Application of DSGE Model:

What Happens if we Switch from Fixed to Flexible Interest Rate Regime?

Two Examples

- Standard monetary policy briefing question:
  - ‘What Happens if We Set the Interest Rate to Fixed Level for *y* Periods?’

- Policy question relevant in some countries today:
  - ‘What Happens if Shocks Drive the Economy into the Zero Lower Bound and are expected to keep the economy there for a while?’
  - ‘Zero Lower Bound’: lower bound on nominal rate of interest.

Model

- Model in linearized form:

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0,$$

  – Here,
    - z_t denotes the list of endogenous variables whose values are determined at time *t*.
    - s_t denotes the list of exogenous variables whose values are determined at time *t*.

$$s_t = P s_{t-1} + \varepsilon_t.$$

- Solution: *A* and *B in*

$$z_t = A z_{t-1} + B s_t,$$

  – where

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0, \qquad (\beta_0 + \alpha_0 B)P + [\beta_1 + (\alpha_0 A + \alpha_1)B] = 0$$

Policy Experiment

- The n^{th} equation in the system is a monetary policy rule (Taylor rule). One of the variables in z_t is the policy interest rate, R_t, in deviation from its non-stochastic steady state value:

$$R_t = \tau' z_t.$$

- τ is composed of *0*'s and a single *1*
- Policy:
  - it is now time $t=T$ and policy is $R_t = \tilde{d}$ from $t=T+1$ to $t=T+y$.
  - For $t>T+y$, policy follows the Taylor rule again.

Convenient to 'Stack' the System to be Conformable with Dynare Notation

- First set of equations is the equilibrium conditions and second set is the exogenous shock process:

$$E_t \left\{ \overbrace{\begin{bmatrix} \alpha_0 & \beta_0 \\ 0 & 0 \end{bmatrix}}^{A_0} \overbrace{\begin{pmatrix} z_{t+1} \\ s_{t+1} \end{pmatrix}}^{Z_{t+1}} + \overbrace{\begin{bmatrix} \alpha_1 & \beta_1 \\ 0 & I \end{bmatrix}}^{A_1} \overbrace{\begin{pmatrix} z_t \\ s_t \end{pmatrix}}^{Z_t} + \overbrace{\begin{bmatrix} \alpha_2 & 0 \\ 0 & -P \end{bmatrix}}^{A_2} \overbrace{\begin{pmatrix} z_{t-1} \\ s_{t-1} \end{pmatrix}}^{Z_{t-1}} + \overbrace{\begin{pmatrix} 0 \\ -\varepsilon_t \end{pmatrix}}^{\epsilon_t} \right\} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

  - or

$$E_t\{A_0 Z_{t+1} + A_1 Z_t + A_2 Z_{t-1} + \epsilon_t\} = 0.$$

  - where ϵ_t is independent over time and in time *t* information set.

- The n^{th} row of the above system corresponds to monetary policy rule.

Fixed *R* Equilibrium Conditions

- Delete monetary policy rule (i.e., n^{th} equation) from system and replace it by $R_t = \tilde{d}$:
  - Let $\hat{A}_0$ and $\hat{A}_2$ denote A_0 and A_2 with their n^{th} rows replaced by *0*'s.
  - Let

$$\hat{A}_1 = \begin{bmatrix} \hat{\alpha}_1 & \hat{\beta}_1 \\ 0 & I \end{bmatrix}$$

    - Where $\hat{\alpha}_1$ is α_1 with its n^{th} row replaced by τ' and $\hat{\beta}_1$ is β_1 with its n^{th} row replaced by *0*'s.
- Equilibrium conditions:

$$E_t\{\hat{A}_0 Z_{t+1} + \hat{A}_1 Z_t + \hat{A}_2 Z_{t-1} + \epsilon_t\} = d.$$

  - d is a column vector zero in all but one location and $\tau' d = \tilde{d}$

Problem

- Equilibrium conditions for *t=T+1,…,T+y*:

$$E_t\{\hat{A}_0 Z_{t+1} + \hat{A}_1 Z_t + \hat{A}_2 Z_{t-1} + \epsilon_t\} = d.$$

- Equilibrium conditions for *t>T+y:*

$$E_t\{A_0 Z_{t+1} + A_1 Z_t + A_2 Z_{t-1} + \epsilon_t\} = 0.$$

  – Solution for *t>T+y:*

$$\overbrace{\begin{pmatrix} z_t \\ s_t \end{pmatrix}}^{Z_t} = \overbrace{\begin{bmatrix} A & BP \\ 0 & P \end{bmatrix}}^{a} \overbrace{\begin{pmatrix} z_{t-1} \\ s_{t-1} \end{pmatrix}}^{Z_{t-1}} + \overbrace{\begin{pmatrix} 0 & -B \\ 0 & -I \end{pmatrix}}^{b} \overbrace{\begin{pmatrix} 0 \\ -\varepsilon_t \end{pmatrix}}^{\epsilon_t}$$

  – or,

$$Z_t = aZ_{t-1} + b\epsilon_t$$

- How is the solution for *t=T+1,…,T+y?*

Solve the Model 'Backward'

- In period *t=T+y*:

$$E_{T+y}\left\{\hat{A}_0 \overbrace{Z_{T+y+1}}^{=aZ_{T+y}+b\epsilon_{T+y+1}} + \hat{A}_1 Z_{T+y} + \hat{A}_2 Z_{T+y-1} + \epsilon_{T+y}\right\} = d$$

  – or

$$(\hat{A}_0 a + \hat{A}_1) Z_{T+y} + \hat{A}_2 Z_{T+y-1} + \epsilon_{T+y} = d$$

$$\rightarrow Z_{T+y} = a_1 Z_{T+y-1} + b_1 \epsilon_{T+y} + d_1$$

$$a_1 \equiv -(\hat{A}_0 a + \hat{A}_1)^{-1} \hat{A}_2$$

$$b_1 \equiv -(\hat{A}_0 a + \hat{A}_1)^{-1}$$

$$d_1 \equiv (\hat{A}_0 a + \hat{A}_1)^{-1} d$$

Backward, cnt'd

- Period *t=T+y-1*:

$$E_{T+y-1}\left\{\hat{A}_0 \overbrace{Z_{T+y}}^{=a_1 Z_{T+y-1}+b_1\epsilon_{T+y}+d_1} + \hat{A}_1 Z_{T+y-1} + \hat{A}_2 Z_{T+y-2} + \epsilon_{T+y-1}\right\} = d.$$

  – or

$$(\hat{A}_0 a_1 + \hat{A}_1) Z_{T+y-1} + \hat{A}_0 d_1 + \hat{A}_2 Z_{T+y-2} + \epsilon_{T+y-1} = d.$$

$$\rightarrow Z_{T+y-1} = a_2 Z_{T+y-2} + b_2 \epsilon_{T+y-1} + d_2$$

$$a_2 = -(\hat{A}_0 a_1 + \hat{A}_1)^{-1} \hat{A}_2$$

$$b_2 = -(\hat{A}_0 a_1 + \hat{A}_1)^{-1}$$

$$d_2 = (\hat{A}_0 a_1 + \hat{A}_1)^{-1} (d - \hat{A}_0 d_1)$$

  – and so on.....

Backwards, cnt'd

- Solution for *t=T+1,…,T+y*.

$$Z_{T+y-j} = a_{j+1} Z_{T+y-j-1} + b_{j+1} \epsilon_{T+y-j},$$

  - for *j=0,1,2,…,y-1*, where

$$a_{j+1} = -(\hat{A}_0 a_j + \hat{A}_1)^{-1} \hat{A}_2,$$

$$b_{j+1} = -(\hat{A}_0 a_j + \hat{A}_1)^{-1}$$

$$d_{j+1} = (\hat{A}_0 a_j + \hat{A}_1)^{-1} (d - \hat{A}_0 d_j),$$

$$a_0 \equiv a,\ b_0 \equiv b,\ d_0 = 0.$$

In Sum

- Future stochastic realization of length, *x*, with interest rate fixed at some specified value for y<x periods….Three steps:
- Backward step:

$$a_0, a_1, \ldots, a_y; b_0, b_1, \ldots, b_y; d_0, d_1, \ldots, d_y$$

- Two forward steps: draw shocks, and simulate

$$\overbrace{\epsilon_{T+1}, \ldots, \epsilon_{T+y}}^{\text{realization of future shocks during fixed interest rate regime}}, \overbrace{\epsilon_{T+y+1}, \ldots, \epsilon_{T+x}}^{\text{realization of shocks after fixed interest rate regime}}$$

$$\begin{aligned}
Z_{T+1} &= a_y Z_T + b_y \epsilon_{T+1} + d_y \\
Z_{T+2} &= a_{y-1} Z_{T+1} + b_{y-1} \epsilon_{T+2} + d_{y-1} \\
&\ldots \\
Z_{T+y} &= a_1 Z_{T+y-1} + b_1 \epsilon_{T+y} + d_1 \\
Z_{T+y+1} &= a Z_{T+y} + b \epsilon_{T+y+1} \\
&\ldots \\
Z_{T+x} &= a Z_{T+x-1} + b \epsilon_{T+x}
\end{aligned}$$

Simple New Keynesian Model

log deviation of actual and natural output ('output gap')

Net rate of inflation
(deviated from natural inflation, which is zero)

$$\beta E_t \pi_{t+1} + \kappa x_t - \pi_t = 0 \text{ (Phillips curve)}$$

Nominal net rate of interest

$$-[r_t - E_t \pi_{t+1} - r_t^*] + E_t x_{t+1} - x_t = 0 \text{ (IS equation)}$$

$$\alpha r_{t-1} + (1-\alpha)\phi_\pi \pi_t + (1-\alpha)\phi_x x_t - r_t = 0 \text{ (policy rule)}$$

a_t is log technology shock, which is AR(1) in first difference with ar coefficient ρ

$$r_t^* - \rho \Delta a_t - \frac{1}{1+\varphi}(1-\lambda)\tau_t = 0 \text{ (definition of natural rate)}$$

an AR(1) shock to disutility of work, with ar coefficient, λ

Natural rate of interest

$\frac{1}{\varphi}$ is Frish labor supply elasticity

Solving the Model

$$s_t = \begin{pmatrix} \Delta a_t \\ \tau_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \lambda \end{bmatrix} \begin{pmatrix} \Delta a_{t-1} \\ \tau_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^{\tau} \end{pmatrix}$$

$$s_t = P s_{t-1} + \epsilon_t$$

$$\begin{bmatrix} \beta & 0 & 0 & 0 \\ \frac{1}{\sigma} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \\ r_{t+1} \\ r_{t+1}^{*} \end{pmatrix} + \begin{bmatrix} -1 & \kappa & 0 & 0 \\ 0 & -1 & -\frac{1}{\sigma} & \frac{1}{\sigma} \\ (1-\alpha)\phi_{\pi} & (1-\alpha)\phi_{x} & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ rr_t^{*} \end{pmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \\ r_{t-1} \\ r_{t-1}^{*} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} s_{t+1} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\sigma\psi\rho & -\frac{1}{\sigma+\varphi}(1-\lambda) \end{pmatrix} s_t$$

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

Model Solution

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

$$s_t - P s_{t-1} - \epsilon_t = 0.$$

- Solution:

$$z_t = A z_{t-1} + B s_t$$

- where:

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0,$$

$$(\beta_0 + \alpha_0 B)P + [\beta_1 + (\alpha_0 A + \alpha_1)B] = 0$$

Simulation

- Parameter values:

$\beta = 0.99,\ \phi_x = 0,\ \phi_\pi = 1.5,\ \alpha = 0,\ \rho = 0,\ \lambda = 0.5,\ \varphi = 1,\ \theta = 0.75$ (Calvo sticky price parameter)

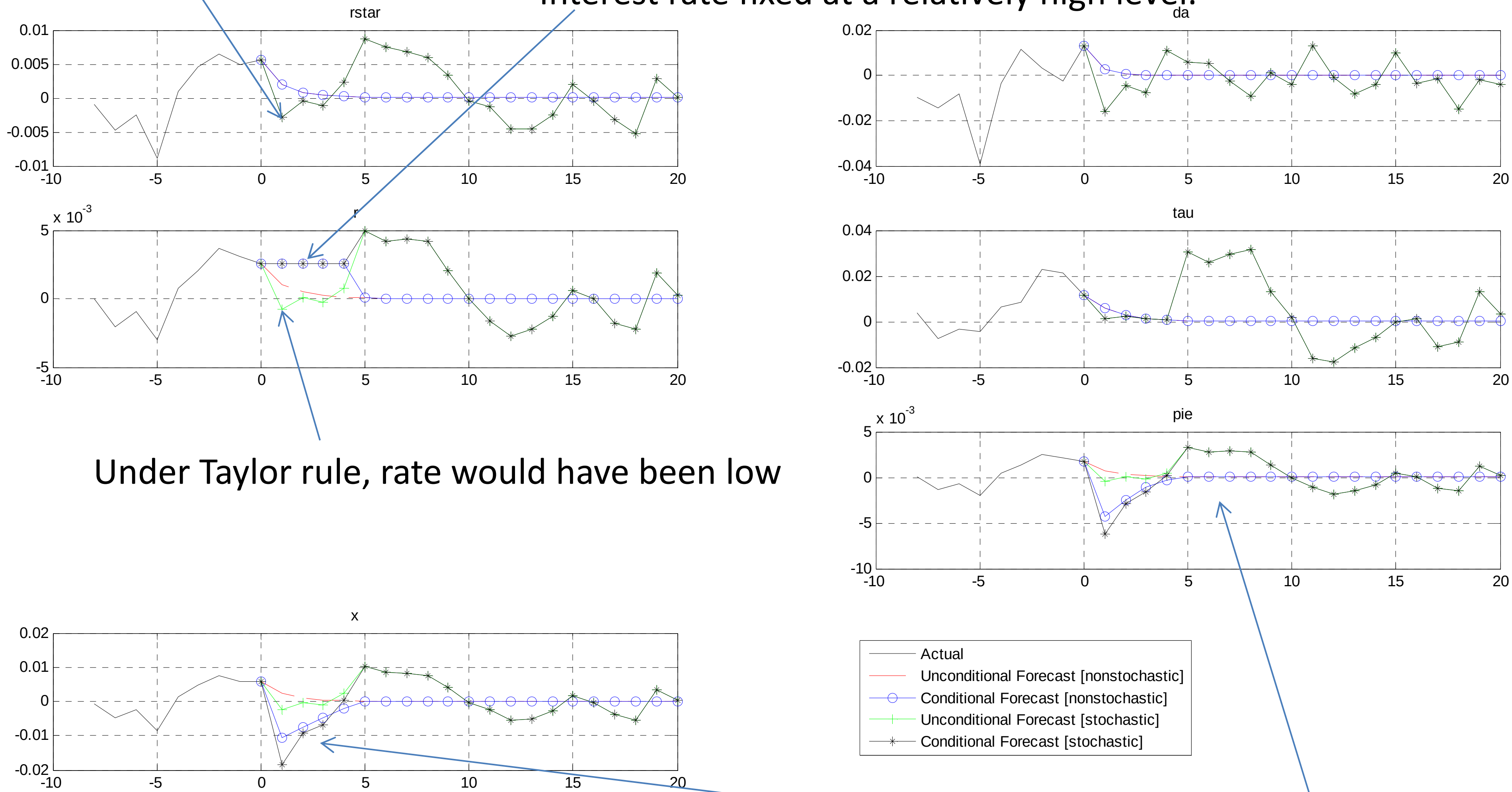
variance, innovation in preference shock $= 0.01^2$, variance, innovation in technology growth $= 0.01^2$

$$\kappa = \frac{(1-\theta)(1-\beta\theta)(1+\varphi)}{\theta} = 0.1717.$$

- Experiment:
  - From periods -8,-7,…,-1,0, economy is stochastically fluctuating with Taylor rule in place.
  - At period 0, monetary authority commits to keeping interest rate fixed in t=1,2,3,4, at the value it took on in t=0. Afterward, return to Taylor rule
  - After period 0, economy continues to be hit by shocks.

Under optimal policy, rate would have been low

Interest rate fixed at a relatively high level.



Under Taylor rule, rate would have been low

Because we consider a high interest gap and inflation are low.

‘Actual’ Taylor rule followed in each period.
‘Unconditional’ follow Taylor rule
‘Conditional’ fix interest rate in t=1,2,3,4.
‘Nonstochastic’ set shocks in t>1 to zero (gives mean prediction as of t=0)
‘Stochastic’ shocks drawn from Normal, mean zero, variance indicated, in all periods

The Zero Lower Bound

- Monetary policy:

$$Z_t = R + \rho(R_{t-1} - R) + (1-\rho)[\alpha_\pi(\pi_t - \pi) + \alpha_y(y_t - y)]$$

$$R_t = \begin{cases} Z_t & Z_t \geq 0 \\ 0 & Z_t < 0 \end{cases}$$

- Here, Z_t (the ‘shadow interest rate’) is the value to which they would ideally like to set the interest rate.

- When $Z_t<0$, then the zero bound ‘binds’.
  - If Z_t is *very* negative then the zero bound binds a lot.
  - In this case, R_t remains zero even with fluctuations in inflation and output.

ZLB, cnt'd

- Ideal way to model ZLB
  - Sometimes binding, sometimes not.
  - Projection method ideal for this case, but difficult.
- Alternative approaches to ZLB.
  - Eggertsson and Woodford: assume we're in the zero bound, use very simple model and slightly unrealistic assumption about how you get out.
    - Can solve what happens while you're in, trivially.
  - With empirically realistic models: assume we're in the zero bound and that you will leave forever at a specific date in the future.
    - That's the case we can handle easily with the preceding approach.

Representation of Equilibrium Conditions

- Now, the vector, z_t, contains Z_t and R_t as variables.
  - One equation is the Taylor rule, determining the shadow interest rate, Z_t.

- There is a second equation, which is one thing when zlb is binding and another when it is not.
  - When zlb not binding, R_t=Z_t.
  - When zlb binds, latter equation replaced by R_t=*0.*

- Simulation
  - Difficult to incorporate stochastic shocks, because it makes exit from zero bound stochastic and this is hard to deal with outside the E-W example.
  - Will want deterministic simulation in response to sequence of deterministic s_t's that push economy into binding zlb.
  - For this, must revert to initial notation for characterizing equilibrium conditions.

Model

- Equilibrium conditions after zlb ceases to bind and shocks are back to deterministic steady state values:

$$\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} = 0$$

  – Here,
    - z_t denotes the list of endogenous variables whose values are determined at time t.

- Solution: A in

$$z_t = A z_{t-1}$$

  – where $\alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0,$

Model

- Exogenous shocks take on values, $s_1, s_2, \ldots, s_T$ and $s_t=0$ for $t>T$.

- For $t=1,\ldots,T$, equilibrium conditions are

$$\alpha_{0,t} z_{t+1} + \alpha_{1,t} z_t + \alpha_{2,t} z_{t-1} + \beta_{0,t} s_{t+1} + \beta_{1,t} s_t = d_t$$

$$\alpha_{i,t} = \begin{cases} \alpha_i & Z_t \geq 0 \\ \hat{\alpha}_i & Z_t < 0 \end{cases}, \; i = 0,1,2, \; d_t = \begin{cases} 0 & Z_t \geq 0 \\ d & Z_t < 0 \end{cases}$$

- Use same algorithm as before to solve 'backwards' for policy rule governing evolution of z_t's for $t=1,2,3\ldots$.
  - Do this based on a conjecture about when zlb is binding.

- With the sequence of policy rules in hand, simulate z_t's, $t=1,2,\ldots$
  - Evaluate conjecture about when zlb is binding. If conjecture verified, stop. Otherwise, change conjecture and redo backward solution and forward simulation.

- For an example, see Christiano-Eichenbaum-Rebelo JPE, 2011.