Simple New Keynesian Model without Capital

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What's It Good For?

- Conveying basic principles of macroeconomics -
  - Concept and measurement of *output gap*:
    - 'difference between the actual economy and where would be if policy was managed as well as possible'.
  - Importance of aggregate demand.
    - problems when it goes awry.
  - Important policy objective: assuring the right level of aggregate demand.
- Thinking through the operating characteristics of policy rules:
  - Inflation targeting, Tax/spending rules, Leverage restrictions on banks.
- Can even use it to learn econometrics
  - how well do standard econometric estimators work?
  - how good is HP filter at estimating output gap?

Our Approach to NK Model

- We will derive the familiar 'three equation NK model', but they will not be our starting point.
  - Start with households, firms, technology, etc....
- Necessary to build the model from scratch -
  - need this to uncover the principles hiding inside it
  - needed to know how to 'go back to the drawing board' and modify the model so it can address interesting questions:
    - how should macro prudential policy be conducted?
    - how might currency mismatch problems affect the usual transmission of exchange rate depreciation to the economy?
    - what should the role of inflation, labor markets, credit growth, stock markets, etc., be in monetary policy?
    - how does an expansion of unemployment benefits in a recession affect the business cycle?

Households

- Problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \; \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}$$

$$\text{s.t. } P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + \text{Profits net of taxes}_t$$

- First order conditions:

$$\begin{aligned} \frac{1}{C_t} &= \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5) \\ \exp(\tau_t) C_t N_t^{\varphi} &= \frac{W_t}{P_t}. \end{aligned}$$

Goods Production

- A homogeneous final good is produced using the following (Dixit-Stiglitz) production function:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon - 1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon - 1}}.$$

- Each intermediate good, $Y_{i,t}$, is produced as follows:

$$Y_{i,t} = \overbrace{A_t}^{=\exp(a_t)} N_{i,t},\ a_t = \rho a_{t-1} + \varepsilon_t^a$$

- Before discussing the firms that operate these production functions, we briefly investigate the socially efficient ('First Best') allocation of labor across i, for given N_t :

$$N_t = \int_0^1 N_{it} di$$

Efficient Sectoral Allocation of Labor

- With Dixit-Stiglitz final good production function, there is a socially optimal allocation of resources to all the intermediate activities, $Y_{i,t}$
  - It is optimal to run them all at the same rate, *i.e.*, $Y_{i,t} = Y_{j,t}$ for all $i, j \in [0, 1]$.
- For given N_t, it is optimal to set $N_{i,t} = N_{j,t}$ for all $i, j \in [0, 1]$
- In this case, final output is given by

$$Y_t = e^{a_t} N_t.$$

- Best way to see this is to suppose that labor is *not* allocated equally to all activities.
  - But, this can happen in a million different ways when there is a continuum of inputs!
  - Explore one simple deviation from $N_{i,t} = N_{j,t}$ for all $i, j \in [0, 1]$.

Suppose Labor *Not* Allocated Equally

- Example:

$$N_{it} = \begin{cases} 2\alpha N_t & i \in \left[0, \frac{1}{2}\right] \\ 2(1-\alpha)N_t & i \in \left[\frac{1}{2}, 1\right] \end{cases},\ 0 \leq \alpha \leq 1.$$

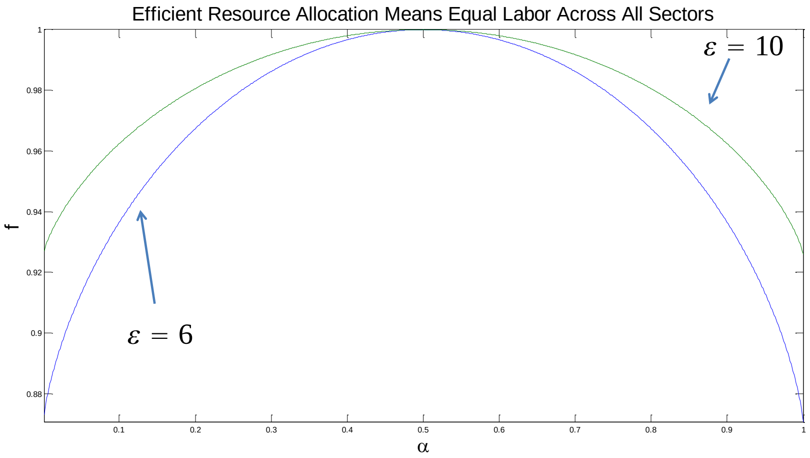
- Note that this is a particular distribution of labor across activities:

$$\int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1-\alpha) N_t = N_t$$

Labor *Not* Allocated Equally, cnt'd

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\int_0^{\frac{1}{2}} Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} (2\alpha N_t)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha)N_t)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\int_0^{\frac{1}{2}} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\frac{1}{2} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t f(\alpha)
\end{aligned}
$$

$$f(\alpha) = \left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$



Final Goods Production

- Final good firms:
  - maximize profits:

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,$$

  subject to:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} .$$

  - Foncs:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon} \rightarrow \overbrace{P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}}^{\text{"cross price restrictions"}}$$

Intermediate Goods Production

- Demand curve for i^{th} monopolist:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon}.$$

- Production function:

$$Y_{i,t} = \exp(a_t) N_{i,t},\ \Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t^a$$

- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}.$$

- Real marginal cost:

$$s_t = \frac{\frac{d\text{Cost}}{d\text{worker}}}{\frac{d\text{output}}{d\text{worker}}} = \frac{\overbrace{(1-\nu)}^{\text{minimize monopoly distortion by setting } = \frac{\varepsilon-1}{\varepsilon}} \frac{W_t}{P_t}}{\exp(a_t)}$$

Optimal Price Setting by Intermediate Goods Producers

- Let

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t},\ \bar{\pi}_t \equiv \frac{P_t}{P_{t-1}}.$$

- First order condition implied by optimal price setting:

$$\tilde{p}_t = \frac{K_t}{F_t},$$

where

$$\begin{aligned} K_t &= \frac{\varepsilon}{\varepsilon - 1} s_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} (1) \\ F_t &= 1 + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1}. (2) \end{aligned}$$

- Note:

$$\begin{aligned} K_t &= \frac{\varepsilon}{\varepsilon - 1} s_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+1} \\ &\quad + (\beta\theta)^2 E_t \bar{\pi}_{t+2}^{\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+2} + \ldots \end{aligned}$$

Price Equilibrium Conditions

- Cross-price restrictions imply, given the Calvo price-stickiness:

$$P_t = \left[(1-\theta)\, \tilde{P}_t^{(1-\varepsilon)} + \theta P_{t-1}^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}} .$$

- Dividing latter by P_t and solving for $\tilde{p}_t$:

$$\tilde{p}_t = \left[\frac{1-\theta \bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}}$$

- Combining with the first order condition for $\tilde{p}_t$:

$$\frac{K_t}{F_t} = \left[\frac{1-\theta \bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \quad (3)$$

Aggregate Inputs and Aggregate Output

- Tack Yun argument:

$$Y_t^* \equiv \int_0^1 Y_{i,t} di \ \left(= \int_0^1 e^{a_t} N_{i,t} di = e^{a_t} N_t \right)$$

$$\overbrace{=}^{\text{demand curve}} \int_0^1 Y_t P_t^{\varepsilon} P_{i,t}^{-\varepsilon} di = Y_t P_t^{\varepsilon} \overbrace{\int_0^1 P_{i,t}^{-\varepsilon} di}^{\equiv (P_t^*)^{-\varepsilon}}$$

$$\rightarrow Y_t = \overbrace{\left(\frac{P_t^*}{P_t} \right)^{\varepsilon}}^{\equiv p_t^*} e^{a_t} N_t$$

$$P_t^* = \left[(1-\theta) \tilde{P}_t^{-\varepsilon} + \theta P_{t-1}^{-\varepsilon} \right]^{\frac{1}{-\varepsilon}}$$

$$\rightarrow \quad p_t^* = \left[(1-\theta) \tilde{p}_t^{-\varepsilon} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1}$$

Goods Equilibrium Conditions

- Relationship between aggregate output and aggregate inputs:

$$C_t = p_t^* A_t N_t, \ (6)$$

where ('Tack Yun distortion')

$$p_t^* = p^* \left(\bar{\pi}_t, p_{t-1}^* \right) \equiv \left[(1-\theta) \left(\frac{1 - \theta \bar{\pi}_t^{(\varepsilon - 1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon - 1}} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \quad ($$

Tack Yun Distortion: a Closer Look

- Distortion:

$$p_t^* = p^*\left(\bar{\pi}_t, p_{t-1}^*\right) \equiv \left[(1-\theta)\left(\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta\frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1}$$

  - Distortion, p_t^*, increasing function of lagged distortion, p_{t-1}^*.
  - Current shocks affect current distortion via $\bar{\pi}_t$ only.

- Derivatives:

$$\begin{aligned} p_1^*\left(\bar{\pi}_t, p_{t-1}^*\right) &= -\left(p_t^*\right)^2 \varepsilon\theta\bar{\pi}_t^{\varepsilon-2}\left[\frac{\bar{\pi}_t}{p_{t-1}^*} - \left(\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}\right] \\ p_2^*\left(\bar{\pi}_t, p_{t-1}^*\right) &= \left(\frac{p_t^*}{p_{t-1}^*}\right)^2 \theta\bar{\pi}_t^{\varepsilon}. \end{aligned}$$

Linear Expansion of Tack Yun Distortion in Undistorted Steady State

- Linearizing about $\bar{\pi}_t = \bar{\pi}$, $p_{t-1}^* = p^*$:

$$dp_t^* = p_1^*\left(\bar{\pi}, p^*\right) d\bar{\pi}_t + p_2^*\left(\bar{\pi}, p^*\right) dp_{t-1}^*,$$

where $dx_t \equiv x_t - x$, for $x_t = p_t^*, p_{t-1}^*, \bar{\pi}_t$.

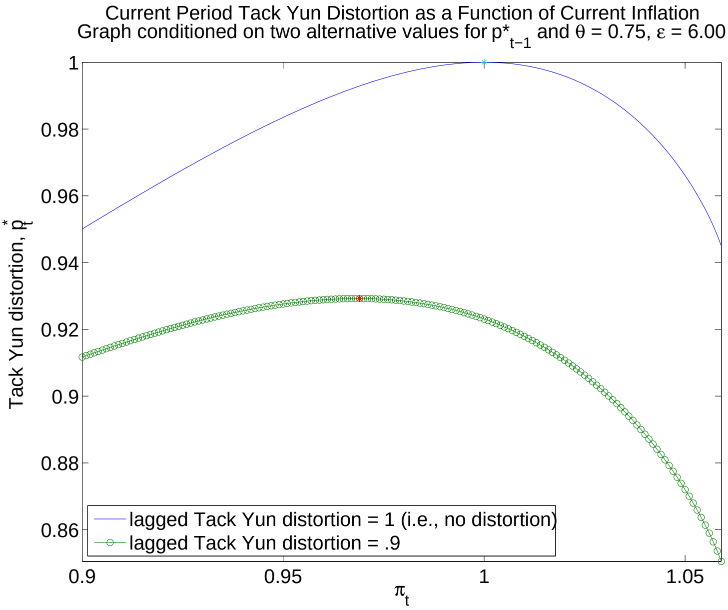
- In an undistorted steady state (i.e., $\bar{\pi}_t = p_t^* = p_{t-1}^* = 1$) :

$$p_1^*\left(1,1\right) = 0,\; p_2^*\left(1,1\right) = \theta.$$

so that

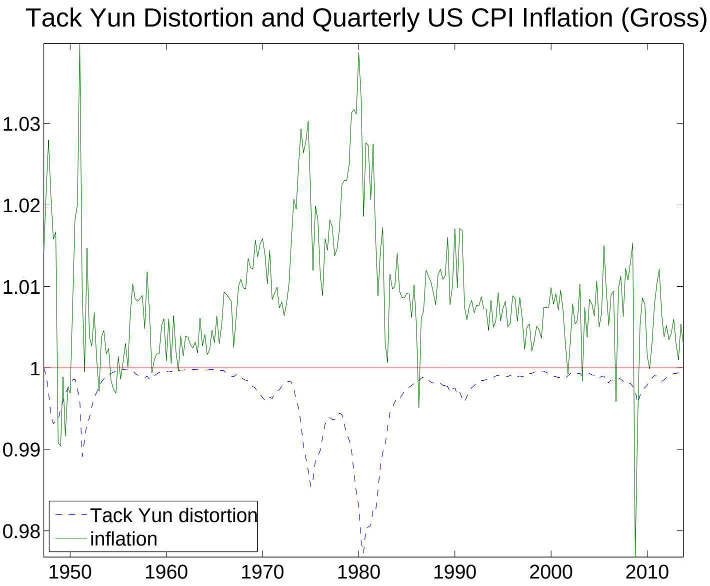
$$\begin{aligned} dp_t^* &= 0 \times d\bar{\pi}_t + \theta dp_{t-1}^* \\ &\to p_t^* = 1 - \theta + \theta p_{t-1}^* \end{aligned}$$

- Often, people that linearize NK model ignore p_t^*.
  - Reflects that they linearize the model around a price-undistorted steady state.



Current Period Tack Yun Distortion as a Function of Current Inflation
Graph conditioned on two alternative values for p^*_{t-1} and θ = 0.75, ε = 6.00
Tack Yun distortion, p^*_t
π_t
1
0.98
0.96
0.94
0.92
0.9
0.88
0.86
0.9
0.95
1
1.05
lagged Tack Yun distortion = 1 (i.e., no distortion)
lagged Tack Yun distortion = .9

Ignoring Tack Yun Distortion, a Mistake?



First Best Consumption, Employment

- 'First Best Consumption and Employment' useful concepts in simple NK model.
  - Ramsey is the appropriate benchmark, but Ramsey and first best coincide in simple NK model.
- Explained above that with socially efficient sectoral allocation of labor,

$$Y_t = \exp(a_t) N_t.$$

- First best level of employment and consumption is solution to

$$N_t^{\text{best}} = \arg\max_N \left\{ \log[\exp(a_t) N] - \exp(\tau_t) \frac{N^{1+\varphi}}{1+\varphi} \right\}$$

so,

$$N_t^{\text{best}} = \exp\left(-\frac{\tau_t}{1+\varphi}\right),\ C_t^{\text{best}} = \exp\left(a_t - \frac{\tau_t}{1+\varphi}\right)$$

Linearizing around Efficient Steady State

- In steady state (assuming $\bar{\pi} = 1$, $1 - \nu = \frac{\varepsilon - 1}{\varepsilon}$)

$$p^* = 1,\ K = F = \frac{1}{1 - \beta\theta},\ s = \frac{\varepsilon - 1}{\varepsilon},\ \Delta a = \tau = 0,\ N = 1$$

- Linearizing the Tack Yun distortion, (4):

$$\boxed{p_t^* = 1,\ t \text{ large enough}}$$

- Denote the *output gap* in ratio form by X_t:

$$X_t \equiv \frac{C_t}{\exp\left(a_t - \frac{\tau_t}{1+\varphi}\right)} = p_t^* N_t \exp\left(\frac{\tau_t}{1+\varphi}\right),$$

  where the denominator is the socially efficient ('First Best') level of consumption.

- Then, with $x_t \equiv \hat{X}_t$ and $\hat{p}_t^* = 0$:

$$\boxed{x_t = \hat{N}_t + \frac{d\tau_t}{1+\varphi}}$$

NK IS Curve, Baseline Model

- The intertemporal Euler equation, (5), after substituting for C_t in terms of X_t:

$$\frac{1}{X_t \exp\left(a_t - \frac{\tau_t}{1+\varphi}\right)} = \beta E_t \frac{1}{X_{t+1} \exp\left(a_{t+1} - \frac{\tau_{t+1}}{1+\varphi}\right)} \frac{R_t}{\bar{\pi}_{t+1}}$$

$$\frac{1}{X_t} = E_t \frac{1}{X_{t+1} R^*_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}},$$

where

$$\overbrace{R^*_{t+1}}^{\text{value of } R_t \text{ in 'first best'}} \equiv \frac{1}{\beta} \exp\left(a_{t+1} - a_t - \frac{\tau_{t+1} - \tau_t}{1+\varphi}\right)$$

then, (using $\widehat{z_t u_t} = \hat{z}_t + \hat{u}_t$, $\widehat{\left(\frac{u_t}{z_t}\right)} = \hat{u}_t - \hat{z}_t$):

$$\hat{X}_t = E_t \left[\hat{X}_{t+1} - \left(\hat{R}_t - \widehat{\bar{\pi}}_{t+1} - \hat{R}^*_{t+1}\right)\right]$$

NK IS Curve, Baseline Model

- Note:

$$\begin{aligned} Z_t &= \exp(z_t), \text{ where } z_t \equiv \log Z_t \\ \hat{Z}_t &\equiv \frac{dZ_t}{Z} = \frac{d\exp(z_t)}{Z} = \frac{Zdz_t}{Z} = dz_t = \log Z_t - \log Z. \end{aligned}$$

- Use this to establish, when the steady state is efficient:

$$\begin{aligned} & E_t\left(\hat{R}_t - \hat{\bar{\pi}}_{t+1} - \hat{R}^*_{t+1}\right) \\ = & \log R_t - E_t \pi_{t+1} - E_t \log R^*_{t+1} \\ = & r_t - E_t \pi_{t+1} - r^*_t \end{aligned}$$

where

$$r_t \equiv \log R_t,\ r^*_t \equiv E_t \log R^*_{t+1},\ \pi_{t+1} \equiv \log \bar{\pi}_{t+1},$$

- and, in efficient steady state:

$$\log R^* = \log R,\ \log \bar{\pi} = 0.$$

NK IS Curve, Baseline Model

- Substituting

$$\hat{X}_t = E_t \left[\hat{X}_{t+1} - \left(\hat{R}_t - \hat{\bar{\pi}}_{t+1} - \hat{R}^*_{t+1} \right) \right],\ x_t \equiv \hat{X}_t,$$

we obtain NK IS curve:

$$\boxed{x_t = E_t x_{t+1} - E_t \left[r_t - \pi_{t+1} - r^*_t \right]}$$

- Also,

$$r^*_t = -\log(\beta) + E_t \left[a_{t+1} - a_t - \frac{\tau_{t+1} - \tau_t}{1 + \varphi} \right].$$

Linearized Marginal Cost in Baseline Model

- Marginal cost (using $da_t = a_t$, $d\tau_t = \tau_t$ because $a = \tau = 0$):

$$
\begin{aligned}
s_t &= (1-\nu)\frac{\bar{w}_t}{A_t},\ \bar{w}_t = \exp(\tau_t) N_t^{\varphi} C_t \\
&\rightarrow \widehat{\bar{w}}_t = \tau_t + a_t + (1+\varphi)\hat{N}_t
\end{aligned}
$$

- Then,

$$
\boxed{\hat{s}_t = \widehat{\bar{w}}_t - a_t = (\varphi+1)\left[\tfrac{\tau_t}{\varphi+1} + \hat{N}_t\right] = (\varphi+1)\,x_t}
$$

Linearized Phillips Curve in Baseline Model

- Log-linearize equilibrium conditions, (1)-(3), around steady state:

$$\boxed{\begin{array}{l} \hat{K}_t = (1-\beta\theta)\,\hat{s}_t + \beta\theta\left(\varepsilon\widehat{\bar{\pi}}_{t+1} + \hat{K}_{t+1}\right)\ (1) \\ \hline \hat{F}_t = \beta\theta\,(\varepsilon-1)\,\widehat{\bar{\pi}}_{t+1} + \beta\theta\hat{F}_{t+1}\ (2) \\ \hline \hat{K}_t = \hat{F}_t + \frac{\theta}{1-\theta}\widehat{\bar{\pi}}_t\ (3) \end{array}}$$

- Substitute (3) into (1)

$$\hat{F}_t + \frac{\theta}{1-\theta}\hat{\pi}_t = (1-\beta\theta)\,\hat{s}_t + \beta\theta\left(\varepsilon\widehat{\bar{\pi}}_{t+1} + \hat{F}_{t+1} + \frac{\theta}{1-\theta}\hat{\pi}_{t+1}\right)$$

- Simplify the latter using (2), to obtain the NK Phillips curve:

$$\boxed{\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{s}_t + \beta\pi_{t+1}}$$

Nonlinear Private Sector Equilibrium Conditions

$$
\begin{aligned}
K_t &= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} (1) \\
F_t &= 1 + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1}. (2) \\
\frac{K_t}{F_t} &= \left[\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \quad (3) \\
p_t^* &= \left[(1-\theta)\left(\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta\frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1} \quad (4) \\
\frac{1}{C_t} &= \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \ (5) \\
C_t &= p_t^* A_t N_t. \ (6)
\end{aligned}
$$

The Linearized Private Sector Equilibrium Conditions

$x_t = E_t x_{t+1} - \left[r_t - E_t \pi_{t+1} - r_t^*\right]$
$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta} \hat{s}_t + \beta E_t \pi_{t+1}$
$\hat{s}_t = (\varphi + 1) x_t$
$r_t^* = -\log(\beta) + E_t \left[a_{t+1} - a_t - \frac{\tau_{t+1} - \tau_t}{1+\varphi}\right]$

Monetary policy rule:

$$r_t = \alpha r_{t-1} + (1 - \alpha) \left[r + \phi_\pi \pi_t + \phi_x x_t\right]$$

Solving the Model

- Vision about evolution of actual data:
  - Nature draws the exogenous shocks.
  - The economy transforms exogenous shocks into realization of endogenous variables, inflation, output, unemployment, etc.
- 'Solving the model':
  - Determining the model's implications for the mapping from exogenous shocks to endogenous variables.
  - Potentially massive problem: current value of endogenous variables a function of past data and *expected future value of endogenous variables*.
- Primary strategy for solving a model:
  - Find a linear representation ('policy rule') of the endogenous variables, z_t, in terms of current and past data only:

    $$z_t = Az_{t-1} + Bs_t$$

    such that the equilibrium conditions (after linearization) are satisfied.

- Exogenous shocks:

$$s_t = \begin{pmatrix} \Delta a_t \\ \tau_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \lambda \end{bmatrix} \begin{pmatrix} \Delta a_{t-1} \\ \tau_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^a \\ \varepsilon_t^\tau \end{pmatrix}$$

$$s_t = P s_{t-1} + \epsilon_t$$

- Equilibrium Conditions:

$$\begin{bmatrix} \beta & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \\ r_{t+1} \\ r_{t+1}^* \end{pmatrix} + \begin{bmatrix} -1 & \frac{(1-\theta)(1-\beta\theta)}{\theta}(1+\varphi) & 0 & 0 \\ 0 & -1 & -1 & 1 \\ (1-\alpha)\phi_\pi & (1-\alpha)\phi_x & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ r_t^* \end{pmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \\ r_{t-1} \\ r_{t-1}^* \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & \frac{1}{1+\varphi} \end{pmatrix} s_{t+1} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{1+\varphi} \end{pmatrix} s_t$$

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

- Collecting:

$$\begin{aligned} E_t\left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t\right] &= 0 \\ s_t - P s_{t-1} - \epsilon_t &= 0. \end{aligned}$$

- Policy rule:

$$z_t = A z_{t-1} + B s_t$$

- As before, want A such that

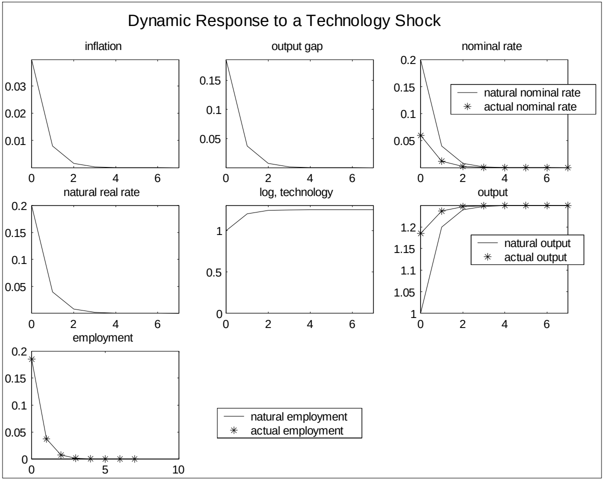
$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0,$$

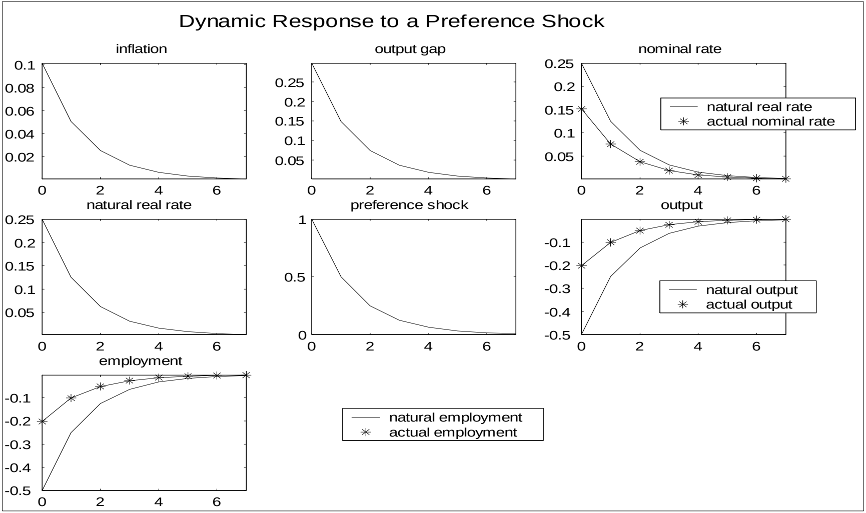
- Want B such that:

$$(\beta_0 + \alpha_0 B)P + [\beta_1 + (\alpha_0 A + \alpha_1)B] = 0$$

- Note: if $\alpha = 0$, then $A = 0$ is one solution (there is another one!).

$\phi_x = 0,\ \phi_\pi = 1.5,\ \beta = 0.99,\ \varphi = 1,\ \rho = 0.2,\ \theta = 0.75,\ \alpha = 0,\ \delta = 0.2,\ \lambda = 0.5.$





Dynamic Response to a Preference Shock
inflation
output gap
nominal rate
natural real rate
actual nominal rate
natural real rate
preference shock
output
natural output
actual output
employment
natural employment
actual employment