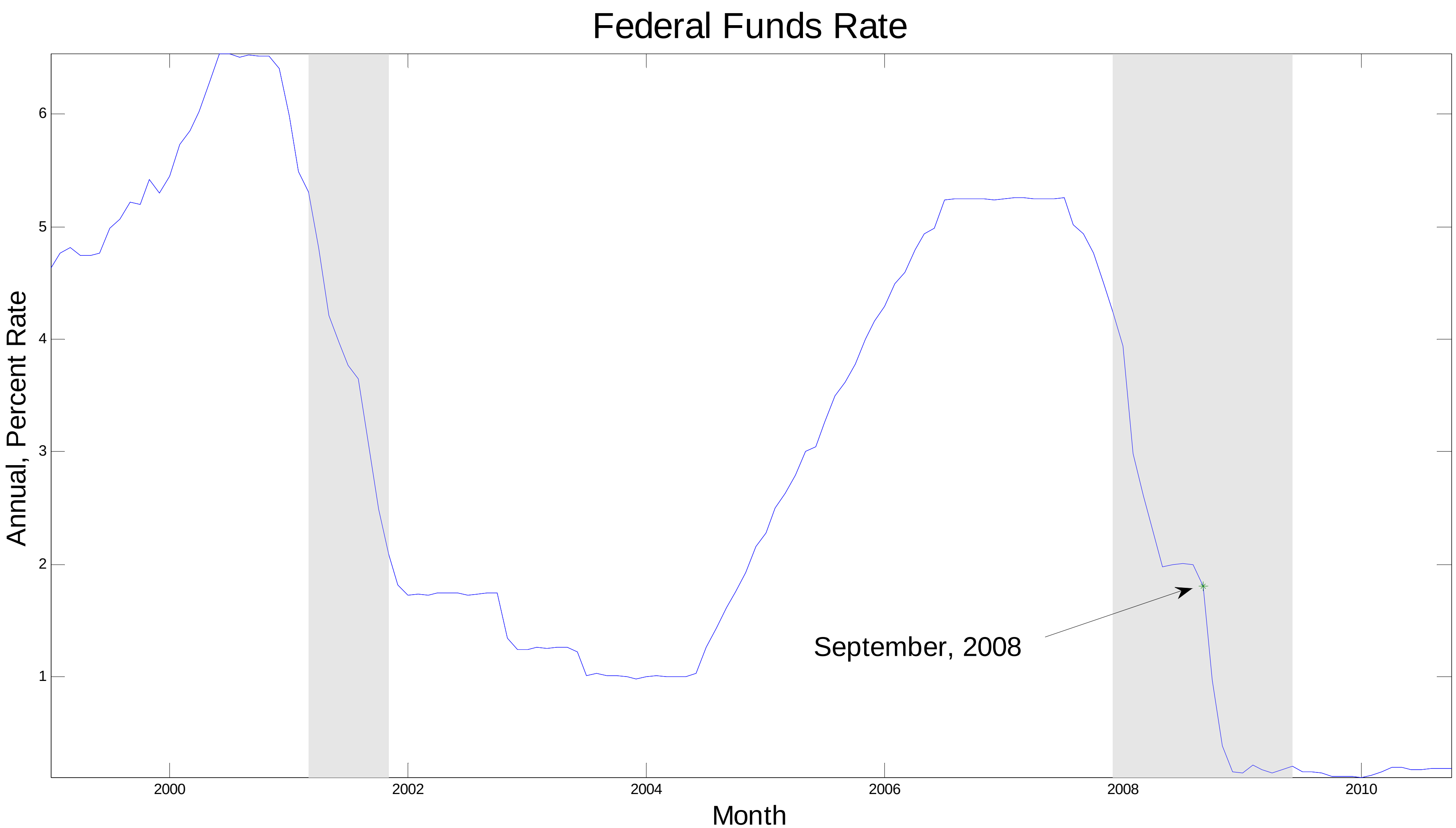
Spread, BAA versus AAA bonds



Economic Activity Shows (tentative) Signs of Recovery June, 2009



Banks’ Cost of Funds Low



Characterization of Crisis to be Explored here

- "…a fall in housing prices and other assets caused a fall in bank net worth. The banking system became dysfunctional as interest rate spreads increased and intermediation and economic activity fell to inefficiently low levels.''

- Various government policies can correct the situation.
  - However, which policies work depend on the details of the financial frictions.

Objective

- Keep analysis simple and on point by:
  - Two periods
  - Minimize complications from agent heterogeneity.
  - Leave out endogeneity of employment.
  - Leave out nominal variables: just look ‘behind the veil of monetary economics’
- Two models:
  - ‘Running away’ model: Gertler-Kiyotaki/Gertler-Karadi.
  - Hidden effort model.

‘Nonlinearities’

- Virtually All Economic Models have equilibrium conditions that are not linear.

- What I have in mind by non-linearity:
  - Model in which relevant equilibrium conditions are different in different points of the state space.

- Two banking models characterized by nonlinearities
  - In ‘normal times’, net worth of banks is high and banking system supports efficient allocations, despite financial frictions.
  - When banking net worth falls below a threshold, allocations cease to be efficient.
    - Banking system is ‘dysfunctional’
  - Equilibrium conditions different when net worth is too low.

Zero Lower Bound

- Zero lower bound restriction on interest rate.
  - Monetary policy:

$$Z_t = \rho R_{t-1} + (1-\rho)[\alpha_\pi \pi^e_{t+1} + \alpha_y y_t]$$

$$R_t = \begin{cases} Z_t & \text{if } Z_t \geq 0 \\ 0 & \text{if } Z_t < 0 \end{cases}$$

- Complementary slackness:

$$\overbrace{R_t}^{\geq 0} \times \overbrace{(R_t - \rho R_{t-1} - (1-\rho)[\alpha_\pi \pi^e_{t+1} + \alpha_y y_t])}^{\geq 0} = 0$$

- Solution strategy: Christiano-Fisher (JEDC, 2000)

Two-period Version of GK Model

- Many identical households, each with a unit measure of members:
  – Some members are 'bankers'
  – Some members are 'workers'
  – Perfect insurance inside households…everyone consumes same amount.
- Period 1
  – Workers endowed with y goods, household makes deposits in a bank
  – Bankers endowed with N goods, take deposits and purchase securities from a firm.
  – Firm issues securities to finance capital used in production in period 2.
- Period 2
  – Household consumes earnings from deposits plus profits from banker.
  – Goods consumed are produced by the firm.

Problem of the Household		
	period 1	period 2
budget constraint	$c + d \leq y$	$C \leq R^d d + \pi$
problem	$\max_{c,C,d}[u(c) + \beta u(C)]$	

Solution to Household Problem	
$\frac{u'(c)}{\beta u'(C)} = R^d$	$c + \frac{C}{R^d} = y + \frac{\pi}{R^d}$

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No change!
(Ricardian-Wallace Irrelevance)

Household budget constraint when government buys private assets using tax dollars

$$c + \frac{C}{R^d} = y - T + \frac{\pi + TR^d}{R^d} = y + \frac{\pi}{R^d}$$

Problem of the Household		
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budget constraint	$c + d \leq y$	$C \leq R^d d + \pi$
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Efficient Benchmark

Problem of the Bank	
period 1	period 2
take deposits, d	pay dR^d to households
buy securities, $s = N + d$	receive sR^k from firms
problem: $\max_d[sR^k - R^d d]$	

Properties of Efficient Benchmark

Equilibrium: R^d, c, C, d, π

(i) household problem solved

(ii) bank problem solved

(iii) market clearing

- Properties:
  - Household faces true social rate of return on saving:

$$R^k = R^d$$

  - Equilibrium is 'first best', i.e., solves

$$\max_{c,C,k,} u(c) + \beta u(C)$$

$$c + k \leq y + N,\ C \leq kR^k$$

Friction

- bank combines deposits, *d*, with net worth, *N*, to purchase *N+d* securities from firms.

- bank has two options:
  - ('no-default') wait until next period when $(N+d)R^k$ arrives and pay off depositors, $R^d d$, for profit:

$$(N+d)R^k - R^d d$$

  - ('default') take $\theta(N+d)$ securities, leave banking forever, refuse to pay depositors and wait until next period when securities pay off:

$$\theta(N+d)R^k$$

Incentive Constraint

- Bank will choose ‘no default’ iff

$$\overbrace{(N+d)R^k - R^d d}^{\text{no default}} \quad \geq \quad \overbrace{\theta(N+d)R^k}^{\text{default}}$$

- Default will never be observed, because banks don’t bother to offer deposits that exceed above limit, as depositors would not put their money into such a bank.

Collapse in Net Worth

- No default condition:

$$\overbrace{(N+d)R^k - R^d d}^{\text{no default}} \quad \geq \quad \overbrace{\theta(N+d)R^k}^{\text{default}}$$

- When condition is non-binding, then $R^k = R^d$ and

$$NR^k \geq \theta(N+d)R^k.$$

- If *N* collapses, then constraint may be violated for *d* associated with $R^d = R^k$
  - Equilibrium requires lower value of *d*
  - Lower *d* requires a spread: $R^d < R^k$
  - Lower *d* is not efficient

Policy Implications

$$\overbrace{(N+d)R^k - R^d d}^{\text{no default}} \geq \overbrace{\theta(N+d)R^k}^{\text{default}}$$

- Make direct tax-financed loans to non-financial firms
  - Works by reducing supply of *d* by households, and eliminating interest rate spread.

- Make loans/equity injections into banks.
  - Government may have an advantage here because it's harder for banks to 'steal' from the government.

- Subsidize bank interest rate costs
  - Raises bank profits and increases confidence of depositors.

Recap

- Basic idea:
  - Bankers can run away with a fraction of bank assets.
  - If banker net worth is high relative to deposits, running away is not in their interest.
  - If banker net worth falls below a certain cutoff, then they must restrict the deposits that they take.
    - To keep deposits at ‘normal level’ would cause depositors to lose confidence and take their business to another bank.
  - Reduced supply of deposits:
    - makes deposit interest rates fall and so spreads rise.
    - Reduced intermediation means investment drops, output drops.

Next: another moral hazard model

- Previous model: bankers can run away with a fraction of bank assets.

- Now: bankers must make an unobserved and costly effort to identify good projects that make a high return for their depositors.
  - Bankers must have the right incentive to make that effort.

- Otherwise, model similar to previous one.

Model Has a Similar Diagnosis of the Financial Crisis as Previous One

- Both models articulate the idea:

- "…a fall in housing prices and other assets caused a fall in bank net worth and initiated a crisis. The banking system became dysfunctional as interest rate spreads increased and intermediation and economic activity was reduced. Various government policies can correct the situation''

Bank Balance Sheet

Assets	Liabilities
One investment project, with return $R^b < R^g$ Good return occurs with probability *p(e), e* ~ unobserved effort. Whether return is good or bad is observed	Deposits Pay return, $R_b^d \leq R_g^d$ Banker net worth

Finding: (a) want banker net worth to be high enough, so that $R_b^d = R_g^d$ is feasible. When net worth falls too low, must have state-contingent deposit rates and this robs bankers of incentive to make effort.

(b) Equity injections have zero impact on the equilibrium outcomes (not helpful).

Two-period Hidden Effort Model

- Many identical households, each with a unit measure of members:
  - Some members are 'bankers'
  - Some members are 'workers'
  - Perfect insurance inside households…everyone consumes same amount.
- Period 1
  - Workers endowed with y goods, household makes deposits in a bank
  - Bankers endowed with N goods, take deposits and **make hidden efforts to identify a firm with a good investment project.**
  - Firm issues securities to finance capital used in production in period 2.
- Period 2
  - Household consumes earnings from deposits plus profits from banker.
  - Goods consumed are produced by the firm.

Problem of the Household		
	period 1	period 2
budget constraint	$c + d \leq y$	$C \leq R^d d + \pi$
problem	$\max_{c,C,d}[u(c) + \beta u(C)]$	

Solution to Household Problem	
$\frac{u'(c)}{\beta u'(C)} = R^d$	$c + \frac{C}{R^d} = y + \frac{\pi}{R^d}$
$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$	$c = \frac{y+\frac{\pi}{R^d}}{1+\frac{\left(\beta R^d\right)^{\frac{1}{\gamma}}}{R^d}}$

Banker Problem

- Bankers combine their net worth, *N*, and deposits, *d*, to acquire the securities of a single firm.
  - Bankers **not diversified**.

- Firms:
  - Good firms: investment project with return, R^g
  - Bad firms: an investment project with return, R^b

- Banker makes a **costly, unobserved effort, *e***, to locate a good firm, and finds one with probability, *p(e)*.
  - *p(e)* increasing in *e*.

Banker Problem, cnt'd

- Mean and variance on banker's asset:

$$\text{mean: } p(e)R^g + (1 - p(e))R^b$$

$$\text{variance: } p(e)[1 - p(e)](R^g - R^b)^2$$

- Note:
  - Mean increases in *e*
  - For *p(e)>1/2,*
    - Variance of the portfolio *decreases* with increase in *e*

$$\text{derivative of variance w.r.t. } e\text{:}$$

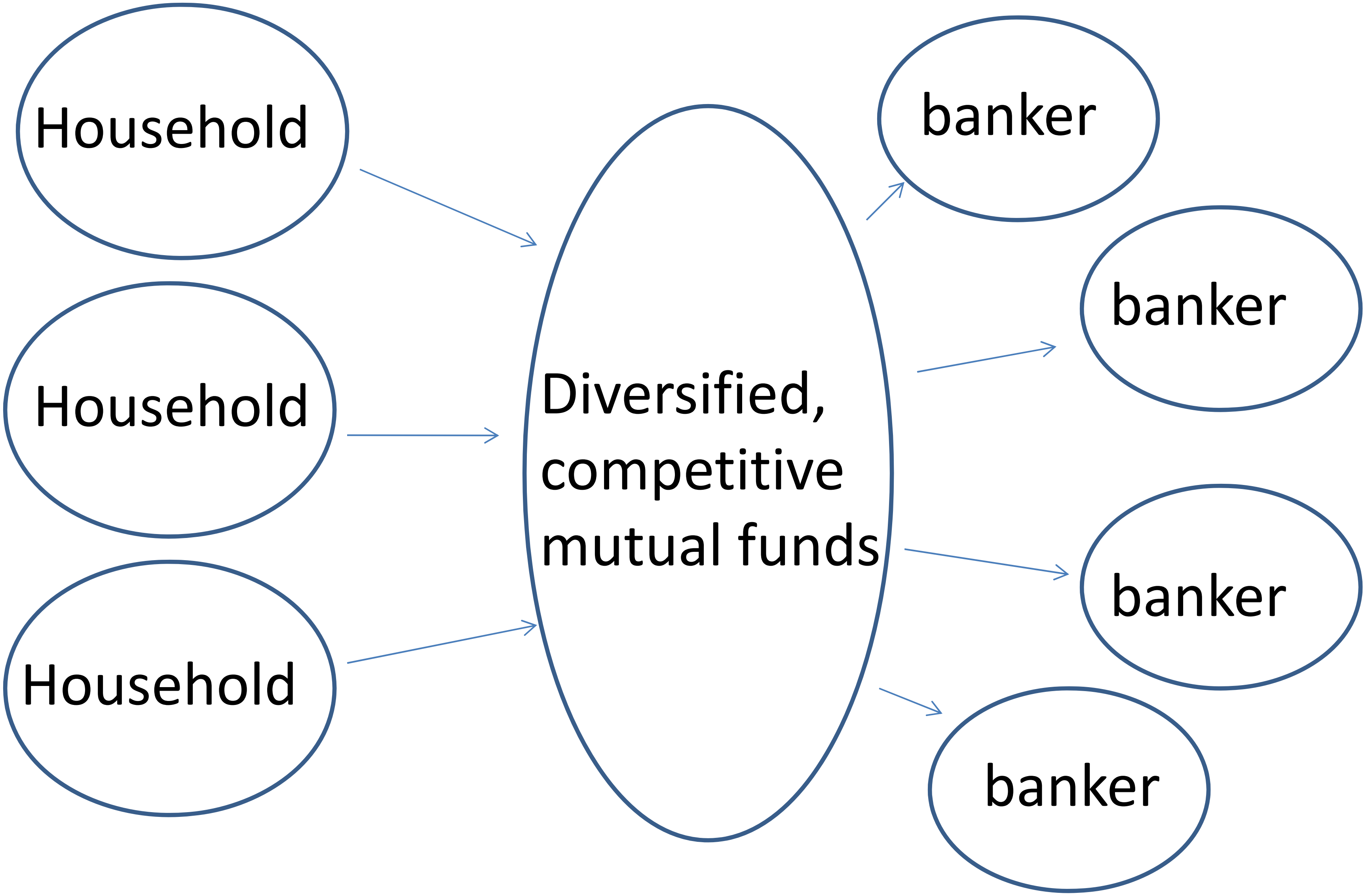
$$[1 - 2p(e)](R^g - R^b)^2 p'(e),$$

Funding for Bankers

- Representative household deposits money into a representative mutual fund.
  - Household receives a certain return, *R*.

- Representative mutual fund acquires deposit, *d*, in each of a diversified set of banks.
  - Mutual fund receives dR_g^d from *p(e)* banks with a good investment.
  - Mutual fund receives dR_b^d from *1-p(e)* banks with a bad investment.

Risky Bankers Funded By Mutual Funds



Arrangement Between Banks and Mutual Funds

- Contract traded in competitive market:

Deposit amount effort Interest rate in good state

$$(d, e, R_g^d, R_b^d)$$

Interest rate in bad state

Two Versions of Model

- No financial frictions: mutual fund observes banker effort.
  - This is the benchmark version.

- Financial frictions: mutual fund does not observe banker effort.
  - This is the interesting version.
  - Use it to think about crisis in 2008-2009, and unconventional monetary policy.

Equilibrium Contract When Effort is Observable

- Competition and free entry among mutual funds:

$$\overbrace{Rd}^{\text{money owed to households by mutual funds}}$$

$$= \overbrace{p(e)}^{\text{fraction of banks with good investments}} R_g^d d + \overbrace{(1-p(e))}^{\text{fraction of banks with bad investments}} R_b^d d$$

- Zero profit condition represents a menu of contracts available to banks.

Contract Selected by Banks in Observable Effort Equilibrium

Marginal value assigned by household to bank profits

$$\max_{e,d,R_g^d,R_b^d} \lambda \overbrace{\{p(e)[R^g(N+d)-R_g^d d]+(1-p(e))[R^b(N+d)-R_b^d d]\}}^{\text{expected bank profits}}$$

$$-\overbrace{\frac{1}{2}e^2}^{\text{utility cost of effort suffered by banker}}$$

$$\text{subject to: } \overbrace{Rd = p(e)R_g^d d + (1-p(e))R_b^d d}^{\text{zero profit condition of mutual funds}}, \quad \overbrace{R^b(N+d) \geq R_b^d d}^{\text{cash flow constraint on banks}}$$

Characterizing Equilibrium Contract

- Substitute out the mutual fund zero profit condition, so that banker problem is:

$$\max_{e,d,R_g^d,R_b^d} \lambda\{p(e)[R^g(N+d)-R_g^d d]+(1-p(e))[R^b(N+d)-R_b^d d]\}-\frac{1}{2}e^2$$

$$\max_{e,d} \lambda\{[p(e)R^g+(1-p(e))R^b](N+d)-Rd\}-\frac{1}{2}e^2$$

- Optimal contract conditions:

$$\begin{aligned}
\text{effort} &: e = \lambda p'(e)(R^g - R^b)(N+d)\\
\text{deposits} &: R = p(e)R^g + (1-p(e))R^b\\
\text{zero profits, mutual fund} &: R = p(e)R_g^d + (1-p(e))R_b^d\\
\text{cash constraint} &: R^b(N+d) \geq R_b^d d
\end{aligned}$$

Properties of Contract

- Banker treats *d* and *N* symmetrically

$$\text{effort}\ :\ e = \lambda p'(e)(R^g - R^b)(N+d)$$

- Other equations:

$$\text{deposits}\ :\ R = p(e)R^g + (1 - p(e))R^b$$

$$\text{zero profits, mutual fund}\ :\ R = p(e)R_g^d + (1 - p(e))R_b^d$$

$$\text{cash constraint}\ :\ R^b(N+d) \geq R_b^d d$$

- Get *e* from first equation, *R* from second.
- Returns on deposits not uniquely pinned down. Cash constraint not binding.
  - *N* large enough relative to *d*, can choose $R_g^d = R_b^d = R$

Observable Effort Equilibrium

Observable Effort Equilibrium: $c,\ C,\ e,\ d,\ R,\ \lambda,\ R_g^d,\ R_b^d$ such that

(i) the household maximization problem is solved

(ii) mutual funds earn zero profits

(iii) the banker problem with e observable, is solved

(iv) markets clear

(v) $c, C, d, e > 0$

Unobservable Effort

- Suppose that the banker has obtained a contract, (d, e, R_g^d, R_b^d), from the mutual fund.
- The mutual fund can observe (d, R_g^d, R_b^d) so that the banker no longer has any choice about these.
- The mutual fund does not observe *e*, and so the bank can still choose *e* freely after the contract has been selected.
- The banker solves

$$\max_e \lambda\{p(e)[R^g(N+d) - R_g^d d] + (1-p(e))[R^b(N+d) - R_b^d d]\} - \frac{1}{2}e^2$$

Incentive Constraint

- Banker choice of *e* after the deposit contract has been selected:

$$\max_e \lambda\{p(e)[R^g(N+d) - R_g^d d] + (1-p(e))[R^b(N+d) - R_b^d d]\} - \frac{1}{2}e^2$$

- First order condition:

$$e = \lambda p'(e)[(R^g - R^b)(N+d) - (R_g^d - R_b^d)d]$$

  – Note: if $R_g^d > R_b^d$ then the banker exerts less effort than in the observable effort equilibrium.

  – Reason is that the banker does not receive the full return on its effort if $R_g^d > R_b^d$

Unobservable Effort Equilibrium

- Mutual funds are only willing to consider contracts, (d, e, R_g^d, R_b^d), that satisfy the following restrictions:

$$\text{zero profits, mutual fund} : R = p(e)R_g^d + (1 - p(e))R_b^d$$

$$\text{cash constraint} : R^b(N + d) \geq R_b^d d$$

$$\text{incentive compatibility:} \quad e = \lambda p'(e)[(R^g - R^b)(N + d) - (R_g^d - R_b^d)d]$$

- There is no point for the mutual fund to consider a contract in which *e* does not satisfy the last condition, since bankers will set *e* according to the last condition in any case.

Contract Selected by Banks in Unobservable Effort Equilibrium

- Solve

$$\max_{e,d,R_g^d,R_b^d} \lambda\{p(e)[R^g(N+d)-R_g^d d]+(1-p(e))[R^b(N+d)-R_b^d d]\} - \frac{1}{2}e^2$$

- Subject to

$$\begin{aligned}
\text{zero profits, mutual fund}: &\quad R = p(e)R_g^d + (1-p(e))R_b^d \\
\text{cash constraint}: &\quad R^b(N+d) \geq R_b^d d \\
\text{incentive compatibility}: &\quad e = \lambda p'(e)[(R^g - R^b)(N+d) - (R_g^d - R_b^d)d]
\end{aligned}$$

Two Unobservable Effort Equilibria

- **Case 1: Banker net worth, *N*, is high enough**
  - Recall the two conditions on deposit returns:

$$\text{zero profits, mutual fund}: R = p(e)R_g^d + (1 - p(e))R_b^d$$

$$\text{cash constraint}: R^b(N + d) \geq R_b^d d$$

  - Suppose that *N* is large enough so that given *d* from the observable effort equilibrium, cash constraint is satisfied with

$$R_g^d = R_b^d = R$$

  - Then, observable effort equilibrium is also an unobservable effort equilibrium.

With N large enough, unobservable effort equilibrium is efficient.

Risk Premium

- R is the risk free rate in the model (i.e., the sure return received by the household).

- Let R_g^d denote the 'bank interest rate on deposits'.
  - This is what the bank pays in the event that its portfolio is 'good'.
- Risk premium: $R_g^d - R$

Result: when *N* is high enough, equilibrium level of intermediation is efficient and risk premium is zero.

Case 2: Banker net worth, *N*, is low

- Recall the two conditions on deposit returns:

$$\text{zero profits, mutual fund} : R = p(e)R_g^d + (1 - p(e))R_b^d$$

$$\text{cash constraint} : R^b(N + d) \geq R_b^d d$$

  - Suppose that *N* is small, so that given *d* from the observable effort equilibrium, cash constraint is **not** satisfied with

$$R_g^d = R_b^d = R$$

  - Then, observable effort equilibrium is **not** an unobservable effort equilibrium.

With N small enough, unobservable effort equilibrium is not efficient.

Unobserved Effort Equilibrium, low *N* Case

- The two conditions on deposit returns:

$$\text{zero profits, mutual fund}: \; R = p(e)R_g^d + (1-p(e))R_b^d$$

$$\text{cash constraint}: \; R^b(N+d) \geq R_b^d d$$

- Suppose, with efficient *d* and *e*, cash constraint is not satisfied for $R_b^d = R$. Then
  - Set $R_b^d < R,\ R_g^d > R$ (still have $R = p(e)R^g + (1-p(e))R^b$)
  - Risk premium positive
  - Incentive constraint implies inefficiently low *e.*
  - Low *e* implies low *R*, which implies low *d*.
    - Banking system ‘dysfunctional’.
  - Mean of bank return goes down, and variance up.

Scenario Rationalized by Model

- Before 2007, when N was high, the banking system supported the efficient allocations and the interest spread was zero.

- The fall in bank net worth after 2007, caused a jump in the risk premium, and a slowdown in intermediation and investment.

- Banking system became dysfunctional because banks did not have enough net worth to cover possible losses.
  - This meant depositors had to take losses in case of a bad investment outcome in banks.
  - Depositors require a high return in good states as compensation: risk premium.
  - Bankers lose incentive to exert high effort. More bad projects are funded, reducing the overall return on saving.
  - Saving falls below its efficient level.

How to Fix the Problem

- One solution: tax the workers and transfer the proceeds to bankers so they have more net worth.
  - In the model, this is a good idea because income distribution issues have been set aside.
  - In practice, income distribution problems could be a serious concern and this policy may therefore not be feasible

- Subsidize the interest rate costs of banks.
  - This increases the chance that bank net worth is sufficient to cover losses, reduces the risk premium and gives bankers an incentive to increase effort.
  - Increased effort increases the return on banker portfolios and reduces their variance.

- Equity injections and loans to banks have zero impact in the model, when it is in a bad equilibrium.
  - Ricardian irrelevance not overturned.
  - the sources of moral hazard matter for whether a particular asset purchase programs is effective!

Conclusion

- Have described two models of moral hazard, that can rationalize the view:
  - Net worth fell, causing interest rate spreads to jump and intermediation to slow down. The banking system is dysfunctional.
- Net worth transfers and interest rate subsidies can revive a dysfunctional banking system in both models.
- However, the models differ in terms of the detailed economic story, as well as in terms of their implications for asset purchases.