The Labor Market in the New Keynesian Model: Incorporating a Simple DMP Version of the Labor Market and Rediscovering the Shimer Puzzle

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Outline

- We present baseline NK model with a Diamond-Mortensen-Pissarides type labor market.
  - First paper to combine NK and DMP is Walsh (2003). Now this is a huge literature.
- The New Keynesian DMP model.
  - Describe equilibrium conditions of household and goods producers
  - These coincide with those in the simple, standard NK model ('Clarida-Gali-Gertler model')
    - Details: http://faculty.wcas.northwestern.edu/~lchrist/course/Korea_2012/intro_NK.pdf
  - Describe the DMP labor market.
- Compare the DMP model with a version in which specification in which labor market is competitive.
  - discover 'Shimer puzzle for monetary models': employment/unemployment respond very little to an expansionary monetary policy.
  - replace hiring with search costs.

Households

- Many identical households, each having a unit measure of workers.
- All workers are sent to the labor market, where they are either employed or unemployed.
- Household problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \log C_t,$$

$$\text{subject to: } P_t C_t + B_{t+1} \leq W_t \times \overbrace{l_t}^{\text{number of employed households}}$$

$$+ \overbrace{(1 - l_t)\, D}^{\text{number of unemployed households, times unemployment benefits}}$$

$$+ R_{t-1} B_t + \text{Profits net of taxes}_t$$

$$\text{First order condition: } \frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}}$$

Obtaining the NK IS Curve

- Let $X_t \equiv C_t \exp(-a_t)$ and substitute into household Fonc:

$$\frac{1}{X_t \exp(a_t)} = \beta E_t \frac{1}{X_{t+1} \exp(a_{t+1})} \frac{R_t}{\bar{\pi}_{t+1}}$$

or,

$$\frac{1}{X_t} = E_t \frac{1}{X_{t+1} R^*_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}},\ R^*_{t+1} \equiv \frac{1}{\beta} \exp(a_{t+1} - a_t).$$

- log-linearizing:

$$\hat{X}_t = E_t \left[\hat{X}_{t+1} - \left(\hat{R}_t - \widehat{\bar{\pi}}_{t+1} - \hat{R}^*_{t+1} \right) \right].$$

- linearizing about a zero inflation steady state:

$$\boxed{\begin{aligned} x_t &= E_t x_{t+1} - E_t \left[r_t - \pi_{t+1} - r^*_t \right] \\ r^*_t &= -\log(\beta) + E_t \left[a_{t+1} - a_t \right]. \end{aligned}}$$

Goods Production

- Final good firms solve

$$\max P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj, \text{ s.t. } Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

- Demand curve for i^{th} monopolist:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon}.$$

- Production function:

$$Y_{i,t} = \exp(a_t) h_{i,t}, \; a_t = \rho_a a_{t-1} + \varepsilon_t^a,$$

  where $h_{i,t}$ is a homogeneous input (not labor!) with after-subsidy nominal price, $(1-\nu)\vartheta_t P_t$.
- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}.$$

Optimal Price Setting

- As in baseline NK model:

$$\begin{aligned}
\tilde{p}_t &= \frac{K_t}{F_t},\ \tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t},\ F_t = 1 + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \text{ (a)} \\
K_t &= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \text{ (b)} \\
\tilde{p}_t &= \left[\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \to \frac{K_t}{F_t} = \left[\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \quad \text{(c)} \\
s_t &= \frac{(1-\nu)\,\vartheta_t}{\exp(a_t)}
\end{aligned}$$

- Log-linearizing (a)-(c) around zero inflation steady state (and, assuming $1-\nu = (\varepsilon-1)/\varepsilon$)

$$\boxed{\begin{array}{l} \pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{s}_t + \beta\pi_{t+1} \\ \hline \pi_t \equiv \bar{\pi}_t - 1. \end{array}}$$

Linearized Equilibrium Conditions for Households and Goods Producing Firms

- Equations

$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{s}_t + \beta\pi_{t+1}$
$x_t = E_t x_{t+1} - E_t\left[r_t - \pi_{t+1} - r_t^*\right]$
$r_t^* = -\log(\beta) + E_t\left[a_{t+1} - a_t\right].$
$\hat{s}_t = \hat{\vartheta}_t - a_t.$

- If we also have a monetary policy rule, would have 5 equations in 6 variables:

$$\pi_t, \hat{s}_t, x_t, r_t, r_t^*, \hat{\vartheta}_t$$

- Need on net, one more equation.
  - In simple model with competitive labor markets, have $\vartheta_t = C_t l_t^{\varphi}$.
    - Here, φ is a utility parameter and this equation is not available now.
  - Equations provided by search/matching framework of DMP.

Labor Market

- Large number of identical and competitive firms; produce homogeneous output using only labor, l_t, for real price, ϑ_t.
- At the start of period t, a firm may post a single vacancy for fixed cost κ, in which case it meets a worker with probability, Q_t.
  - Immediately after they meet, worker and firm agree on a wage and begin employment.
  - Match survives into period $t+1$ with fixed, exogenous probability ρ.
- Number of workers searching for work at the end of period $t-1$:
  - $1-l_{t-1}$ unemployed workers plus the $(1-\rho)\,l_{t-1}$ employed workers that separate.
  - total searching workers

$$1-l_{t-1}+(1-\rho)\,l_{t-1}=1-\rho l_{t-1}.$$

Notation and Identities

- Probability a searching worker finds a period t job denoted f_t.
  - Law of motion of employment is:

$$\begin{aligned} l_t &= \overbrace{\rho l_{t-1}}^{\text{workers that remain attached to their firm}} \\ &+ \overbrace{f_t\left(1-\rho l_{t-1}\right)}^{\text{flow from unempl to empl plus flow from empl to empl}} \end{aligned}$$

  - Alternative law of motion of employment:

$$l_t = \left(\rho + x_t\right) l_{t-1},$$

    where x_t denotes the hiring rate, as a proportion of l_{t-1}.

- For the two laws of motion to be compatible, require:

$$f_t = \frac{x_t l_{t-1}}{1-\rho l_{t-1}} = \frac{\text{net new jobs}}{\text{number of workers searching}}.$$

Labor Market Tightness

- Probability a firm meets a worker is denoted Q_t:

$$Q_t = \frac{\text{net new jobs}}{\text{number of firms posting vacancies}} = \frac{x_t l_{t-1}}{v_t l_{t-1}} = \frac{x_t}{v_t},$$

  where v_t denotes the vacancy rate, as a proportion of l_{t-1}.
- Matching function

$$\overbrace{x_t l_{t-1}}^{\#\text{ new hires}} = \sigma_m \left(\overbrace{1-\rho l_{t-1}}^{\#\text{ people searching}} \right)^{\sigma} \left(\overbrace{l_{t-1} v_t}^{\#\text{ vacancies}} \right)^{1-\sigma}.$$

  – divide by $1-\rho l_{t-1}$:

$$f_t = \frac{x_t l_{t-1}}{1-\rho l_{t-1}} = \sigma_m \left(\frac{l_{t-1} v_t}{1-\rho l_{t-1}} \right)^{1-\sigma} = \sigma_m \Theta_t^{1-\sigma},$$

  where Θ_t denotes labor market tightness.

  – note, $f_t = Q_t \Theta_t$, so

$$Q_t = \sigma_m \Theta_t^{-\sigma}$$

Value Functions

- J_t is the value to a firm of an employed worker:

$$J_t = \vartheta_t - \bar{w}_t + \rho E_t m_{t+1} J_{t+1}, \ \bar{w}_t \equiv \frac{W_t}{P_t}.$$

- ϑ_t and m_{t+1} are given to firm and worker.
- Value of employment to a worker:

$$V_t = \bar{w}_t + E_t m_{t+1} \left[\rho V_{t+1} + (1-\rho)\left(f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} \right) \right]$$

- Value of unemployment to a worker:

$$U_t = D + E_t m_{t+1} \left[f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} \right]$$

- Free entry and zero profits dictate: $\kappa = Q_t J_t$.

Nash Bargaining

- Workers and firms choose a wage that solves a joint optimization problem, taking as given future wages.
- Surplus of worker:

$$\begin{aligned} S_t &\equiv \bar{w}_t + \tilde{V}_t - U_t, \\ \tilde{V}_t &\equiv E_t m_{t+1} \left[\rho V_{t+1} + (1-\rho) \left(f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} \right) \right] \end{aligned}$$

- Surplus of firm:

$$\tilde{J}_t - \bar{w}_t,\ \tilde{J}_t \equiv \vartheta_t + \rho E_t m_{t+1} J_{t+1}$$

- Agreed-upon wage rate solves

$$\max_{\bar{w}_t} \left(\bar{w}_t + \tilde{V}_t - U_t \right)^{\eta} \left(\tilde{J}_t - \bar{w}_t \right)^{1-\eta},$$

taking $\tilde{V}_t$ and $\tilde{J}_t$ as given. FONC (*Nash sharing rule*):

$$J_t = \frac{1-\eta}{\eta} \left(V_t - U_t \right).$$

Labor and Goods Market Clearing

- Total purchases of homogeneous inputs by intermediate good producer:

$$h_t = \int_0^1 h_{i,t} di.$$

- Labor market clearing

$$h_t = l_t.$$

- Goods market clearing

$$C_t + v_t l_{t-1} \kappa = p_t^* \exp\left(a_t\right) l_t,$$

where

$$p_t^* = \left[(1-\theta) \left(\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{1-\varepsilon}} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1}.$$

Equilibrium Conditions from Labor Market

13 equilibrium conditions associated with 12 new variables:

$$l_t, x_t, Q_t, J_t, V_t, U_t, m_t, f_t, \Theta_t, Y_t, v_t, \bar{w}_t$$

$$(1)\ l_t = (\rho + x_t)\, l_{t-1},\ (2)\ \kappa = Q_t J_t$$

$$(3)\ J_t = \vartheta_t - \bar{w}_t + \rho E_t m_{t+1} J_{t+1}$$

$$(4)\ V_t = \bar{w}_t$$

$$+E_t m_{t+1} \left[\rho V_{t+1} + (1-\rho)\left(f_{t+1} V_{t+1} + (1 - f_{t+1})\, U_{t+1}\right)\right]$$

$$(5)\ U_t = D + E_t m_{t+1} \left[f_{t+1} V_{t+1} + (1 - f_{t+1})\, U_{t+1}\right]$$

$$(6)\ J_t = \frac{1-\eta}{\eta}\left(V_t - U_t\right),\ (7)\ Q_t = \sigma_m \Theta_t^{-\sigma},\ (8)\ \Theta_t = \frac{l_{t-1} v_t}{1 - \rho l_{t-1}}$$

$$(9)\ m_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)},\ (10)\ C_t + \kappa v_t l_{t-1} = Y_t,$$

$$(11)\ f_t = \frac{x_t l_{t-1}}{1 - \rho l_{t-1}}\ (12)\ Y_t = p_t^* \exp(a_t)\, l_t,\ (13)\ Q_t = \frac{x_t}{v_t}$$

with $p_t^* = 1$ bec we linearly approximate around zero inflation ss.

Solving the Model and Assigning Values to Parameters

- Shortage of one equation in goods and household sector.
- The labor market added, on net, one equation (i.e., 12 extra variables and 13 extra equations).
- The model has the following 10 parameters

$$\overbrace{\beta,\rho_a,\sigma_a,\theta,\varepsilon}^{\text{household and goods production}},\overbrace{D,\kappa,\eta,\sigma_m,\sigma}^{\text{labor market}}$$

- Interestingly, ε plays no role in the solution to the model, to a first order approximation
  - This reflects, in part, how we set the subsidy to monopolists.
  - Also reflects constant marginal costs of intermediate good producers (not obvious).
- Parameter values of goods and household sector easy to choose, since there is a lot of experience with them
  - $\beta = 0.99$, $\rho_a = 0.95$, $\sigma_a = 0.01$, $\theta = 0.75$.

Monetary Policy

- Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\alpha} \times \exp\left\{(1-\alpha)\left[\phi_\pi\left(\bar{\pi}_t - 1\right) + \phi_x \log\left(C_t/C\right)\right] - u_t\right\},$$

  where u_t is an *iid* (expansionary) policy shock.

- Parameters were set to the following (fairly conventional) values:

$$\alpha = 0.80,\ \phi_\pi = 1.5,\ \phi_x = 0.05.$$

Calibration of Labor Market Parameters

Generally, follow Ravenna-Walsh (2008) and Walsh (2003), but also Shimer (2012) and Hall-Milgrom (2008).

Variable name	symbol	value	end/ex
unemployment rate	$1-l$	0.05	endog
replacement ratio	$\frac{D}{\bar{w}}$	0.4	endog
search cost/gross output of homog good	$\kappa v l/Y$	0.01	endog
vacancy filling rate	Q	0.7	endog
elasticity of matching w.r.t. unemp	σ	0.5	exog
discount factor	β	0.99	exog
match survival rate	ρ	0.9	exog

Since we exogenously set the values of four endogenous variables, we must allow four exogenous parameters to be endogenous:

$$D, \kappa, \eta, \sigma_m.$$

- Steady state computations
  - first, go down first column, then go down second.
  - In each case, which equilibrium condition that is used is indicated.

$R = 1/\beta$ (intertemporal Euler)	
(9) $m = \beta$	$\kappa = (\kappa v l/Y) \times Y/(lv)$
(1) $x = 1 - \rho$	(2) $J = \frac{\kappa}{Q}$
(8) $\Theta = \frac{xl}{Q(1-\rho l)}$	(3) $\bar{w} = \vartheta + \rho\beta J - J$
(11) $f = \frac{xl}{1-\rho l}$	$D = \frac{D}{\bar{w}} \times \bar{w}$
(7) $\sigma_m = Q\Theta^{\sigma}$	(10) $C = l - \kappa v l$
$\vartheta = 1$ (pricing equations)	(4)-(5) $V - U = \frac{\bar{w} - D}{1 - \beta\rho(1-f)}$
(12) $Y = l$	(5) $U = \frac{D + \beta f(V-U)}{1-\beta}$
(13) $v = x/Q$	(6) $\eta = \frac{V-U}{V-U+J}$.

Comparing the DMP Labor Market with Competitive Labor Markets

- The DMP version of the model was described in detail above.
- The competitive labor market version of the model is obtained by deleting equations 1-13 above and:
  - Interpret ϑ_t as the real wage.
  - Allow households to set employment, l_t, on the intensive margin and equate

    $$\vartheta_t = C_t l_t^{\varphi},$$

    where $1/\varphi$ denotes the Frisch labor supply elasticity (see the first set of lectures on the labor market in the New Keynesian model for details).

Comparing...results

- Run the Dynare file, DMP.mod, to see responses of the system to an expansionary monetary policy shock.
- "Shimer puzzle" for monetary economics:
  - note that the response of output is weaker in the DMP model than it is in the (already very weak) version of the model in which the labor market is competitive.
  - The intuition is that in an expansion,
    - the labor market tightens (i.e., Θ_t rises) and the probability that a vacancy draws a worker, Q_t, decreases,
    - with costs fixed in terms of vacancies, the implied cost of meeting a worker rises,
    - this negative feedback loop limits the expansion itself.

Hiring versus Search Costs

- One response to this 'Shimer puzzle' is to replace the *search costs* of meeting a worker in the DMP model with *hiring costs*.
  - Approach taken in New Keynesian model versions of DMP by GT and GST.
- Hiring cost specification: firm pays κ to meet a worker.
  - vacancies must be posted too, but they are costless.
  - the only actual cost of meeting a worker is κ.
- Replace equations (2) and (10) above by

$$\begin{aligned} \kappa &= J_t \ \ (2)' \\ C_t + \kappa x_t l_{t-1} &= Y_t \ \ (10)' \end{aligned}$$

  - do same replacement in the steady state equations, also replace the calibration equation for κ :

$$\kappa = (\kappa x l / Y) \times Y / (l x)$$

The Effect of Switching to Hiring Costs from Search

- The switch cancels the negative feedback loop mentioned earlier.
- With a monetary expansion, the rise in employment is greater and inflation, smaller.
  - This can be confirmed by running the Dynare software, DMP.mod.
- Unfortunately, this change is not sufficient to allow the model to display the response to shocks observed in the data.
  - in practice, exogenous frictions in the adjustment of wages are still required (see GT, GST and others).
  - Christiano-Eichenbaum-Trabandt (2013) stress this point too, and suggest an alternative to exogenous stickiness in sticky wages.
    - this is the subject of the next handout.