# Two Financial Friction Models

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Motivation

- Beginning in 2007 and then accelerating in 2008:
  - Asset values (particularly for banks) collapsed.
  - Intermediation slowed and investment/output fell.
  - Interest rates spreads over what the US Treasury and highly safe private firms had to pay, jumped.
  - US central bank initiated unconventional measures (loans to financial and non-financial firms, very low interest rates for banks, etc.)
- In 2009 – the worst parts of 2007-2008 began to turn around.

Collapse in Asset Values and Investment

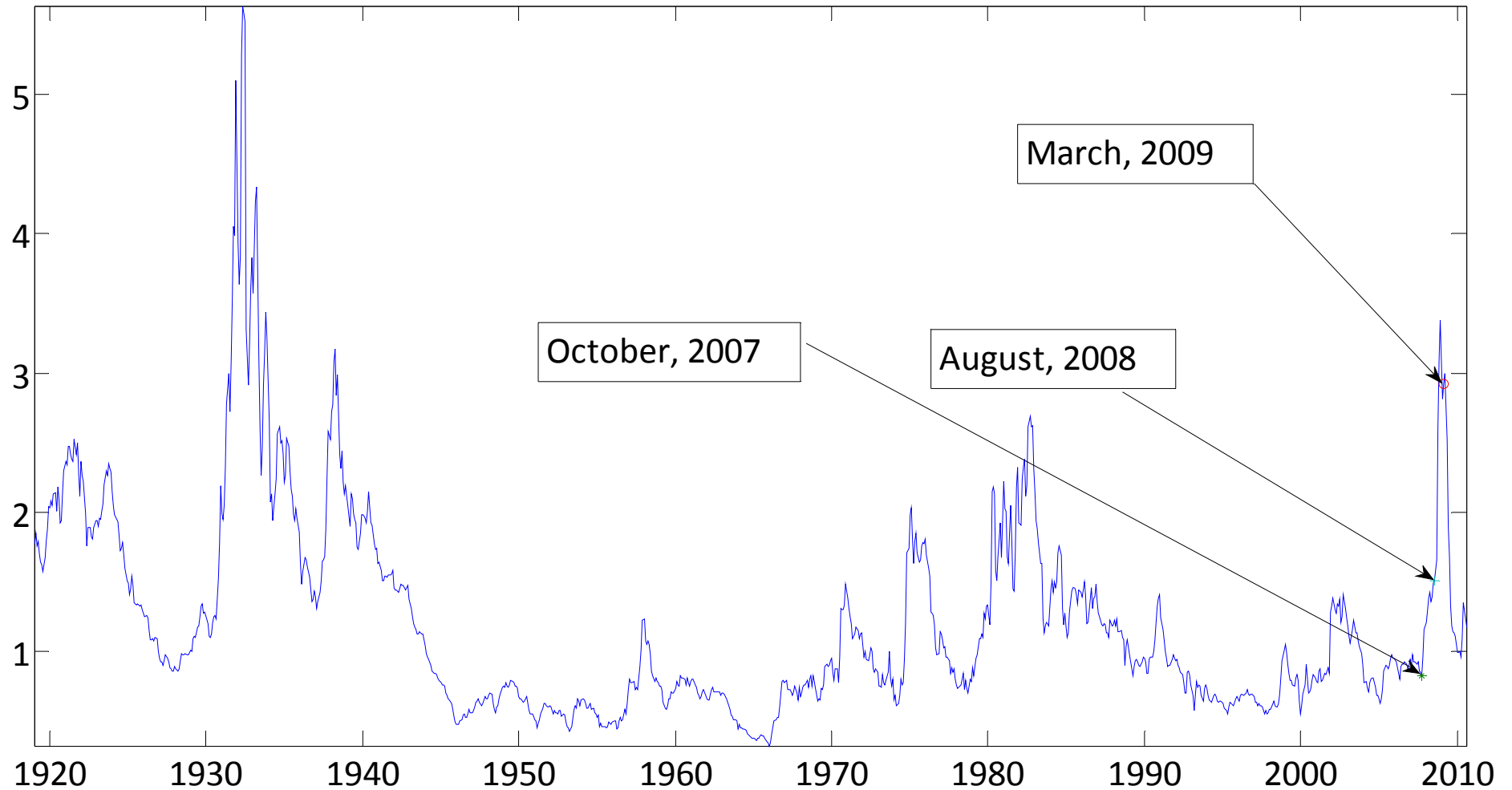Log, real Stock Market Index, real Housing Prices and real Investment
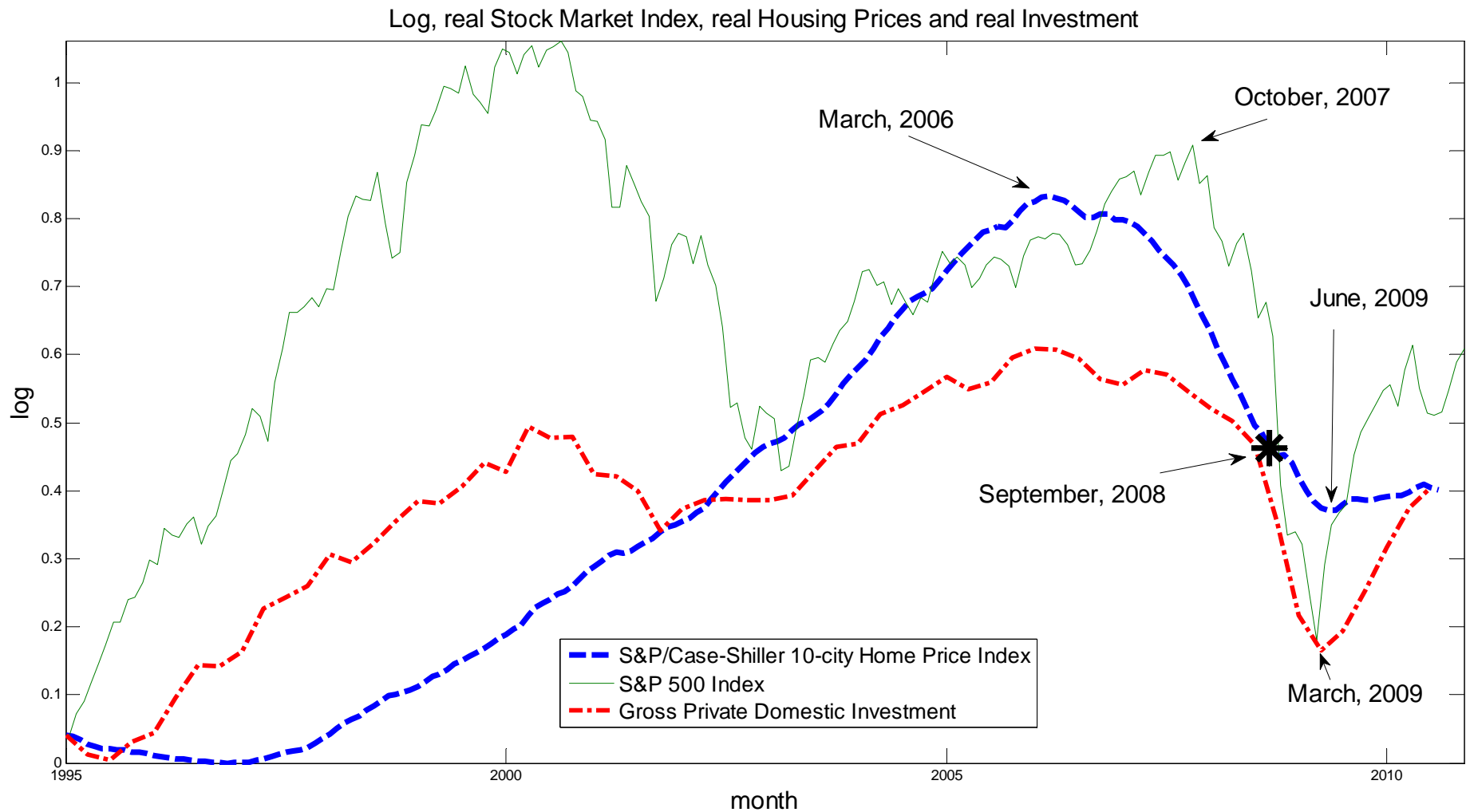
Spreads for 'Risky' Firms Shot Up in Late 2008

Interest Rate Spread on Corporate Bonds of Various Ratings Over Rate on AAA Corporate Bonds
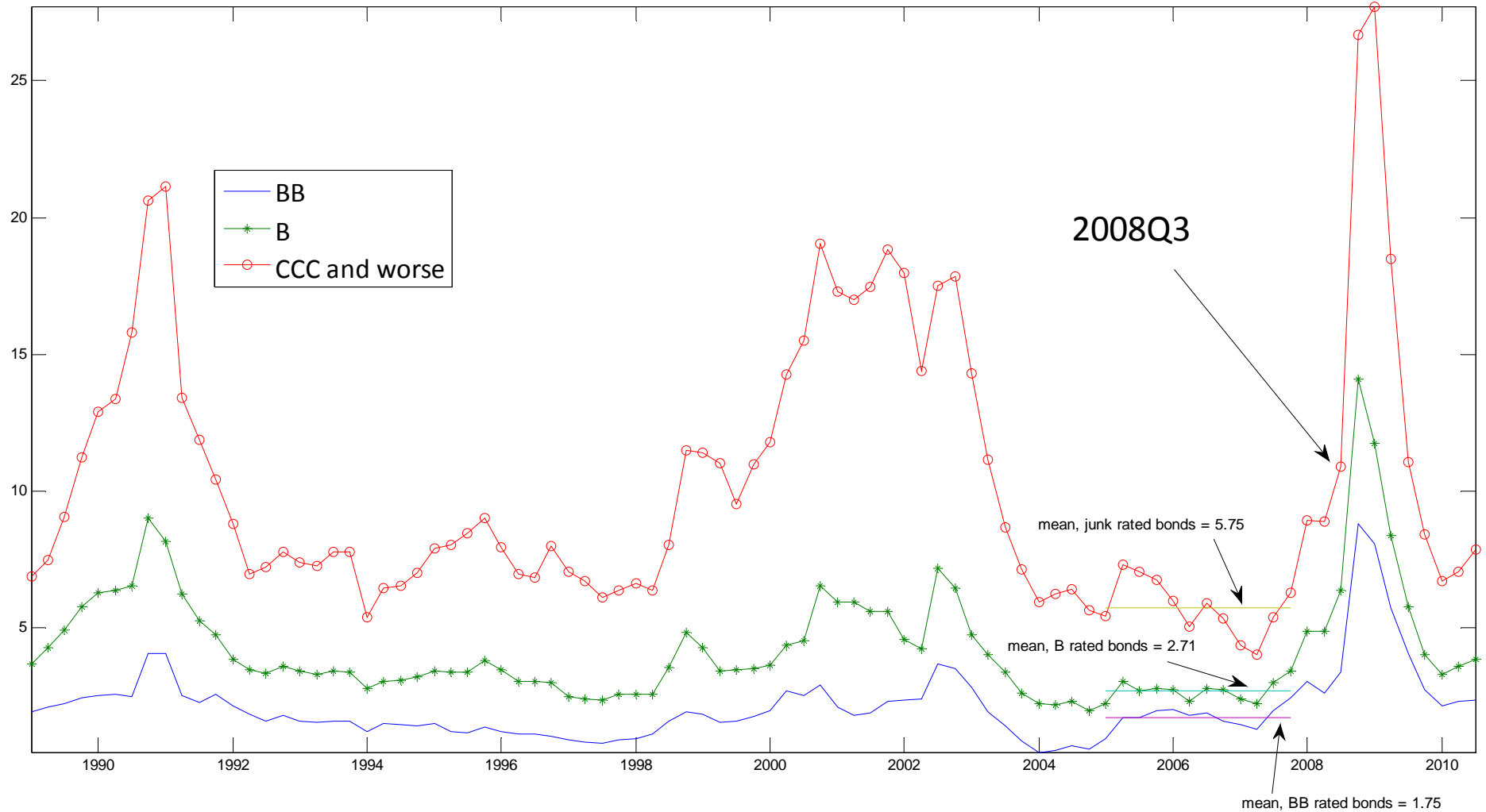
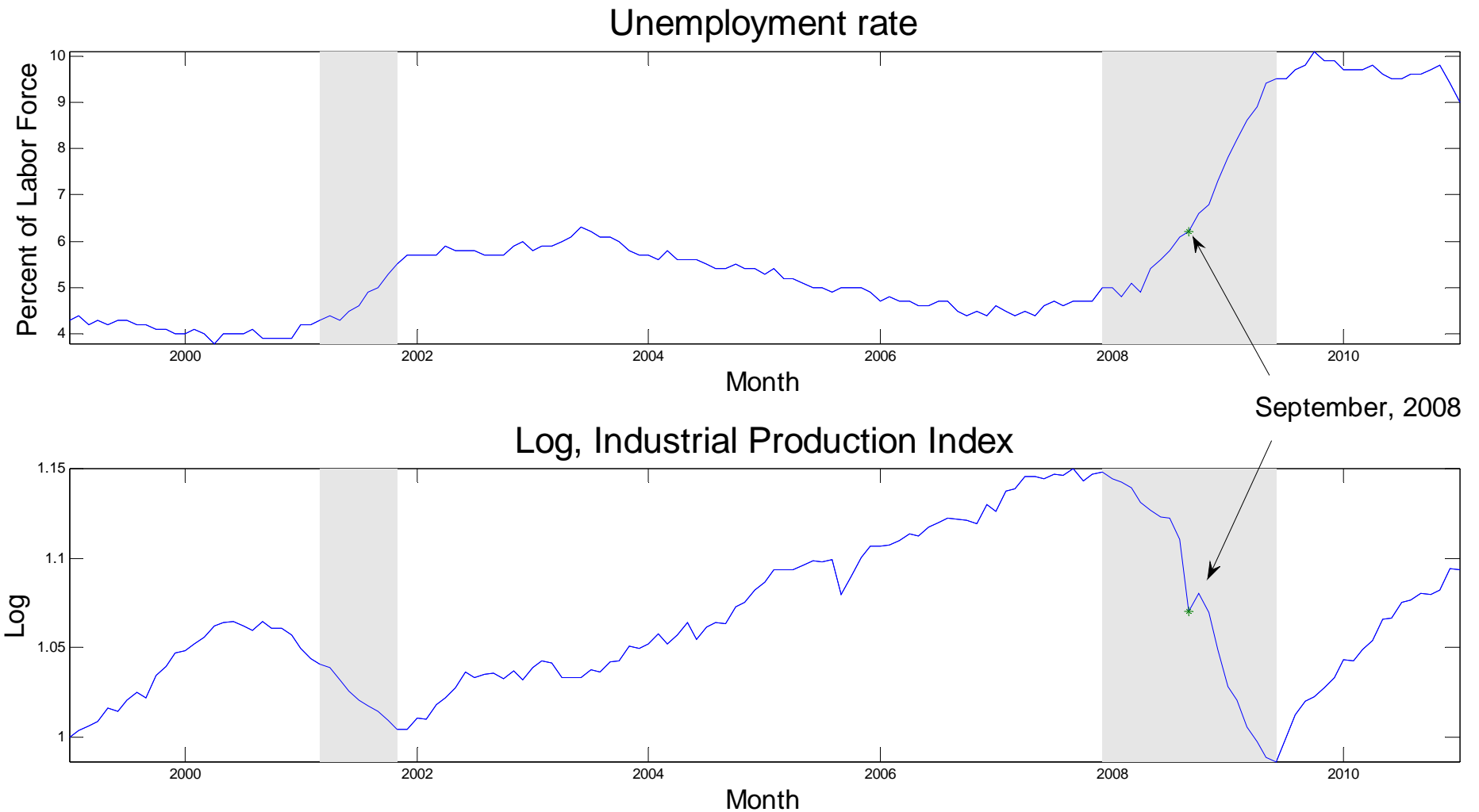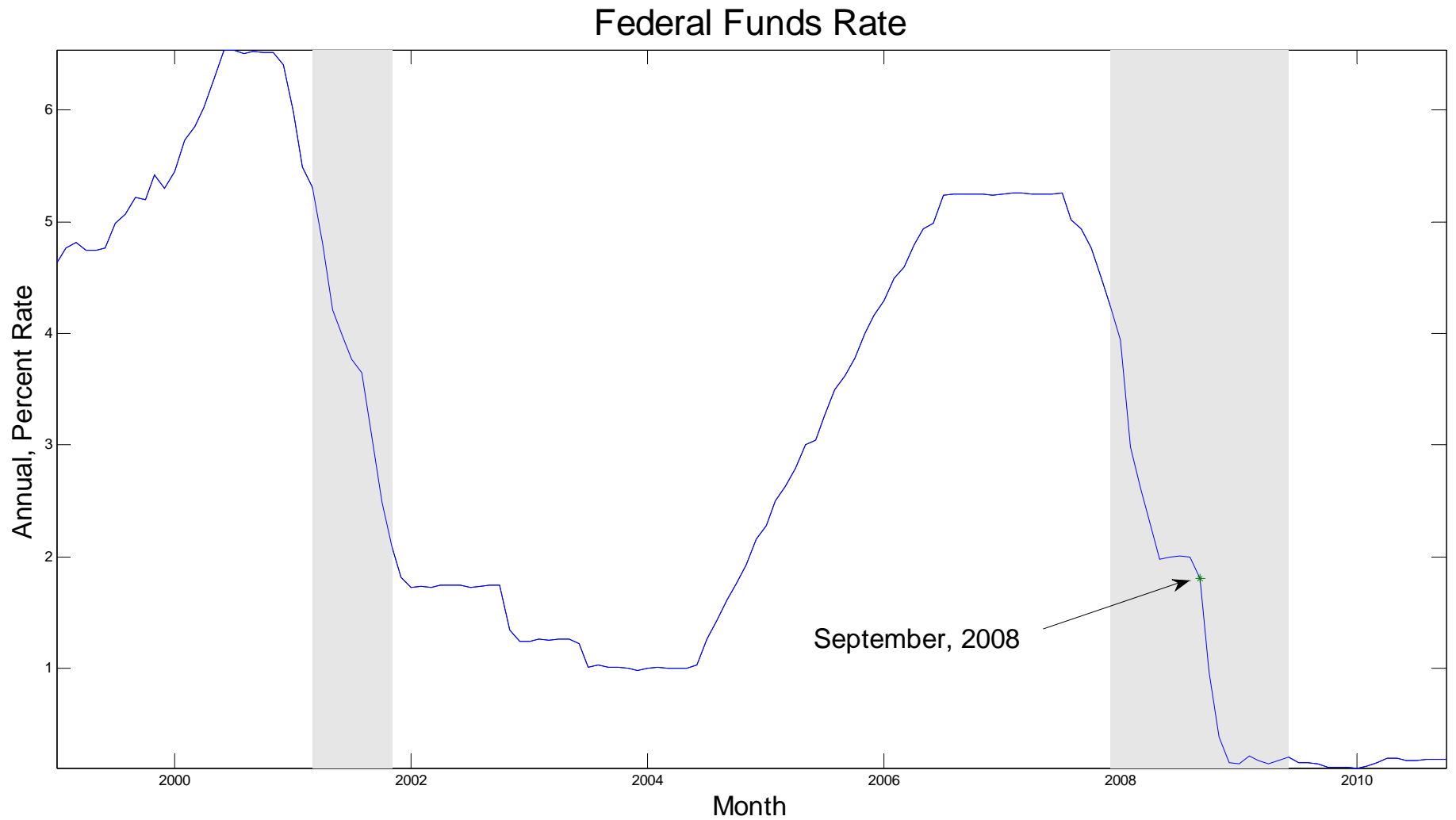# Must Go Back to Great Depression to See Spreads as Large as the Recent Ones

Spread, BAA versus AAA bonds

March, 2009

October, 2007

August, 2008

Economic Activity Shows (tentative) Signs of Recovery June, 2009

Banks' Cost of Funds Low

Federal Funds Rate

September, 2008

Characterization of Crisis to be Explored Here

- Bank Asset Values Fell.
- Banking System Became 'Dysfunctional'
  - Interest rate spreads rose.
  - Intermediation and economy slowed.
- Monetary authority:
  - Transferred funds on various terms to private companies and to banks.
  - Sharply reduced cost of funds to banks.
- Economy in (tentative) recovery.
- Seek to construct models that links these observations together.

Objective

- Keep analysis simple and on point by:
  - Two periods
  - Minimize complications from agent heterogeneity.
  - Leave out endogeneity of employment.
  - Leave out nominal variables: just look 'behind the veil of monetary economics'

- Two models:
  - Moral hazard I: Gertler-Kiyotaki/Gertler-Karadi
  - Moral hazard II: hidden effort by bankers.

Two-period Version of GK Model

- Many identical households, each with a unit measure of members:
  - Some members are 'bankers'
  - Some members are 'workers'
  - Perfect insurance inside households...everyone consumes same amount.
- Period 1
  - Workers endowed with y goods, household makes deposits, d, in a bank
  - Bankers endowed with N goods, take deposits and purchase securities, d, from a firm.
  - Firm issues securities, s, to produce sR^k in period 2.
- Period 2
  - Household consumes earnings from deposits plus profits, π, from banker.
  - Goods consumed are produced by the firm.

Problem of the Household		
	period 1	period 2
budget constraint	$c + d \leq y$	$C \leq R^d d + \pi$
problem	$\max_{c,C,d}\left[u(c) + \beta u(C)\right]$	

Solution to Household Problem	
$\dfrac{u'(c)}{\beta u'(C)} = R^d$	$c + \dfrac{C}{R^d} = y + \dfrac{\pi}{R^d}$

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Household budget constraint when gov't buys private assets using tax receipts, T, and gov't gets the same rate of return, R^d, as households:

No change!
(Ricardian-Wallace Irrelevance)

$$c + \frac{C}{R^d} = y - T + \frac{\pi + TR^d}{R^d} = y + \frac{\pi}{R^d}$$

Problem of the Household		
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Household Supply of Deposits

- For given π, d rises or falls with R^d, depending on parameter values.

- But, in equilibrium $\pi = R^k(N+d) - R^d d$.

- Substituting into the expression for c and solving for d:

$$d = \frac{(\beta R^d)^{\frac{1}{\gamma}} - \frac{N}{y} R^k}{(\beta R^d)^{\frac{1}{\gamma}} + R^k} y$$

R^d

Upward-sloping deposit supply

d

Household Supply of Deposits

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- But, in equilibrium $\pi = R^k(N+d) - R^d d$.

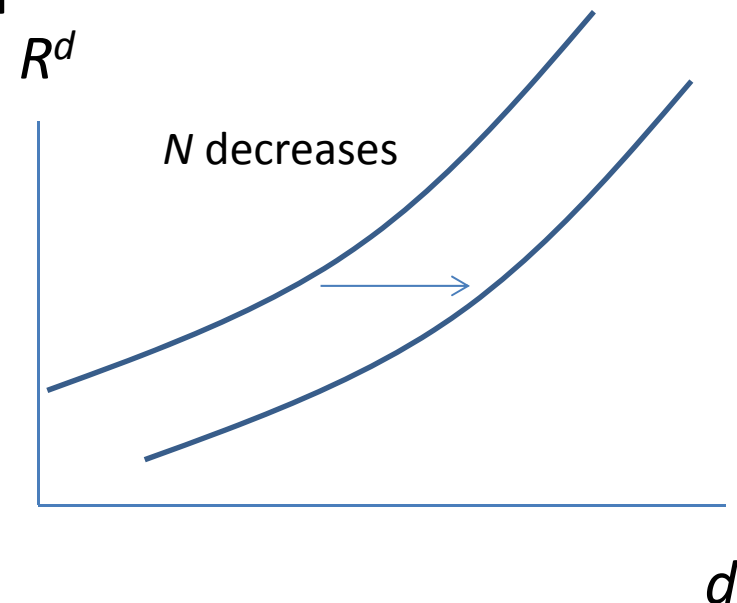- Substituting into the expression for *c* and solving for *d*:

$$d = \frac{(\beta R^d)^{\frac{1}{\gamma}} - \frac{N}{y}R^k}{(\beta R^d)^{\frac{1}{\gamma}} + R^k}y$$

R^d

N decreases

d

Properties of Equilibrium Household Supply of Deposits

- Deposits increasing in R^d.

- Shifts right with decrease in N because of wealth effect operating via bank profits, π.
  - rise in deposit supply smaller than decrease in N.

$$\frac{\partial d}{\partial N} = - \overbrace{\left[\frac{R^k}{(\beta R^d)^{\frac{1}{\gamma}} + R^k} \right]}^{>0,\ <1}$$

Efficient Benchmark

Problem of the Bank	
period 1	period 2
take deposits, d	pay dR^d to households
buy securities, $s = N + d$	receive sR^k from firms
problem: $\max_d [sR^k - R^d d]$	

Bank demand for *d*

R^d

Supply of *d* by households

Demand for *d* by banks

R^k

Equilibrium *d*

d
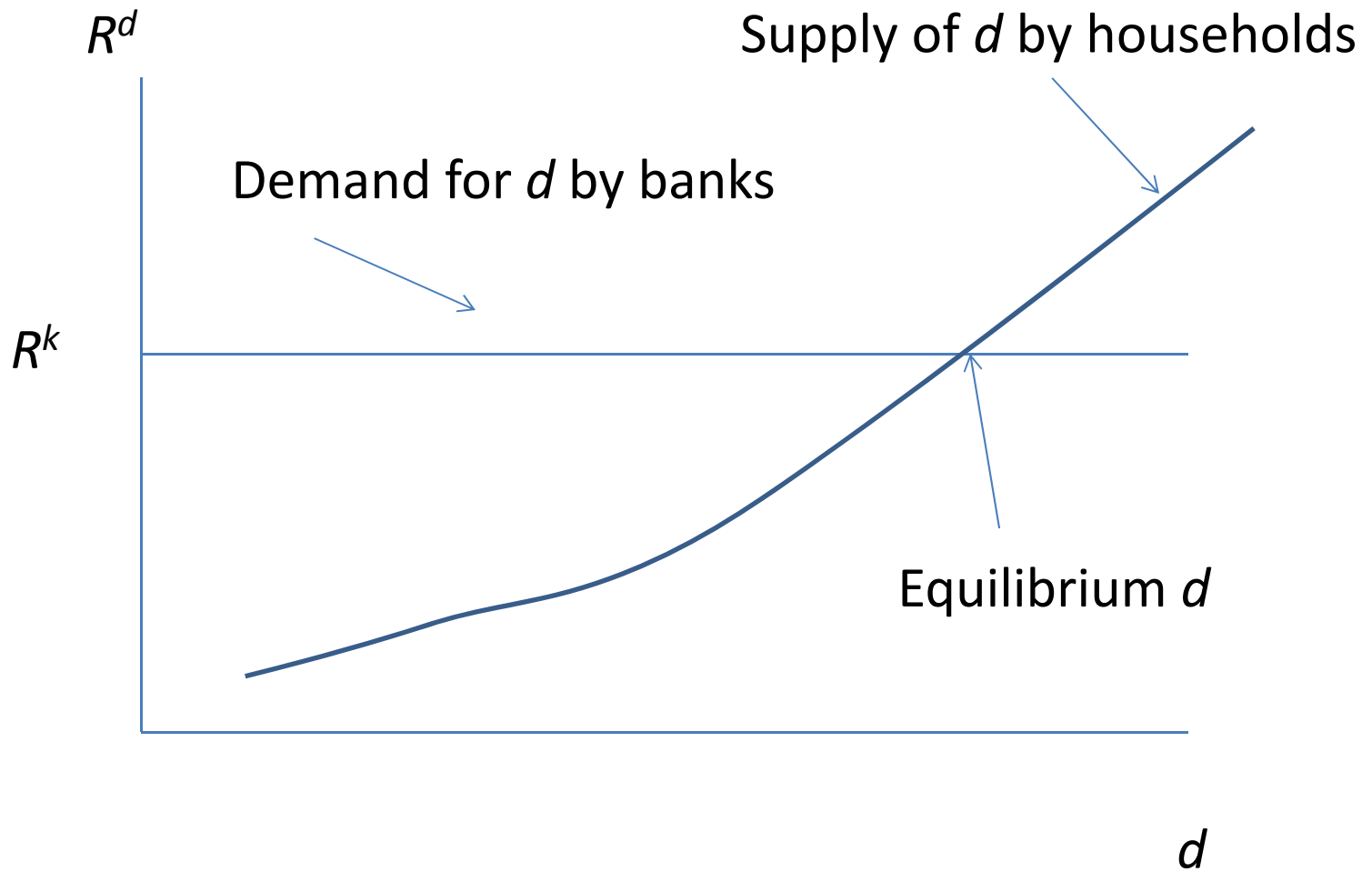
Equilibrium in Absence of Frictions

Interior Equilibrium: R^d, π, d, c, C

(i) $c, d, C > 0$

(ii) household problem is solved

(iii) bank problem is solved

(iv) goods and financial markets clear

- Properties:

  – Household faces true social rate of return on saving:

  $$R^k = R^d$$

  – Equilibrium is 'first best', i.e., solves

  $$\max_{c,C,k,} u(c) + \beta u(C)$$
  $$c + k \leq y + N, \; C \leq kR^k$$

Friction

- bank combines deposits, *d*, with net worth, *N*, to purchase *N+d* securities from firms.

- bank has two options:
  - ('no-default') wait until next period when $(N + d)R^k$ arrives and pay off depositors, $R^d d$, for profit:

    $$(N + d)R^k - R^d d$$

  - ('default') take $\theta(N + d)$ securities, refuse to pay depositors and wait until next period when securities pay off:
    $$\theta(N + d)R^k$$

  - Bank must announce what value of *d* it will choose at the beginning of a period.

Incentive Constraint

- Recall, banks maximize profits

- Choose 'no default' iff

$$\overbrace{(N + d)R^k - R^d d}^{\text{no default}} \quad \geq \quad \overbrace{\theta(N + d)R^k}^{\text{default}}$$

- Next: derive banking system's demand for deposits in presence of financial frictions.

Result for a no-default equilibrium:

- Consider an individual bank that contemplates deviating.
- It sets a d that implies default,
  $R^k(N+d) - R^d d < \theta R^k(d+N)$, or

$$\underbrace{R^d}_{\text{what the household gets in the other banks}} \quad > \quad \underbrace{\frac{(1-\theta)R^k(d+N)}{d}}_{\text{what the household gets in the defaulting bank}}$$

- A deviating bank will in fact receive no deposits.
- **An optimizing bank would never default**
  - Can verify this is so if $R^d > R^k$, $R^d = R^k$, $R^d < R^k$.
  - Assume that in the case of indifference, they do not default.

Problem of the bank in no-default, interior equilibrium

- Maximize, by choice of d,

$$R^k(N+d) - R^d d$$

subject to:

$$R^k(N+d) - R^d d - R^k \theta(N+d) \geq 0,$$
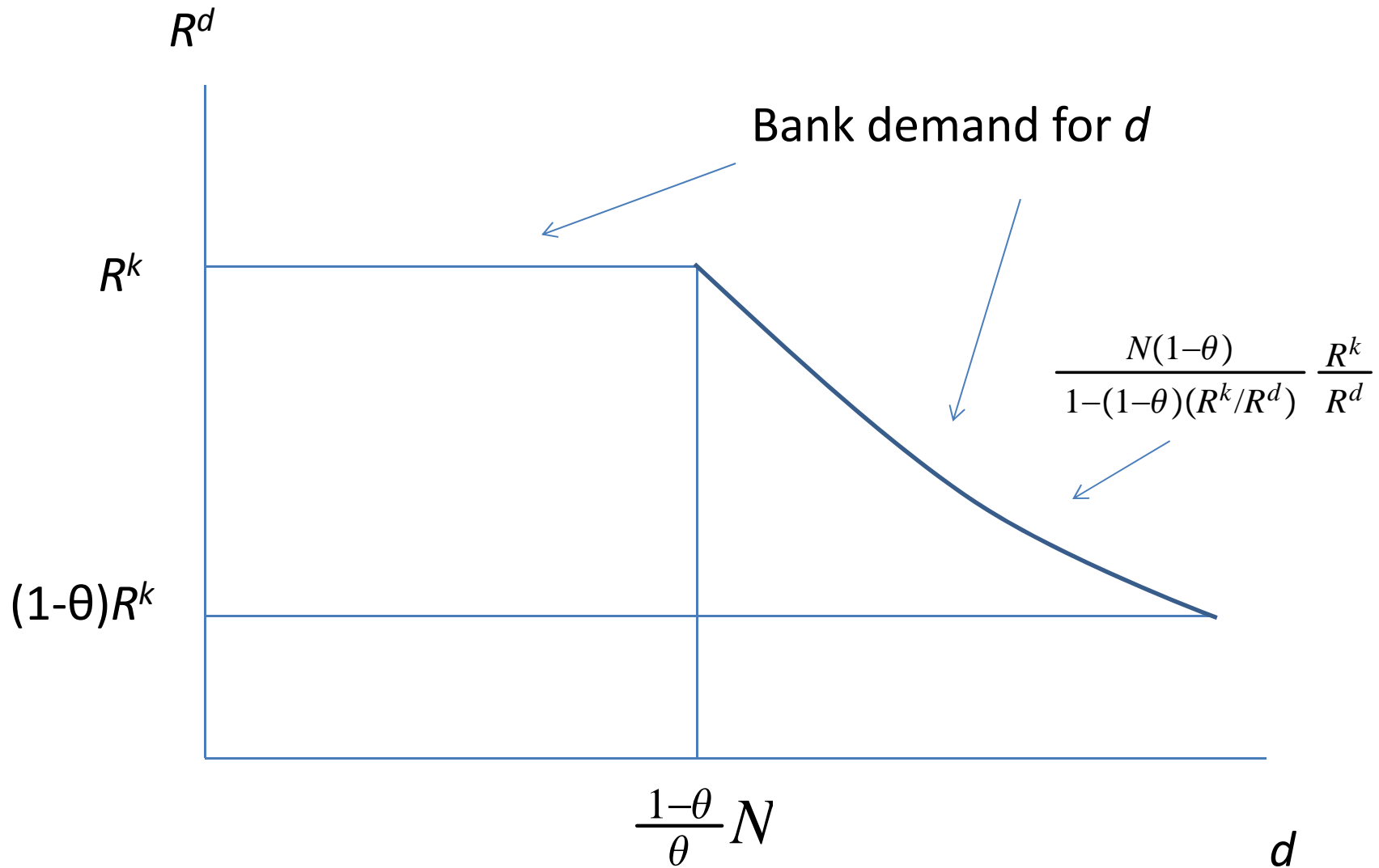
or,

$$(1-\theta)R^k N - [R^d - (1-\theta)R^k]d \geq 0.$$

- Note that $0 < d < \infty$ requires

if not, then $d=\infty$ if not, then $d=0$

$$(1-\theta)R^k \quad \overset{\frown}{<} \quad R^d \quad \overset{\frown}{\leq} \quad R^k.$$

Problem of the bank in no-default, interior equilibrium, cnt'd

- For $R^d = R^k$
  - a bank makes no profits on d so – absent default considerations - it is indifferent over all values of $0 \leq d$
  - Taking into account default, a bank is indifferent over $0 \leq d \leq N(1\text{-}\theta)/\theta$

- For $(1\text{-}\theta)R^k < R^d < R^k$
  - Bank wants d as large as possible, subject to incentive constraint.
  - So, $d = R^k N(1\text{-}\theta)/(R\text{-}(1\text{-}\theta)R^k)$

Bank demand for d

R^d

Bank demand for d

R^k

$\dfrac{N(1-\theta)}{1-(1-\theta)(R^k/R^d)}\dfrac{R^k}{R^d}$

$(1-\theta)R^k$

$\dfrac{1-\theta}{\theta}N$

d

Interior, no default equilibrium

R^d

R^k

Household supply

Bank demand

In this equilibrium, $R^d = R^k$ and first-best allocations occur. Banking system is highly effective in allocating resources efficiently.

d
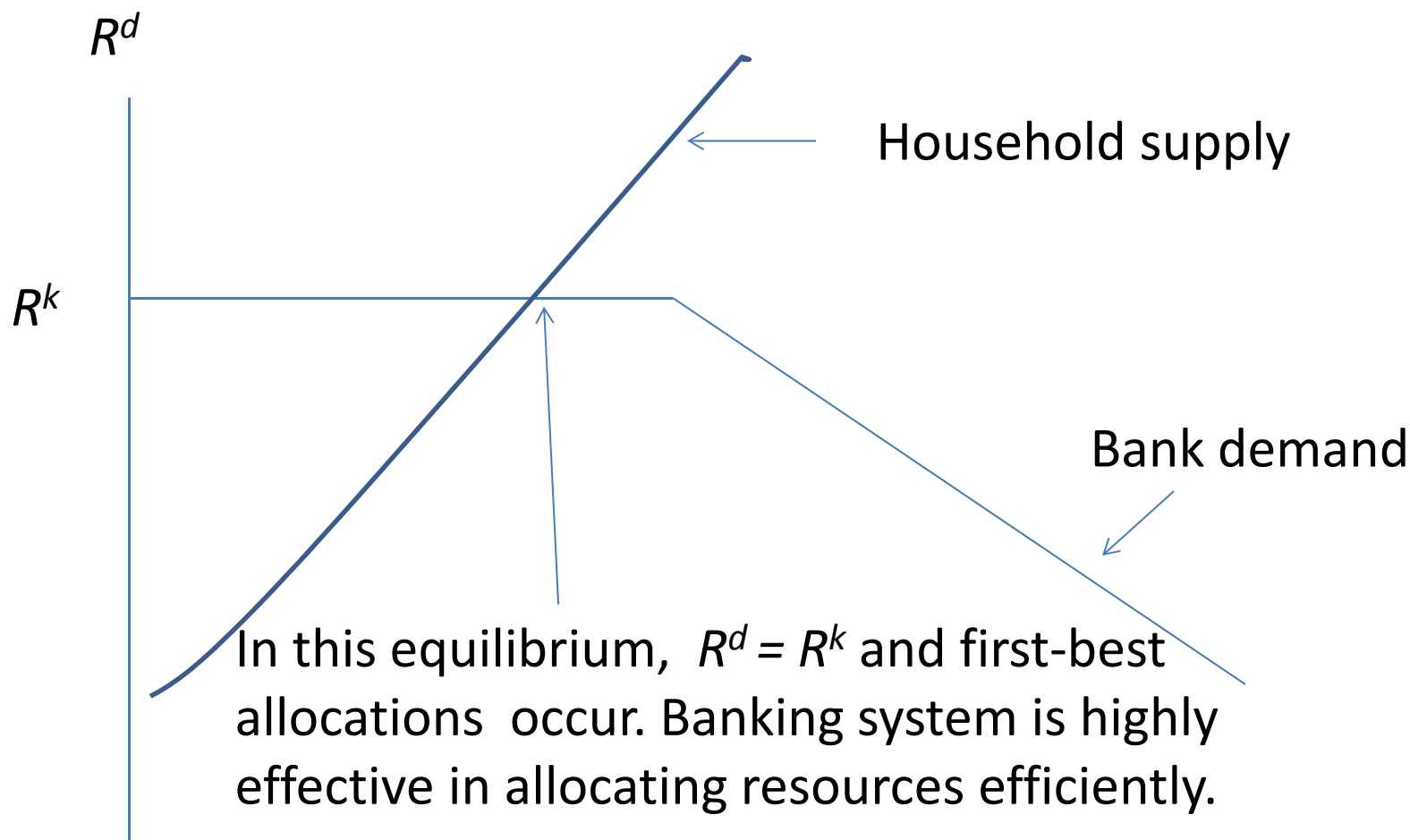
Collapse in Bank Net Worth

- Suppose that the economy is represented by a sequence of repeated versions of the above model.

- In the periods before the 2007-2008 crisis, net worth was high and the equilibrium was like it is on the previous slide: efficient, with zero interest rate spreads.

  – In practice, spreads are always positive, but that reflects various banking costs that are left out of this model.

- With the crisis, *N* dropped a lot, shifting demand and supply to the left.

  – But, supply shifts more than demand, according to the model.

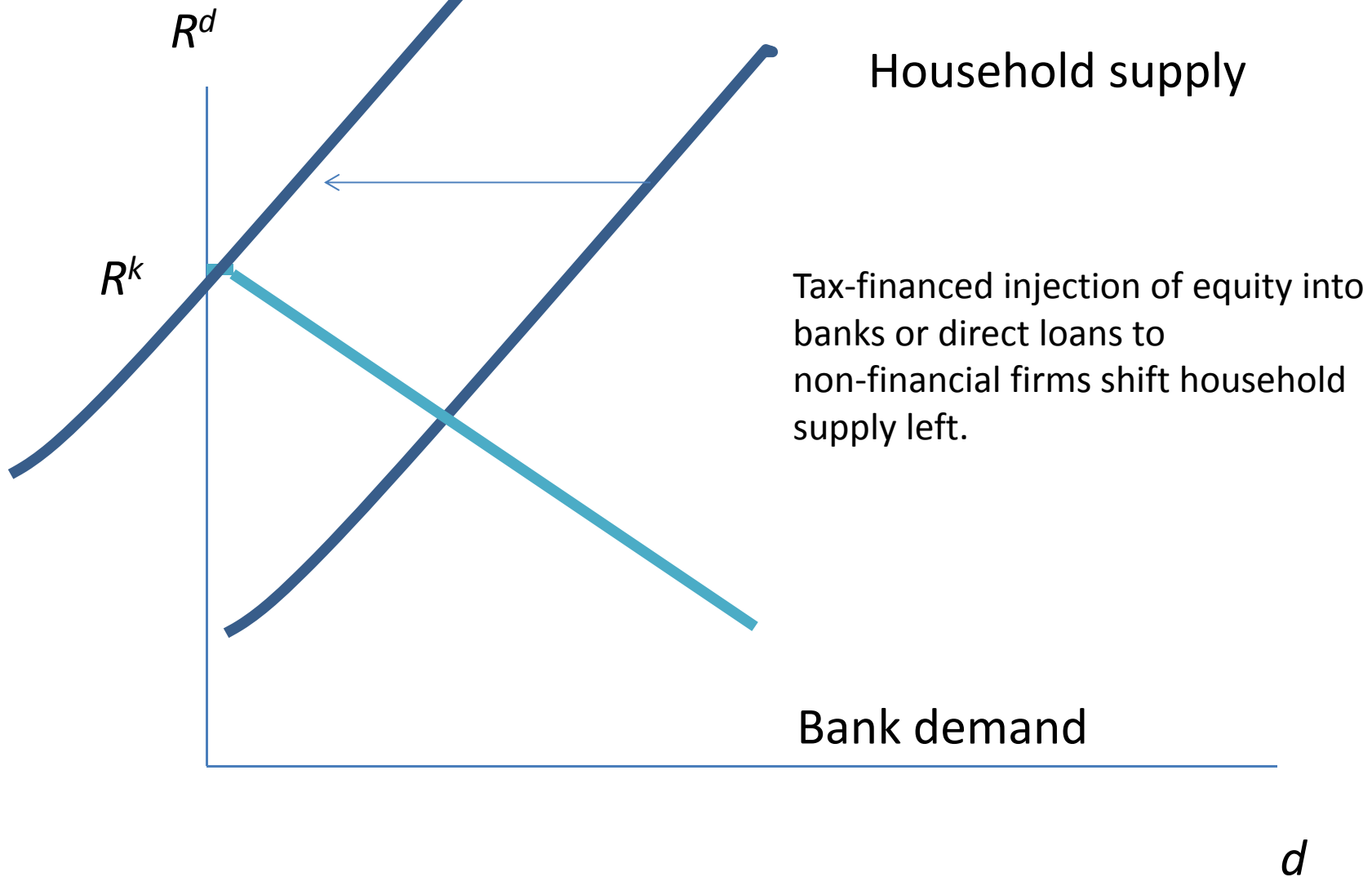# Effect of Substantial Drop in Bank Net Worth

R^d

Initial, efficient equilibrium

R^k

Household supply

Bank demand
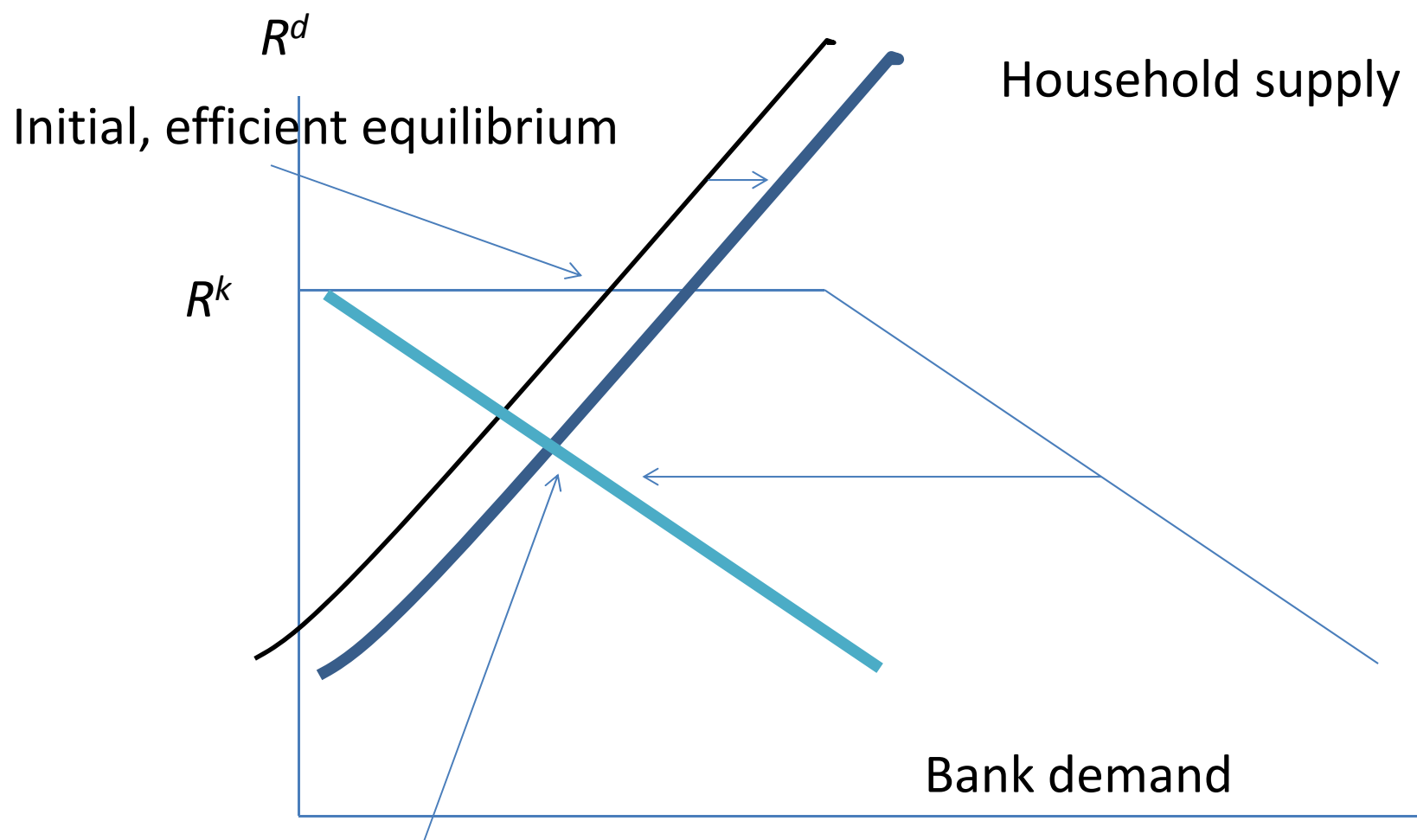
Equilibrium after N drops is inefficient because $R^d < R^k$.

d

Government Intervention

- Equity injection.
  - Government raises T in period 1, provides proceeds to banks and demands $R^k T$ in return at start of period 2.
  - Rebates earnings to households in 2.

- Has no impact on demand for deposits by banks (no impact on default incentive or profits).

- Reduces supply of deposits by households.

- Direct, tax-financed government loans to firms work in the same way.

- An interest rate subsidy to banks will shift their demand for deposits to the right….it will also shift supply to the left.

Equity Injection and Drop in *N*

R^d

R^k

Household supply

Tax-financed injection of equity into
banks or direct loans to
non-financial firms shift household
supply left.

Bank demand

d

Recap

- Basic idea:
  - Bankers can run away with a fraction of bank assets.
  - If banker net worth is high relative to deposits, friction not a factor and banking system efficient.
  - If banker net worth falls below a certain cutoff, then banker must restrict the deposits.
    - Otherwise, depositors to lose confidence and take their business to another bank.
  - Reduced supply of deposits:
    - makes deposit interest rates fall and so spreads rise.
    - Reduced intermediation means investment drops, output drops.

Next: another moral hazard model

- Previous model: bankers can run away with a fraction of bank assets.

- Now: bankers must make an unobserved and costly effort to identify good projects that make a high return for their depositors.
  - Bankers must have the right incentive to make that effort.

- Otherwise, model similar to previous one.

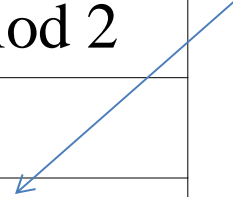# Model Has a Similar Diagnosis of the Financial Crisis as Moral Hazard I

- Both models articulate the idea:

- "…a fall in housing prices and other assets caused a fall in bank net worth and initiated a crisis. The banking system became dysfunctional as interest rate spreads increased and intermediation and economic activity was reduced. Various government policies can correct the situation''

Two-period Hidden Effort Model

- Many identical households, each with a unit measure of members:
  - Some members are 'bankers'
  - Some members are 'workers'
  - Perfect insurance inside households...everyone consumes same amount.
- Period 1
  - Workers endowed with y goods, household makes deposits in a bank
  - Bankers endowed with N goods, take deposits and **make hidden efforts to identify a firm with a good investment project.**
  - Firm issues securities to finance capital used in production in period 2.
- Period 2
  - Household consumes earnings from deposits plus profits from banker.
  - Goods consumed are produced by the firm.

Problem of the Household		
	period 1	period 2
budget constraint	$c + d \leq y$	$C \leq Rd + \pi$
problem	$\max_{c,C,d}[u(c) + \beta u(C)]$	

slight change
in notation.

Household problem in hidden banker effort
model is same as in moral hazard I

Problem of the Household		
	period 1	period 2
budget constraint	$c + d \leq y$	$C \leq Rd + \pi$
problem	$\max_{c,C,d}\left[u(c) + \beta u(C)\right]$	

Solution to Household Problem	
$\dfrac{u'(c)}{\beta u'(C)} = R$	$c + \dfrac{C}{R} = y + \dfrac{\pi}{R}$
$u(c) = \dfrac{c^{1-\gamma}}{1-\gamma}$	$c = \dfrac{y + \dfrac{\pi}{R}}{1 + \dfrac{(\beta R)^{\frac{1}{\gamma}}}{R}}$

Banker Problem

- Bankers combine their net worth, *N*, and deposits, *d*, to acquire the securities of a single firm.
  - Bankers **not diversified**.

- Firms:
  - Good firms: investment project with return, R^g
  - Bad firms: an investment project with return, R^b

- Banker makes a **costly, unobserved effort, *e*,** to locate a good firm, and finds one with probability, *p(e)*.
  - *p(e)* increasing in *e*.

Banker Problem, cnt'd

- Mean and variance on banker's asset:

$$\text{mean: } p(e)R^g + (1 - p(e))R^b$$

$$\text{variance: } p(e)[1 - p(e)](R^g - R^b)^2$$

- Note:
  - Mean increases in *e*
  - For *p(e)>1/2,*
    - Variance of the portfolio *decreases* with increase in *e*
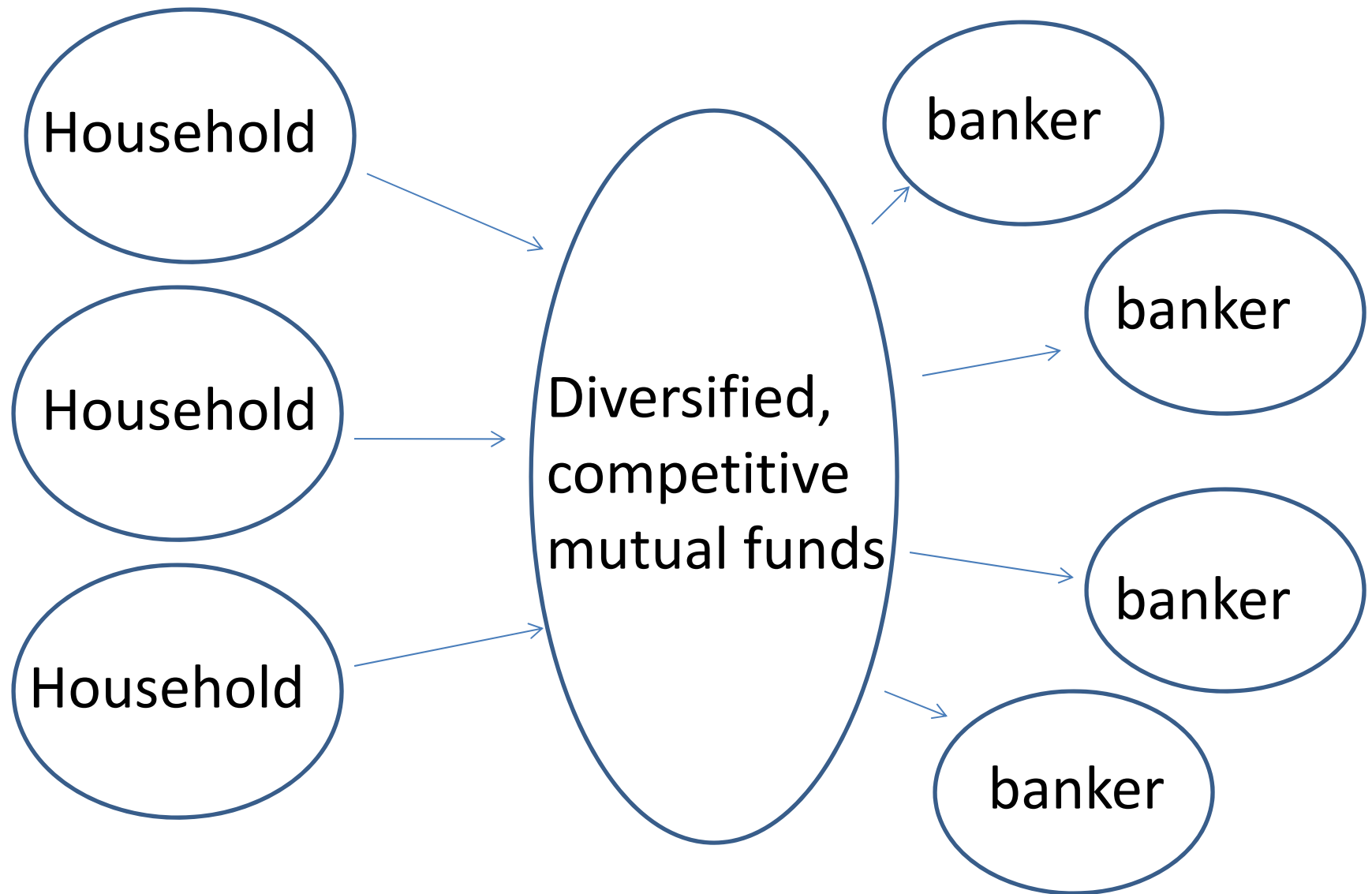
      derivative of variance w.r.t. e:

      $$[1 - 2p(e)](R^g - R^b)^2 p'(e),$$
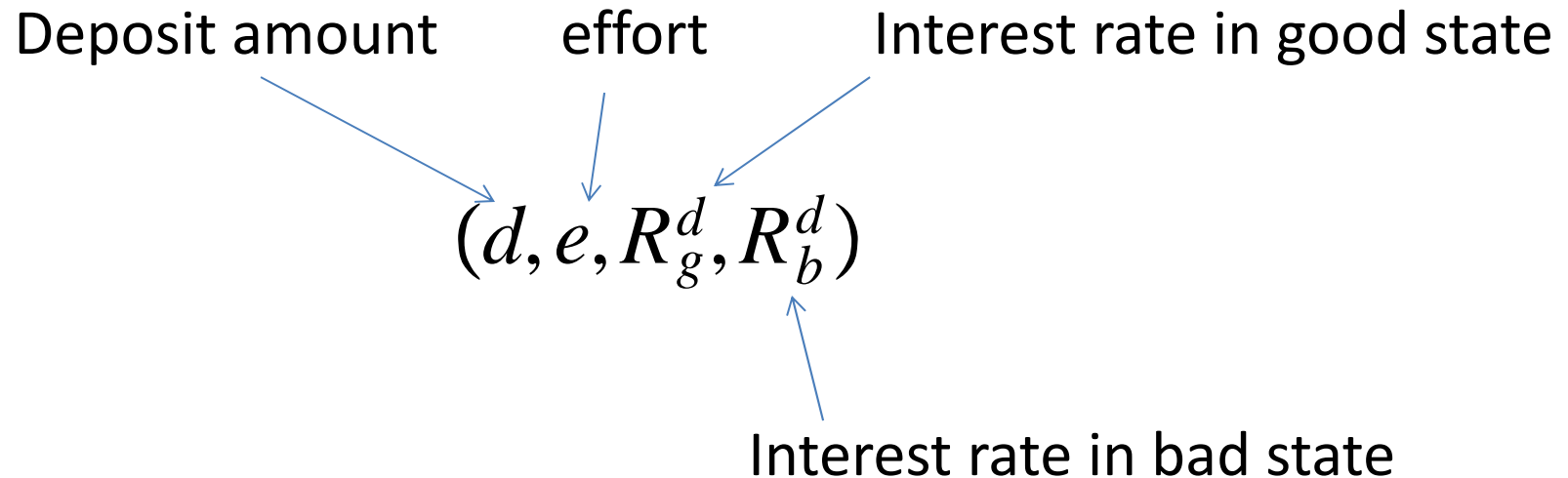
Funding for Bankers

- Representative household deposits money into a representative mutual fund.
  - Household receives a certain return, *R*.

- Representative mutual fund acquires deposit, *d*, in each of a diversified set of banks.
  - Mutual fund receives dR_g^d from *p(e)* banks with a good investment.
  - Mutual fund receives dR_b^d from *1-p(e)* banks with a bad investment.

Risky Bankers Funded By Mutual Funds

Household → Diversified, competitive mutual funds → banker

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Household → Diversified, competitive mutual funds → banker

banker

Arrangement Between Banks and Mutual Funds

- Contract traded in competitive market:

Deposit amount   effort   Interest rate in good state

$$(d, e, R_g^d, R_b^d)$$

Interest rate in bad state

Two Versions of Model

- No financial frictions: mutual fund observes banker effort.

  – This is the benchmark version.

- Financial frictions: mutual fund does not observe banker effort.

  – This is the interesting version.

  – Use it to think about crisis in 2008-2009, and unconventional monetary policy.

Equilibrium Contract When Effort is Observable

- Competition and free entry among mutual funds:

money owed to households by mutual funds

$$\overbrace{Rd}$$

fraction of banks with good investments   fraction of banks with bad investments

$$= \quad \overbrace{p(e)} \qquad R_g^d d + \qquad \overbrace{(1 - p(e))} \qquad R_b^d d$$

- Zero profit condition represents a menu of contracts available to banks.

Contract Selected by Banks in Observable Effort Equilibrium

Marginal value assigned by household to bank profits

expected bank profits

$$\max_{e,d,R_g^d,R_b^d} \lambda \overbrace{\{p(e)[R^g(N+d) - R_g^d d] + (1-p(e))[R^b(N+d) - R_b^d d]\}}$$

utility cost of effort suffered by banker

$$- \qquad \overbrace{\frac{1}{2}e^2}$$
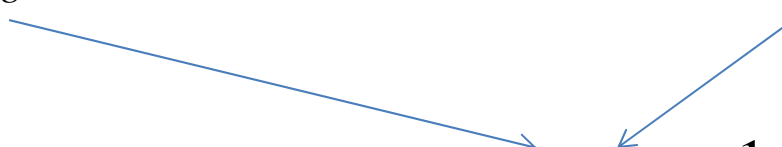
zero profit condition of mutual funds   cash flow constraint on banks

$$\text{subject to: } \overbrace{Rd = p(e)R_g^d d + (1-p(e))R_b^d d}, \quad \overbrace{R^b(N+d) \geq R_b^d d}$$

Characterizing Equilibrium Contract

- Substitute out the mutual fund zero profit condition, so that banker problem is:

$$\max_{e,d,R_g^d,R_b^d} \lambda\{p(e)[R^g(N+d) - R_g^d d] + (1 - p(e))[R^b(N+d) - R_b^d d]\} - \frac{1}{2}e^2$$

$$\max_{e,d} \lambda\{[p(e)R^g + (1 - p(e))R^b](N+d) - Rd\} - \frac{1}{2}e^2$$

- Optimal contract conditions:

$$\text{effort}: e = \lambda p'(e)(R^g - R^b)(N+d)$$

$$\text{deposits}: R = p(e)R^g + (1 - p(e))R^b$$

$$\text{zero profits, mutual fund}: R = p(e)R_g^d + (1 - p(e))R_b^d$$

$$\text{cash constraint}: R^b(N+d) \geq R_b^d d$$

Properties of Contract

- Banker treats *d* and *N* symmetrically

$$\text{effort} : e = \lambda p'(e)(R^g - R^b)(N + d)$$

- Other equations:

$$\text{deposits} : R = p(e)R^g + (1 - p(e))R^b$$

$$\text{zero profits, mutual fund} : R = p(e)R^d_g + (1 - p(e))R^d_b$$

$$\text{cash constraint} : R^b(N + d) \geq R^d_b d$$

- Algorithm:
  - Fix *R*, get *c, C, d* from household problem
  - Compute *e* from effort equation (use $p(e) = a + be,\ b > 0$.)
  - Adjust *R* until deposits equation is satisfied.

- Returns on deposits not uniquely pinned down. Cash constraint not binding.
  - *N* large enough relative to *d*, can choose $R^d_g = R^d_b = R$

Observable Effort Equilibrium

Observable Effort Equilibrium: c, C, e, d, R, λ, R_g^d, R_b^d such that

(i) the household maximization problem is solved

(ii) mutual funds earn zero profits

(iii) the banker problem with e observable, is solved

(iv) markets clear

(v) $c, C, d, e > 0$

Unobservable Effort

- Suppose that the banker has obtained a contract, (d, e, R_g^d, R_b^d), from the mutual fund.

- The mutual fund can observe (d, R_g^d, R_b^d) so that the banker no longer has any choice about these.

- The mutual fund does not observe e, and so the bank can still choose e freely after the contract has been selected.

- The banker solves

$$\max_e \lambda \{ p(e)[R^g(N+d) - R_g^d d] + (1 - p(e))[R^b(N+d) - R_b^d d] \} - \frac{1}{2} e^2$$

Incentive Constraint

- Banker choice of e after the deposit contract has been selected:

$$\max_{e} \lambda \{ p(e)[R^g(N+d) - R_g^d d] + (1 - p(e))[R^b(N+d) - R_b^d d] \} - \frac{1}{2}e^2$$

- First order condition:

$$e = \lambda p'(e)[(R^g - R^b)(N+d) - (R_g^d - R_b^d)d]$$

   - Note: if $R_g^d > R_b^d$ then the banker exerts less effort than in the observable effort equilibrium.
   - Reason is that the banker does not receive the full return on its effort if $R_g^d > R_b^d$

Unobservable Effort Equilibrium

- Mutual funds are only willing to consider contracts, (d, e, R_g^d, R_b^d), that satisfy the following restrictions:

zero profits, mutual fund : $R = p(e)R_g^d + (1 - p(e))R_b^d$

cash constraint : $R^b(N + d) \geq R_b^d d$

incentive compatibility: $e = \lambda p'(e)[(R^g - R^b)(N + d) - (R_g^d - R_b^d)d]$

- There is no point for the mutual fund to consider a contract in which e does not satisfy the last condition, since bankers will set e according to the last condition in any case.

Contract Selected by Banks in Unobservable Effort Equilibrium

- Solve

$$\max_{e,d,R_g^d,R_b^d} \lambda\{p(e)[R^g(N+d) - R_g^d d] + (1-p(e))[R^b(N+d) - R_b^d d]\}$$

$$- \frac{1}{2}e^2$$

- Subject to

$$\text{zero profits, mutual fund} : R = p(e)R_g^d + (1-p(e))R_b^d$$

$$\text{cash constraint} : R^b(N+d) \geq R_b^d d$$

$$\text{incentive compatibility:} \quad e = \lambda p'(e)[(R^g - R^b)(N+d) - (R_g^d - R_b^d)d]$$

Two Unobservable Effort Equilibria

- **Case 1: Banker net worth, *N*, is high enough**
  - Recall the two conditions on deposit returns:

$$\text{zero profits, mutual fund}: R = p(e)R_g^d + (1 - p(e))R_b^d$$

$$\text{cash constraint}: R^b(N + d) \geq R_b^d d$$

  - Suppose that *N* is large enough so that given *d* from the observable effort equilibrium, cash constraint is satisfied with

$$R_g^d = R_b^d = R$$

  - Then, observable effort equilibrium is also an unobservable effort equilibrium.

> **With N large enough, unobservable effort equilibrium is efficient.**

Risk Premium

- R is the risk free rate in the model (i.e., the sure return received by the household).

- Let R_g^d denote the 'bank interest rate on deposits'.
    - This is what the bank pays in the event that its portfolio is 'good'.

- Risk premium: $R_g^d - R$

Result: when *N* is high enough, equilibrium level of intermediation is efficient and risk premium is zero.

Case 2: Banker net worth, *N*, is low

- Recall the two conditions on deposit returns:

$$\text{zero profits, mutual fund} : R = p(e)R_g^d + (1 - p(e))R_b^d$$

$$\text{cash constraint} : R^b(N + d) \geq R_b^d d$$

  – Suppose that *N* is small, so that given *d* from the observable effort equilibrium, cash constraint is **not** satisfied with
$$R_g^d = R_b^d = R$$

  – Then, observable effort equilibrium is **not** an unobservable effort equilibrium.

With N small enough, unobservable effort equilibrium is not efficient.

Unobserved Effort Equilibrium, low *N* Case

- The two conditions on deposit returns:

$$\text{zero profits, mutual fund} : R = p(e)R_g^d + (1 - p(e))R_b^d$$

$$\text{cash constraint} : R^b(N + d) \geq R_b^d d$$

- Suppose, with efficient *d* and *e*, cash constraint is not satisfied for $R_b^d = R$. Then

  - Set $R_b^d < R$, $R_g^d > R$ (still have $R = p(e)R^g + (1 - p(e))R^b$)
  - Risk premium positive
  - Incentive constraint implies inefficiently low *e*.
  - Low *e* implies low *R*, which implies low *d*.
    - Banking system 'dysfunctional'.
  - Mean of bank return goes down, and variance up.

Scenario Rationalized by Model

- Before 2007, when N was high, the banking system supported the efficient allocations and the interest spread was zero.

- The fall in bank net worth after 2007, caused a jump in the risk premium, and a slowdown in intermediation and investment.

- Banking system became dysfunctional because banks did not have enough net worth to cover possible losses.
  - This meant depositors had to take losses in case of a bad investment outcome in banks.
  - Depositors require a high return in good states as compensation: risk premium.
  - Bankers lose incentive to exert high effort. More bad projects are funded, reducing the overall return on saving.
  - Saving falls below its efficient level.

How to Fix the Problem

- One solution: tax the workers and transfer the proceeds to bankers so they have more net worth.
    - In the model, this is a good idea because income distribution issues have been set aside.
    - In practice, income distribution problems could be a serious concern and this policy may therefore not be feasible

- Subsidize the interest rate costs of banks.
    - This increases the chance that bank net worth is sufficient to cover losses, reduces the risk premium and gives bankers an incentive to increase effort.
    - Increased effort increases the return on banker portfolios and reduces their variance.

- Equity injections and loans to banks have zero impact in the model, when it is in a bad equilibrium.
    - Ricardian irrelevance not overturned.
    - the sources of moral hazard matter for whether a particular asset purchase programs is effective!

Conclusion

- Have described two models of moral hazard, that can rationalize the view:
  - Bank net worth fell, causing interest rate spreads to jump and intermediation to slow down. The banking system is dysfunctional.

- Net worth transfers and interest rate subsidies can revive a dysfunctional banking system in both models.

- However, the models differ in terms of the detailed economic story, as well as in terms of their implications for asset purchases.