Fiscal Policy

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Outline

- Analysis is entirely in ‘normal times’, when the zero lower bound constraint on the interest rate can be ignored.
- First, consider the case of lump sum taxes.
- Second, consider the case when (most) taxes are distortionary and they are set by a fiscal rule.

Derivation of Model Equilibrium Conditions

- Households
  - First order conditions
- Firms:
  - final goods and intermediate goods
  - marginal cost of intermediate good firms
- Aggregate resources
- Monetary policy
- Three linearized equilibrium conditions:
  - Intertemporal, Pricing, Monetary policy
- Results

Model

King-Plosser-Rebelo (KPR) preferences.

- Household preferences and constraints:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left[C_t^{\gamma}(1-N_t)^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma} + v(G_t) \right]$$

$$P_t C_t + B_{t+1} \leq W_t N_t + (1+R_t)B_t + T_t,\ T_t \text{ ~lump sum taxes and profits}$$

- Optimality conditions

$$\overbrace{u_{c,t}}^{\text{marginal cost of giving up one unit of consumption to save}} = \overbrace{E_t\ \beta u_{c,t+1} \overbrace{\frac{1+R_{t+1}}{1+\pi_{t+1}}}^{\text{extra goods tomorrow from saving more today}}}^{\text{marginal benefit tomorrow from saving more today}},$$

$$\overbrace{\frac{-u_{N,t}}{u_{c,t}}}^{\text{marginal cost (in units of goods) of labor effort}} = \overbrace{\frac{W_t}{P_t}}^{\text{marginal benefit of labor effort}}$$

Linearized Intertemporal Equation

- Inter-temporal Euler equation

$$E_t\left[u_{c,t} - \beta u_{c,t+1}\frac{1+R_{t+1}}{1+\pi_{t+1}}\right] = 0$$

- In zero inflation no growth steady state:

$$1 = \beta(1+R)$$

- Totally differentiate:

$$du_{c,t} - [\beta(1+R)du_{c,t+1} + \beta u_c dR_{t+1} - \beta u_c(1+R)d\pi_{t+1}] = 0$$

  – Log-differentiation:

$$u_c\hat{u}_{c,t} - \beta(1+R)u_c\left[\hat{u}_{c,t+1} + \frac{1}{1+R}dR_{t+1} - d\pi_{t+1}\right] = 0$$

  – Finally:

$$\hat{u}_{c,t} - [\hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1}] = 0$$

Linearized intertemporal , cnt'd

- Repeat:

$$\hat{u}_{c,t} - [\hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1}] = 0$$

$$u = \frac{\left[C_t^{\gamma}(1-N_t)^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma} \rightarrow u_{c,t} = \gamma C_t^{\gamma(1-\sigma)-1}(1-N_t)^{(1-\gamma)(1-\sigma)}$$

$$\hat{u}_{c,t} = [\gamma(1-\sigma)-1]\hat{C}_t - \frac{(1-\gamma)(1-\sigma)N}{1-N}\hat{N}_t$$

Firms

- Final, homogeneous good

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 1$$

  – Efficiency condition:

$$P_t(i) = P_t \left(\frac{Y_t}{Y_t(i)}\right)^{\frac{1}{\varepsilon}}$$

- i-th intermediate good

$$Y_t(i) = N_t(i)$$

  – Optimize price with probability *1*-θ, otherwise

$$P_t(i) = P_{t-1}(i)$$

Intermediate Good Firm Marginal Cost

- Marginal cost:

$$MC_t = \frac{\frac{dCost_t}{dWorker_t}}{\frac{dOutput_t}{dWorker_t}} = \frac{W_t \overbrace{(1-\nu)}^{\text{subsidy to undo effects of monopoly power }=(\varepsilon-1)/\varepsilon}}{MP_{L,t}}$$

$$= W_t(1-\nu) = P_t \overbrace{\frac{-u_{N,t}}{u_{c,t}}}^{\text{household first order condition}} (1-\nu)$$

- Real marginal cost

$$s_t \equiv \frac{MC_t}{P_t} = \frac{-u_{N,t}}{u_{c,t}}(1-\nu) \overbrace{=}^{\text{in steady state}} \frac{\varepsilon-1}{\varepsilon}$$

$$\overbrace{\frac{-u_{N,t}}{u_{c,t}}}^{\text{marginal cost to household of providing one more unit of labor}} \overbrace{=}^{\text{in steady state}} \overbrace{1}^{\text{marginal benefit of one extra unit of labor}}$$

Aggregate Resources

- Resource relation:

$$C_t + G_t = Y_t = p_t^* N_t$$

  – p_t^* is 'Tak Yun' distortion

  – recall, distortion = 1 to first order:

$$\hat{Y}_t = \hat{N}_t$$

- Log-linear expansion:

$$(1-g)\hat{C}_t + g\hat{G}_t = \hat{Y}_t,\ g \equiv \frac{G}{Y}$$

- Consumption:

$$\hat{C}_t = \frac{1}{1-g}\hat{Y}_t - \frac{g}{1-g}\hat{G}_t$$

Simplifying Marginal Utility of *C*

$$\frac{-u_{N,t}}{u_{c,t}} \overbrace{=}^{\text{in steady state}} 1 \rightarrow \frac{1-\gamma}{1-N} = \frac{\gamma}{C}$$

$$\hat{u}_{c,t} = [\gamma(1-\sigma)-1]\hat{C}_t - \frac{(1-\gamma)(1-\sigma)N}{1-N}\hat{N}_t$$

$$= [\gamma(1-\sigma)-1]\hat{C}_t - \frac{\gamma(1-\sigma)N}{C}\hat{N}_t$$

$$= [\gamma(1-\sigma)-1]\hat{C}_t - \frac{\gamma(1-\sigma)}{1-g}\hat{N}_t$$

$$= [\gamma(1-\sigma)-1]\left[\frac{1}{1-g}\hat{Y}_t - \frac{g}{1-g}\hat{G}_t\right] - \frac{\gamma(1-\sigma)}{1-g}\hat{Y}_t$$

$$= -\frac{1}{1-g}\hat{Y}_t - [\gamma(1-\sigma)-1]\frac{g}{1-g}\hat{G}_t$$

Simplify Intertemporal Equation

- Intertemporal Euler equation:

$$\hat{u}_{c,t} = \hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1}$$

- Substitute out marginal utility of consumption:

$$-\frac{1}{1-g}\hat{Y}_t - [\gamma(1-\sigma)-1]\frac{g}{1-g}\hat{G}_t$$
$$= -\frac{1}{1-g}\hat{Y}_{t+1} - [\gamma(1-\sigma)-1]\frac{g}{1-g}\hat{G}_{t+1} + \beta dR_{t+1} - d\pi_{t+1}$$

- Rearranging:

$$\hat{Y}_t + [\gamma(1-\sigma)-1]g\hat{G}_t$$
$$= \hat{Y}_{t+1} + [\gamma(1-\sigma)-1]g\hat{G}_{t+1} - (1-g)[\beta dR_{t+1} - d\pi_{t+1}]$$

Phillips Curve

- Equilibrium condition associated with price setting just like before:

$$\pi_t = \beta\pi_{t+1} + \kappa\widehat{s}_t,$$

$$\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

- Marginal cost:

$$\widehat{s}_t = \widehat{\frac{(1-\gamma)C_t}{\gamma(1-N_t)}} = \hat{C}_t - \widehat{(1-N_t)} = \hat{C}_t + \frac{N}{1-N}\hat{N}_t$$

$$\overbrace{=}^{\left(\hat{C}_t = \frac{1}{1-g}\hat{Y}_t - \frac{g}{1-g}\hat{G}_t,\ \hat{N}_t = \hat{Y}_t\right)} \left[\frac{1}{1-g} + \frac{N}{1-N}\right]\hat{Y}_t - \frac{g}{1-g}\hat{G}_t$$

Monetary Policy

- Monetary policy rule (after linearization)

$$dR_{t+1} = \rho_R dR_t + (1-\rho_R)\left[\frac{\phi_1}{\beta}\pi_{t+k} + \frac{\phi_2}{\beta}\hat{Y}_{t+l}\right]$$

$$dR_{t+1} \equiv R_{t+1} - R,\ R = \frac{1}{\beta} - 1$$

$$\hat{Y}_t \equiv \frac{Y_t - Y}{Y}$$

$$k, l = 0, 1.$$

Pulling All the Equations Together

- IS equation:

$$\hat{Y}_t + [\gamma(1-\sigma)-1]g\hat{G}_t$$
$$= \hat{Y}_{t+1} + [\gamma(1-\sigma)-1]g\hat{G}_{t+1} - (1-g)[\beta dR_{t+1} - d\pi_{t+1}]$$

- Phillips curve:

$$\pi_t = \beta\pi_{t+1} + \kappa\left[\left(\tfrac{1}{1-g} + \tfrac{N}{1-N}\right)\hat{Y}_t - \tfrac{g}{1-g}\hat{G}_t\right]$$

- Monetary policy rule:

$$dR_{t+1} = \rho_R dR_t + (1-\rho_R)\left[\tfrac{\phi_1}{\beta}\pi_{t+k} + \tfrac{\phi_2}{\beta}\hat{Y}_{t+l}\right]$$

The Equations in Matrix Form

$$\begin{bmatrix} -\frac{1}{1-g} & -1 & 0 \\ 0 & \beta & 0 \\ l(1-\rho_R)\frac{\phi_2}{\beta} & k(1-\rho_R)\frac{\phi_1}{\beta} & 0 \end{bmatrix} \begin{pmatrix} \hat{Y}_{t+1} \\ \pi_{t+1} \\ dR_{t+2} \end{pmatrix}$$

$$+ \begin{bmatrix} \frac{1}{1-g} & 0 & \beta \\ \kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right) & -1 & 0 \\ (1-l)(1-\rho_R)\frac{\phi_2}{\beta} & (1-k)(1-\rho_R)\frac{\phi_1}{\beta} & -1 \end{bmatrix} \begin{pmatrix} \hat{Y}_t \\ \pi_t \\ dR_{t+1} \end{pmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \rho_R \end{bmatrix} \begin{pmatrix} \hat{Y}_{t-1} \\ \pi_{t-1} \\ dR_t \end{pmatrix}$$

$$+ \begin{pmatrix} \frac{g[\gamma(\sigma-1)+1]}{1-g} \\ 0 \\ 0 \end{pmatrix} \hat{G}_{t+1} + \begin{pmatrix} -\frac{g[\gamma(\sigma-1)+1]}{1-g} \\ -\frac{\kappa g}{1-g} \\ 0 \end{pmatrix} \hat{G}_t,$$

- or, $\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t = 0.$

$$s_t = P s_{t-1} + \varepsilon_t,\ s_t \equiv \hat{G}_t,\ P = \rho$$

Solution:

- Undetermined coefficients, *A* and *B*:

$$z_t = Az_{t-1} + Bs_t$$

- *A* and *B* must satisfy:

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 = 0$$
$$\alpha_0(AB + BP) + \alpha_1 B + \beta_0 P + \beta_1 = 0.$$

- When $\rho_R = 0,\ \alpha_2 = 0 \rightarrow A = 0$ works .

Results

- Fiscal spending multiplier small, but can easily be bigger than unity (i.e., *C* rises in response to *G* shock)

- Contrasts with standard results in which multiplier is less than unity
  - Typical preferences in estimated models:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\gamma}}{1+\gamma} + v(G_t) \right], \ \psi, \gamma, \sigma > 0.$$

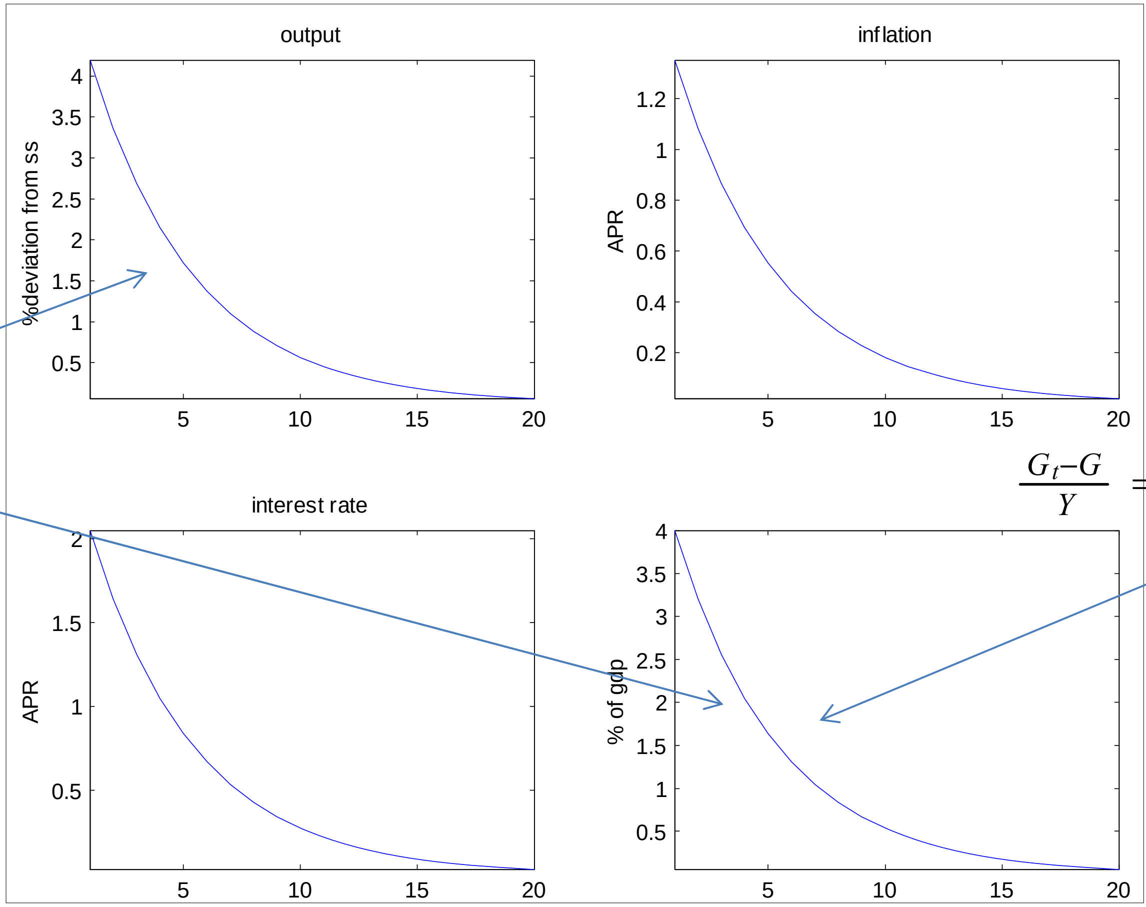
  - Marginal utility of *C* independent of *N* for CGG
  - Marginal utility of *C* increases in *N* for KPR.

Simulation Results

- Benchmark parameter values:

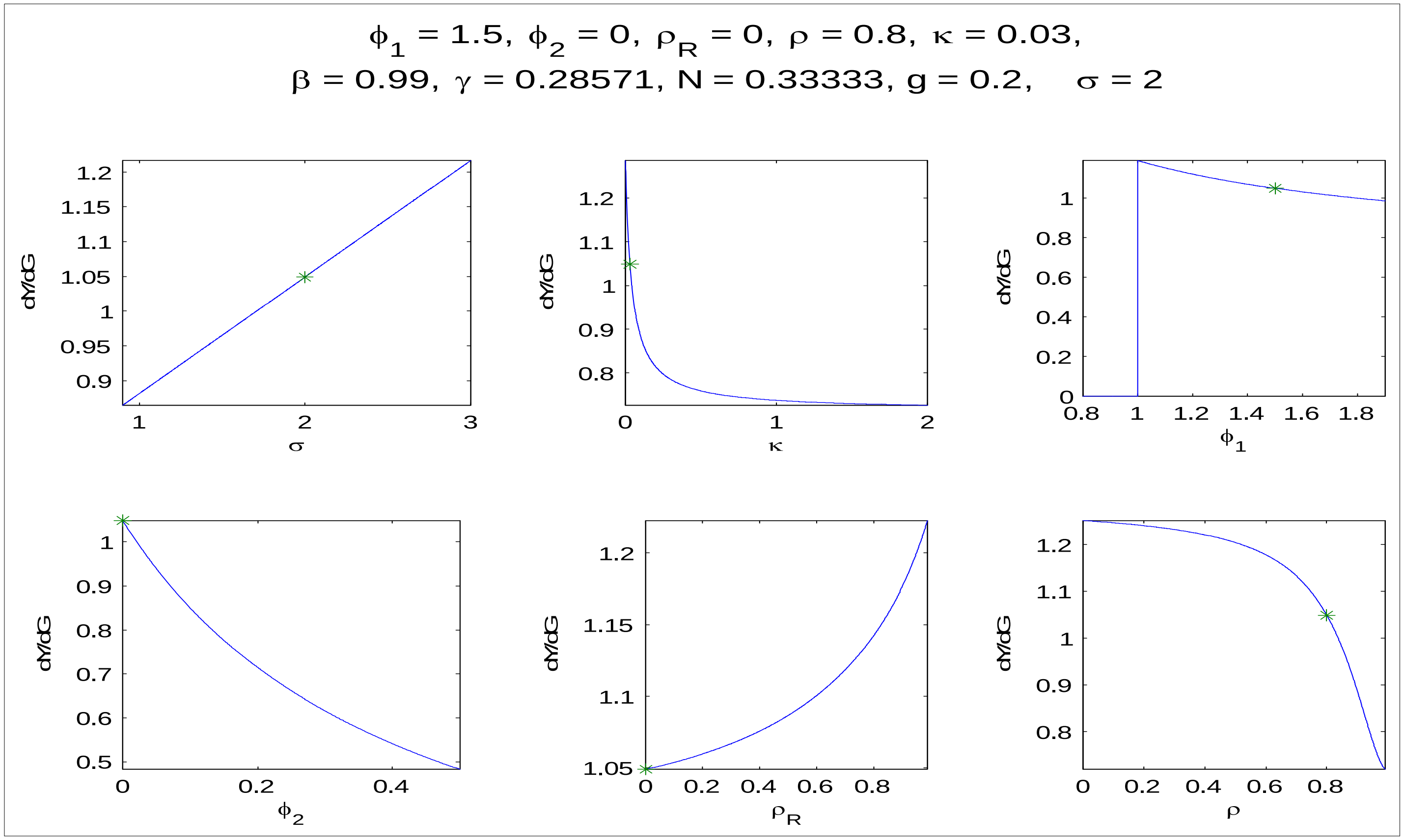
$$\beta = 0.99,\ \sigma = 2,\ g = 0.20,\ N = 0.33,\ \phi_1 = 1.5,\ \phi_2 = 0.0,\ \rho_R = 0,\ \kappa = 0.03,\ \rho = 0.8$$

Multiplier = 1.05, constant.



$$\frac{G_t - G}{Y} = \frac{G_t - G}{G}\frac{G}{Y} = \hat{G}_t g$$

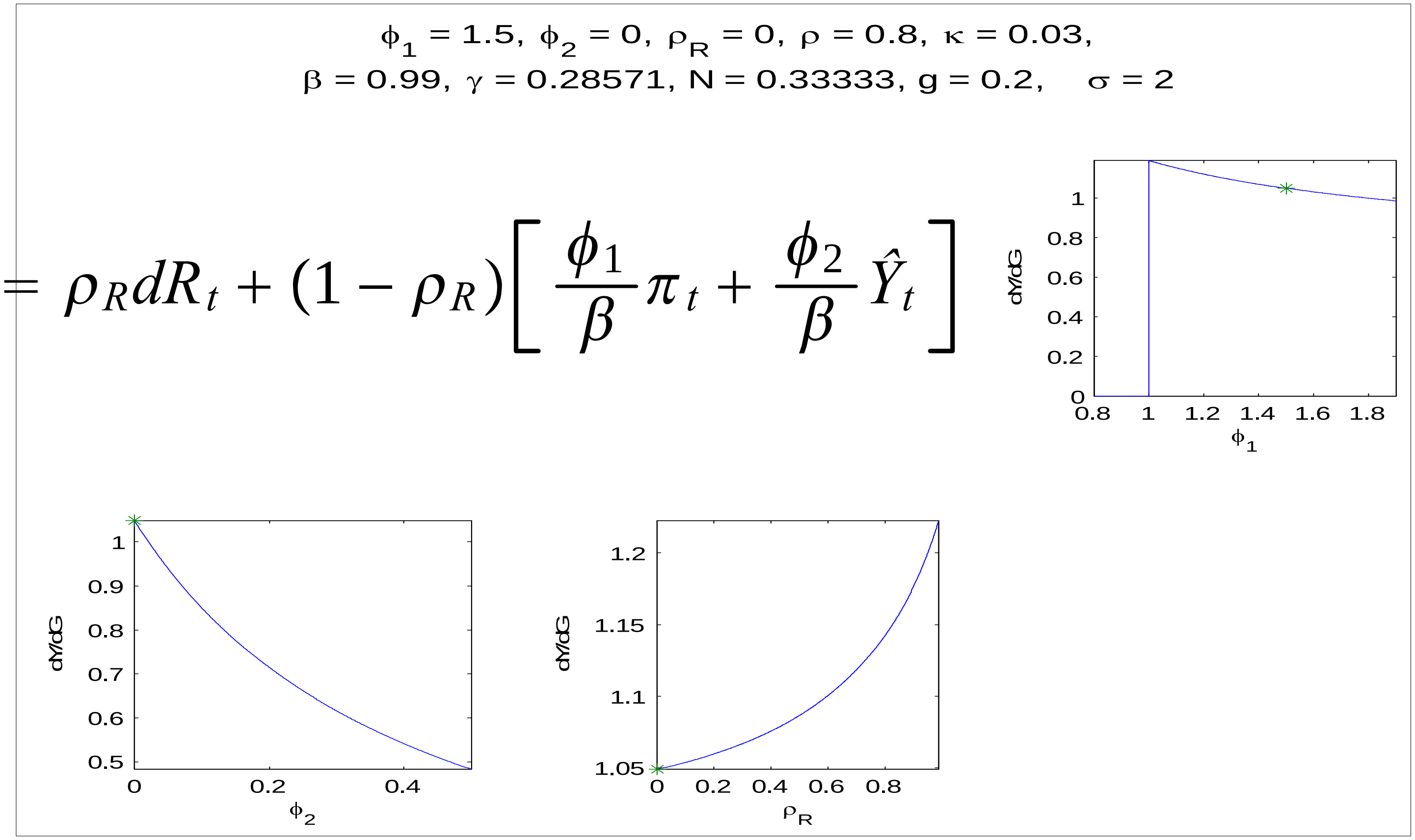
Multiplier for Alternative Parameter Values



- Results: multiplier bigger
  - the less monetary policy allows *R* to rise.
  - the more complementary are consumption and labor (i.e., the bigger is σ).
  - the smaller the negative income effect on consumption (i.e., the smaller is ρ).
  - smaller values of κ (i.e., more sticky prices)

Multiplier for Alternative Parameter Values

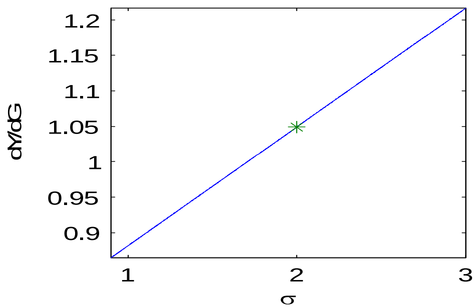
$$dR_{t+1} = \rho_R dR_t + (1-\rho_R)\left[\frac{\phi_1}{\beta}\pi_t + \frac{\phi_2}{\beta}\hat{Y}_t\right]$$



- Results: multiplier bigger
  - the less monetary policy allows *R* to rise.

Multiplier for Alternative Parameter Values

$\phi_1 = 1.5,\ \phi_2 = 0,\ \rho_R = 0,\ \rho = 0.8,\ \kappa = 0.03,$
$\beta = 0.99,\ \gamma = 0.28571,\ N = 0.33333,\ g = 0.2,\quad \sigma = 2$



$$u(C_t, N_t) = \frac{\left[C_t^{\gamma}(1-N_t)^{1-\gamma}\right]^{1-\sigma} - 1}{1-\sigma}$$

$$u_{c,t} = C_t^{[\gamma(1-\sigma)-1]}(1-N_t)^{(1-\gamma)(1-\sigma)}$$

- Results: multiplier bigger
  - the more complementary are consumption and labor (i.e., the bigger is σ).

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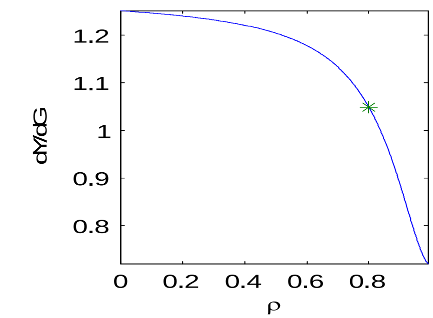
$\beta = 0.99,\ \gamma = 0.28571,\ N = 0.33333,\ g = 0.2,\quad \sigma = 2$

$$\hat{G}_t = \rho \hat{G}_{t-1} + \varepsilon_t$$



- Results: multiplier bigger
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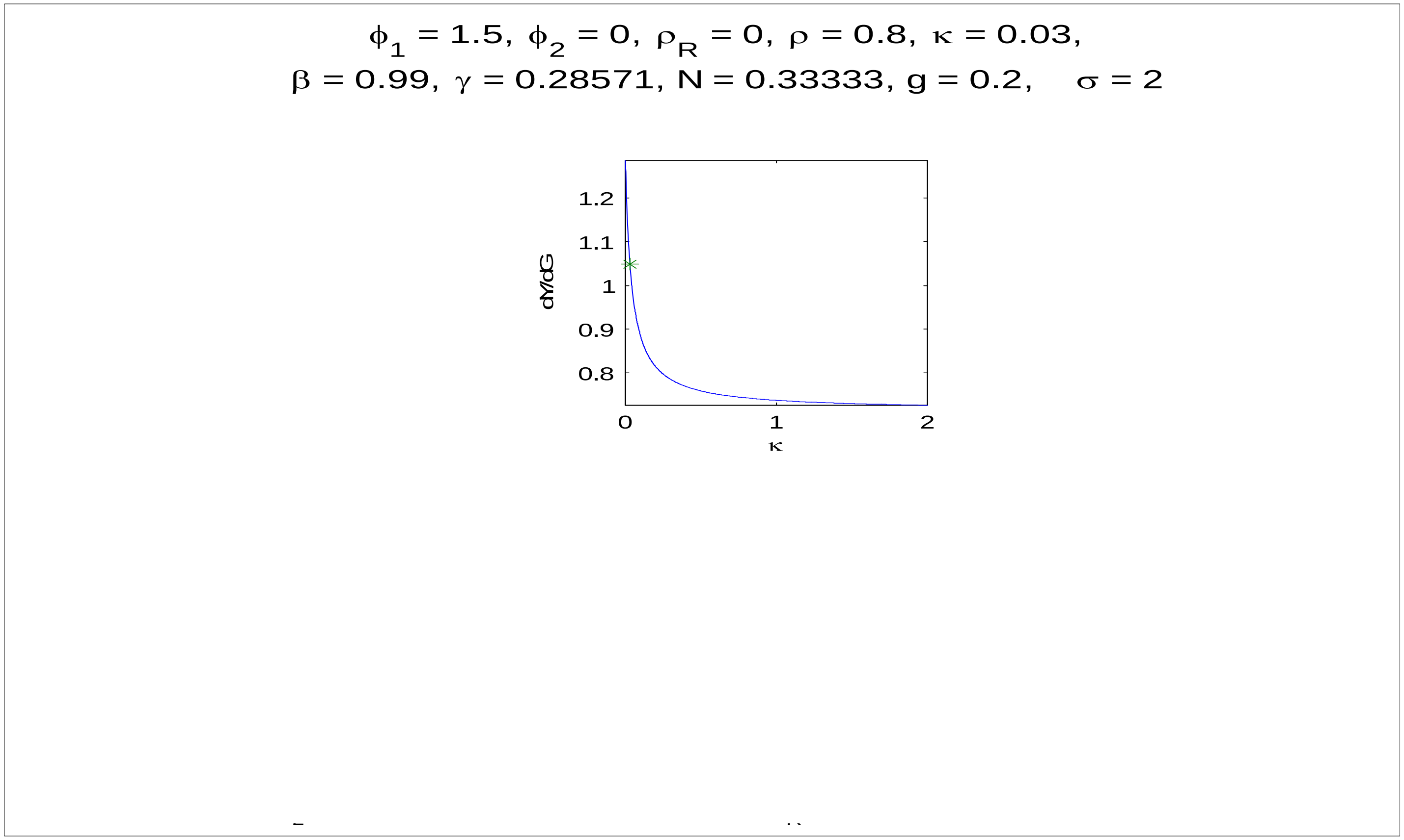
Multiplier for Alternative Parameter Values



- Results: multiplier bigger
  - smaller values of κ (i.e., more sticky prices)

Household and Government Budget Constraints in Previous Case

- A sort of ‘odd’ thing:
  - We seem to never have made explicit use of budget constraints, bond market clearing or taxes in the previous analysis.

- Resolution:
  - Taxes were ‘lump sum’, so it’s not surprising they don’t appear in the household first order conditions.
  - By imposing the resource constraint (which we do), we implicitly impose all budget constraints and bond market clearing, by Walras’ law.

Walras' Law

- Government budget constraint:

subsidy to intermediate good producers $= \nu \int_0^1 W_t N_{i,t} di$

$$\overbrace{\nu W_t N_t} + P_t G_t + (1+R_t) B_t^g = \overbrace{T_t^g}^{\text{lump sum taxes}} + \overbrace{B_{t+1}^g}^{\text{government debt}}$$

- Household budget constraint:

$$P_t C_t + B_{t+1} = W_t N_t + (1+R_t) B_t + \overbrace{T_t}^{\text{profits, net of taxes}}$$

Walras' Law, cnt'd

- Profits for final good producers are zero, so that revenues = costs:

$$\overbrace{P_t Y_t}^{\text{revenues}} = \overbrace{\int_0^1 P_{i,t} Y_{i,t} di}^{\text{costs}}.$$

- Profits of all intermediate good producers:

$$\int_0^1 [P_{i,t} Y_{i,t} - (1-\nu) W_t N_{i,t}] di$$

$$= P_t Y_t - (1-\nu) W_t N_t$$

Walras' Law, cnt'd

- Profits net of taxes are:

$$T_t = \overbrace{P_t Y_t - (1-\nu) W_t N_t}^{\text{integral of profits of all intermediate good producers}} - \overbrace{\left[(1+R_t)B_t^g + \nu W_t N_t + P_t G_t - B_{t+1}^g\right]}^{=T_t^g \text{ using government budget constraint}}$$

- Substituting this into the household budget constraint:

$$\begin{aligned} P_t C_t + B_{t+1} &= W_t N_t + (1+R_t)B_t + T_t \\ &= W_t N_t + (1+R_t)B_t + P_t Y_t - (1-\nu)W_t N_t \\ &\quad - \left[(1+R_t)B_t^g + \nu W_t N_t + P_t G_t - B_{t+1}^g\right] \\ &= (1+R_t)B_t + P_t Y_t - (1+R_t)B_t^g + B_{t+1}^g - P_t G_t \end{aligned}$$

Walras' Law, cnt'd

- Household budget constraint after making use of gov't budget constraint:

$$P_t(C_t + G_t) + (B_{t+1} - B_{t+1}^g) = (1 + R_t)(B_t - B_t^g) + P_t Y_t$$

- Bond market clearing:

$$B_{t+1} - B_{t+1}^g = 0 \text{ for all } t$$

- Thus, market clearing and household/gov't budget constraints imply resource constraint:

$$C_t + G_t = Y_t$$

- We in effect impose budget clearing and bond market clearing for everyone when we impose the resource constrain in the solution.
  - We can 'back out' the budget constraints after solving the model ignoring them and imposing only the resource constraint.
  - Timing of government taxes and debt have no impact on equilibrium allocations ('Ricardian equivalence').

Distortionary Taxes

- When taxes are related to scale of market activity, then they enter first order conditions.
  - Now, the 'fiscal rule' matters.
  - Fiscal rule:

    Strategy for setting tax rates, government spending, as a function of government debt, the state of the economy, etc.

  - The form of the fiscal rule matters.
    - Example: balanced budget requirement.
      - Could create unnecessary economic instability.
      - Suppose balanced budget implies raising taxes in recessions.
      - If people think there will be a recession, then they reason that taxes will be high and they cut back their labor effort, creating the recession and high taxes that they expected. (Laffer curve.)

Fiscal Rules

- One can imagine a variety of fiscal rules.
  - We will consider a fiscal rule in which government consumption remains exogenous, as in previous analysis
    - Perhaps, *G* can best be thought as the outcome of a political process that has only a small connection to aggregate *GDP*, consumption, etc.
- The tax rule raises distortionary labor income tax when government debt increases.
- We will do this in a very simple model.
  - My purpose is to show how it is done using a simple example.
  - A more interesting analysis would have to be done in a more empirically appealing model.

Incorporating A Tax Rule

- Household budget constraint:

$$P_t C_t + B_{t+1} = (1-\tau_t) W_t N_t + (1+R_t) B_t + \overbrace{T_t}^{\text{profits - } T_t^g}$$

- Government budget constraint:

$$\nu W_t N_t + P_t G_t + (1+R_t) B_t^g = \overbrace{T_t^g}^{\text{lump sum tax still in use}} + \tau_t W_t N_t + B_{t+1}^g$$

- Fiscal rule:

$$\tau_t = \tau + \eta \left(\frac{B_t^g}{P_{t-1}} - b^g \right)$$

Modifying the Equilibrium Conditions

- In lump-sum case, derived three equilibrium conditions: Phillips curve, IS curve, Taylor rule.
  - No change to IS curve, Taylor rule.

- The tax rate affects the household's first order condition for labor.
  - We used this to substitute out for the real wage in the definition of marginal cost for intermediate good firm.
  - So, the tax rule requires us to modify the **Phillips curve** because it has marginal cost.

- Because tax rule is a function of government debt
  - must bring in **government budget constraint**.

Modifying the Equilibrium Conditions

- Household first order condition:

$$\overbrace{\frac{(1-\gamma)C_t}{\gamma(1-N_t)}}^{\text{using our functional form}} = \frac{-u_{N,t}}{u_{c,t}} = \frac{W_t}{P_t}(1-\tau_t)$$

- Real marginal cost of intermediate good firm:

$$s_t = \frac{\frac{W_t}{P_t}(1-\nu)}{MP_{L,t}} = \frac{W_t}{P_t}(1-\nu) \overbrace{=}^{\text{household Euler}} \frac{1-\nu}{1-\tau_t}\frac{-u_{N,t}}{u_{c,t}} \overbrace{=}^{\text{in steady state}} \frac{\varepsilon-1}{\varepsilon}$$

- Efficient setting of government subsidy:

$$1-\nu = (1-\tau)\frac{\varepsilon-1}{\varepsilon}$$

Modifying the Equilibrium Conditions

- Real marginal cost:

$$s_t = \frac{1-\nu}{1-\tau_t}\frac{-u_{N,t}}{u_{c,t}} = \frac{1-\nu}{1-\tau_t}\frac{(1-\gamma)C_t}{\gamma(1-N_t)}$$

- Log-linearized representation:

$$\hat{s}_t = \hat{C}_t + \frac{\tau}{1-\tau}\hat{\tau}_t + \frac{N}{1-N}\hat{N}_t$$

- Log-linearized resource constraint (as before):

$$\hat{C}_t = \frac{1}{1-g}\hat{Y}_t - \frac{g}{1-g}\hat{G}_t$$

- Substituting:

$$\hat{s}_t = \left[\frac{1}{1-g} + \frac{N}{1-N}\right]\hat{Y}_t - \frac{g}{1-g}\hat{G}_t + \frac{\tau}{1-\tau}\hat{\tau}_t$$

Modifying the Equilibrium Conditions

- Fiscal rule:
  - Let $b_{t+1}^g \equiv \frac{B_{t+1}^g}{P_t},\ t_t^g \equiv \frac{T_t^g}{P_t}$
  - Then

$$\tau_t = \tau + \eta(b_t^g - b^g) \rightarrow \hat{\tau}_t = \frac{b^g}{\tau}\eta \hat{b}_t^g$$

- Gov't budget constraint (provides law of motion of debt):

$$v\frac{W_t}{P_t}N_t + G_t + (1+R_t)\overbrace{\frac{b_t^g}{1+\pi_t}}^{=\frac{B_t^g}{P_{t-1}}\times\frac{P_{t-1}}{P_t}} = t_t^g + \tau_t \overbrace{\frac{W_t}{P_t}}^{\frac{(1-\gamma)C_t}{\gamma(1-N_t)}\frac{1}{1-\tau_t}} N_t + b_{t+1}^g$$

Modifying the Equilibrium Conditions

- Government budget constraint:

$$G_t + \frac{1+R_t}{1+\pi_t} b_t^g = t_t^g + \frac{\tau_t - \nu}{1-\tau_t} \frac{(1-\gamma)C_t}{\gamma(1-N_t)} N_t + b_{t+1}^g$$

- Linearize about steady state (using log-linearized resource constraint and $\hat{Y}_t = \hat{N}_t$)

$$\left[1 + \frac{\tau - \nu}{1-\tau} \frac{1}{1-g}\right] g\hat{G}_t + \frac{1}{\beta} r^b \hat{b}_t^g + r^b dR_t - \frac{1}{\beta} r^b d\pi_t$$

$$= \frac{t^g}{N} \hat{t}_t^g + \frac{\tau - \nu}{1-\tau} \left[\frac{1}{1-g} + \frac{1}{1-N}\right] \hat{Y}_t + \frac{\tau}{1-\tau} \frac{1-\nu}{1-\tau} \hat{\tau}_t + r^b \hat{b}_{t+1}^g$$

Steady State

- To work with the log-linearized equilibrium conditions, must have values for:

$$\underbrace{N}_{GDP\text{=employment}}, \underbrace{g}_{G/GDP}, \underbrace{r^b}_{\text{debt-to-GDP ratio}}, \underbrace{t^g}_{\text{real lump-sum taxes}}$$

$$\gamma, \nu, \beta, \varepsilon, \tau, \sigma, \kappa, \rho_R, \phi_1, \phi_2.$$

- For comparability with previous (lump sum) calculations

$$\overbrace{\varepsilon = 6,\ \tau = \tfrac{1}{3}}^{\text{'reasonable values, given US data'}}, \quad \overbrace{\beta = 0.99,\ \sigma = 2,\ \kappa = 0.03,\ \rho_R = 0,\ \phi_1 = 1.5,\ \phi_2 = 0.}^{\text{set for comparability with lump sum example}}$$

Steady State

- Specify *N=1/3, g=0.20* and this delivers a value for γ

$$\frac{(1-\gamma)C}{\gamma(1-N)} = \frac{(1-\gamma)(1-g)N}{\gamma(1-N)} = 1$$

- Given labor tax rate, can compute subsidy rate from

$$1 - \nu = (1 - \tau)\frac{\varepsilon-1}{\varepsilon}$$

- Given $r^b = b^g/N$ can compute t^g from steady state government budget constraint

$$b^g = \frac{t^g + \left(\frac{\tau-\nu}{1-\tau} - g\right)N}{\frac{1}{\beta} - 1}$$

Linearized system

f=1 means tax rate a function of lagged debt

$$
E_t\Bigg\{
\begin{bmatrix}
-\frac{1}{1-g} & -1 & 0 & 0 & 0 \\
0 & \beta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -r^b & 0
\end{bmatrix}
\begin{pmatrix}
\hat{Y}_{t+1} \\ \pi_{t+1} \\ dR_{t+2} \\ \hat{b}^g_{t+1} \\ \hat{\tau}_{t+1}
\end{pmatrix}
$$

$$
+\begin{bmatrix}
\frac{1}{1-g} & 0 & \beta & 0 & 0 \\
\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right) & -1 & 0 & 0 & \kappa\frac{\tau}{1-\tau} \\
(1-\rho_R)\frac{\phi_2}{\beta} & (1-\rho_R)\frac{\phi_1}{\beta} & -1 & 0 & 0 \\
0 & 0 & 0 & -(1-f)\frac{b^g}{\tau}\eta & 1 \\
-\frac{\tau-\nu}{1-\tau}\left(\frac{1}{1-g}+\frac{1}{1-N}\right) & -\frac{1}{\beta}r^b & 0 & \frac{1}{\beta}r^b & -\frac{\tau}{1-\tau}\frac{1-\nu}{1-\tau}
\end{bmatrix}
\begin{pmatrix}
\hat{Y}_t \\ \pi_t \\ dR_{t+1} \\ \hat{b}^g_t \\ \hat{\tau}_t
\end{pmatrix}
$$

$$
+\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_R & 0 & 0 \\
0 & 0 & 0 & -f\frac{b^g}{\tau}\eta & 0 \\
0 & 0 & r^b & 0 & 0
\end{bmatrix}
\begin{pmatrix}
\hat{Y}_{t-1} \\ \pi_{t-1} \\ dR_t \\ \hat{b}^g_{t-1} \\ \hat{\tau}_{t-1}
\end{pmatrix}
$$

$$
+\begin{pmatrix}
\frac{g[\gamma(\sigma-1)+1]}{1-g} & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
\hat{G}_{t+1} \\ \hat{t}^g_{t+1}
\end{pmatrix}
+\begin{pmatrix}
-\frac{g[\gamma(\sigma-1)+1]}{1-g} & 0 \\
-\frac{\kappa g}{1-g} & 0 \\
0 & 0 \\
0 & 0 \\
\left[1+\frac{\tau-\nu}{1-\tau}\frac{1}{1-g}\right]g & -\frac{t^g}{N}
\end{pmatrix}
\begin{pmatrix}
\hat{G}_t \\ \hat{t}^g_t
\end{pmatrix}
\Bigg\} = 0,
$$

$$
s_t = \begin{bmatrix} \rho & 0 \\ 0 & \rho_T \end{bmatrix} s_{t-1} + \begin{pmatrix} \varepsilon^G_t \\ \varepsilon^{t^g}_t \end{pmatrix}
$$

$$
E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0
$$

Solving the Model

- Look for a solution of the following form:

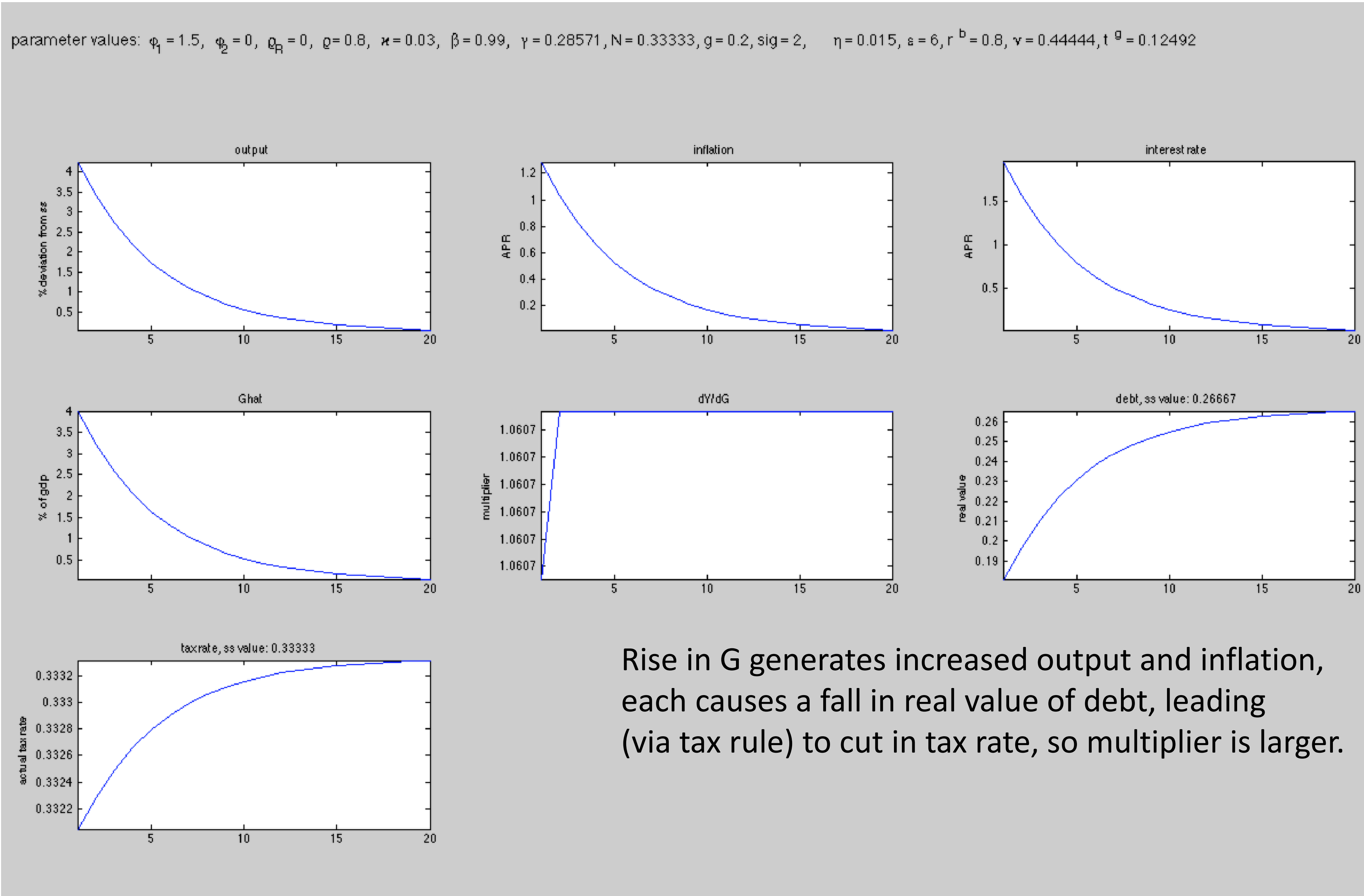
$$z_t = Az_{t-1} + Bs_t$$

- Choose *A* and *B* so that linearized Euler equations are satisfied. This requires

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 = 0$$

$$\alpha_0(AB + BP) + \alpha_1 B + \beta_0 P + \beta_1 = 0.$$

Parameter, η, in tax rule set to 0.015 (small)



Rise in G generates increased output and inflation, each causes a fall in real value of debt, leading (via tax rule) to cut in tax rate, so multiplier is larger.