## Leverage Restrictions in a Business Cycle Model

Lawrence J. Christiano Northwestern University summary of work with: Daisuke Ikeda, Bank of Japan

## Background

- Wish to address the following sorts of questions:
- What restrictions should be placed on bank borrowing?
- How should those restrictions be varied over the business cycle?
- Want an environment with the following properties:
- model includes problem that restrictions on bank borrowing are supposed to solve.
- if you want an interesting model to help think about how many umbrellas to build, there had better be rain in the model.
- riskiness of banks varies over time.

FRED ${ }_{\sim}^{w}$


Source: Federal Reserve Bank of St. Louis
Shaded areas indicate US recessions - 2015 research.stlouisfed.org

## What We Do

- Modify a standard medium-sized DSGE model to include a banking sector.

| Assets | Liabilities |
| :--- | :--- |
| Loans and other securities | Deposits |
|  | Banker net worth |

- Job of bankers is to identify and finance good investment projects.
- doing this requires exerting costly effort.
- Agency problem between bank and its creditors:
- banker effort is not observable.
- Consequence: borrowing restrictions on banks may generate substantial welfare gains.


## Outline

- Model
- first, without borrowing restriction
- observable effort benchmark
- unobservable case
- potential welfare gains of borrowing restriction
- Dynamics




## Standard Model with Banking



## Standard Model with Banking



## Standard Model with Banking



## Entrepreneurs

- bad entrepreneur: 1 unit, raw capital $\rightarrow e^{b_{t}}$ units, effective capital
- good entrepreneur: 1 unit, raw capital $\rightarrow e^{g_{t}}>e^{b_{t}}$ units, effective capital
- return to capital enjoyed by entrepreneurs:

$$
\begin{gathered}
R_{t+1}^{g}=e^{g_{t}} R_{t+1}^{k}, R_{t+1}^{b}=e^{b_{t}} R_{t+1}^{k} \\
R_{t+1}^{k} \equiv \frac{r_{t+1}^{k} P_{t+1}+(1-\delta) P_{k^{\prime}, t+1}}{P_{k^{\prime} t}}
\end{gathered}
$$

## Bankers

- each has net worth, $N_{t}$.
- a banker can only invest in one entrepreneur (asset side of banker balance sheet is risky).
- by exerting effort, $e_{t}$, a banker finds a good entrepreneur with probability $p$ :

$$
p\left(e_{t}\right)=\bar{a}+\bar{b} e_{t}
$$

- in $t$, bankers seek to optimize:

$$
\begin{aligned}
& E_{t} \lambda_{t+1}\left\{p\left(e_{t}\right)\left[R_{t+1}^{g}\left(N_{t}+d_{t}\right)-R_{d, t+1}^{g} d_{t}\right]\right. \\
& \left.+\left(1-p\left(e_{t}\right)\right)\left[R_{t+1}^{b}\left(N_{t}+d_{t}\right)-R_{d, t+1}^{b} d_{t}\right]\right\}-\frac{1}{2} e_{t}^{2}
\end{aligned}
$$

- Bankers have a cash constraint:

$$
R_{t+1}^{b}\left(N_{t}+d_{t}\right) \geq R_{d, t+1}^{b} d_{t}
$$

## Bankers and their Creditors

- Bankers and Mutual Funds interact in competitive markets for loan contracts:

$$
\left(d_{t}, e_{t}, R_{d, t+1}^{g}, R_{d, t+1}^{b}\right)
$$

- Free entry and competition among mutual funds implies:

$$
p\left(e_{t}\right) R_{d, t+1}^{g}+\left(1-p\left(e_{t}\right)\right) R_{d, t+1}^{b}=R_{t}
$$

- Two scenarios:
- banker effort, $e_{t}$, is observed by mutual fund
- banker effort, $e_{t}$, is unobserved.


## Observed Effort Benchmark

- Set of contracts available to bankers is the $\left(d_{t}, e_{t}, R_{d, t+1}^{g}, R_{d, t+1}^{b}\right)$ 's that satisfy

MF zero profits: $\quad p\left(e_{t}\right) R_{d, t+1}^{g}+\left(1-p\left(e_{t}\right)\right) R_{d, t+1}^{b}=R_{t}$,
cash constraint: $\quad R_{t+1}^{b}\left(N_{t}+d_{t}\right) \geq R_{d, t+1}^{b} d_{t}$

- Each banker chooses the most preferred contract from the menu.
- Key feature of observed effort equilibrium:

$$
e_{t}=E_{t} \lambda_{t+1} p^{\prime}\left(e_{t}\right)\left(R_{t+1}^{g}-R_{t+1}^{b}\right)\left(N_{t}+d_{t}\right)
$$

## Unobserved Effort

- In this case, banker always sets $e_{t}$ to its privately optimal level, whatever $e_{t}$ is specified in the loan contract:
incentive: $\quad e_{t}=E_{t} \lambda_{t+1} p^{\prime}\left(e_{t}\right)\left[\left(R_{t+1}^{g}-R_{t+1}^{b}\right)\left(N_{t}+d_{t}\right)\right.$

$$
\left.-\left(R_{d, t+1}^{g}-R_{d, t+1}^{b}\right) d_{t}\right]
$$

- Set of contracts available to bankers is the $\left(d_{t}, e_{t}, R_{d, t+1}^{g}, R_{d, t+1}^{b}\right)$ 's that satisfy 'incentive' in addition to:

MF zero profits: $\quad p\left(e_{t}\right) R_{d, t+1}^{g}+\left(1-p\left(e_{t}\right)\right) R_{d, t+1}^{b}=R_{t}$, cash constraint: $\quad R_{t+1}^{b}\left(N_{t}+d_{t}\right) \geq R_{d, t+1}^{b} d_{t}$

- One factor that can make $e_{t}$ inefficiently low:
$-R_{d, t+1}^{g}>R_{d, t+1}^{b}$.


## Source of Inefficiency in Unobserved Effort Model

- The presence of a market interest rate in the incentive constraint creates a 'pecuniary externality'.
- Basic idea:
- Private cost to bank of higher funds, $d$ :
- interest paid on deposits, R.
- Social cost of higher $d$ :
- $R$ plus damage to bank incentives when $R$ rises with bigger $d$.
- Consequence: equilibrium $d$ may be too high, in which case limit on $d$ is desirable.
- Most straightforward to see in a simple two-period setting.
- Grateful to Saki Bigio and Emmanuel Fahri for bringing following argument to our attention.


## Desirability of Borrowing Restrictions in Two Period Version of Model

- Bankers and workers live in large, identical households.
- as in Gertler-Karadi, Gertler-Kiyotaki.
- Representative household's problem:

$$
\begin{aligned}
& \max _{\left\{c_{0}, c_{1}, d\right\}} u\left(c_{0}\right)+c_{1}, \\
& \text { s.t. } \quad c_{0}+d= \\
& y, c_{1}=R d+\pi
\end{aligned}
$$

where $\pi$ denotes the profits brought home by bankers:

$$
\pi=p(e)\left[R^{g}(N+d)-R_{d}^{g} d\right]+(1-p(e))\left[R^{b}(N+d)-R_{d}^{b} d\right]
$$

Optimality condition for deposits:

$$
R=u^{\prime}(y-d)
$$

## Desirability of Borrowing Restrictions in Two Period Version of Model

- Banker problem (with potentially binding borrowing restriction):

$$
\begin{aligned}
& \max _{\left\{R_{d}^{g}, R_{d}^{b}, d, e\right\}} p(e)\left[R^{g}(N+d)-R_{d}^{g} d\right] \\
& +(1-p(e))\left[R^{b}(N+d)-R_{d}^{b} d\right]-\frac{1}{2} e^{2} \\
& + \\
& +v\left[R^{b}(N+d)-R_{d}^{b} d\right] \\
& +\eta\left\{p^{\prime}(e)\left[\left(R^{g}-R^{b}\right)(N+d)-\left(R_{d}^{g}-R_{d}^{b}\right) d\right]-e\right\} \\
& +
\end{aligned} \mu(\bar{d}-d)^{\text {( }-d)}
$$

subject to zero profit condition on loan contract.

## Desirability of Borrowing Restrictions in Two Period Version of Model

- Use zero profit condition and binding cash constraint to simplify banker problem (drop $\left.R_{d}^{g}, R_{d}^{b}\right)$ :

$$
\begin{aligned}
& \max _{\{d, e\}} p(e) R^{g}(N+d)+(1-p(e)) R^{b}(N+d)-R d-\frac{1}{2} e^{2} \\
& +\eta\left\{p^{\prime}(e)\left[\left(R^{g}-R^{b}\right)(N+d)-\frac{R d-R^{b}(N+d)}{p(e)}\right]-e\right\} \\
& +\mu(\bar{d}-d)
\end{aligned}
$$

- Optimality condition for $d$ :

$$
p(e) R^{g}+(1-p(e)) R^{b}=R+\frac{\mu}{1+\eta p^{\prime}(e) / p(e)}
$$

borrowing restriction raises cost of funds above $R$.

## Ramsey Problem in Two Period Version of Model

- After substituting out zero profit condition, cash constraint and deposit supply

$$
\begin{aligned}
& \max _{e, d}\{\overbrace{u(y-d)}^{\text {utility of period } 0 \text { consumption }}+c_{1}-\frac{1}{2} e^{2} \\
& +\eta\left(p^{\prime}(e)\left[\left(R^{g}-R^{b}\right)(N+d)-\frac{u^{\prime}(y-d) d-R^{b}(N+d)}{p(e)}\right]-e\right)
\end{aligned}
$$

s.t.
$c_{1}=u^{\prime}(y-d) d+p(e) R^{g}(N+d)+(1-p(e)) R^{b}(N+d)-u^{\prime}(y-d) d$

## Ramsey Problem in Two Period Version of Model

- Optimality condition for $d$ :

$$
\begin{aligned}
& p(e) R^{g}+(1-p(e)) R^{b} \\
= & R+\overbrace{\frac{\eta p^{\prime}(e) / p(e)\left(-u^{\prime \prime}(y-d)\right) d}{1+\eta p^{\prime}(e) / p(e)}}^{\text {extra marginal cost associated with extra } d} .
\end{aligned}
$$

- To get the private $d$ decision to coincide with Ramsey-optimal decision, must choose $\bar{d}$ so that multiplier, $\mu$, on private problem satisfies:

$$
\mu=\eta p^{\prime}(e) / p(e)\left(-u^{\prime \prime}(y-d)\right) d>0
$$

## Back to Dynamic Model

- Model dynamics requires law of motion for banker net worth.
- Introduces an additional borrowing consideration.


## Law of Motion of Net Worth

- Bankers live in a large representative household, with workers.
- Bankers pool their net worth at the end of each period (we avoid worrying about banker heterogeneity)
- Law of motion of banker net worth

$$
\begin{aligned}
N_{t+1}= & \gamma_{t+1}\{p\left(e_{t}\right) \overbrace{\left[R_{t+1}^{g}\left(N_{t}+d_{t}\right)-R_{d, t+1}^{g} d_{t}\right]}^{\text {profits of banks with good assets }} \\
& +\left(1-p\left(e_{t}\right)\right) \overbrace{\left[R_{t+1}^{b}\left(N_{t}+d_{t}\right)-R_{d, t+1}^{b} d_{t}\right]}^{\text {profits of banks with bad assets }}
\end{aligned}
$$

lump sum transfer, households to their bankers

## Model Assumption that Banks Don't

 Systematically Rely on Equity Issues to Finance Assets- Evidence from two sources provide support for this assumption as a description of the data.
- Adrian and Shin's examination of the assets and liabilities of two large French financial firms.
- US flow of funds data on assets and liabilities of financial corporations.
- Adrian and Shin, 'Procyclical Leverage and Value-at-Risk'
- Changes in financial firm equity not systematically related to their assets.
- Changes in financial firm debt moves one-for-one with changes in assets.

Material taken from the work of Adrian Shin.
Displays a scatter plot change in equity and debt on the horizontal axis against change in assets on the horizontal axis. Note that the slope of changes in debt against changes in assets is essentially unity, while the slope of changes in equity against changes in assets has a slope of zero.
The results are consistent with the notion that this financial company headquartered in Paris finances changes in assets with changes in debt and not changes in equity.
BNP Paribas: annual change in assets, equity and debt
(1999-2010)


Figure 3. BNP Paribas: annual change in assets, equity and debt (1999-2010) (Source: Bankscope)

Societe Generale: annual changes in assets, equity and debt
(1999-2010)


Figure 4. Société Générale: annual change in assets, equity and debt (1999-2010) (Source: Bankscope)

- The model assumes that when bankers want funds, issuing equity is not an option.


This shows how major debt instruments were used at private depository institutions in the wake of the crisis.

- The model assumes that when bankers want funds, issuing equity is not an option.


Equity as a source of funds, Private Depository Institutions (F.109, F of F)


## 'Crisis'

- Suppose something makes banker net worth, $N_{t}$, drop.
- For given $d_{t}$, bank cash constraint gets tighter:

$$
R_{t+1}^{b}\left(N_{t}+d_{t}\right) \geq R_{d, t+1}^{b} d_{t} .
$$

- So, $R_{d, t+1}^{b}$ has to be low
- when $N_{t}$ is low, banks with bad assets cannot cover their own losses and creditors must share in losses.
- then, creditors require $R_{d, t+1}^{g}$ high
- So, interest rate spread, $R_{d, t+1}^{g}-R_{t}$, high, banker effort low.
- Banks get riskier (cross sectional mean return down, standard deviation up).


## Endogenous Risk

- Rate of return on equity, good banks and bad banks:
$p\left(e_{t}\right)$ good banks : $\frac{R_{t+1}^{g}\left(N_{t}+d_{t}\right)-R_{d, t+1}^{g} d_{t}}{N_{t}}$,

$$
1-p\left(e_{t}\right) \text { bad banks : } \frac{R_{t+1}^{b}\left(N_{t}+d_{t}\right)-R_{d, t+1}^{b} d_{t}}{N_{t}}=0
$$

- Mean, $E_{t+1}^{b}$, and cross sectional standard deviation, $s_{t+1}^{b}$, of return on equity across banks:

$$
\begin{aligned}
s_{t+1}^{b} & =\left[p\left(e_{t}\right)\left(1-p\left(e_{t}\right)\right)\right]^{1 / 2} \frac{R_{t+1}^{g}\left(N_{t}+d_{t}\right)-R_{d, t+1}^{g} d_{t}}{N_{t}} \\
E_{t+1}^{b} & =p\left(e_{t}\right) \frac{R_{t+1}^{g}\left(N_{t}+d_{t}\right)-R_{d, t+1}^{g} d_{t}}{N_{t}}
\end{aligned}
$$

- In a crisis, risk rises and mean return falls.


## Macro Model

- Sticky wages and prices
- Investment adjustment costs
- Habit persistence in consumption
- Monetary policy rule


## Calibration targets

| Table 2: Steady state calibration targets for baseline model |  |  |  |
| :--- | :--- | :--- | :---: |
| Variable meaning | variable name | magnitude |  |
| Cross-sectional standard deviation of quarterly non-financial firm equity returns | $s^{b}$ | 0.20 |  |
| Fnancial firm interest rate spreads (APR) | $400\left(R_{g}^{d}-R\right)$ | 0.60 |  |
| Financial firm leverage | $L$ | 20.00 |  |
|  |  |  |  |

## Data behind calibration targets

Figure 1: Cross-section standard deviation financial firm quarterly return on equity, HP-filtered US real GDP


## Parameter Values

| Table 1: Baseline Model Parameter Values |  |  |
| :---: | :---: | :---: |
| Meaning | Name | Value |
| Panel A: financial parameters |  |  |
| return parameter, bad entrepreneur | $b$ | -0.09 |
| return parameter, good entrepreneur | $g$ | 0.00 |
| constant, effort function | $\bar{a}$ | 0.83 |
| slope, effort function | $\bar{b}$ | 0.30 |
| lump-sum transfer from households to bankers | $\tilde{T}$ | 0.38 |
| fraction of banker net worth that stays with bankers | $\gamma$ | 0.85 |
| Panel B: Parameters that do not affect steady state |  |  |
| steady state inflation (APR) | $400(\pi-1)$ | 2.40 |
| Taylor rule weight on inflation | $\alpha_{\pi}$ | 1.50 |
| Taylor rule weight on output growth | $\alpha_{\Delta y}$ | 0.50 |
| smoothing parameter in Taylor rule | $\rho_{p}$ | 0.80 |
| curvature on investment adjustment costs | $S^{\prime \prime}$ | 5.00 |
| Calvo sticky price parameter | $\xi_{p}$ | 0.75 |
| Calvo sticky wage parameter | $\xi_{w}$ | 0.75 |
| Panel C: Nonfinancial parameters |  |  |
| steady state gdp growth (APR) | $\mu_{z^{*}}$ | 1.65 |
| steady state rate of decline in investment good price (APR) | $\Upsilon$ | 1.69 |
| capital depreciation rate | $\delta$ | 0.03 |
| production fixed cost | $\Phi$ | 0.89 |
| capital share | $\alpha$ | 0.40 |
| steady state markup, intermediate good producers | $\lambda_{f}$ | 1.20 |
| habit parameter | $b_{u}$ | 0.74 |
| household discount rate | $100\left(\beta^{-4}-1\right)$ | 0.52 |
| steady state markup, workers | $\lambda_{w}$ | 1.05 |
| Frisch labor supply elasticity | $1 / \sigma_{L}$ | 1.00 |
| weight on labor disutility | $\psi_{L}$ | 1.00 |
| steady state scaled government spending | $\tilde{g}$ | 0.89 |

## Impact of Loss of Bank Net Worth



## Borrowing Restrictions

- Banks taxed for issuing deposits $d_{t}$
- $1.2 \%$ AR (versus $3 \%$ AR on the risk free nominal rate).
- revenues redistributed back to banks in lump-sum form.
- What is the consequence of this restriction?
- With less $d_{t}$, banks with bad assets more able to cover losses
- interest rate spread falls, so banker effort rises.
- Second effect of borrowing restriction,
- borrowing restriction in effect implements collusion among bankers
- allows them to behave as monopsonists
- make profits on demand deposits....lots of profits:

$$
\left[p\left(e_{t}\right)\left(R_{t+1}^{g}-R_{d, t+1}^{g}\right)+\left(1-p\left(e_{t}\right)\right)\left(R_{t+1}^{b}-R_{d, t+1}^{b}\right)\right] \overbrace{\frac{d_{t}}{N_{t}}}^{\overbrace{\text { big }}}
$$

- makes $N_{t}$ grow, offseting incentive effects of decline in $d_{t}$.


## Impact of Loss of Bank Net Worth



## Conclusion

- Described a model in which there is a problem that is mitigated by the introduction of borrowing restrictions.
- Currently exploring what are the optimal dynamic properties of leverage.
- the cyclical behavior of the tax on leverage depends on which shock drives the cycle.
- if driven by permanent technology shocks, then act to discourage debt in a boom.

