Extensions on the Basic Model

- Open economy (based on work of Adolfson-Laséen-LindéVillani and Christiano-Trabandt-Walentin (CTW))

- Financial frictions (based on work of Bernanke-Gertler-Gilchrist and Christiano-Motto-Rostagno, CTW)
  (Separate Handout)

Basic Model

- Results from closed economy model
  - Household preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ \left(u\left(C_t\right) - \exp\left(\tau_t\right) \frac{N_t^{1+\varphi}}{1+\varphi} \right),\ u\left(C_t\right) \equiv \log C_t$$

  - Aggregate resources and household intertemporal optimization:

$$Y_t = p_t^* A_t N_t,\ u_{c,t} = \beta E_t u_{c,t+1} \frac{R_t}{\bar{\pi}_{t+1}}$$

  - Law of motion of price distortion:

$$p_t^* = \left((1-\theta) \left(\frac{1 - \theta \left(\bar{\pi}_t\right)^{\varepsilon - 1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right)^{-1}. \tag{1}$$

– Equilibrium conditions associated with price setting:

$$1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta \theta F_{t+1} = F_t \tag{2}$$

$$F_t \left[\frac{1 - \theta \bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} = K_t \tag{3}$$

$$\begin{aligned} & \frac{\varepsilon}{\varepsilon-1} \overbrace{(1-\nu_t) \overbrace{\frac{\exp(\tau_t) N_t^{\varphi}}{u_{c,t}}}^{=\frac{W_t}{P_t} \text{ by household optimization}} \frac{1-\psi+\psi R_t}{A_t}}^{= \text{intermediate good rm marginal cost}} + E_t \beta \theta \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \\ = \ & K_t \end{aligned} \tag{4}$$

Extension to Small Open Economy

- Outline
  - the equilibrium conditions of the open economy model
    - system jumps from 5-6 equations in basic model to 16 equations in 16 variables!
    - additional variables:

$$\overbrace{s_t, x_t, a_t^f, p_t^x, q_t}^{\text{rate of depreciation, exports, real foreign assets, terms of trade, real exchange rate, respectively}},$$

$$\overbrace{p_t^c}^{\text{price of domestic consumption (now, } c \text{ is a composite of domestically produced goods and imports)}},$$

$$\overbrace{p_t^{m,c}}^{\text{price of imports}}, \quad \overbrace{\pi_t^c}^{\text{consumption price in ation}},$$

$$\overbrace{\Phi_t}^{\text{reduced form object to (i) achieve technical objective, (ii) correct a fundamental failing of open economy models}},$$

$$\overbrace{R_t, \bar{\pi}_t, N_t, c_t, K_t, F_t, p_t^*}^{\text{closed economy variables}}.$$

Extension to Small Open Economy ...

– computing the steady state

– the ‘uncovered interest parity puzzle’, and the role of Φ_t in addressing the puzzle.

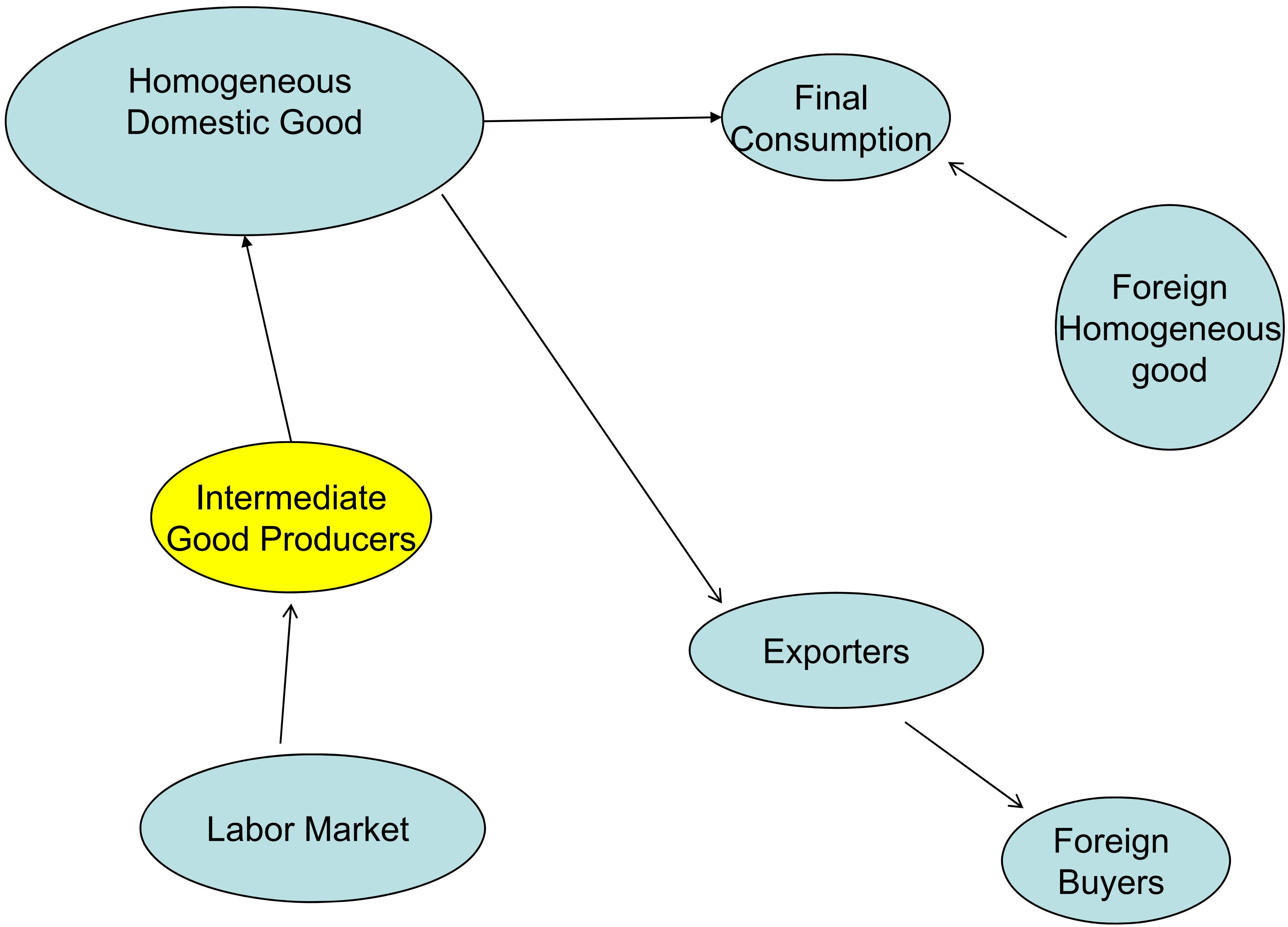
– summary of the endogenous and exogenous variables of the model, as well as the equations.

– several computational experiments to illustrate the properties of the model.

Extension to Small Open Economy ...

- Modi cations to basic model to create open economy
  - unchanged:
    - household preferences
    - production of (domestic) homogeneous good, Y_t $(= A_t p_t^* N_t)$
    - three Calvo price friction equations
  - changes:
    - household budget constraint includes opportunity to acquire foreign assets/liabilities.
    - intertemporal Euler equation changed as a reduced form accommodation of evidence on uncovered interest parity.
    - $Y_t = C_t$ no longer true.
    - introduce exports, imports, current account.
    - exchange rate,

$$\boxed{S_t = \text{domestic currency price of one unit of foreign currency} = \frac{\text{domestic money}}{\text{foreign money}}.}$$


Homogeneous Domestic Good
Final Consumption
Foreign Homogeneous good
Intermediate Good Producers
Exporters
Labor Market
Foreign Buyers

Extension to Small Open Economy ...

- Monetary policy: three approaches
  - Taylor rule

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\, E_t[r_\pi \log\left(\frac{\pi^c_{t+1}}{\bar{\pi}^c}\right) + r_y \log\left(\frac{y_{t+1}}{y}\right)] + \varepsilon_{R,t}, \quad (5)$$

where (could also add exchange rate, real exchange rate and other things):

$\bar{\pi}^c$ ~target consumer price in ation

$\varepsilon_{R,t}$ ~iid, mean zero monetary policy shock

y_t ~Y_t/A_t

R_t ~'risk free' nominal rate of interest

  - Svensson-style policy that solves Ramsey problem with the following preferences:

$$E_t \sum_{j=0}^{\infty} \beta^j \{ \left(100 \left[\pi^c_t \pi^c_{t-1} \pi^c_{t-2} \pi^c_{t-3} - (\bar{\pi}^c_t)^4\right]\right)^2 + \lambda_y \left(100 \log\left(\frac{y_t}{y}\right)\right)^2$$
$$+ \lambda_{\Delta R} \left(400 \left[R_t - R_{t-1}\right]\right)^2 + \lambda_s \left(S_t - \bar{S}\right)^2 \}$$

  - straight Ramsey policy that maximizes domestic social welfare.

Extension to Small Open Economy ...

- Household budget constraint

$$S_t A^f_{t+1} + P_t C_t + B_{t+1}$$
$$\leq \; B_t R_{t-1} + S_t \left[\Phi_{t-1} R^f_{t-1}\right] A^f_t + W_t N_t + Transfers\, and\, profits_t$$

- Domestic bonds

B_t ~beginning of period t stock of loans

R_t ~rate of return on bonds

- Foreign assets

A^f_t ~beginning-of-period t net stock of foreign assets (liabilities, if negative) held by domestic residents.

$\Phi_t R^f_t$ ~rate of return on A^f_t

Φ_t ~premium on foreign asset returns, over foreign risk free rate, R^f_t

Extension to Small Open Economy ...

- optimality of household foreign asset decision (verify this by solving Lagrangian)

$$\overbrace{\frac{u_{c,t}S_t}{P_t^c}}^{\text{utility cost of one unit of foreign currency}=S_t\text{ units of domestic currency, which is }S_t/P_t^c\text{ units of }C_t}$$

$$= \beta E_t \overbrace{u_{c,t+1}}^{\text{conversion into utility units}} \times \overbrace{\frac{S_{t+1}\overbrace{R_t^f\Phi_t}^{\text{foreign currency payoff next period from one unit of foreign currency today}}}{P_{t+1}^c}}^{\text{quantity of domestic consumption goods that can be purchased from the payoff of one unit of foreign currency}}$$

or

$$\frac{S_t}{P_t^c C_t} = \beta E_t \frac{S_{t+1} R_t^f \Phi_t}{P_{t+1}^c C_{t+1}}$$

or

$$\frac{1}{c_t} = \beta E_t \frac{s_{t+1} R_t^f \Phi_t}{\pi_{t+1}^c c_{t+1} \exp\left(\Delta a_{t+1}\right)},\ s_t \equiv \frac{S_t}{S_{t-1}},\ c_t = \frac{C_t}{A_t}. \tag{6}$$

- Optimality of household domestic bond decision:

$$\frac{1}{P_t^c C_t} = \beta E_t \frac{R_t}{P_{t+1}^c C_{t+1}}$$

  – after scaling:

$$\frac{1}{c_t} = \beta E_t \frac{R_t}{\pi_{t+1}^c c_{t+1} \exp\left(\Delta a_{t+1}\right)}. \tag{7}$$

Extension to Small Open Economy ...

- Final domestic consumption goods, C_t
  - produced by representative, competitive   rm using:

$$C_t \;=\; \left[(1-\omega_c)^{\frac{1}{\eta_c}}\left(C_t^d\right)^{\frac{(\eta_c-1)}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}}\left(C_t^m\right)^{\frac{(\eta_c-1)}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}}$$

C_t^d ~one-for-one transformation on domestic homogeneous output good, price P_t

C_t^m ~imported good, with price $P_t^{m,c}$

$C_t\sim$ nal consumption good, with price, P_t^c

η_c ~elasticity of substitution between domestic and foreign goods.

  - Pro t maximization leads to:

$$\begin{aligned} C_t^d &= (1-\omega_c)\left(p_t^c\right)^{\eta_c} C_t \\ C_t^m &= \omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta c} C_t. \\ p_t^c &= \left[(1-\omega_c) + \omega_c\left(p_t^{m,c}\right)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}} \qquad (8) \\ p_t^c &\equiv \frac{P_t^c}{Pt},\; p_t^{m,c} \equiv \frac{P_t^{m,c}}{P_t} \end{aligned}$$

– C_t^m is produced by competitive rm, which converts foreign homogeneous output one-for-one into C_t^m.

∗ Setting price equal to marginal cost:

$$P_t^{m,c} = S_t P_t^f \left(1 - \psi^f + \psi^f R_t^f\right), \; P_t^f \text{ ~foreign currency price of foreign good.}$$

or,

$$p_t^{m,c} = \frac{P_t^c}{P_t^c} \frac{S_t P_t^f}{P_t} \left(1 - \psi^f + \psi^f R_t^f\right) = p_t^c \overbrace{q_t}^{\text{real exchange rate} \equiv \frac{S_t P_t^f}{P_t^c}} \left(1 - \psi^f + \psi^f R_t^f\right). \tag{9}$$

– Consumption good in ation:

$$\pi_t^c \equiv \frac{P_t^c}{P_{t-1}^c} = \frac{P_t p_t^c}{P_{t-1} p_{t-1}^c} = \bar{\pi}_t \left[\frac{(1-\omega_c) + \omega_c \left(p_t^{m,c}\right)^{1-\eta_c}}{(1-\omega_c) + \omega_c \left(p_{t-1}^{m,c}\right)^{1-\eta_c}} \right]^{\frac{1}{1-\eta_c}}. \tag{10}$$

- Exports, X_t
  - foreign demand for exports

$$X_t = \left(\frac{P_t^x}{P_t^f}\right)^{-\eta_f} Y_t^f = (p_t^x)^{-\eta_f} Y_t^f \tag{11}$$

Y_t^f ~foreign output, P_t^f ~price of foreign good, P_t^x ~ price of export

  - X_t is produced one-for-one using the domestic homogeneous good by a representative, competitive producer. Equating price, $S_t P_t^x$, to marginal cost:

$$S_t P_t^x = P_t \left(\nu^x R_t + 1 - \nu^x\right),$$

where $\nu^x = 1$ if all inputs must be nanced in advance. Rewriting

$$q_t p_t^x p_t^c = \nu^x R_t + 1 - \nu^x, \tag{12}$$

where

$$\overbrace{q_t}^{\text{real exchange rate}} \equiv \frac{S_t P_t^f}{P_t^c}, \quad \overbrace{p_t^x}^{\text{terms of trade}} = \frac{P_t^x}{P_t^f}$$

$$\frac{q_t}{q_{t-1}} = s_t \frac{\pi_t^f}{\pi_t^c}, \; s_t = \frac{S_t}{S_{t-1}}. \tag{13}$$

Extension to Small Open Economy ...

- Clearing in domestic homogeneous goods market:

$$
\begin{aligned}
& \text{output of domestic homogeneous good, } Y_t \\
= \; & \text{uses of domestic homogeneous goods} \\
= \; & \overbrace{C_t^d}^{\text{goods used in production of nal consumption, } C_t} + \overbrace{X_t}^{\text{exports}} + \overbrace{G_t}^{\text{government}} \\
= \; & (1-\omega_c)\,(p_t^c)^{\eta_c}\, C_t + X_t + G_t .
\end{aligned}
$$

- Substituting out for Y_t:

$$
A_t p_t^* N_t = (1-\omega_c)\,(p_t^c)^{\eta_c}\, C_t + X_t + G_t,
$$

or,

$$
p_t^* N_t = (1-\omega_c)\,(p_t^c)^{\eta_c}\, c_t + x_t + g_t, \tag{14}
$$

$$
c_t \equiv \frac{C_t}{A_t},\; x_t \equiv \frac{X_t}{A_t},\; g_t \equiv \frac{G_t}{A_t}.
$$

Extension to Small Open Economy ...

- Current Account
  - equality of international demand and supply for currency:

$$\overbrace{\overbrace{S_t A_{t+1}^f}^{\text{acquisition of new net foreign assets, in domestic currency units}} + \text{expenses on imports}_t}^{\text{currency owing out of the country}}$$

$$= \overbrace{\text{receipts from exports}_t + \overbrace{S_t R_{t-1}^f \Phi_{t-1} A_t^f}^{\text{receipts from existing stock of net foreign assets}}}^{\text{currency owing into the country}},$$

  - The pieces:

$$\text{expenses on imports}_t = S_t P_t^f \left(1 - \psi^f + \psi^f R_t^f\right) \omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta c} C_t$$

$$\text{receipts from exports}_t = S_t P_t^x X_t.$$

  - Current account:

$$S_t A_{t+1}^f + S_t P_t^f \left(1 - \psi^f + \psi^f R_t^f\right) \omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta_c} C_t = S_t P_t^x X_t + S_t R_{t-1}^f \Phi_{t-1} A_t^f.$$

– Divide current account by $P_t A_t$:

$$\frac{S_t A_{t+1}^f}{P_t A_t} + \frac{S_t P_t^f}{P_t}\left(1 - \psi^f + \psi^f R_t^f\right)\omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta_c} c_t = \frac{S_t P_t^x}{P_t} x_t + \frac{S_t R_{t-1}^f \Phi_{t-1} A_t^f}{P_t A_t},$$

or, using (9):

$$a_t^f + p_t^{m,c}\omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta_c} c_t = p_t^c q_t p_t^x x_t + \frac{s_t R_{t-1}^f \Phi_{t-1} a_{t-1}^f}{\bar{\pi}_t \exp\left(\Delta a_t\right)}, \qquad (15)$$

where a_t^f is ‘scaled real, domestic value of foreign assets’:

$$a_t^f = \frac{S_t A_{t+1}^f}{P_t A_t}$$

Extension to Small Open Economy ...

- 'Risk' adjustments

$$\Phi_t = \Phi\left(a_t^f, R_t^f, R_t, \tilde{\phi}_t\right) = \qquad (16)$$
$$\exp\left(-\tilde{\phi}_a\left(a_t^f - \bar{a}\right) - \tilde{\phi}_s\left(R_t^f - R_t - \left(R^f - R\right)\right) + \tilde{\phi}_t\right)$$
$$\tilde{\phi}_a > 0, \text{ small and not important for dynamics}$$
$$\tilde{\phi}_s > 0, \text{ important}$$
$$\tilde{\phi}_t \sim \text{mean zero, iid.}$$

- Discussion of $\tilde{\phi}_a$.
  - $\tilde{\phi}_a > 0$ implies (i) if $a_t^f > \bar{a}$, then return on foreign assets low and $a_t^f \downarrow$; (ii) if $a_t^f < \bar{a}$, then return on foreign assets high and $a_t^f \uparrow$
  - implication: $\tilde{\phi}_a > 0$ is a force that drives $a_t^f \rightarrow \bar{a}$ in steady state, independent of initial conditions.
  - logic is same as reason why steady state stock of capital in neoclassical growth model is unique, independent of initial conditions.
  - in practice, $\tilde{\phi}_a$ is tiny, so that its only effect is to pin down a_t^f in steady state and not affect dynamics (see Schmitt-Grohe and Uribe).

Extension to Small Open Economy ...

- Discussion of $\tilde{\phi}_t$
  - Captures, informally, the possibility that there is a shock to the required return on domestic assets. Perhaps this could be a crude stand-in for a 'sub-prime mortgage crisis', because it implies that people require a higher return on domestic assets if they are to hold them.

- Discussion of $\tilde{\phi}_s$.
  - $\tilde{\phi}_s$ is an important reduced form feature, designed to correct an important aw in models of international nance. It represents a quick x for the problem, not a substitute for a longer-run solution.
  - to better explain this, it is convenient to rst solve for the model's steady state.

- Steady state

  - household intertemporal efficiency conditions:

$$0 = E_t\left[\frac{1}{c_t} - \beta\frac{s_{t+1}R_t^f\Phi_t}{\pi_{t+1}^c c_{t+1}\exp\left(\Delta a_{t+1}\right)}\right], \text{ steady state: } 1 = \beta\frac{sR^f\Phi}{\pi^c} \quad (17)$$

$$0 = E_t\left[\frac{1}{c_t} - \beta\frac{1}{c_{t+1}}\frac{R_t}{\pi_{t+1}^c\exp\left(\Delta a_{t+1}\right)}\right], \text{ steady state: } 1 = \beta\frac{R}{\pi^c} \quad (18)$$

  - assumption about foreign households:

$$1 = \beta\frac{R^f}{\pi^f} \quad (19)$$

$$\pi_t^f \equiv \frac{P_t^f}{P_{t-1}^f} \text{ (exogenous)}$$

Extension to Small Open Economy ...

– Taylor rule:

$$\pi^c = \bar{\pi}^c \text{ (central bank's in ation target).} \tag{20}$$

– from (10):

$$\pi^c = \bar{\pi} \equiv \frac{P_t}{P_{t-1}}. \tag{21}$$

– using price friction equilibrium conditions:

$$p^* = \frac{\frac{1-\theta\bar{\pi}^{\varepsilon}}{1-\theta}}{\left(\frac{1-\theta(\bar{\pi})^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}}, \text{ (no distortion if } \bar{\pi} = 1, \text{)} \tag{22}$$

$$F = \frac{1}{1-\beta\theta\bar{\pi}^{\varepsilon-1}}, \text{ (don't allow } \bar{\pi}^{\varepsilon-1}\beta\theta < 1\text{)} \tag{23}$$

$$K = \frac{\frac{\varepsilon}{\varepsilon-1}(1-\nu)\ \exp(\tau) N^{\varphi+1} p^* (1-\psi+\psi R)}{1-\beta\theta\bar{\pi}^{\varepsilon}}, \ (\beta\theta\bar{\pi}^{\varepsilon} < 1) \tag{24}$$

$$K = F\left[\frac{1-\theta\bar{\pi}^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}. \tag{25}$$

– other steady state conditions:

$$p^c = \left[(1-\omega_c) + \omega_c \left(p^{m,c}\right)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}} \tag{26}$$

$$p^{m,c} = p^c q \left(1 - \psi^f + \psi^f R^f\right) \tag{27}$$

$$q p^x p^c = \nu^x R + 1 - \nu^x \tag{28}$$

$$p^* N = (1-\omega_c)\left(p^c\right)^{\eta_c} c + x + g \tag{29}$$

$$a^f + p^c q \left(1 - \psi^f + \psi^f R^f\right) \omega_c \left(\frac{p^c}{p^{m,c}}\right)^{\eta_c} c \; = \; p^c q p^x x + \frac{s R^f \Phi a^f}{\bar{\pi}} \tag{30}$$

$$x = \left(p^x\right)^{-\eta_f} y^f \tag{31}$$

– 15 equations: (17)-(31), 15 unknowns:

$$p^c, p^{m,c}, q, p^x, c, x, a^f, R, \Phi, \bar{\pi}, K, F, p^*, \pi^c, s$$

– for convenience, set exogenous variables, g, $\bar{a}$,

$$g = \eta_g y, \;\; \bar{a} = \eta_a y, \;\; \text{where } y = p^* N.$$

– algorithm for solving for the steady state:

  * p^*, F, K can be computed from (21), (22), (23) and (25).
  * solve (24) for N.
  * solve

$$\begin{aligned} g &= \eta_g p^* N \\ a^f &= \bar{a} = \eta_a p^* N. \end{aligned}$$

  * (18), (19) imply

$$\frac{R^f}{\pi^f} = \frac{R}{\pi^c} \tag{32}$$

  * steady state depreciation, s, can be computed from the in ation differential:

$$q_t \to q \text{ implies (see (13)) } s\pi^f = \pi^c.$$

  * (17), (18) imply

$$sR^f\Phi = R, \tag{33}$$

  or after multiplication by π^f and rearranging,

$$\frac{R^f}{\pi^f}\Phi = \frac{R}{\pi^c}, \text{ so (see (32)) } \Phi = 1 \text{ and } a_t = \bar{a} \text{ (see (16))}$$

– rest of the algorithm solves a single non-linear equation in a single unknown.

– set

$$\tilde{\varphi} = p^c q.$$

– use (27), (28), (26):

$$\begin{aligned} p^{m,c} &= \tilde{\varphi} R^{\nu,*} \\ p^x &= \frac{R^x}{\tilde{\varphi}}, \\ p^c &= \left[(1-\omega_c) + \omega_c \left(p^{m,c} \right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \\ q &= \frac{\tilde{\varphi}}{p^c}. \end{aligned}$$

– solve the resource constraint, (29), for c in terms of x :

$$c = \frac{p^c q p^x x + \frac{s R^f a^f}{\bar{\pi}} - a^f}{p^c q \left(1 - \psi^f + \psi^f R^f\right) \omega_c \left(\frac{p^c}{p^{m,c}} \right)^{\eta_c}}.$$

– use the latter to substitute out for c in the current account, (30):

$$
\begin{aligned}
& a^f + p^c q \left(1 - \psi^f + \psi^f R^f\right) \omega_c \left(\frac{p^c}{p^{m,c}}\right)^{\eta_c} \frac{p^c q p^x x + \frac{sR^f a^f}{\bar{\pi}} - a^f}{p^c q \left(1 - \psi^f + \psi^f R^f\right) \omega_c \left(\frac{p^c}{p^{m,c}}\right)^{\eta_c}} \\
= \; & p^c q p^x x + \frac{sR^f a^f}{\bar{\pi}},
\end{aligned}
$$

which can be solved linearly for x.

– evaluate (31) and adjust $\tilde{\varphi}$ until it is satis ed. In practice, we set $\tilde{\varphi} = 1$ and used (31) to de ne y^f.

- Uncovered interest rate parity puzzle and Φ_t^b
  - subtract equations (17) and (18):
$$E_t\left[\frac{R_t - s_{t+1}R_t^f\Phi_t}{c_{t+1}\pi_{t+1}^c\exp\left(\Delta a_{t+1}\right)}\right] = 0. \tag{34}$$
  - totally differentiate the object in square brackets, and evaluate in steady state

$$d\frac{R_t - s_{t+1}R_t^f\Phi_t}{c_{t+1}\pi_{t+1}^c\exp\left(\Delta a_{t+1}\right)} = \frac{dR_t}{c\pi^c} - \frac{1}{c\pi^c}\left[sR^f d\Phi_t + sdR_t^f + R^f ds_{t+1}\right]$$
$$-\frac{R - sR^f}{\left[c\pi^c\right]^2}d\left[c_{t+1}\pi_{t+1}^c\exp\left(\Delta a_{t+1}\right)\right],$$

so that, taking into account (33), (34) is, to a rst approximation:

$$\hat{R}_t = E_t\hat{s}_{t+1} + \hat{R}_t^f + \hat{\Phi}_t, \qquad \hat{x}_t = \log\left(x_t\right) - \log\left(x\right) = \frac{x_t - x}{x}.$$

– Note:

$$\hat{R}_t = \log R_t - \log(R) \simeq r_t - \log R, \qquad \hat{R}_t^f = \log R_t^f - \log\left(R^f\right) \simeq r_t^f - \log R^f$$
$$R_t \equiv 1 + r_t,\ R_t^f \equiv 1 + r_t^f,$$

so that:

$$r_t - \log(R) = \log S_{t+1} - \log S_t - \log s + r_t^f - \log R^f + \hat{\Phi}_t.$$

It follows from:

$$\log(R) - \log s - \log R^f = \log\left(\frac{R}{sR^f}\right) = 0,$$

that

$$\boxed{r_t = E_t \log S_{t+1} - \log S_t + r_t^f + \hat{\Phi}_t} \tag{35}$$

$$\hat{\Phi}_t = \log \Phi_t = -\tilde{\phi}_a\left(a_t^f - \bar{a}\right) - \tilde{\phi}_s\left(r_t^f - r_t - \left(r^f - r\right)\right) + \tilde{\phi}_t$$

which is our log-linear expansion of (34).

Extension to Small Open Economy ...

– Uncovered Interest Parity (UIP)

  * Under UIP, $\hat{\Phi}_t \equiv 0$ and

$r_t > r_t^f \rightarrow$ there must be an anticipated depreciation (instantaneous appreciation) of the currency for people to be happy holding the existing stock of net foreign assets.

  * Consider the standard 'UIP regression' ($\tilde{\phi}_a \simeq 0,\ \tilde{\phi}_t = 0$), involving risk free rate differentials:

$$\log S_{t+1} - \log S_t = \alpha + \beta \left(r_t - r_t^f \right) + u_t.$$

  * Substitute out for $\log S_{t+1} - \log S_t$ from (35) and make use of the fact that a (rational expectations) forecast error is orthogonal to date t information, to obtain:

$$\hat{\beta} = \frac{cov\left(\log S_{t+1} - \log S_t, r_t - r_t^f\right)}{var\left(r_t - r_t^f\right)} = 1 - \tilde{\phi}_s,$$

Extension to Small Open Economy ...

$*$ In data,

· $\hat{\beta} \simeq -.75$, so UIP rejected (that's the *UIP puzzle*)
· $\tilde{\phi}_s = 1.75 \rightarrow \hat{\beta} = -0.75.$

$*$ VAR impulse responses by Eichenbaum and Evans (QJE, 1992)

· data: $r_t \uparrow$ after monetary policy shock $\rightarrow \log S_{t+j}$ falls slowly for $j = 1, 2, 3, ...$.

· UIP puzzle: $r_t \uparrow$ *and* expected appreciation of the currency represents a double-boost to the return on domestic assets. On the face of it, it appears that there is an irresistible pro t opportunity. Why doesn't the double-boost to domestic returns launch an avalanche of pressure to buy the domestic currency? In standard models, this pressure produces a greater instantaneous appreciation in the exchange rate, until the familiar UIP overshooting result emerges - the pressure to buy the currency leads to such a large appreciation, that expectations of depreciation emerge. In this way, UIP leads to the counterfactual prediction that a higher r_t will be followed (after an instantaneous appreciation) by a period of time during which the currency depreciates.

· model's resolution of the UIP puzzle: when $r_t \uparrow$ the return required for people to hold domestic bonds rises. This is why the double-boost to domestic returns does not create an appetite to buy large amounts of domestic assets. Possibly this is a reduced form way to capture the notion that increases in r_t make the domestic economy more risky. (However, the precise mechanism by which the domestic required return rises - earnings on *foreign* assets go up - may be dif cult to interpret. An alternative speci cation was explored, with risk-premia affecting domestic bonds, but this resulted in indeterminacy problems.)

- Model dynamics
  - 16 equations: price setting, (1), (2),(3) and (4); monetary policy rule, (5); household intertemporal Euler equations (6), (7); relative price equations (13), (8), (9), (10), (12); aggregate resource condition, (14); current account, (15); risk term, (16); demand for exports (11).
  - 16 endogenous variables: $p_t^c, p_t^{m,c}, q_t, R_t, \bar{\pi}_t, \pi_t^c, p_t^x, N_t, p_t^*, a_t^f, \hat{\Phi}_t, s_t, x_t, c_t, K_t, F_t$.
  - exogenous variables: $R_t^f, y_t^f, \tilde{\phi}_t, g_t, \varepsilon_{R,t}, \Delta a_t, \tau_t, \pi_t^f$.
    for the purpose of numerical calculations, these were modeled as independent scalar AR(1) processes.
  - the model was solved in the manner described above:
    * compute the steady state using the formulas described above
    * log-linearize the 16 equations about steady state
    * solve the log-linearized system
    * these calculations were made easy by implementing them in Dynare.

Extension to Small Open Economy ...

- Numerical examples
- Parameter values:

$$\overbrace{\bar{\pi}^c = \pi^f}^{\text{foreign and domestic in ation same}} = 1.005,\ \overbrace{\psi = \psi^f = \nu^x = 0}^{\text{no nancial frictions}},\ \overbrace{\tilde{\phi}_a = 0.03}^{\text{small weight on } a_t^f \text{ in } \Phi_t},$$

$$\beta = 1.03^{-1/4},\ \overbrace{\theta = 3/4}^{\text{prices unchanged on average for 1 year}},\ \overbrace{\varphi = 1}^{1/\varphi \text{ Frisch elasticity}},$$

$$\overbrace{\varepsilon = 6}^{\text{modest elasticity of demand for domestic intermediate goods}},\ \overbrace{1 - \nu_t = \frac{\varepsilon - 1}{\varepsilon}}^{\text{subsidy extinguishes monopoly power in labor margin}},$$

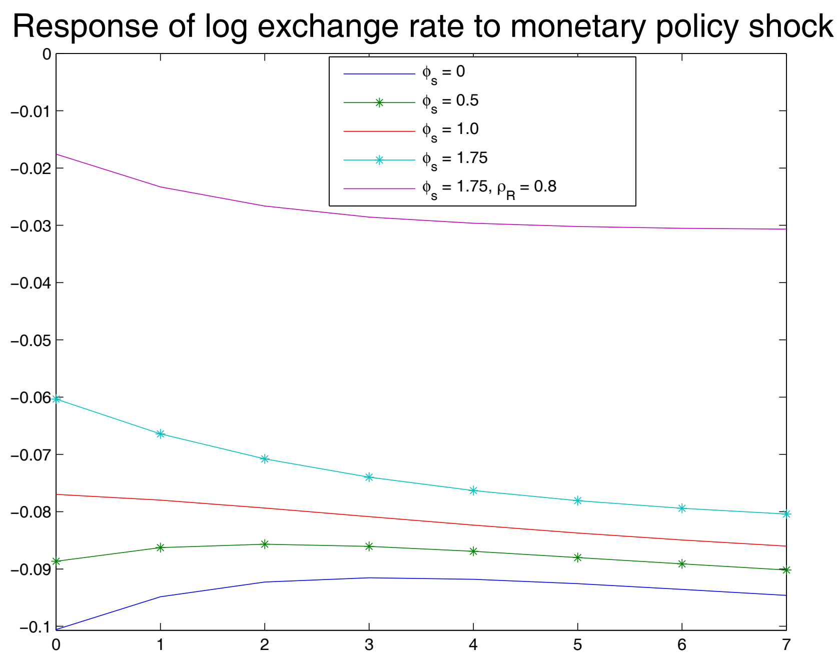
$$\overbrace{\eta_c = 5}^{\text{elasticity of substitution between domestic and foreign inputs in producing nal consumption}}$$

$$\overbrace{\omega_c = 0.4}^{\text{roughly 60\% of domestic nal consumption is composed of domestic content}},\ \overbrace{\eta_g = 0.3}^{\text{share of } g \text{ in } y},\ \overbrace{\eta_a = 0}^{\text{net foreign assets/y}},$$

$$\overbrace{\eta_f = 1.5}^{\text{elasticity of demand for exports as function of relative price paid by foreigners}},\ \overbrace{\rho_R = 0.9,\ r_\pi = 1.5, r_y = 0.15}^{\text{monetary policy rule parameters}}$$

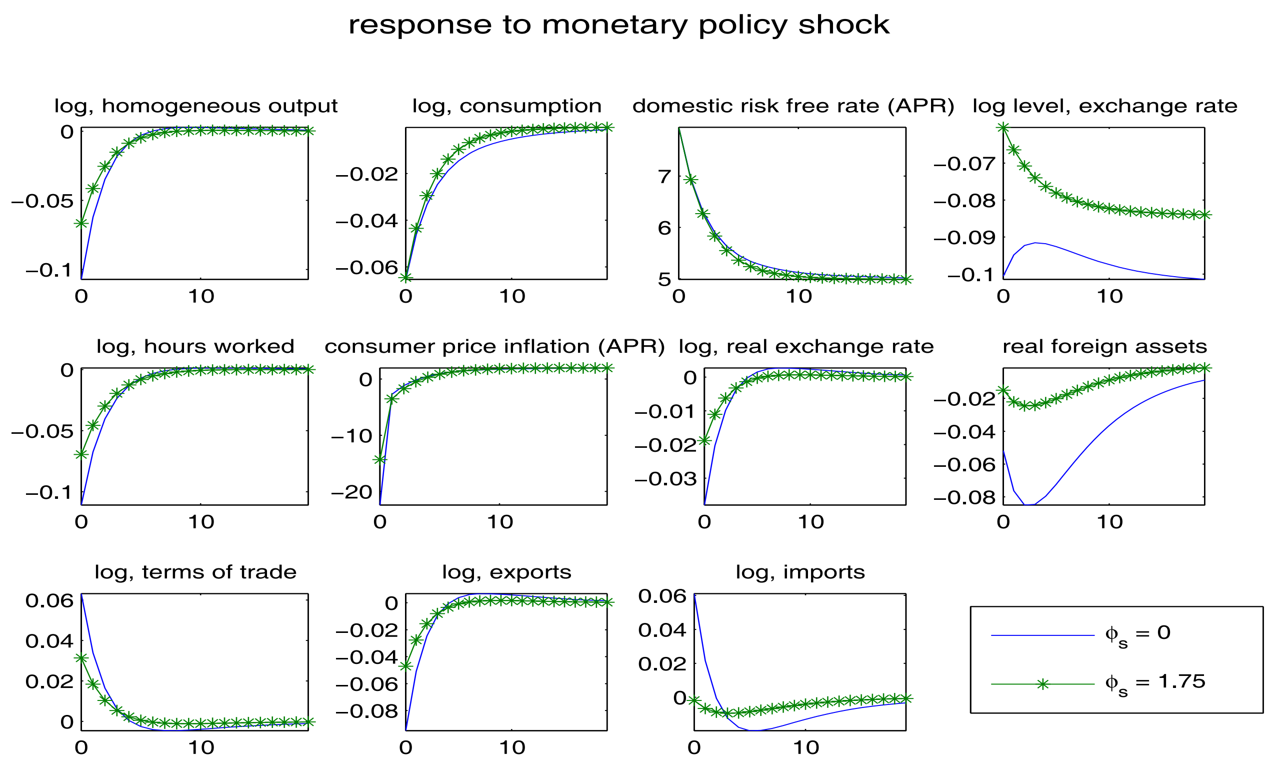
Extension to Small Open Economy ...

- iid shock, 0.01, to $\varepsilon_{R,t}$.
  - $\tilde{\phi}_s = 0$ $\rightarrow$after instantaneous appreciation, positive $\varepsilon_{R,t}$ shock followed by depreciation.
  - for higher $\tilde{\phi}_s$, shock followed by appreciation.
  - long run appreciation is increasing function of persistence of ρ_R.



Extension to Small Open Economy ...

We now consider a monetary policy shock, $\varepsilon_{R,t} = 0.01$. According to (5), implies a four percentage point (at an annual rate) policy-induced jump in R_t. The dynamic effects are displayed in the following gure, for $\tilde{\phi}_s = 0$, $\tilde{\phi}_s = 1.75$.

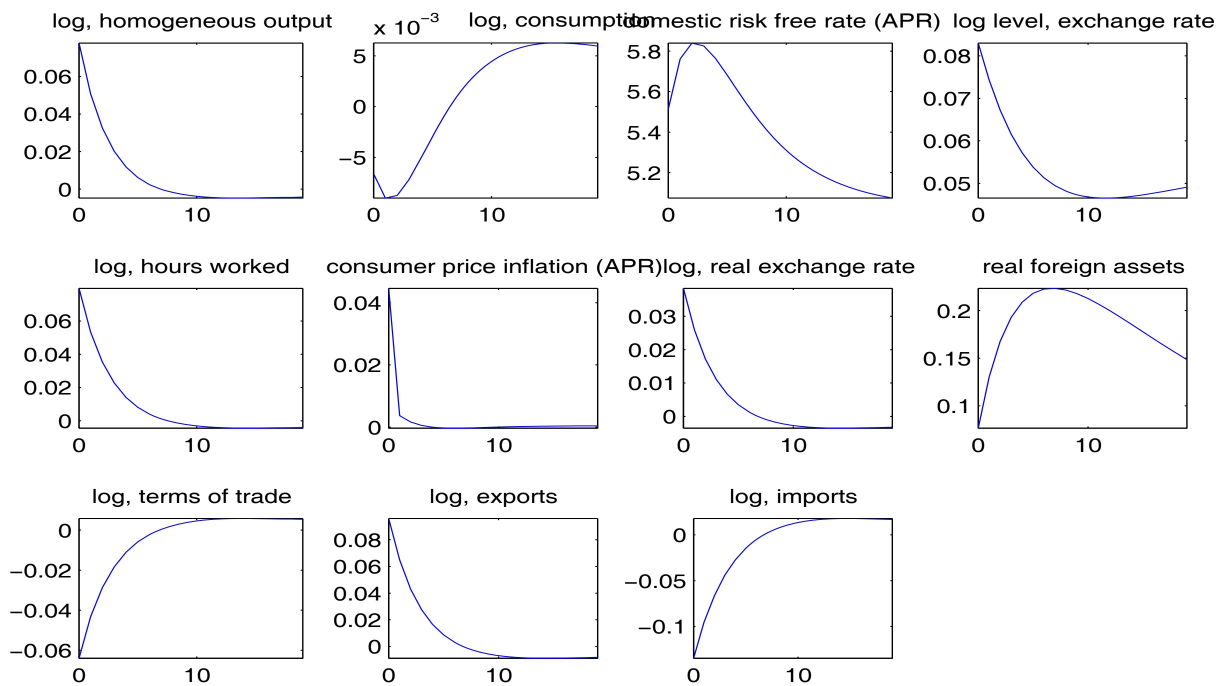


Note: (i) appreciation smaller, though a more drawn out, when $\tilde{\phi}_s$ is big; (ii) smaller appreciation results in smaller drop in net exports, so less of a drop in demand, so less fall in output and in ation; (iii) smaller drop in net exports results in smaller drop in real foreign assets.

Extension to Small Open Economy ...

Consider now a domestic economy risk premium shock, a jump in the innovation to $\tilde{\phi}_t$ equal to 0.01.

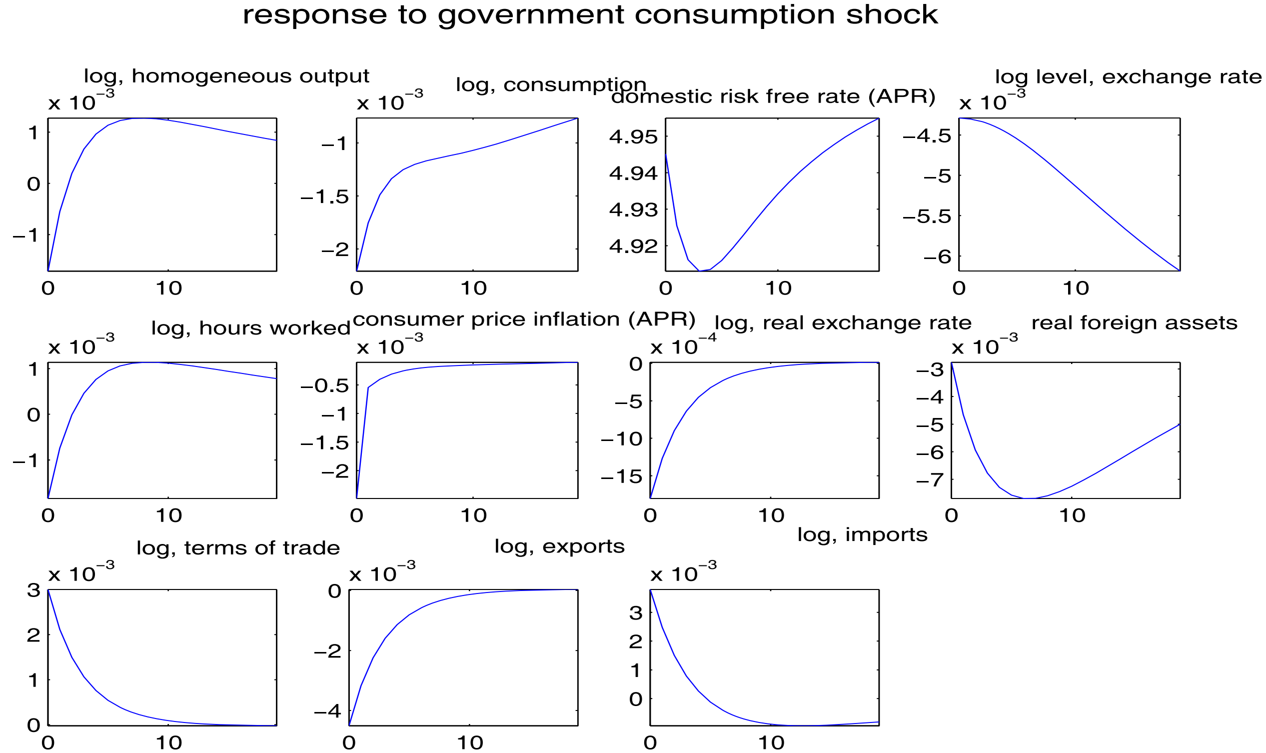
response to country risk premium shock



With the reduced interest in domestic assets, (i) the currency depreciates, (ii) net exports rise, (iii) hours and output rise, (iv) the upward pressure on costs associated with higher output leads to a rise in prices.

Extension to Small Open Economy ...

Next we consider a 0.01 innovation in log, government consumption, g_t.



After a delay, the higher g_t leads to a rise in output. However, there is so much crowding out in the short run that output actually falls. There is crowding out of net exports and consumption because of the effects created by a higher interest rate. The higher interest rate directly reduces consumption, and by making the currency appreciate, it produces a fall in net exports. The initial drop in government spending in the wake of a rise in government spending is interesting.