

# Financial Frictions Under Asymmetric Information and Costly State Verification

# General Idea

- Standard dsge model assumes borrowers and lenders are the same people..no conflict of interest.
- Financial friction models suppose borrowers and lenders are different people, with conflicting interests.
- Financial frictions: features of the relationship between borrowers and lenders adopted to mitigate conflict of interest.

# Discussion of Financial Frictions

- Simple model to illustrate the basic costly state verification (csv) model.
  - Original analysis of Townsend (1978), Gale-Helwig.
- Then: integrate the csv model into a full-blown dsge model.
  - Follows the lead of Bernanke, Gertler and Gilchrist (1999).
  - Empirical analysis of Christiano, Motto and Rostagno (2003,2009,2011).
- After fitting model to data, find that a new shock, ‘risk shock’, appears to be important in business cycles.

# Simple Model

- There are entrepreneurs with all different levels of wealth,  $N$ .
  - Entrepreneur have different levels of wealth because they experienced different idiosyncratic shocks in the past.
- For each value of  $N$ , there are many entrepreneurs.
- In what follows, we will consider the interaction between entrepreneurs with a specific amount of  $N$  with competitive banks.
- Later, will consider the whole population of entrepreneurs, with every possible level of  $N$ .

# Simple Model, cont'd

- Each entrepreneur has access to a project with rate of return,

$$(1 + R^k)\omega$$

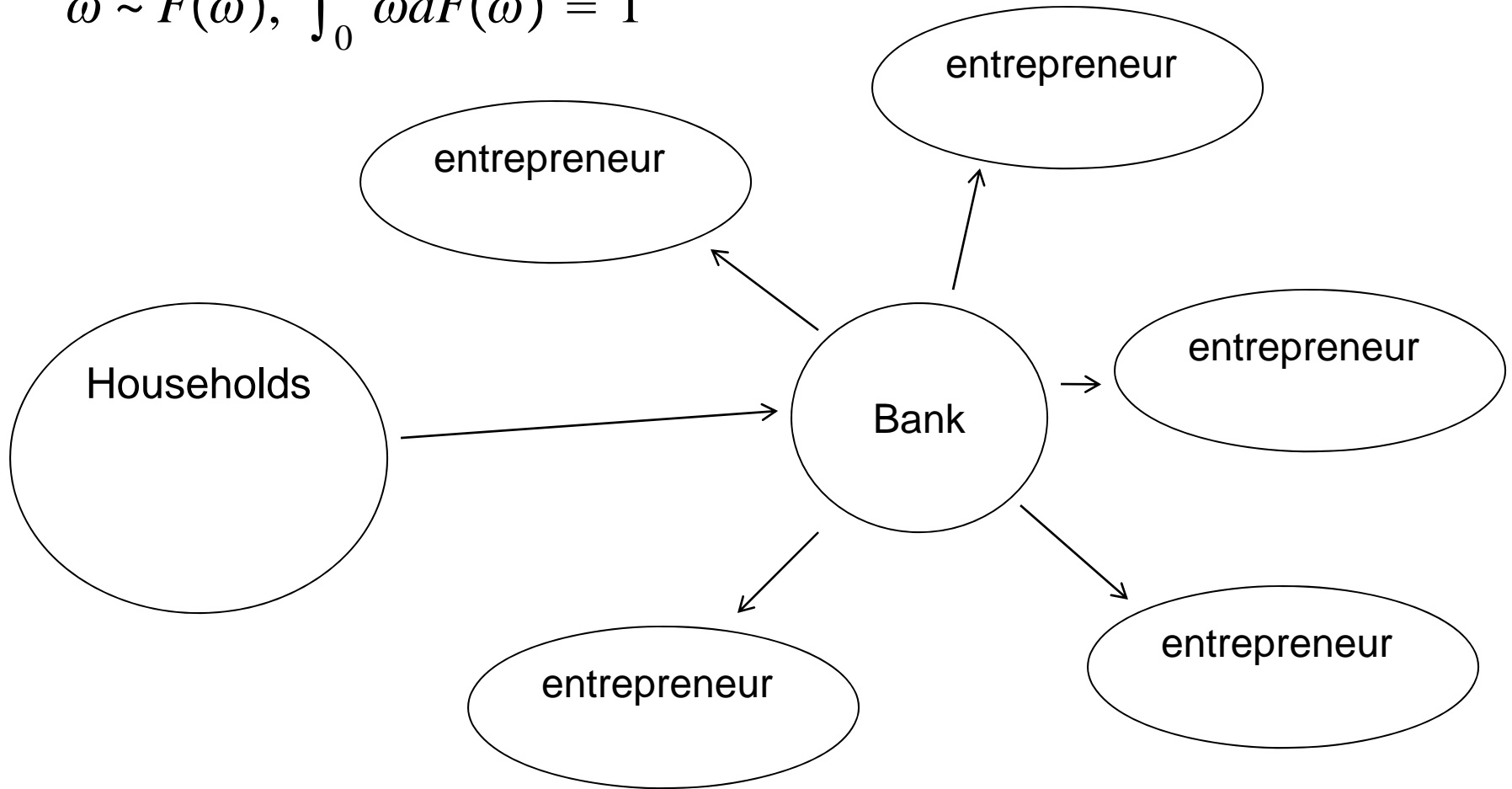
- Here,  $\omega$  is a unit mean, idiosyncratic shock experienced by the individual entrepreneur after the project has been started,

$$\int_0^{\infty} \omega dF(\omega) = 1$$

- The shock,  $\omega$ , is privately observed by the entrepreneur.
- $F$  is lognormal cumulative distribution function.

# Banks, Households, Entrepreneurs

$$\omega \sim F(\omega), \int_0^\infty \omega dF(\omega) = 1$$



Standard debt contract

- Entrepreneur receives a contract from a bank, which specifies a rate of interest,  $Z$ , and a loan amount,  $B$ .
  - If entrepreneur cannot make the interest payments, the bank pays a monitoring cost and takes everything.

- Total assets acquired by the entrepreneur:

$$\overbrace{A}^{\text{total assets}} = \overbrace{N}^{\text{net worth}} + \overbrace{B}^{\text{loans}}$$

- Entrepreneur who experiences sufficiently bad luck,  $\omega \leq \bar{\omega}$ , loses everything.

- Expected return to entrepreneur, over opportunity cost of funds:

Expected payoff for entrepreneur

$$\frac{\int_{\bar{\omega}}^{\infty} [(1+R^k)\omega A - ZB] dF(\omega)}{N(1+R)}$$

For lower values of  $\omega$ , entrepreneur receives nothing 'limited liability'.

opportunity cost of funds



- Rewriting entrepreneur's rate of return:

$$\frac{\int_{\bar{\omega}}^{\infty} [(1 + R^k)\omega A - ZB] dF(\omega)}{N(1 + R)} = \frac{\int_{\bar{\omega}}^{\infty} [(1 + R^k)\omega A - (1 + R^k)\bar{\omega}A] dF(\omega)}{N(1 + R)}$$

$$= \int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \left( \frac{1 + R^k}{1 + R} \right) L$$

$$\bar{\omega} = \frac{Z}{(1+R^k)} \frac{L-1}{L} \rightarrow_{L \rightarrow \infty} \frac{Z}{(1+R^k)}$$

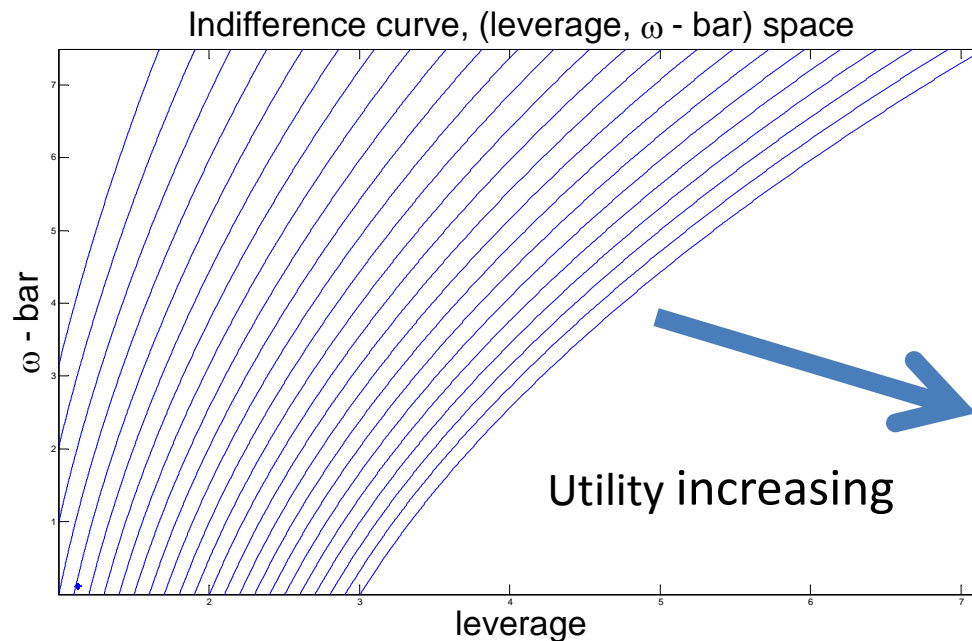
- Entrepreneur's return unbounded above
  - Risk neutral entrepreneur would always want to borrow an infinite amount (infinite leverage).

- If given a fixed interest rate, entrepreneur with risk neutral preferences would borrow an unbounded amount.
- In equilibrium, bank can't lend an infinite amount.
- This is why a loan contract must specify *both* an interest rate,  $Z$ , and a loan amount,  $B$ .

# 'Indifference Curves' of Entrepreneurs

- Think of the loan contract in terms of the loan amount (or, leverage,  $(N+B)/N$ ) and the cutoff,  $\bar{\omega}$

$$\frac{\int_{\bar{\omega}}^{\infty} [(1+R^k)\omega A - ZB] dF(\omega)}{N(1+R)} = \int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \left( \frac{1+R^k}{1+R} \right) L$$



$$L = \frac{A}{N} = \frac{N+B}{N}$$

# Banks

- Source of funds from households, at fixed rate,  $R$
- Bank borrows  $B$  units of currency, lends proceeds to entrepreneurs.
- Provides entrepreneurs with standard debt contract,  $(Z, B)$

# Banks, cont'd

- Monitoring cost for bankrupt entrepreneur

with  $\omega < \bar{\omega}$

Bankruptcy cost parameter

$$\mu(1 + R^k)\omega A$$

- Bank zero profit condition

fraction of entrepreneurs with  $\omega > \bar{\omega}$

quantity paid by each entrepreneur with  $\omega > \bar{\omega}$

$$\overbrace{[1 - F(\bar{\omega})]}$$

$$\overbrace{ZB}$$

quantity recovered by bank from each bankrupt entrepreneur

$$+ \overbrace{(1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega) (1 + R^k) A}$$

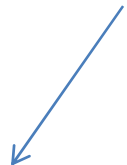
amount owed to households by bank

$$= \overbrace{(1 + R)B}$$

# Banks, cont'd

- Simplifying zero profit condition:

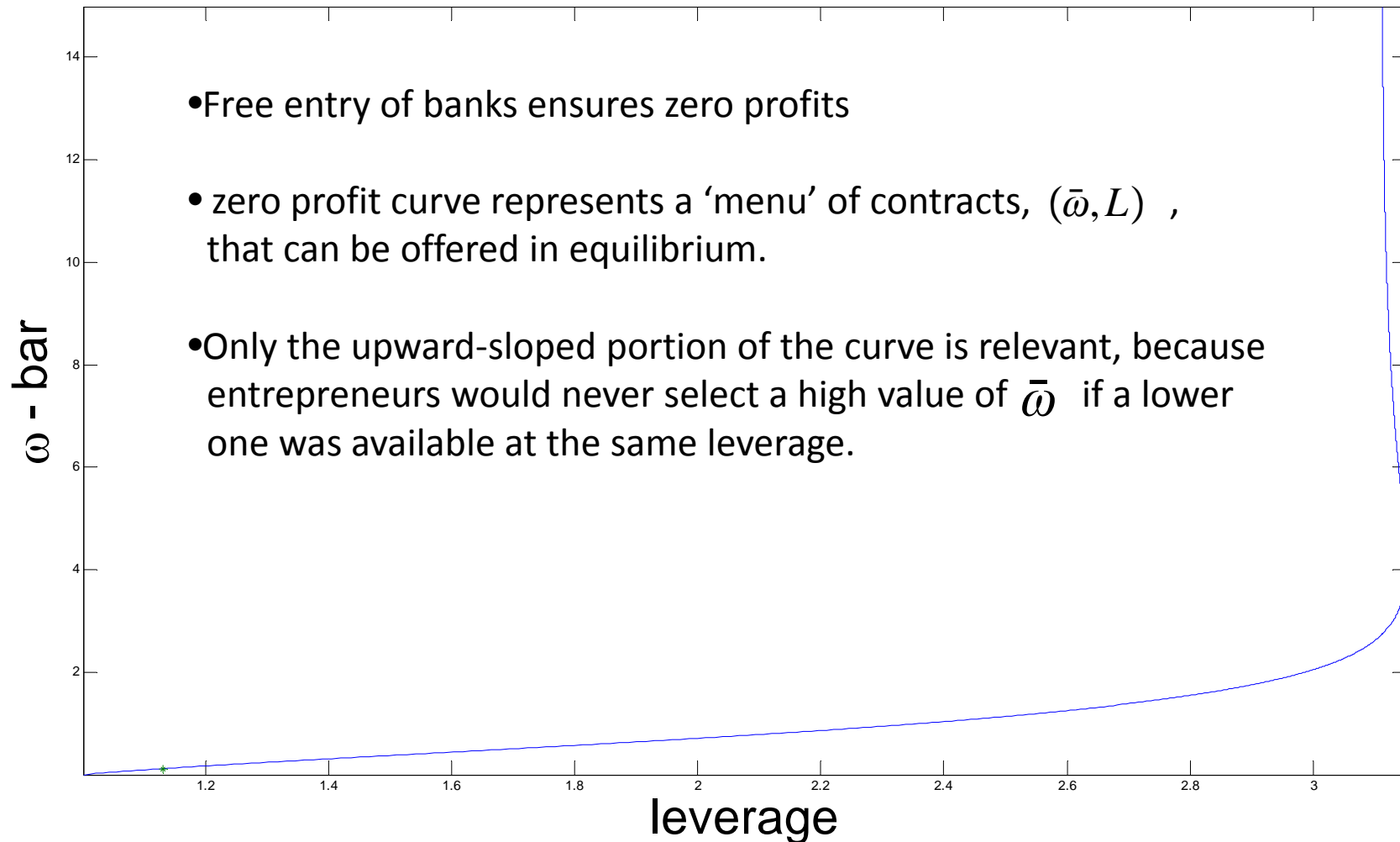
$$[1 - F(\bar{\omega})]ZB + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A = (1 + R)B$$


$$[1 - F(\bar{\omega})]\bar{\omega}(1 + R^k)A + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A = (1 + R)B$$

$$\begin{aligned} [1 - F(\bar{\omega})]\bar{\omega} + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega) &= \frac{1 + R}{1 + R^k} \frac{B/N}{A/N} \\ &= \frac{1 + R}{1 + R^k} \frac{L - 1}{L} \end{aligned}$$

- Expressed naturally in terms of  $(\bar{\omega}, L)$

## Bank zero profit condition, in (leverage, $\bar{\omega}$ ) space



# Some Notation and Results

- Let

expected value of  $\omega$ , conditional on  $\omega < \bar{\omega}$

$$G(\bar{\omega}) = \overbrace{\int_0^{\bar{\omega}} \omega dF(\omega)}^{\text{expected value of } \omega, \text{ conditional on } \omega < \bar{\omega}}, \quad \Gamma(\bar{\omega}) = \bar{\omega}[1 - F(\bar{\omega})] + G(\bar{\omega}),$$

- Results:

$$G'(\bar{\omega}) = \frac{d}{d\bar{\omega}} \int_0^{\bar{\omega}} \omega dF(\omega) \quad \underbrace{\quad}_{\text{Leibniz's rule}} \quad \bar{\omega}F'(\bar{\omega})$$

$$\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega}) - \bar{\omega}F'(\bar{\omega}) + G(\bar{\omega}) = 1 - F(\bar{\omega})$$



# Moving Towards Equilibrium Contract

- Entrepreneurial utility:

$$\int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \frac{1 + R^k}{1 + R} L$$

$$= (1 - G(\bar{\omega}) - \bar{\omega}[1 - F(\bar{\omega})]) \frac{1 + R^k}{1 + R} L$$

share of entrepreneur return going to entrepreneur

$$= \overbrace{[1 - \Gamma(\bar{\omega})]} \frac{1 + R^k}{1 + R} L$$

# Moving Towards Equilibrium Contract, cn't

- Bank profits:

share of entrepreneurial profits (net of monitoring costs) given to bank

$$\overbrace{(1 - F(\bar{\omega}))\bar{\omega} + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega)} = \frac{1 + R}{1 + R^k} \frac{L - 1}{L}$$

$$\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = \frac{1 + R}{1 + R^k} \frac{L - 1}{L}$$

$$L = \frac{1}{1 - \frac{1+R^k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}$$

# Equilibrium Contract

- Entrepreneur selects the contract is optimal, given the available menu of contracts.
- The solution to the entrepreneur problem is the  $\bar{\omega}$  that solves:

$$\log \left\{ \overbrace{\int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \frac{1+R^k}{1+R}}^{\text{profits, per unit of leverage, earned by entrepreneur, given } \bar{\omega}} \times \overbrace{\frac{1}{1 - \frac{1+R^k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}}^{\text{leverage offered by bank, conditional on } \bar{\omega}} \right\}$$

$$= \log \overbrace{[1 - \Gamma(\bar{\omega})]}^{\text{higer } \bar{\omega} \text{ drives share of profits to entrepreneur down (bad!)}} + \log \frac{1+R^k}{1+R} \overbrace{-\log \left( 1 - \frac{1+R^k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] \right)}^{\text{higher } \bar{\omega} \text{ drives leverage up (good!)}}$$

# Computing the Equilibrium Contract

- Solve first order optimality condition uniquely for the cutoff,  $\bar{\omega}$ :

$$\overbrace{\frac{1 - F(\bar{\omega})}{1 - \Gamma(\bar{\omega})}}^{\text{elasticity of entrepreneur's expected return w.r.t. } \bar{\omega}} = \overbrace{\frac{\frac{1+R^k}{1+R} [1 - F(\bar{\omega}) - \mu F'(\bar{\omega})]}{1 - \frac{1+R^k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}}^{\text{elasticity of leverage w.r.t. } \bar{\omega}}$$

- Given the cutoff, solve for leverage:

$$L = \frac{1}{1 - \frac{1+R^k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}$$

- Given leverage and cutoff, solve for risk spread:

$$\text{risk spread} \equiv \frac{Z}{1+R} = \frac{1+R^k}{1+R} \bar{\omega} \frac{L}{L-1}$$

# Result

- Leverage,  $L$ , and entrepreneurial rate of interest,  $Z$ , **not a function of net worth,  $N$ .**
- Quantity of loans proportional to net worth:

$$L = \frac{A}{N} = \frac{N+B}{N} = 1 + \frac{B}{N}$$

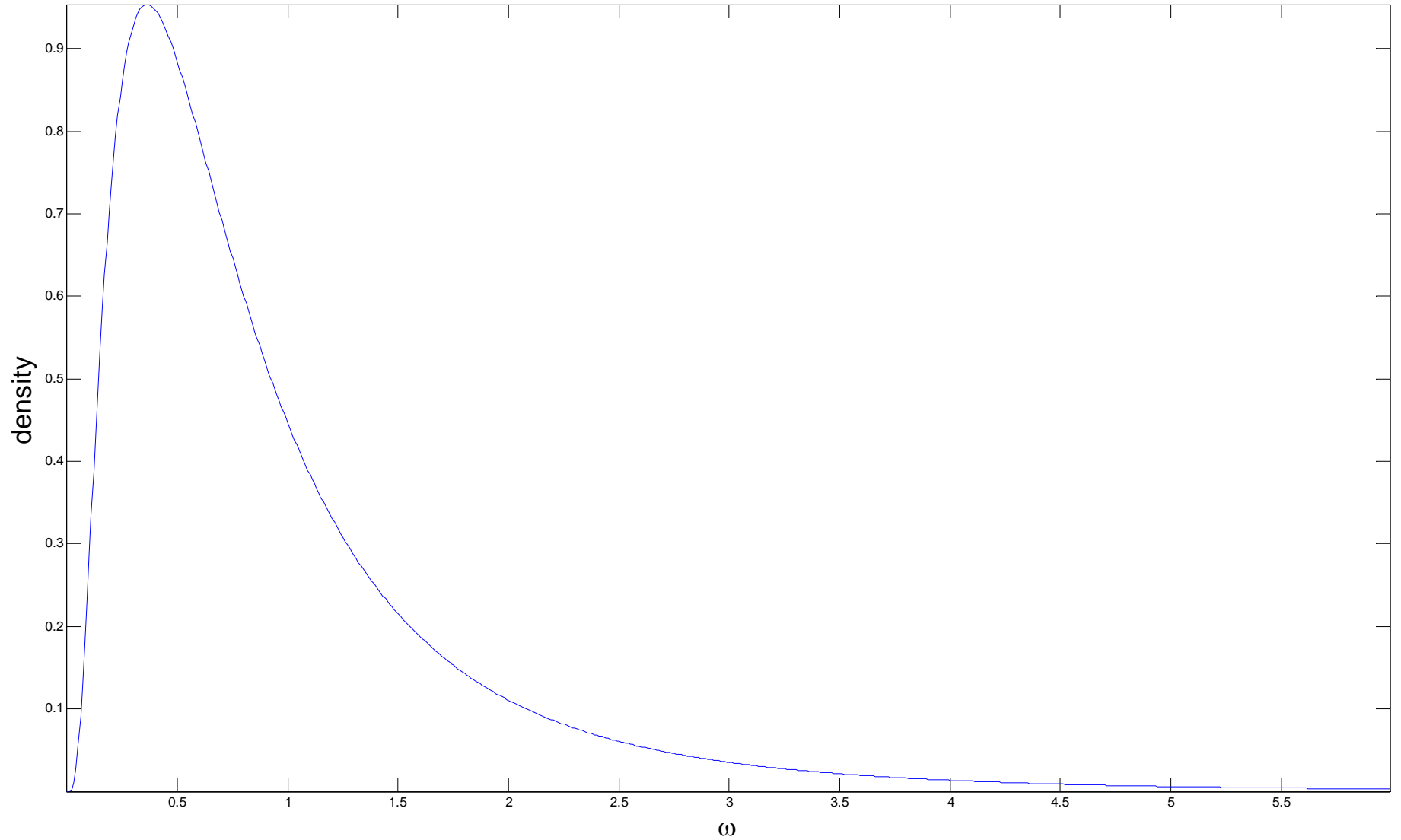
$$B = (L - 1)N$$

- To compute  $L$ ,  $Z/(1+R)$ , must make assumptions about  $F$  and parameters.

$$\frac{1 + R^k}{1 + R}, \mu, F$$

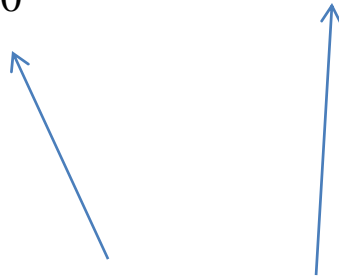
# The Distribution, $F$

Log normal density function,  $E_{\omega} = 1$ ,  $\sigma = 0.82155$



# Results for log-normal

- Need:  $G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega dF(\omega), F'(\omega)$



Can get these from the pdf and the cdf of the standard normal distribution.

These are available in most computational software, like MATLAB.

Also, they have simple analytic representations.

# Results for log-normal

- Need:  $G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega dF(\omega), F'(\omega)$

$$\int_0^{\bar{\omega}} \omega dF(\omega) \quad \underbrace{\qquad\qquad}_{\text{change of variables, } x=\log \omega} \quad \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log \bar{\omega}} e^x e^{-\frac{(x-E_x)^2}{2\sigma_x^2}} dx$$

$$\underbrace{E\omega=1 \text{ requires } E_x=-\frac{1}{2}\sigma_x^2}_{\qquad\qquad\qquad} \quad \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log \bar{\omega}} e^x e^{-\frac{(x+\frac{1}{2}\sigma_x^2)^2}{2\sigma_x^2}} dx$$

$$\underbrace{\text{combine powers of } e \text{ and rearrange}}_{\qquad\qquad\qquad} \quad \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log \bar{\omega}} e^{-\frac{(x-\frac{1}{2}\sigma_x^2)^2}{2\sigma_x^2}} dx$$

$$\underbrace{\text{change of variables, } v=\frac{x-\frac{1}{2}\sigma_x^2}{\sigma_x}}_{\qquad\qquad\qquad} \quad \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}-\sigma_x} \exp^{-\frac{v^2}{2}} \sigma_x dv$$

$$= \text{prob} \left[ v < \frac{\log(\bar{\omega}) + \frac{1}{2}\sigma_x^2}{\sigma_x} - \sigma_x \right] \leftarrow \text{cdf for standard normal}$$



# Results for log-normal, cnt'd

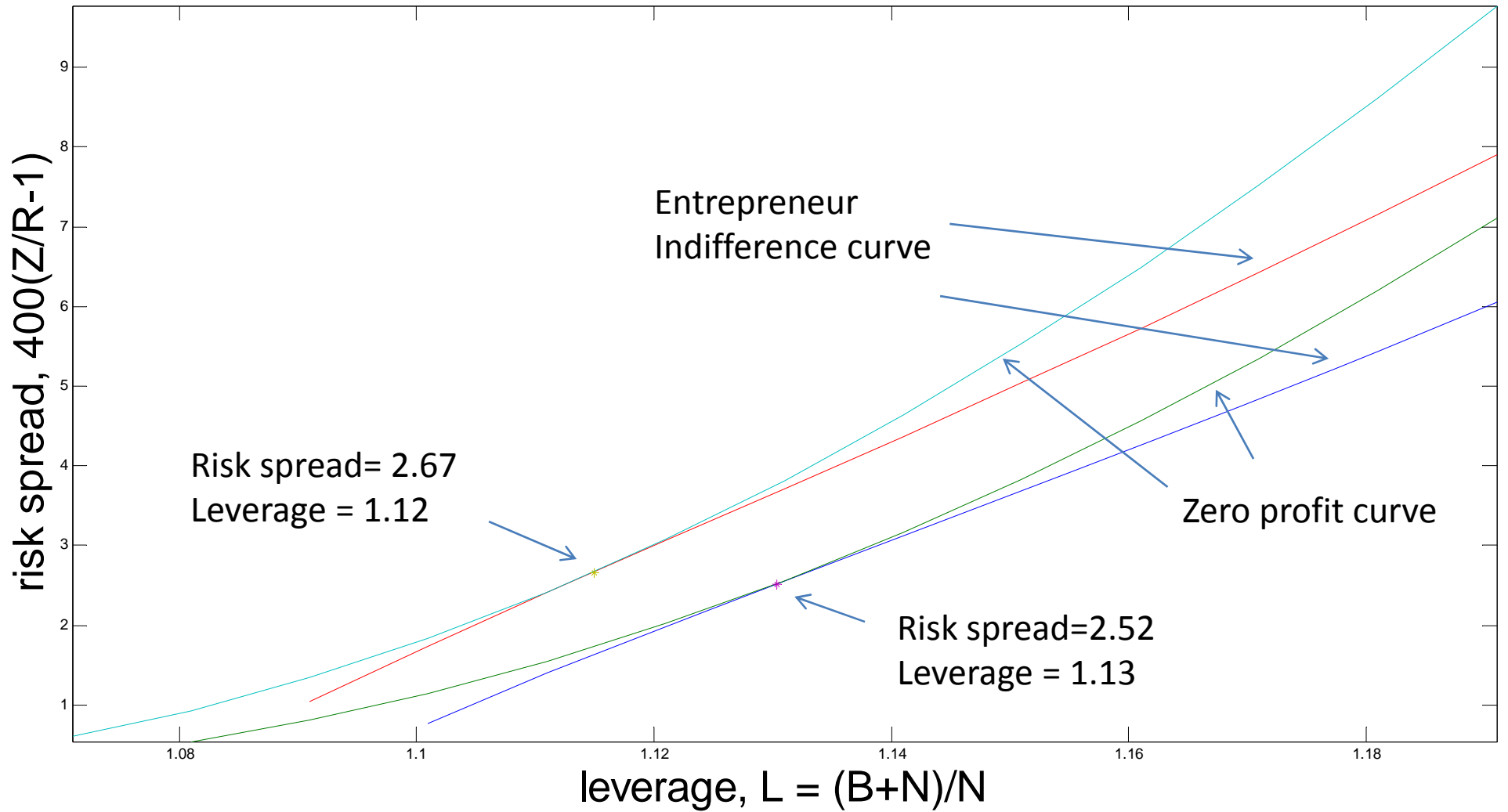
- The log-normal cumulative density:

$$F(\bar{\omega}) = \int_0^{\bar{\omega}} dF(\omega) = \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log \bar{\omega}} e^{-\frac{(x + \frac{1}{2}\sigma_x^2)^2}{2\sigma_x^2}} dx$$

- Differentiating (using Leibniz's rule):

$$\begin{aligned} F_{\bar{\omega}}(\omega; \sigma) &= \frac{1}{\bar{\omega}\sigma} \frac{1}{\sqrt{2\pi}} \exp \frac{-\left[\frac{\log(\bar{\omega}) + \frac{1}{2}\sigma^2}{\sigma}\right]^2}{2} \\ &= \frac{1}{\bar{\omega}\sigma} \text{Standard Normal pdf} \left( \frac{\log(\bar{\omega}) + \frac{1}{2}\sigma^2}{\sigma} \right) \end{aligned}$$

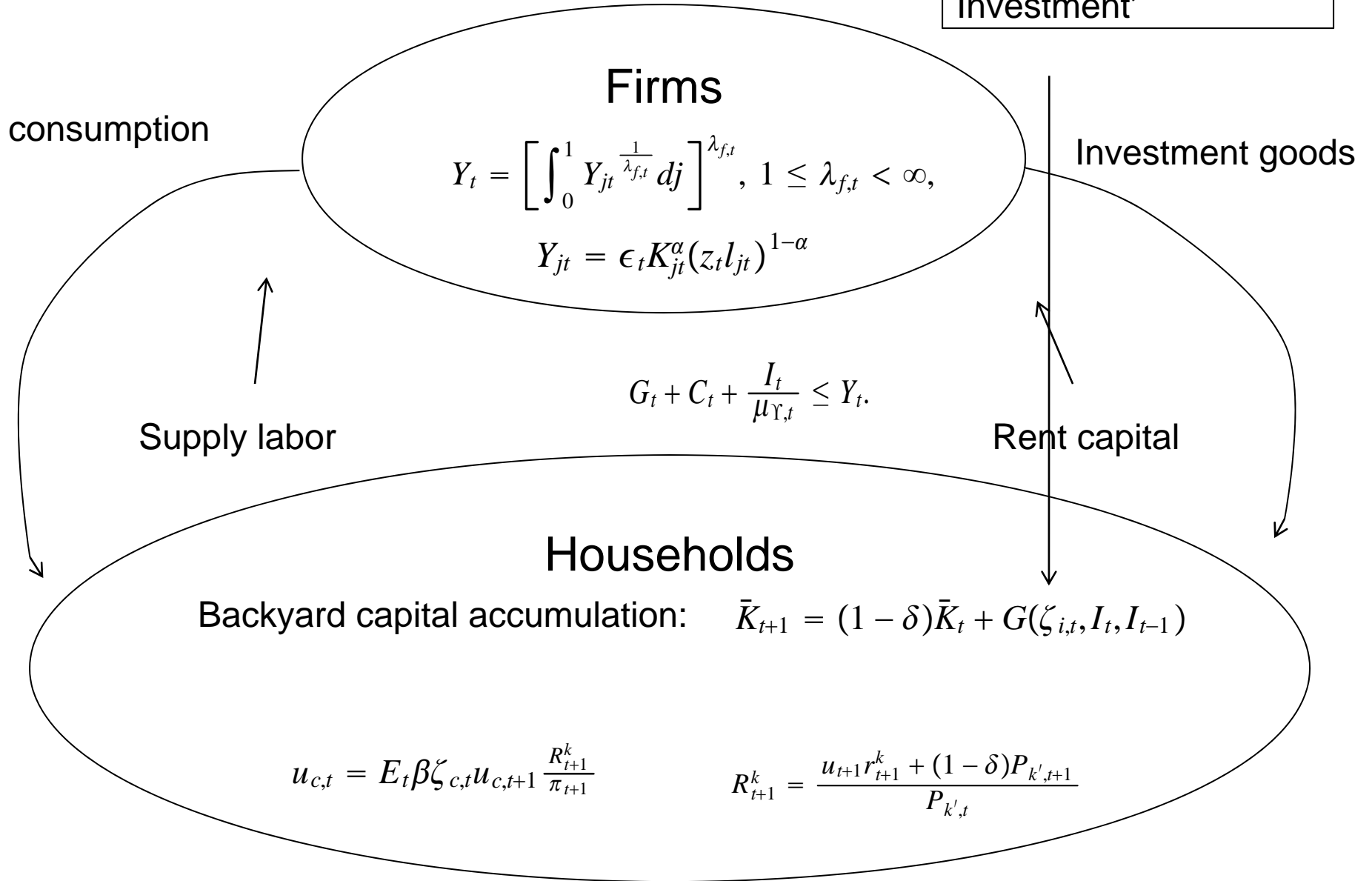
Figure : Impact on standard debt contract of a 5% jump in  $\sigma$



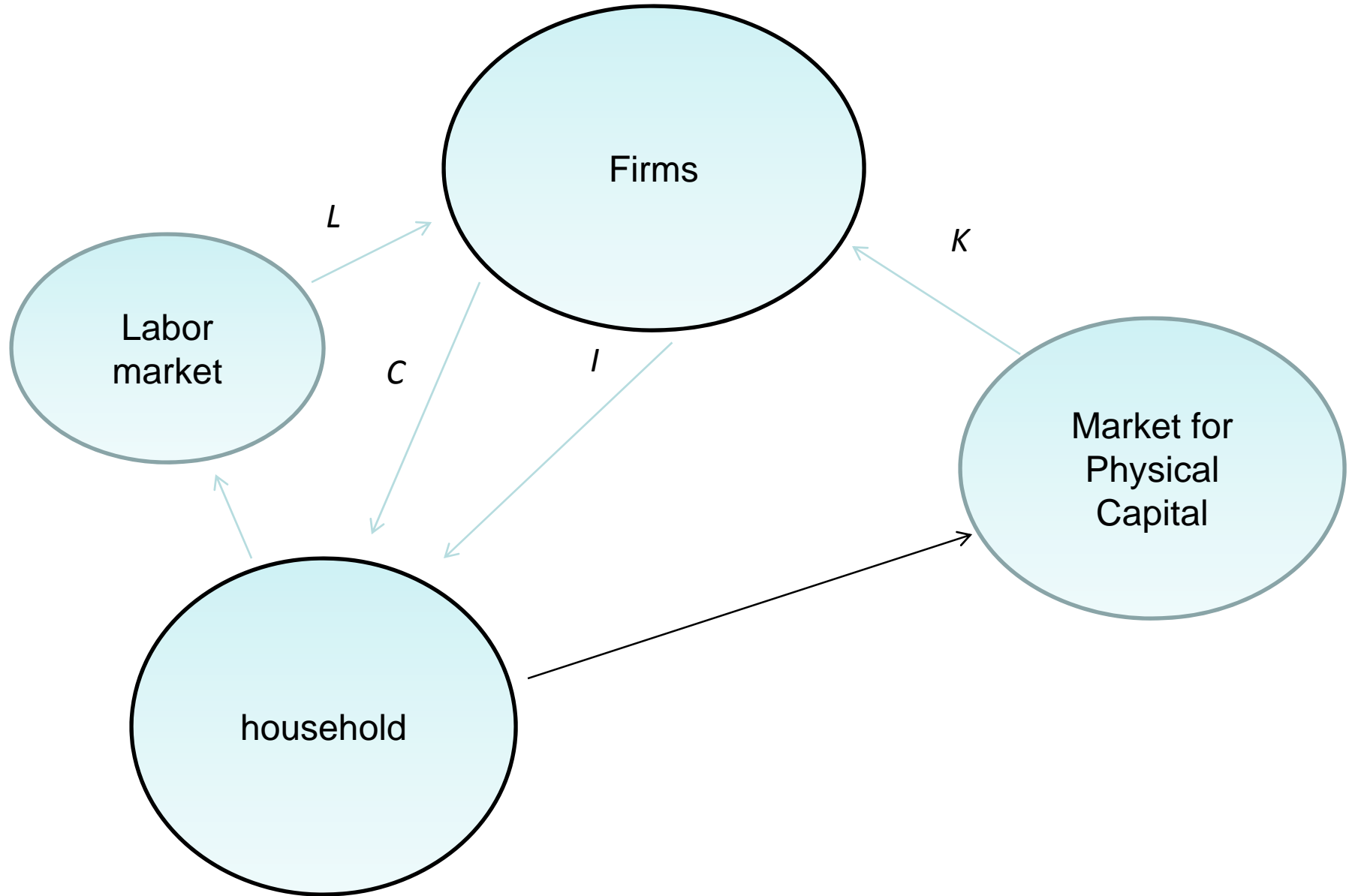
Put this Into DSGE Model

# Standard Model

'Marginal Efficiency of Investment'



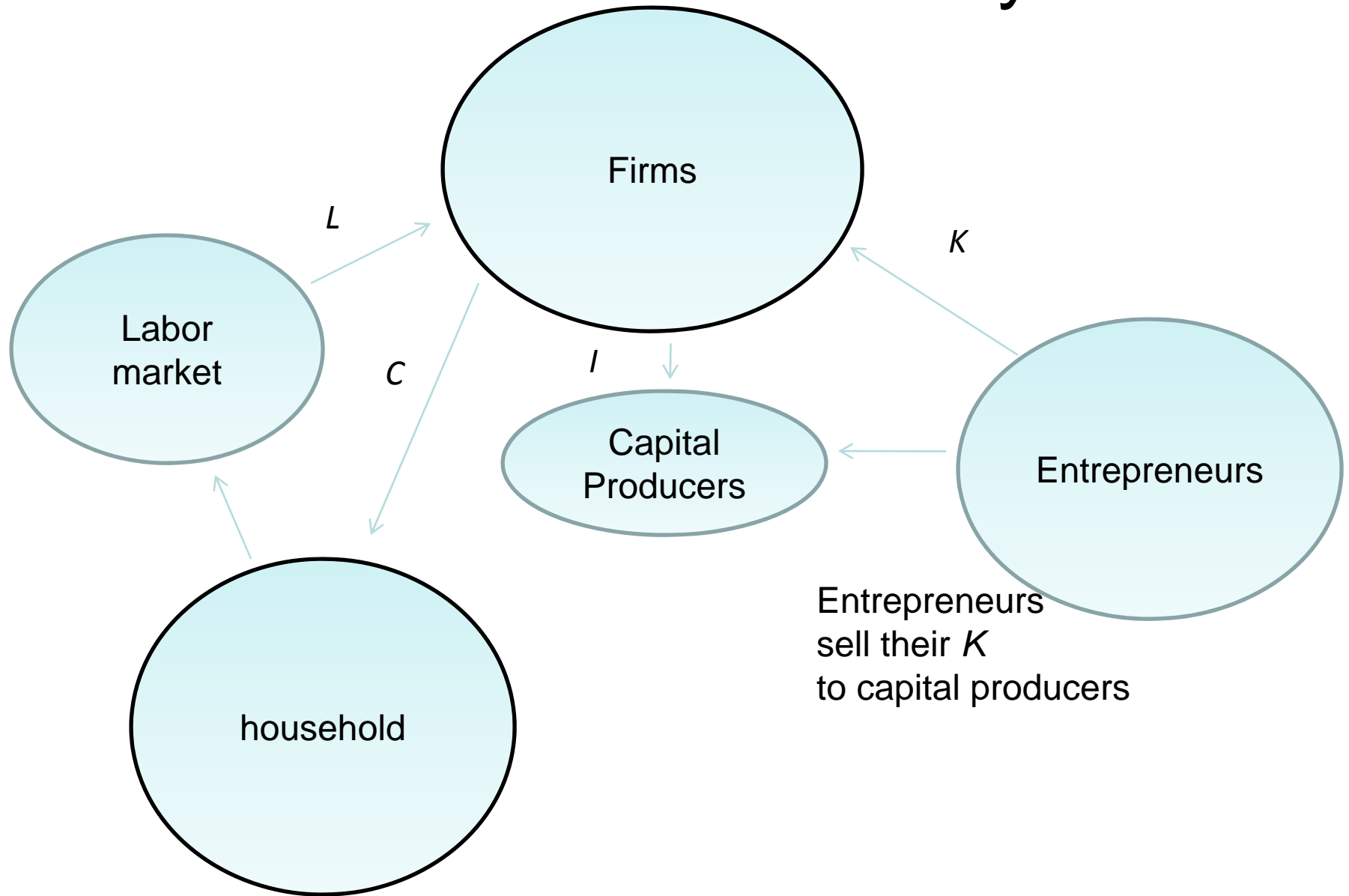
# Standard Model



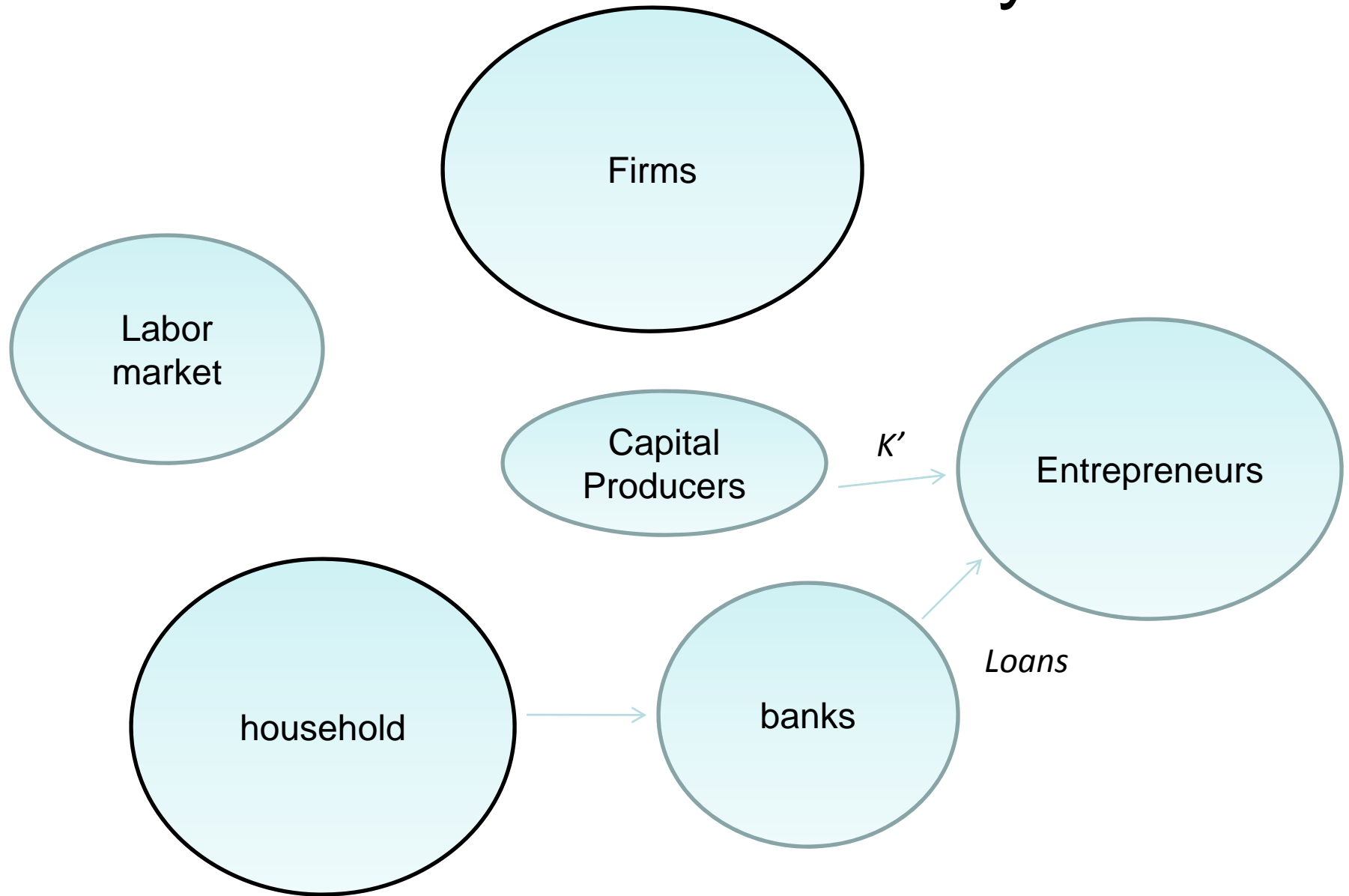
# Financing

- In the standard model, already have borrowing by firms for working capital.
  - will now have banks intermediate this borrowing between households and firms.
- In standard model, ‘putting capital to work’ is completely straightforward and is done by households. They just rent capital into a homogeneous capital market.
- Now: ‘putting capital to work’ involves a special kind of creativity that only some households – entrepreneurs – have.
  - Entrepreneurs finance the acquisition of capital in part by themselves, and in part by borrowing from regular ‘households’.
  - Conflict of interest, because there is asymmetric information about the payoff from capital.
  - Standard sharing contract between entrepreneur and household not feasible.

# Financial Frictions with Physical $K$



# Financial Frictions with Physical $K$

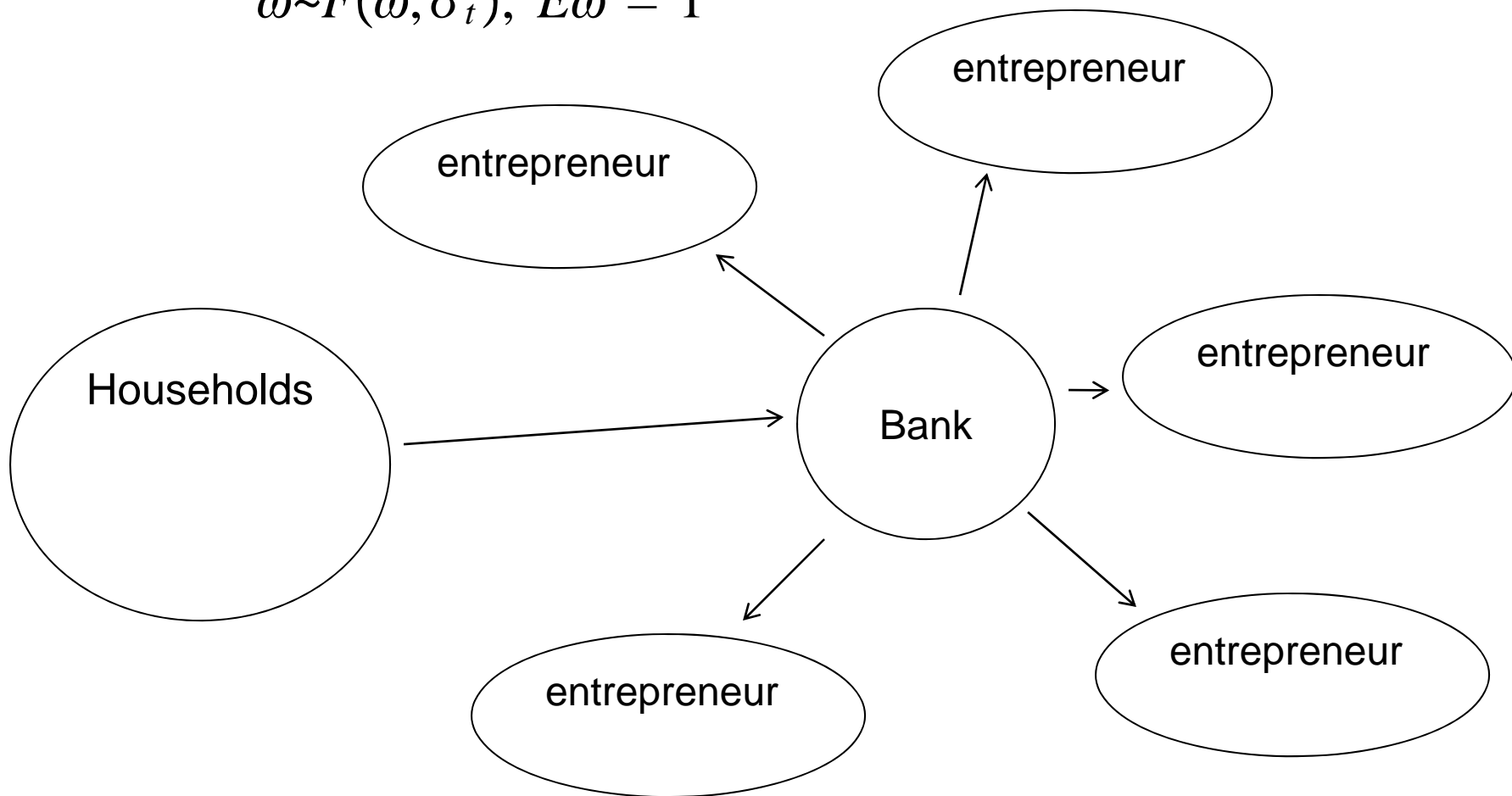




Accounts for nearly 50% of GDP

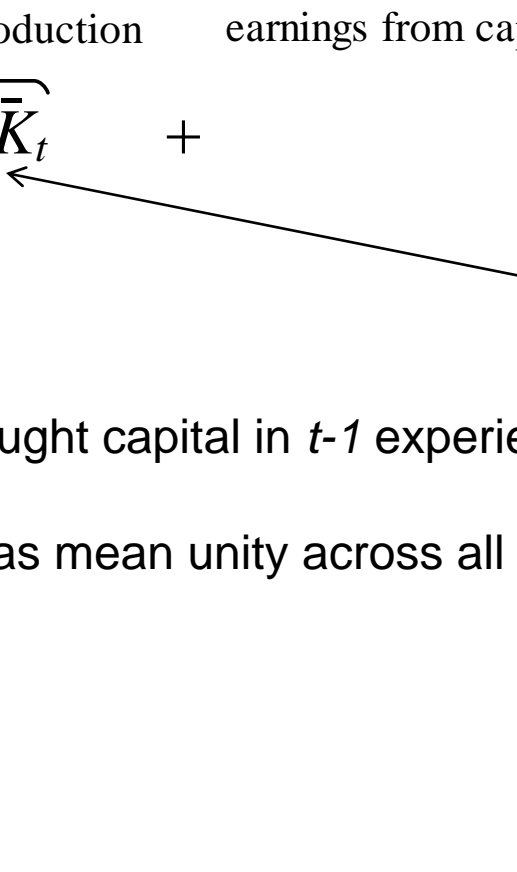
# Banks, Households, Entrepreneurs

$$\omega \sim F(\omega, \sigma_t), E\omega = 1$$



Standard debt contract

- Net worth of an entrepreneur who goes to the bank to receive a loan in period  $t$ .

$$n_t = \overbrace{P_{k',t}(1 - \delta)\omega\bar{K}_t}^{\text{value of capital after production}} + \overbrace{r_t^k \omega\bar{K}_t}^{\text{earnings from capital after utilization costs}} - B_{t-1} \frac{Z_{t-1}}{\pi_t}$$


An entrepreneur who bought capital in  $t-1$  experienced an idiosyncratic shock,  $\omega$ .

This log-normal shock has mean unity across all entrepreneurs,  $\omega \sim F(\omega, \sigma_t)$ .

# Five Adjustments to Standard DSGE Model for CSV Financial Frictions

- Drop: household intertemporal equation for capital.
- Add: characterization of the loan contracts that can be offered in equilibrium (zero profit condition for banks).
- Add: efficiency condition associated with entrepreneurial choice of contract.
- Add: Law of motion for entrepreneurial net worth (source of accelerator and Fisher debt-deflation effects).
- Introduce: bankruptcy costs in the resource constraint.

# Risk Shock and News

- Assume

iid, univariate innovation to  $\hat{\sigma}_t$

$$\hat{\sigma}_t = \rho_1 \hat{\sigma}_{t-1} + \underbrace{u_t}$$

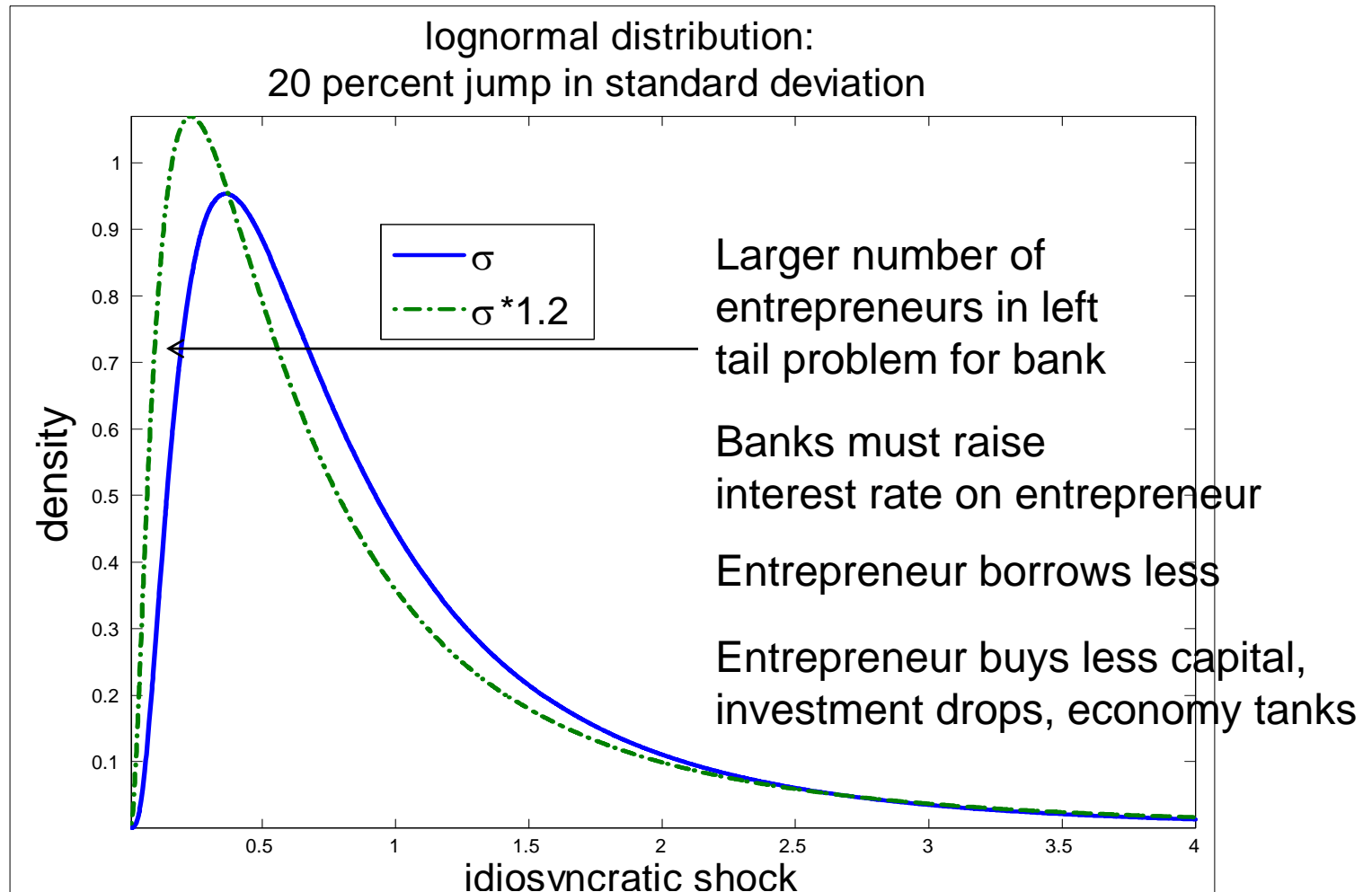
- Agents have advance information about pieces of  $u_t$

$$u_t = \xi_t^0 + \xi_{t-1}^1 + \dots + \xi_{t-8}^8$$

$$\xi_{t-i}^i \sim \text{iid}, E(\xi_{t-i}^i)^2 = \sigma_i^2$$

$$\xi_{t-i}^i \sim \text{piece of } u_t \text{ observed at time } t - i$$

# Economic Impact of Risk Shock



# Monetary Policy

- Nominal rate of interest function of:
  - Anticipated level of inflation and change.
  - Slowly moving inflation target.
  - Deviation of output growth from ss path.
  - Monetary policy shock.

# Estimation

- Use standard macro data: consumption, investment, employment, inflation, GDP, price of investment goods, wages, Federal Funds Rate.
- Also some financial variables: BAA-AAA corporate bond spreads, value of DOW, credit to nonfinancial business.
- Data: 1985Q1-2008Q4

# Key Result

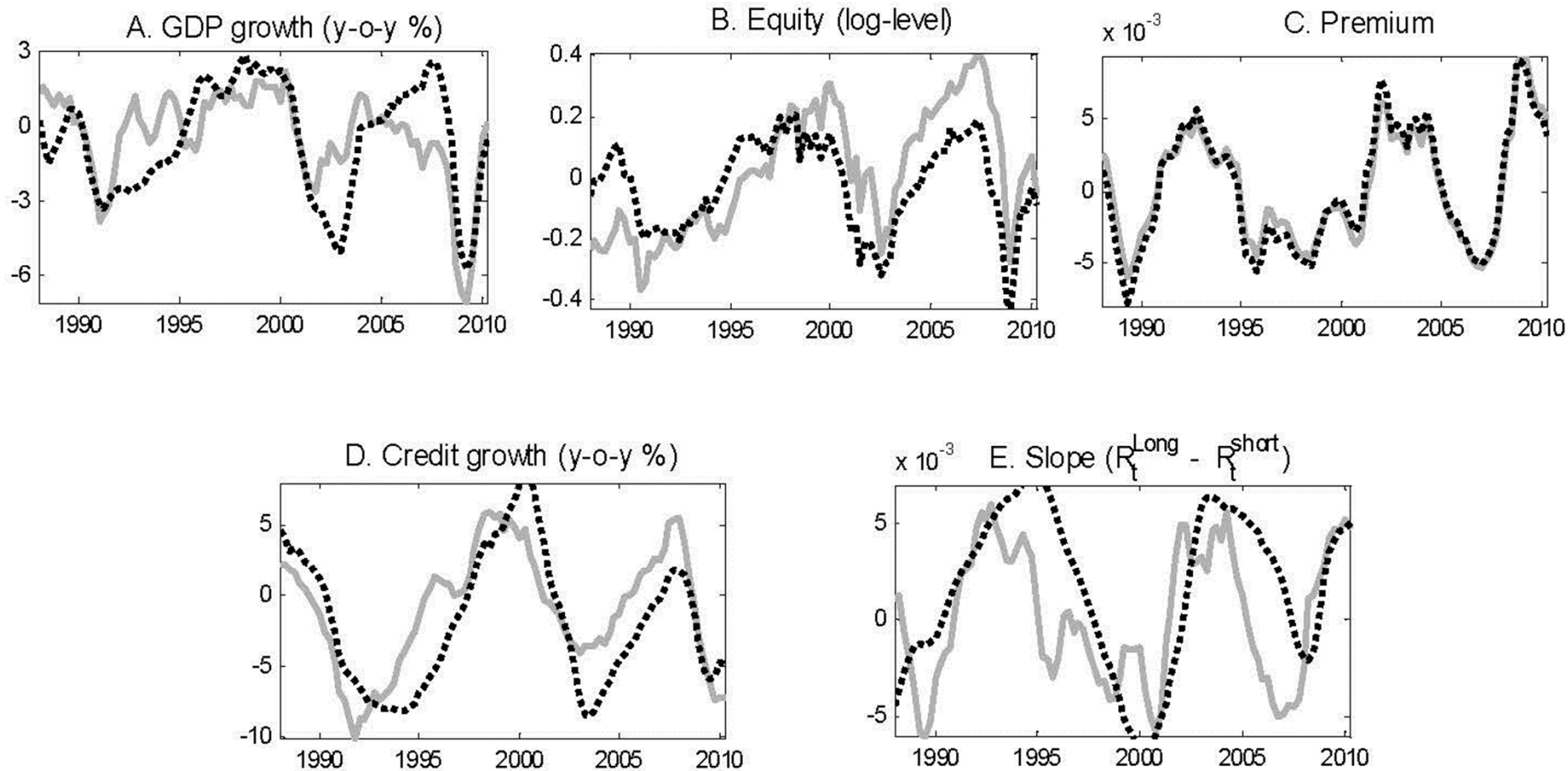
- Risk shocks:
  - important source of fluctuations.
- Out-of-Sample evidence suggests the model deserves to be taken seriously.



# Risk Shocks

- Important
- Why are they important?
- What shock do they displace, and why?

## Role of the Risk Shock in Macro and Financial Variables



Notes: The grey solid line represents the (two-sided) fitted data. The dotted black line is the model simulations.

# Variance Decomposition In Business Cycle Frequencies

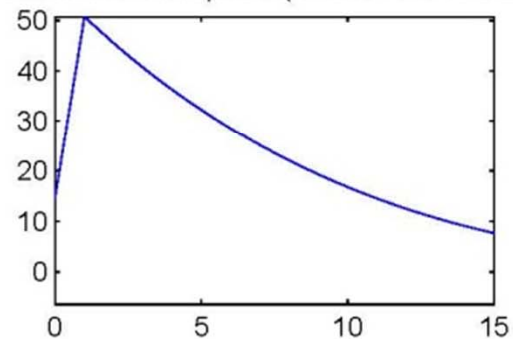
<b>Risk shock, <math>\sigma_t</math></b>
Output
49
Credit
63
Slope of Term Structure
33
Risk spread
98
Real Value of Stock Market
76

# Why Risk Shock is so Important

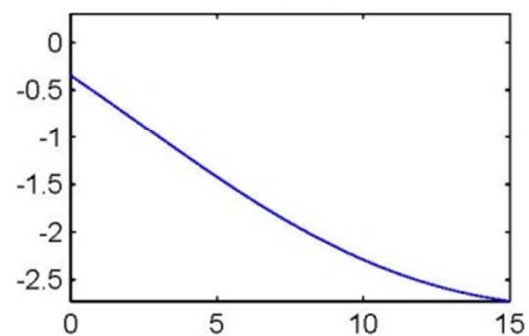
- A. Our econometric estimator ‘thinks’  
risk spread  $\sim$  risk shock.
- B. In the data: the risk spread is strongly negatively correlated with output.
- C. In the model: bad risk shock generates a response that resembles a recession.
- A+B+C suggests risk shock important.

Figure 6: Dynamic Responses to Two Shocks

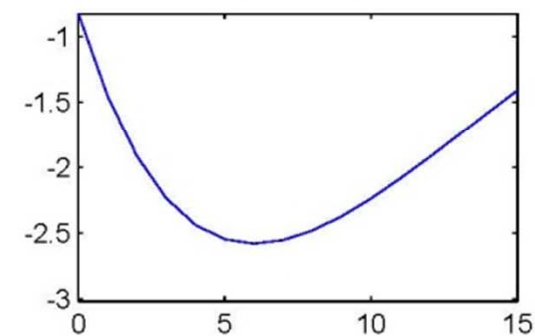
A: interest rate spread (Annual Basis Points)



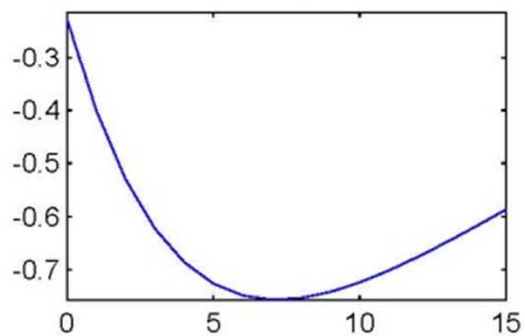
B: credit



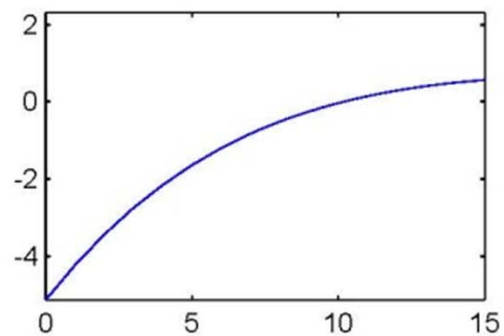
C: investment



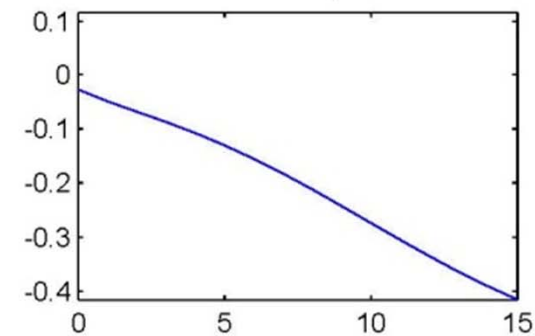
D: output



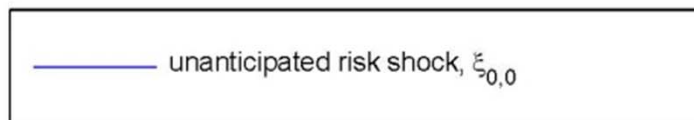
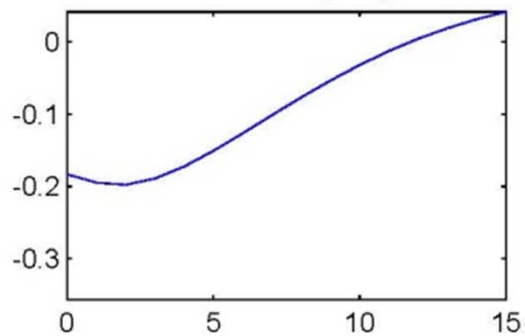
E: net worth



F: consumption



G: inflation (APR)

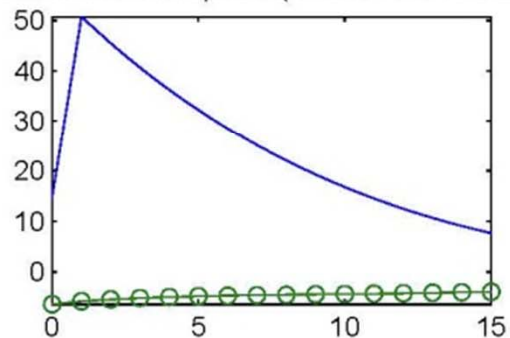


# What Shock Does the Risk Shock Displace, and why?

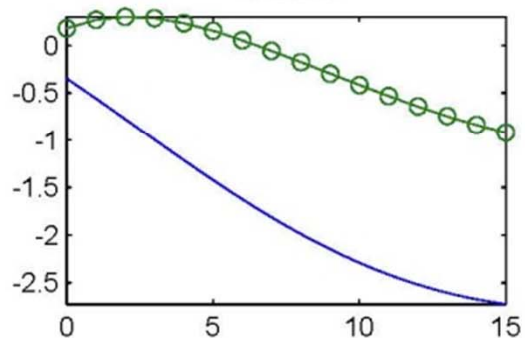
- The risk shock crowds out some of the role of the marginal efficiency of investment shock.

Figure 6: Dynamic Responses to Two Shocks

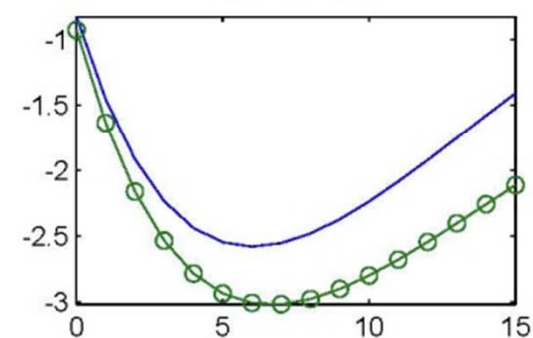
A: interest rate spread (Annual Basis Points)



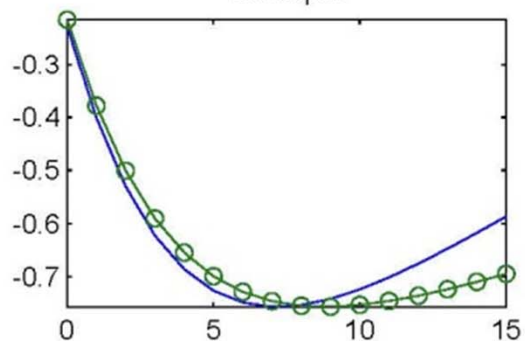
B: credit



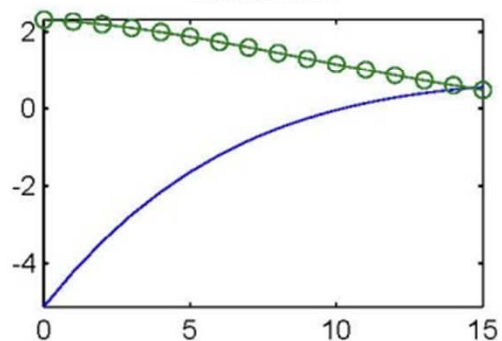
C: investment



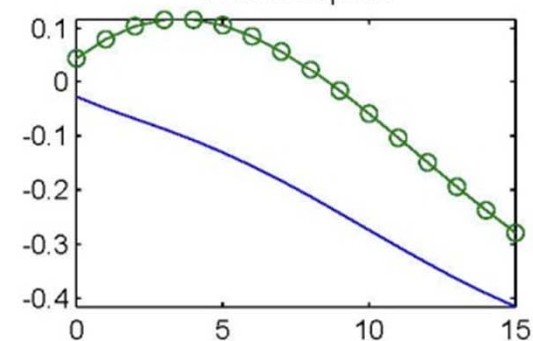
D: output



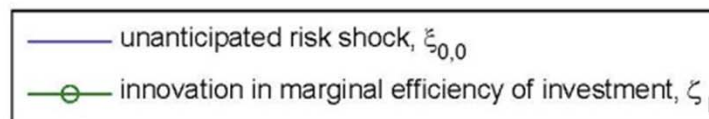
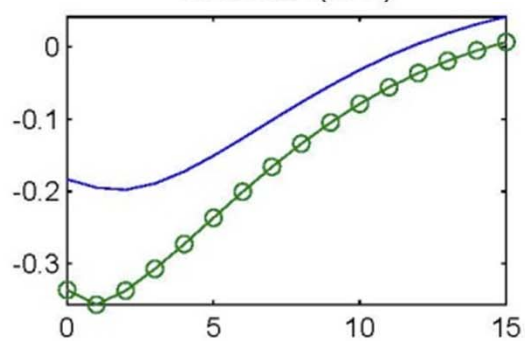
E: net worth



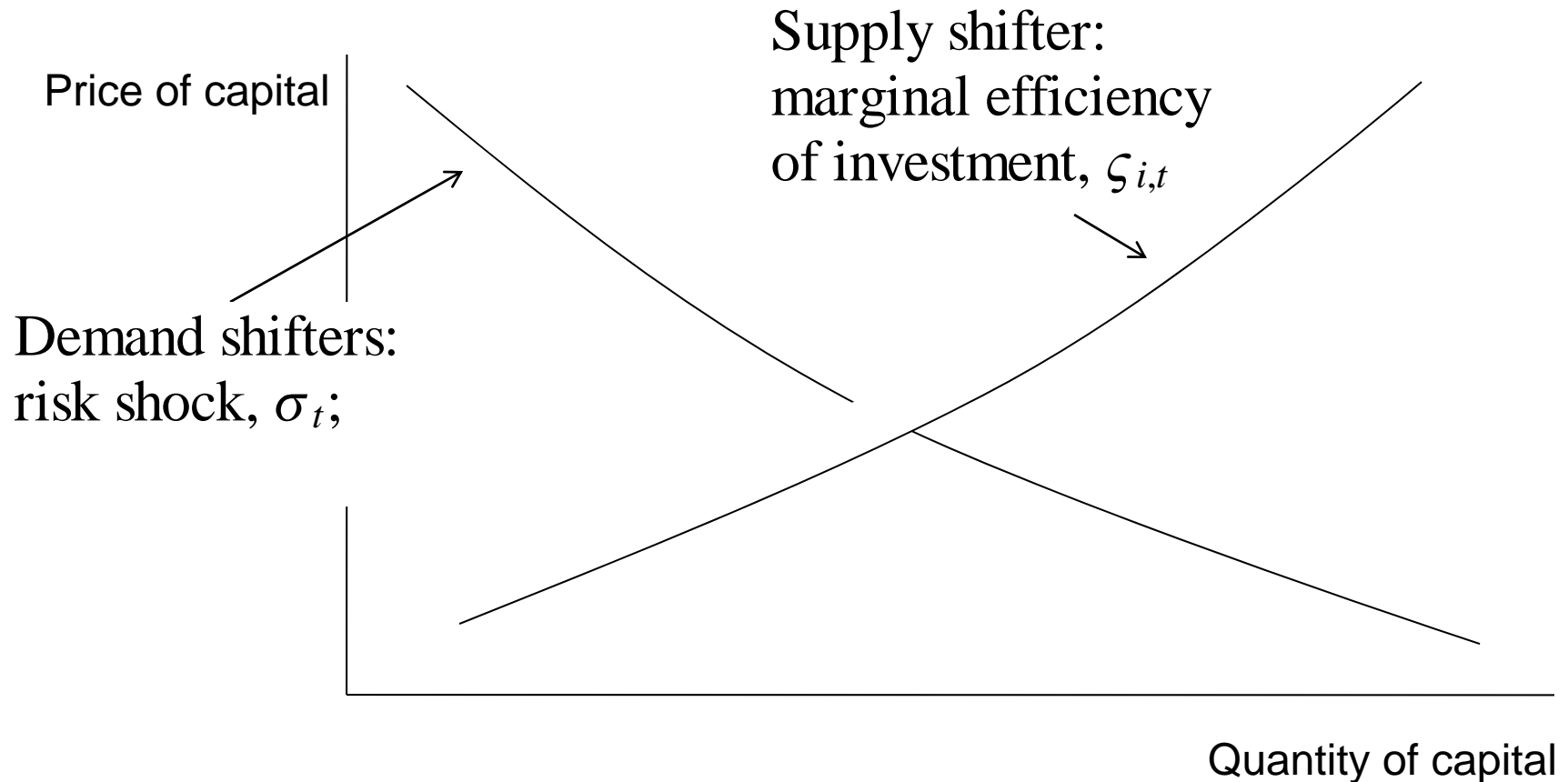
F: consumption



G: inflation (APR)



# Why does Risk Crowd out Marginal Efficiency of Investment?





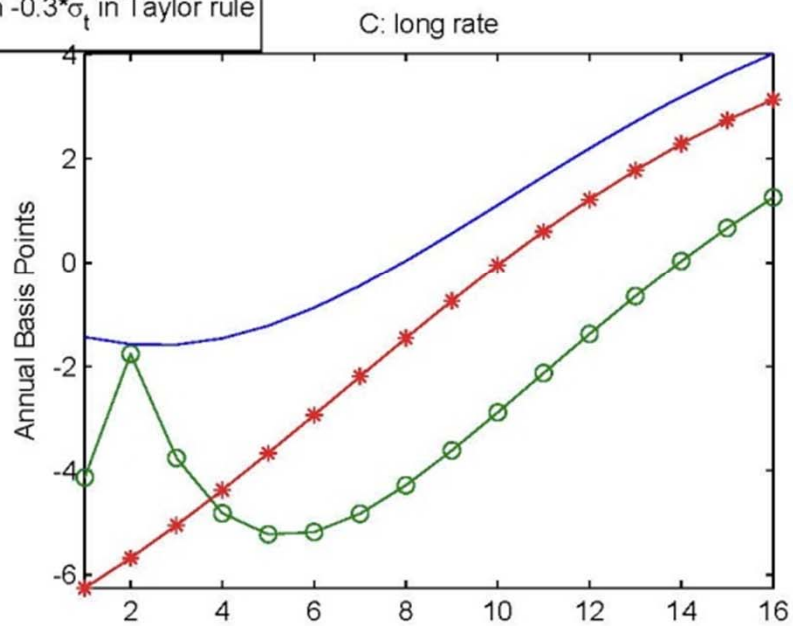
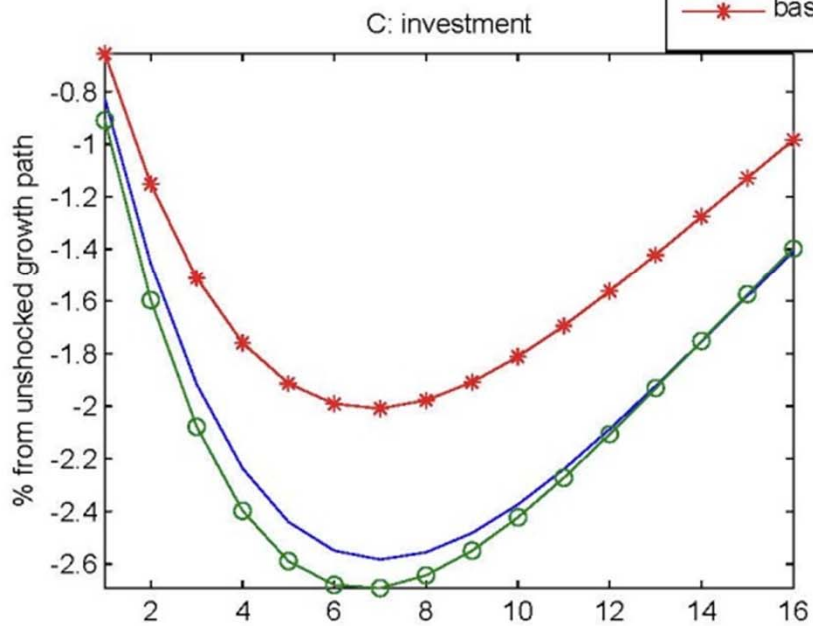
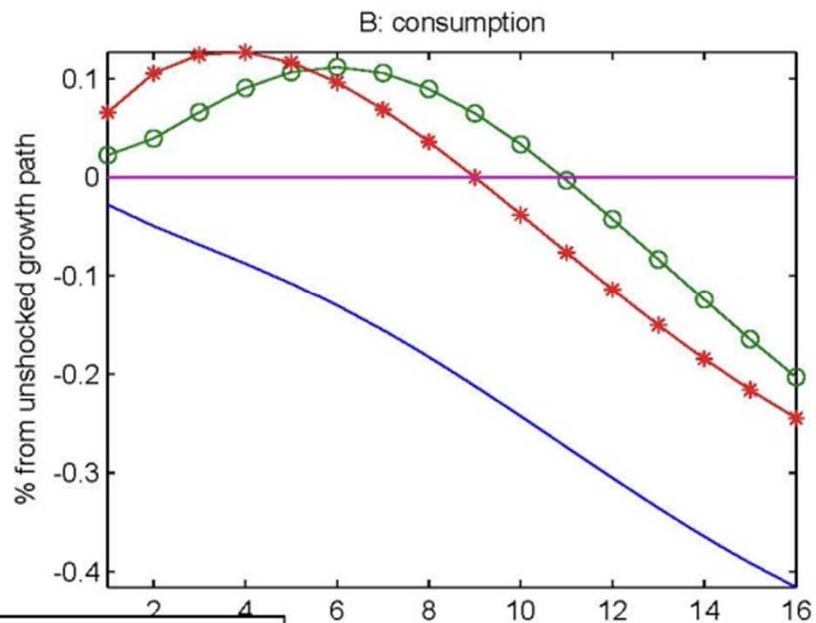
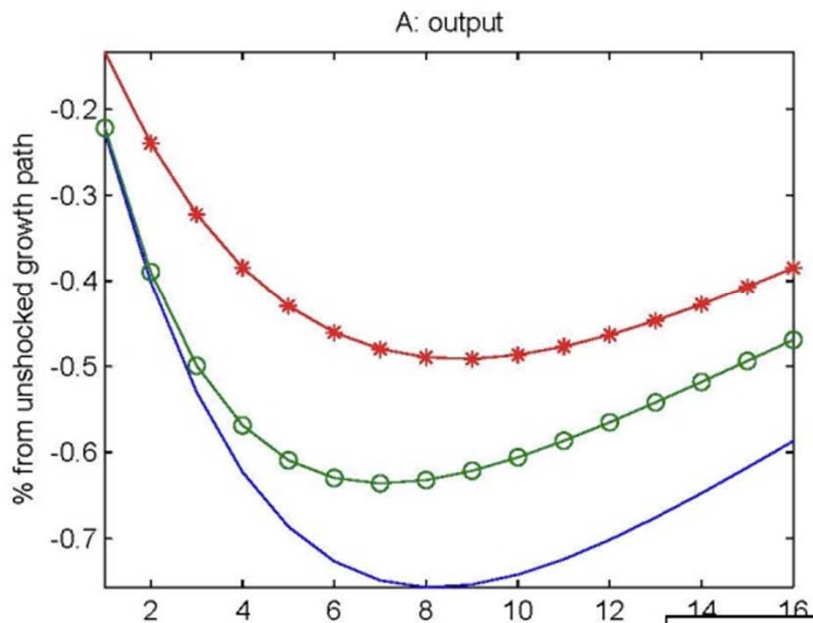
- Marginal efficiency of investment shock can account well for the surge in investment and output in the 1990s, *as long as the stock market is not included in the analysis.*
- When the stock market is included, then explanatory power shifts to financial market shocks.

# CKM Challenge

- CKM argue that risk shocks (actually, any intertemporal shock) cannot be important in business cycles.
- Idea: a shock that hurts the intertemporal margin will induce substitution away from investment and to other margins, such as consumption and leisure.
- CKM argument probably right in RBC model.
- Not valid in New Keynesian models.

# Failure of Comovement Between C & I in RBC Models With Risk Shocks

- In RBC model, jump in risk discourages investment.
- Reduction in demand leads to reduction of price of current goods relative to future goods, i.e., real interest rate.
- Real interest rate decline induces surge in demand, partially offsetting drop in investment.
- This Mechanism does not necessarily work in NK model because real rate not fully market determined there.



— baseline  
 —○— flexible wages and prices  
 —\*— baseline with  $-0.3\sigma_t$  in Taylor rule

# 'Out of Sample Evidence'

- Out of sample forecasting performance good.
- Predictions for aggregate bankruptcy rate good.
- Correlates well with Bloom evidence on cross-sectional uncertainty.

# Conclusion

- Much of the dynamics of past data can be explained as reflecting a risk shock.
- In this analysis, shock is treated as exogenous.
- Interesting to investigate mechanisms that make that 'shock' endogenous.