Optimal Policy

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Optimal Policy

- Suppose
  - we have a model economy with N equations, one of which is a monetary policy rule.
  - The model has a unique solution.
- Throw away the monetary policy rule.
  - Now the system is under-determined: many equilibria (i.e., settings for the variables that satisfy the equilibrium conditions).
  - Choose the 'best' equilibrium.
    - Ramsey – best optimizes social welfare function
    - Riksbank, Norges Bank: best optimizes objectives of MP committee.
- Policy is reported in the form of the sequence of variables in the best equilibrium.

Risk: Time Inconsistency

Action, *X*

now future

- Now: in selecting and announcing *X*, take into account impact on private economy now. (*X* could be high future inflation, which stimulates economy today).
- Future: 'now' is past history, so *X* no longer affects private economy now. Will want to choose different ('time inconsistent') value for *X*.
- Temptation to be time-inconsistent must be avoided.
  - Otherwise, credibility is lost and economy may slide down a slippery slope (wage-price spiral).
  - Policymakers must have discipline to be time-consistent.
  - Technically: remember your multipliers! Adopt timeless perspective

Outline

- Will do two examples: 'toy example' and the general case.

- In earlier discussion of simple New Keynesian model, carefully solved for Ramsey-optimal policy.

- Longer version of these notes go into more detail.
  - Assignment 8 provides computer exercises.

Example #1: Optimal Monetary Policy - Toy Example

- Setup
  - Model
    - One equation characterizing private sector behavior:

$$\pi_t - \beta\pi_{t+1} - \gamma y_t = 0,\ t = 0, 1, 2, ... \tag{1}$$

    - Another equation characterizes policy.
  - Want to do *optimal* policy, so throw away policy equation.
  - System is now under-determined: one equation in two variables, π_t and y_t.

Example #1: Optimal Monetary Policy - Toy Example ...

– Optimization delivers the other equations.

* optimize objective:

$$\sum_{t=0}^{\infty} \beta^t u(\pi_t, y_t)$$

subject to (1).

* If objective corresponds to social welfare function, this is called *Ramsey* optimal problem

* Objective may be preferences of policy maker. (Norges Bank, Riksbank)

Example #1: Optimal Monetary Policy - Toy Example ...

- Lagrangian representation of problem:

$$\max_{\{\pi_t, y_t; t=0,1,\ldots\}} \sum_{t=0}^{\infty} \beta^t \left\{ u(\pi_t, y_t) + \lambda_t \left[\pi_t - \beta \pi_{t+1} - \gamma y_t \right] \right\}$$

$$= \max_{\{\pi_t, y_t; t=0,1,\ldots\}} \left\{ u(\pi_0, y_0) + \lambda_0 \left[\pi_0 - \beta \pi_1 - \gamma y_0 \right] \right.$$

$$\left. + \beta u(\pi_1, y_1) + \beta \lambda_1 \left[\pi_1 - \beta \pi_2 - \gamma y_1 \right] + \ldots \right\}$$

- First order necessary conditions for optimization:

$$\begin{aligned}
u_\pi(\pi_0, y_0) + \lambda_0 &= 0 \text{ (*)} \\
u_\pi(\pi_1, y_1) + \lambda_1 - \lambda_0 &= 0 \\
&\ldots \\
u_y(\pi_0, y_0) - \gamma \lambda_0 &= 0 \\
u_y(\pi_1, y_1) - \gamma \lambda_1 &= 0 \\
&\ldots \\
\pi_0 - \beta \pi_1 - \gamma y_0 &= 0 \\
\pi_1 - \beta \pi_2 - \gamma y_1 &= 0 \\
&\ldots
\end{aligned}$$

Constraints also part of the necessary conditions!

Example #1: Optimal Monetary Policy - Toy Example ...

- These equations 'look' different than the ones we've seen before
  - They are not stationary, (*) is different from the others.
    - reflects that at time 0 there is a constraint 'missing'

Looks like our perturbation/projection solution methods may not apply!
These require that the system of equilibrium conditions be time-invarient.

Example #1: Optimal Monetary Policy - Toy Example ...

- These equations 'look' different than the ones we've seen before
  - They are not stationary, (*) is different from the others.
    - reflects that at time 0 there is a constraint 'missing'
    - no need to respect what people were expecting you to do as of time -1

Heart of time inconsistency problem!

Example #1: Optimal Monetary Policy - Toy Example ...

- These equations 'look' different than the ones we've seen before
  - They are not stationary, (*) is different from the others.
    - reflects that at time 0 there is a constraint 'missing'
    - no need to respect what people were expecting you to do as of time -1
    - do need to respect what they expect you to do in the future, because that affects current behavior.
    - that's the source of the 'time inconsistency of optimal plans'.
- Can trick the problem into being stationary (see, e.g., Kydland and Prescott (JEDC, 1990s) and Levin, Onatski, Williams, and Williams, Macro Annual, 2005). Then, apply standard log-linearization solution method.

Example #1: Optimal Monetary Policy - Toy Example ...

- Consider:

$$v\left(\pi_t, \pi_{t+1}, y_t, \lambda_t, \lambda_{t-1}\right) = \begin{bmatrix} u_\pi\left(\pi_t, y_t\right) + \lambda_t - \lambda_{t-1} \\ u_y(\pi_t, y_t) - \gamma\lambda_t \\ \pi_t - \beta\pi_{t+1} - \gamma y_t \end{bmatrix}, \text{ for all } t.$$

  – time t ‘endogenous variables’: λ_t, π_t, y_t

  – time t ‘state variable’: λ_{t-1}.

  – ‘solution’:

$$\lambda_t = \lambda\left(\lambda_{t-1}\right), \ \pi_t = \pi\left(\lambda_{t-1}\right), \ y_t = y\left(\lambda_{t-1}\right),$$

  such that

$$v\left(\pi\left(\lambda_{t-1}\right), \pi\left(\lambda\left(\lambda_{t-1}\right)\right), y\left(\lambda_{t-1}\right), \lambda\left(\lambda_{t-1}\right), \lambda_{t-1}\right) = 0, \text{ for all possible } \lambda_{t-1}.$$

Example #1: Optimal Monetary Policy - Toy Example ...

- In general, solving this problem exactly is intractable.
- But, can log-linearize!

  - **Step 1**: find π^*, y^*, λ^* such that following three equations are satisfied:

$$v(\pi^*, \pi^*, y^*, \lambda^*, \lambda^*) = \underbrace{0}_{3\times 1}.$$

  - **Step 2**: log-linearly expand v about steady state

$$v(\pi_t, \pi_{t+1}, y_t, \lambda_t, \lambda_{t-1}) \simeq v_1\pi^*\hat{\pi}_t + v_2\pi^*\hat{\pi}_{t+1} + v_3 y^*\hat{y}_t + v_4\Delta\hat{\lambda}_t + v_5\Delta\hat{\lambda}_{t-1},$$

    where

$\Delta\hat{\lambda}_t \equiv \lambda_t - \lambda^*$ (play it safe, don't divide by something that could be zero!)

  - **Step 3**: Posit

$$\Delta\hat{\lambda}_t = A_\lambda\Delta\hat{\lambda}_{t-1},\ \hat{\pi}_t = A_\pi\Delta\hat{\lambda}_{t-1},\ \hat{y}_t = A_y\Delta\hat{\lambda}_{t-1},$$

    and find A_λ, A_π, A_y that solve

$$[v_1\pi^* A_\pi + v_2\pi^* A_\pi A_\lambda + v_3 y^* A_y + v_4 A_\lambda + v_5]\,\Delta\hat{\lambda}_{t-1} = \underbrace{0}_{3\times 1}$$

    for all $\Delta\hat{\lambda}_{t-1}$.

Example #1: Optimal Monetary Policy - Toy Example ...

- What does the stationary solution have to do with the original non-stationary problem?

  – Do we have a solution to the period 0 problem, (*)?

$$u_\pi(\pi_0, y_0) + \lambda_0 = 0.$$

  – Yes! Just pretend that this equation really has the following form:

$$u_\pi(\pi_0, y_0) + \lambda_0 - \lambda_{-1} = 0.$$

  Expression (*) does have this form, if we set $\lambda_{-1} = 0$. Then,

$$\pi_0 = \pi(0), \; y_0 = y(0), \; \lambda_0 = \lambda(0).$$

Example #1: Optimal Monetary Policy - Toy Example ...

- The situation is exactly what it is in the neoclassical model when we want to know what happens when initial capital is away from steady state.
  - Plug k_0 into the stationary rule

$$k_1 = g(k_0).$$

- Possible computational pitfall: if $\lambda_{-1} = 0$ is far from λ^*, then linearized solution might be highly inaccurate

Example #1: Optimal Monetary Policy - Toy Example ...

- Optimal policy in real time.

- Suppose today is date zero.

  – Solve for $\lambda(\cdot)$, $y(\cdot)$, $\pi(\cdot)$

  – set $\lambda_{-1} = 0$

  – Compute and present in charts:

$$
\begin{aligned}
\lambda_0 &= \lambda(\lambda_{-1}),\ y_0 = y(\lambda_{-1}),\ \pi_0 = \pi(\lambda_{-1}) \\
\lambda_1 &= \lambda(\lambda_0),\ y_1 = y(\lambda_0),\ \pi_1 = \pi(\lambda_0) \\
&\ldots \\
\lambda_t &= \lambda(\lambda_{t-1}),\ y_t = y(\lambda_{t-1}),\ \pi_t = \pi(\lambda_0) \\
&\ldots.
\end{aligned}
$$

Example #1: Optimal Monetary Policy - Toy Example ...

- The optimal policy program may break down if policy makers succumb to the temptation to restart the Ramsey problem at a later date.
  - there is a temptation in period 1 when π_1 is determined, to ignore a constraint that went into determining the announcement made about π_1 in period 0:

$$\pi_0 - \beta\pi_1 - \gamma y_0 \text{ (*)}$$

  - If (*) is ignored at date 1, then π_1 computed in date 1 solves a different problem than π_1 computed at date 0 and there will be time inconsistency.

Example #1: Optimal Monetary Policy - Toy Example ...

- Honoring past announcements is equivalent to 'always respect the past multipliers'.
  - 'Remembering λ_0' in period 1 ensures that constraint

$$\pi_0 - \beta\pi_1 - \gamma y_0 \, (*)$$

    is incorporated in period 1. In this case, π_1 solves the same problem in period 1 that it did in period 0.

Example #1: Optimal Monetary Policy - Toy Example ...

– Example:

date 0 meeting : $y_0 = y(0), \; y_1 = y(\lambda(\lambda_{-1})), y_2 = y(\lambda(\lambda(\lambda_{-1}))), \ldots$

date 1 meeting : **YES -** $y_1 = y(\lambda(\lambda_{-1})), \; y_2 = y(\lambda(\lambda(\lambda_{-1}))), \ldots$

'Timeless perspective' – today is not 'date zero', that occurred at an unspecified time in the past.

Because we do not say when date zero occurred, we are taking a timeless perspective.

Example #1: Optimal Monetary Policy - Toy Example ...

– Example:

date 0 meeting : $y_0 = y(0),\ y_1 = y(\lambda(\lambda_{-1})), y_2 = y(\lambda(\lambda(\lambda_{-1}))), \ldots$

date 1 meeting :
YES - $y_1 = y(\lambda(\lambda_{-1})),\ y_2 = y(\lambda(\lambda(\lambda_{-1}))), \ldots$
NO - $y_1 = y(0),\ y_2 = y(\lambda_1(0)), \ldots$

If you restart the optimization problem today , then today is 'date zero' and you set the multipliers to zero. But, in this case, you are time Inconsistent.

Example #1: Optimal Monetary Policy - Toy Example ...

– Example:

date 0 meeting : $y_0 = y(0)\,,\ y_1 = y(\lambda(\lambda_{-1}))\,, y_2 = y(\lambda(\lambda(\lambda_{-1})))\,, \ldots$

date 1 meeting :
YES - $y_1 = y(\lambda(\lambda_{-1}))\,,\ y_2 = y(\lambda(\lambda(\lambda_{-1})))\,, \ldots$
NO - $y_1 = y(0)\,,\ y_2 = y(\lambda_1(0))\,, \ldots$

– If Central Bank selects the bad ('**NO**') option people will see the temporal inconsistency of policy, and CB will lose credibility.

Example #1: Optimal Monetary Policy - Toy Example ...

– Example:

$$\text{date 0 meeting} : \quad y_0 = y(0), \; y_1 = y(\lambda(\lambda_{-1})), y_2 = y(\lambda(\lambda(\lambda_{-1}))), \ldots$$

$$\text{date 1 meeting} : \quad \begin{array}{ll} \textbf{YES -} & y_1 = y(\lambda(\lambda_{-1})), \; y_2 = y(\lambda(\lambda(\lambda_{-1}))), \ldots \\ \textbf{NO -} & y_1 = y(0), \; y_2 = y(\lambda_1(0)), \ldots \end{array}$$

– If Central Bank selects the bad ('**NO**') option people will see the temporal inconsistency of policy, and CB will lose credibility.

– Any differences in charts from one meeting to the next must be fully explicable in terms of new information.

Example #2: Optimal Monetary Policy - More General Discussion

- The equilibrium conditions of a model

$$E_t \underbrace{f\left(z_{t-1}, z_t, z_{t+1}, s_t, s_{t+1}\right)}_{(N-1)\times 1} = 0, \text{ for all } \underbrace{z_{t-1}}_{N\times 1} \text{ (endogenous), } s_t \text{ (exogenous)}$$

$$s_t = P s_{t-1} + \varepsilon_t.$$

- Preferences:

$$E_t \sum_{t=0}^{\infty} \beta^t U\left(z_t, s_t\right).$$

  – Could include discounted utility in f :

$$v\left(z_{t-1}, z_t, s_t\right) = U\left(z_t, s_t\right) + \beta E_t v\left(z_t, z_{t+1}, s_{t+1}\right)$$

Example #2: Optimal Monetary Policy - More General Discussion ...

- Optimum problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(z_t, s_t) + \underbrace{\lambda_t'}_{1\times(N-1)} E_t \underbrace{f(z_{t-1}, z_t, z_{t+1}, s_t, s_{t+1})}_{(N-1)\times 1} \right\}.$$

- N first order conditions:

$$\begin{aligned}
&\underbrace{U_1(z_t, s_t)}_{1\times N} + \underbrace{\lambda_t'}_{1\times(N-1)} E_t \underbrace{f_2(z_{t-1}, z_t, z_{t+1}, s_t, s_{t+1})}_{(N-1)\times N} \\
&\quad +\beta^{-1} \underbrace{\lambda_{t-1}'}_{1\times(N-1)} \underbrace{f_3(z_{t-2}, z_{t-1}, z_t, s_{t-1}, s_t)}_{(N-1)\times N} \\
&\quad +\beta \underbrace{\lambda_{t+1}'}_{1\times(N-1)} E_t \underbrace{f_1(z_t, z_{t+1}, z_{t+2}, s_{t+1}, s_{t+2})}_{(N-1)\times N} = \underbrace{0}_{1\times N}
\end{aligned}$$

  – Endogenous variables: z_t (N), λ_t $(N-1)$

  – Equations: Ramsey optimality conditions (N), equilibrium condition $(N-1)$

Example #2: Optimal Monetary Policy - More General Discussion ...

- First order conditions of optimum problem have exactly the same form as the type of problem we solved using linearization methods.

- Seem much more cumbersome:

  – must differentiate f (includes private first order conditions that have already involved differentiation!)

  – good news: LOWW wrote a program that takes U, f as input and writes Dynare code for solving the system

  – solving policy optimum problem is no harder than solving original problem.

- Dynare now also has the ability to solve for optimal equilibrium. However, unlike LOWW, Dynare does not provide user with the equilibrium conditions. This can be problem if you want to compute higher-order perturbation solution, because Dynare (now) only does first order perturbation on Ramsey. With LOWW you get the actual equilibrium conditions and so can do any order perturbation.

Conclusion

- Although we often speak of policy as a Taylor rule, probably some version of previous approach is taken, either explicitly or implicitly.

- This requires discipline and not reneging on previous announcements:
  - Technically, must remember your multipliers.
  - In this case, you are adopting the ‘timeless perspective’.