

Simple New Keynesian Open Economy Model

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Outline

- Standard Simple Closed Economy Model
- Extend Model to Open Economy
 - Equilibrium conditions
 - Indicate complications to bring the model to the data.
 - Similar in spirit to Rames I model
(Adolfson-Laséen-Lindé-Villan) at Riksbank
- Brief Discussion of Introducing Financial Frictions
 - Christiano-Trabandt-Walentin Model, Ramses II model.
 - Mihai Copaciu of Romanian Central Bank.

Households

- Problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \quad \tau_t = \lambda \tau_{t-1} + \varepsilon_t^\tau$$

s.t. $P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + \text{Profits-taxes}_t$

- First order conditions ($\bar{\pi}_t \equiv P_t/P_{t-1}$):

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}}$$
$$\exp(\tau_t) C_t N_t^\varphi = \frac{W_t}{P_t}.$$

Production

- Final good firms:
 - maximize profits:

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,$$

subject to technology:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

- Foncs:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^\varepsilon \rightarrow P_t = \overbrace{\left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}}^{\text{"cross price restrictions"}}$$

Intermediate Good Producers

- Demand curve for monopoly producer of $Y_{i,t}$:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^\varepsilon .$$

- Production function:

$$Y_{i,t} = \exp(a_t) N_{i,t}, \quad \Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t^a$$

- Competitive in labor market:

- Wage rate, W_t , taken as given.
- (Real) marginal cost of production:

$$s_t = \frac{(1 - \nu) (1 - \psi + \psi R_t) W_t / P_t}{e^{a_t}},$$

where

- ν is a government subsidy to firms.
- ψ is fraction of input costs that must be financed in advance.

Intermediate Good Producers

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where

- ν is a government subsidy to firms.
- ψ is fraction of input costs that must be financed in advance.
- Under flexible prices, optimizing monopolist sets price as a markup, $\varepsilon / (\varepsilon - 1)$, over marginal cost:

Intermediate Good Producers

- Calvo price setting frictions
 - with probability θ cannot change price: $P_{i,t} = P_{i,t-1}$
 - with probability $1 - \theta$ set price optimally: $P_{i,t} = \tilde{P}_t$.
- Price optimizers set price as a function of current and *future* marginal costs:

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t} = \frac{K_t}{F_t},$$

$$K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta \theta E_t (\bar{\pi}_t)^\varepsilon K_{t+1}$$

$$F_t = 1 + \beta \theta E_t (\bar{\pi}_{t+1})^{\varepsilon-1} F_{t+1}.$$

Aggregate Price Restrictions

$$\begin{aligned}P_t &= \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}} \\&= \left[(1-\theta) \tilde{P}_t^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \\&\rightarrow 1 = \left[(1-\theta) \tilde{p}_t^{1-\varepsilon} + \theta \left(\frac{1}{\bar{\pi}_t} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \\&\rightarrow \tilde{p}_t = \left[\frac{1 - \theta (\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}}\end{aligned}$$

Aggregate Output

- Want an aggregate output expression, relating Y_t to N_t and A_t .
- Consider

$$Y_t^* = \int_0^1 Y_{i,t} di = \int_0^1 A_t N_{i,t} di = A_t N_t.$$

- Note also

$$\begin{aligned} Y_t^* &= \int_0^1 Y_{i,t} di = \int_0^1 Y_t \left(\frac{P_t}{P_{i,t}} \right)^\varepsilon di = Y_t P_t^\varepsilon \int_0^1 P_{i,t}^{-\varepsilon} di \\ &= Y_t P_t^\varepsilon (P_t^*)^{-\varepsilon}, \end{aligned}$$

say, where

$$P_t^* = \left[\int_0^1 P_{i,t}^{-\varepsilon} di \right]^{\frac{-1}{\varepsilon}} = \left[(1 - \theta) \tilde{P}_t^{-\varepsilon} + \theta (P_{t-1}^*)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}$$

Aggregate Output

- Then,

$$\begin{aligned} Y_t &= p_t^* Y_t^* = p_t^* A_t N_t, \\ p_t^* &= \left(\frac{P_t^*}{P_t} \right)^\varepsilon = \left[(1 - \theta) \tilde{p}_t^{-\varepsilon} + \theta \left(\frac{P_{t-1}^*}{P_t} \right)^{-\varepsilon} \right]^{-1} \\ &= \left[(1 - \theta) \tilde{p}_t^{-\varepsilon} + \theta \left(\frac{P_{t-1}}{P_t} \frac{P_{t-1}^*}{P_{t-1}} \right)^{-\varepsilon} \right]^{-1} \\ &= \left[(1 - \theta) \tilde{p}_t^{-\varepsilon} + \theta \frac{\bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1} \\ &= \left[(1 - \theta) \left(\frac{1 - \theta (\bar{\pi}_t)^{\varepsilon-1}}{1 - \theta} \right)^{\frac{-\varepsilon}{1-\varepsilon}} + \theta \frac{\bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1} \end{aligned}$$

Basic Closed Economy Model

- Results from closed economy model

- Household preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t) - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right\}, \quad u(C_t) \equiv \log C_t$$

- Aggregate resources and household intertemporal optimization:

$$Y_t = p_t^* A_t N_t, \quad u_{c,t} = \beta E_t u_{c,t+1} \frac{R_t}{\bar{\pi}_{t+1}}$$

- Law of motion of price distortion:

$$p_t^* = \left((1 - \theta) \left(\frac{1 - \theta (\bar{\pi}_t)^{\varepsilon-1}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right)^{-1}. \quad (1)$$

Basic Model ...

– Equilibrium conditions associated with price setting:

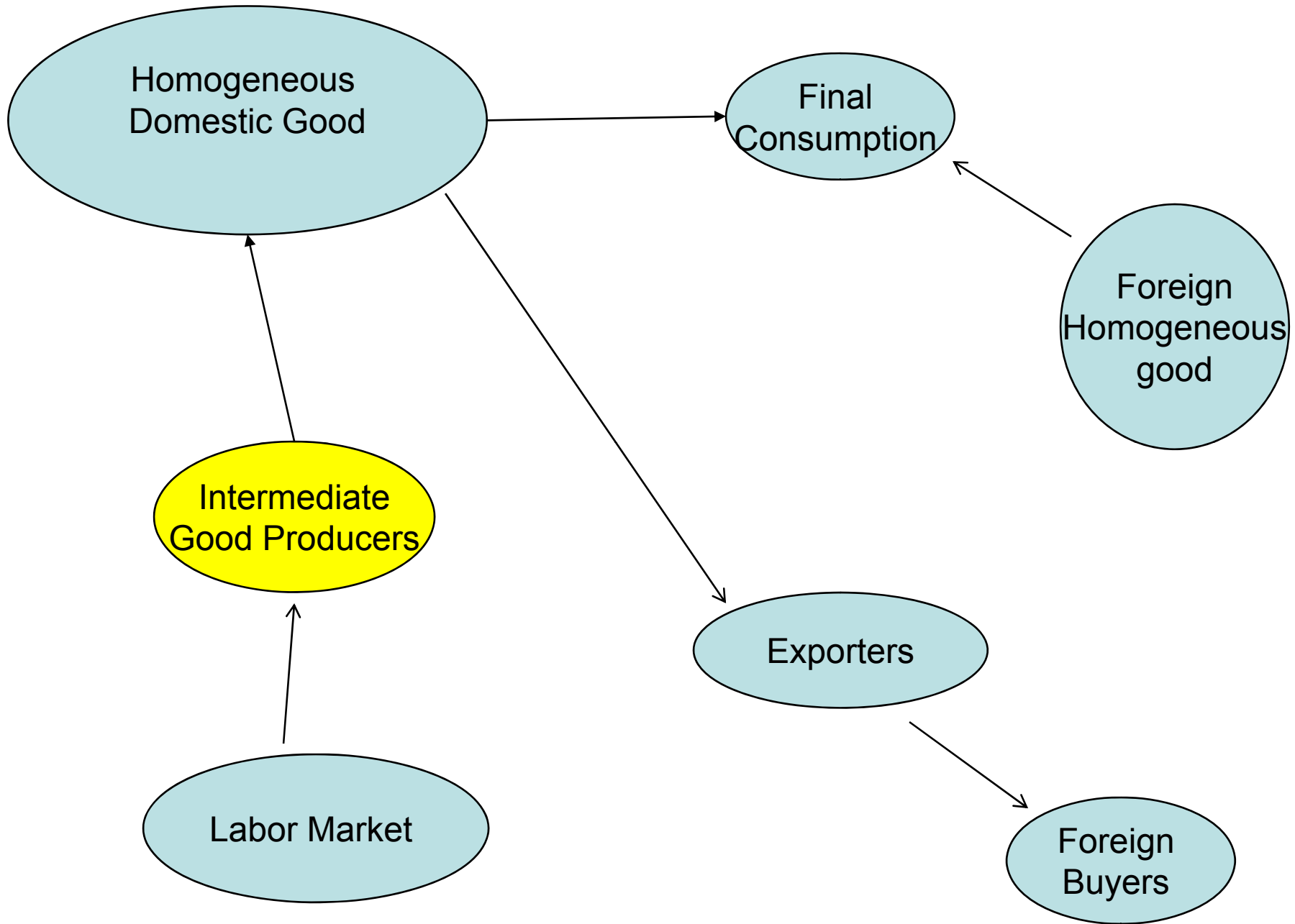
$$1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta \theta F_{t+1} = F_t \quad (2)$$

$$F_t \left[\frac{1 - \theta \bar{\pi}_t^{\varepsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} = K_t \quad (3)$$

= intermediate good marginal cost

= $\frac{W_t}{P_t}$ by household optimization

$$= K_t \frac{\frac{\varepsilon}{\varepsilon - 1} (1 - \nu_t)}{\frac{\overbrace{\exp(\tau_t) N_t^\varphi}}{u_{c,t}} \frac{1 - \psi + \psi R_t}{A_t} + E_t \beta \theta \bar{\pi}_{t+1}^\varepsilon K_{t+1}} \quad (4)$$



Extension to Small Open Economy

- Outline

- the equilibrium conditions of the open economy model

- * system jumps from 6 equations in basic model to 16 equations in 16 variables!

- * additional variables:

rate of depreciation, exports, real foreign assets, terms of trade, real exchange rate, respectively

$$\overbrace{s_t, x_t, a_t^f, p_t^x, q_t},$$

price of domestic consumption (now, c is a composite of domestically produced goods and imports)

$$\overbrace{p_t^c},$$

price of imports consumption price in ation

$$\overbrace{p_t^{m,c}},$$

$$\overbrace{\pi_t^c},$$

reduced form object to (i) achieve technical objective, (ii) correct a fundamental failing of open economy models

$$\overbrace{\Phi_t},$$

closed economy variables

$$\overbrace{R_t, \bar{\pi}_t, N_t, c_t, K_t, F_t, p_t^*}.$$

Extension to Small Open Economy ...

- computing the steady state
- the ‘uncovered interest parity puzzle’, and the role of Φ_t in addressing the puzzle.
- summary of the endogenous and exogenous variables of the model, as well as the equations.
- several computational experiments to illustrate the properties of the model.

Extension to Small Open Economy ...

- Modifications to basic model to create open economy

- unchanged:

- * household preferences
- * production of (domestic) homogeneous good, $Y_t (= A_t p_t^* N_t)$
- * three Calvo price friction equations

- changes:

- * household budget constraint includes opportunity to acquire foreign assets/liabilities.
- * intertemporal Euler equation changed as a reduced form accommodation of evidence on uncovered interest parity.
- * $Y_t = C_t$ no longer true.
- * introduce exports, imports, current account.
- * exchange rate,

$S_t = \text{domestic currency price of one unit of foreign currency} = \frac{\text{domestic money}}{\text{foreign money}} .$

Extension to Small Open Economy ...

- Monetary policy: three approaches

- Taylor rule

$$\log \left(\frac{R_t}{R} \right) = \rho_R \log \left(\frac{R_{t-1}}{R} \right) + (1 - \rho_R) E_t \left[r_\pi \log \left(\frac{\pi_{t+1}^c}{\bar{\pi}^c} \right) + r_y \log \left(\frac{y_{t+1}}{y} \right) \right] + \varepsilon_{R,t}, \quad (5)$$

where (could also add exchange rate, real exchange rate and other things):

$\pi^{-c} \sim$ target consumer price inflation

$\varepsilon_{R,t} \sim$ iid, mean zero monetary policy shock

$y_t \sim Y_t / A_t$

$R_t \sim$ ‘risk free’ nominal rate of interest

- a policy that solves Ramsey problem with the following preferences:

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \left(100 \left[\pi_t^c \pi_{t-1}^c \pi_{t-2}^c \pi_{t-3}^c - (\bar{\pi}^c)^4 \right] \right)^2 + \lambda_y \left(100 \log \left(\frac{y_t}{y} \right) \right)^2 + \lambda_{\Delta R} (400 [R_t - R_{t-1}])^2 + \lambda_s (S_t - \bar{S})^2 \right\}$$

- straight Ramsey policy that maximizes domestic social welfare.

Extension to Small Open Economy ...

- Household budget constraint

$$S_t A_{t+1}^f + P_t C_t + B_{t+1} \\ \leq B_t R_{t-1} + S_t \left[\Phi_{t-1} R_{t-1}^f \right] A_t^f + W_t N_t + \text{Transfers and profits}_t$$

- Domestic bonds

B_t ~beginning of period t stock of loans

R_t ~rate of return on bonds

- Foreign assets

A_t^f ~beginning-of-period t net stock of foreign assets

(liabilities, if negative) held by domestic residents.

$\Phi_t R_t^f$ ~rate of return on A_t^f

Φ_t ~premium on foreign asset returns, over foreign risk free rate, R_t^f

Extension to Small Open Economy ...

- optimality of household foreign asset decision (verify this by solving Lagrangian)

utility cost of one unit of foreign currency = S_t units of domestic currency, which is S_t/P_t^c units of C_t

$$\begin{aligned}
 & \frac{\overbrace{u_{c,t} S_t}}{P_t^c} \\
 = & \beta E_t \overbrace{u_{c,t+1}}^{\text{conversion into utility units}} \\
 & \underbrace{\text{quantity of domestic consumption goods that can be purchased from the payoff of one unit of foreign currency}}_{\text{foreign currency payoff next period from one unit of foreign currency today}} \\
 \times & \frac{S_{t+1} \overbrace{R_t^f \Phi_t}}{P_{t+1}^c}
 \end{aligned}$$

or

$$\frac{S_t}{P_t^c C_t} = \beta E_t \frac{S_{t+1} R_t^f \Phi_t}{P_{t+1}^c C_{t+1}}$$

or

$$\frac{1}{c_t} = \beta E_t \frac{s_{t+1} R_t^f \Phi_t}{\pi_{t+1}^c c_{t+1} \exp(\Delta a_{t+1})}, \quad s_t \equiv \frac{S_t}{S_{t-1}}, \quad c_t = \frac{C_t}{A_t}. \quad (6)$$

Extension to Small Open Economy ...

- Optimality of household domestic bond decision:

$$\frac{1}{P_t^c C_t} = \beta E_t \frac{R_t}{P_{t+1}^c C_{t+1}}$$

– after scaling:

$$\frac{1}{c_t} = \beta E_t \frac{R_t}{\pi_{t+1}^c c_{t+1} \exp(\Delta a_{t+1})}. \quad (7)$$

Extension to Small Open Economy ...

- Final domestic consumption goods, C_t
 - produced by representative, competitive firm using:

$$C_t = \left[(1 - \omega_c)^{\frac{1}{\eta_c}} (C_t^d)^{\frac{(\eta_c-1)}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} (C_t^m)^{\frac{(\eta_c-1)}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}$$

C_t^d ~ one-for-one transformation on domestic homogeneous output good, price P_t

C_t^m ~ imported good, with price $P_t^{m,c}$

C_t ~ final consumption good, with price, P_t^c

η_c ~ elasticity of substitution between domestic and foreign goods.

– Profit maximization leads to:

$$C_t^d = (1 - \omega_c) (p_t^c)^{\eta_c} C_t$$

$$C_t^m = \omega_c \left(\frac{p_t^c}{p_t^{m,c}} \right)^{\eta_c} C_t.$$

$$p_t^c = \left[(1 - \omega_c) + \omega_c (p_t^{m,c})^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \quad (8)$$

$$p_t^c \equiv \frac{P_t^c}{P_t}, \quad p_t^{m,c} \equiv \frac{P_t^{m,c}}{P_t}$$

Extension to Small Open Economy ...

- C_t^m is produced by competitive firm, which converts foreign homogeneous output one-for-one into C_t^m .

* Setting price equal to marginal cost:

$$P_t^{m,c} = S_t P_t^f \left(1 - \psi^f + \psi^f R_t^f \right), \quad P_t^f \sim \text{foreign currency price of foreign good.}$$

or,

$$p_t^{m,c} = \frac{P_t^c S_t P_t^f}{P_t^c P_t} \left(1 - \psi^f + \psi^f R_t^f \right) = p_t^c \underbrace{\frac{S_t P_t^f}{P_t}}_{q_t} \left(1 - \psi^f + \psi^f R_t^f \right).$$

(9)

- Consumption good inflation:

$$\pi_t^c \equiv \frac{P_t^c}{P_{t-1}^c} = \frac{P_t p_t^c}{P_{t-1} p_{t-1}^c} = \bar{\pi}_t \left[\frac{(1 - \omega_c) + \omega_c (p_t^{m,c})^{1-\eta_c}}{(1 - \omega_c) + \omega_c (p_{t-1}^{m,c})^{1-\eta_c}} \right]^{\frac{1}{1-\eta_c}}. \quad (10)$$

Extension to Small Open Economy ...

- Exports, X_t
 - foreign demand for exports

$$X_t = \left(\frac{P_t^x}{P_t^f} \right)^{-\eta_f} Y_t^f = (p_t^x)^{-\eta_f} Y_t^f \quad (11)$$

Y_t^f ~ foreign output, P_t^f ~ price of foreign good, P_t^x ~ price of export

- X_t is produced one-for-one using the domestic homogeneous good by a representative, competitive producer. Equating price, $S_t P_t^x$, to marginal cost:

where $\nu^x = 1$ if all inputs must be financed in advance. Rewriting

$$q_t p_t^x p_t^c = \nu^x R_t + 1 - \nu^x, \quad (12)$$

where

$$\underbrace{q_t}_{\text{real exchange rate}} \equiv \frac{S_t P_t^f}{P_t^c}, \quad \underbrace{p_t^x}_{\text{terms of trade}} = \frac{P_t^x}{P_t^f}$$

$$\frac{q_t}{q_{t-1}} = s_t \frac{\pi_t^f}{\pi_t^c}, \quad s_t = \frac{S_t}{S_{t-1}}. \quad (13)$$

Extension to Small Open Economy ...

- Clearing in domestic homogeneous goods market:

$$\begin{aligned}
 & \text{output of domestic homogeneous good, } Y_t \\
 = & \text{uses of domestic homogeneous goods} \\
 = & \text{goods used in production of } \underbrace{C_t^d}_{\text{domestic consumption, } C_t} + \underbrace{X_t}_{\text{exports}} + \underbrace{G_t}_{\text{government}} \\
 = & (1 - \omega_c) (p_t^c)^{\eta_c} C_t + X_t + G_t.
 \end{aligned}$$

- Substituting out for Y_t :

$$A_t p_t^* N_t = (1 - \omega_c) (p_t^c)^{\eta_c} C_t + X_t + G_t,$$

or,

$$p_t^* N_t = (1 - \omega_c) (p_t^c)^{\eta_c} c_t + x_t + g_t, \quad (14)$$

$$c_t \equiv \frac{C_t}{A_t}, \quad x_t \equiv \frac{X_t}{A_t}, \quad g_t \equiv \frac{G_t}{A_t}.$$

Extension to Small Open Economy ...

● Balance of Payments

- equality of international demand and supply for currency:

$$\underbrace{\text{acquisition of new net foreign assets, in domestic currency units}}_{S_t A_{t+1}^f} \quad \text{currency owing out of the country} \quad + \text{expenses on imports}_t$$

$$\underbrace{\text{receipts from existing stock of net foreign assets}}_{S_t R_{t-1}^f \Phi_{t-1} A_t^f} \quad \text{currency owing into the country}$$

$$= \text{receipts from exports}_t + \quad ,$$

- The pieces:

$$\text{expenses on imports}_t = S_t P_t^f \left(1 - \psi^f + \psi^f R_t^f \right) \omega_c \left(\frac{p_t^c}{p_t^{m,c}} \right)^{\eta_c} C_t$$

$$\text{receipts from exports}_t = S_t P_t^x X_t.$$

- Balance of payments:

$$S_t A_{t+1}^f + S_t P_t^f \left(1 - \psi^f + \psi^f R_t^f \right) \omega_c \left(\frac{p_t^c}{p_t^{m,c}} \right)^{\eta_c} C_t = S_t P_t^x X_t + S_t R_{t-1}^f \Phi_{t-1} A_t^f.$$

Extension to Small Open Economy ...

– Divide balance of payments by $P_t A_t$:

$$\frac{S_t A_{t+1}^f}{P_t A_t} + \frac{S_t P_t^f}{P_t} \left(1 - \psi^f + \psi^f R_t^f\right) \omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta_c} c_t = \frac{S_t P_t^x}{P_t} x_t + \frac{S_t R_{t-1}^f \Phi_{t-1} A_t^f}{P_t A_t},$$

or, using (9):

$$a_t^f + p_t^{m,c} \omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta_c} c_t = p_t^c q_t p_t^x x_t + \frac{s_t R_{t-1}^f \Phi_{t-1} a_{t-1}^f}{\bar{\pi}_t \exp(\Delta a_t)}, \quad (15)$$

where a_t^f is ‘scaled real, domestic value of foreign assets’:

$$a_t^f = \frac{S_t A_{t+1}^f}{P_t A_t}$$

Extension to Small Open Economy ...

- ‘Risk’ adjustments

$$\begin{aligned}\Phi_t &= \Phi \left(a_t^f, R_t^f, R_t, \tilde{\phi}_t \right) = & (16) \\ &\exp \left(-\tilde{\phi}_a \left(a_t^f - \bar{a} \right) - \tilde{\phi}_s \left(R_t^f - R_t - \left(R^f - R \right) \right) + \tilde{\phi}_t \right) \\ &\tilde{\phi}_a > 0, \text{ small and not important for dynamics} \\ &\tilde{\phi}_s > 0, \text{ important} \\ &\tilde{\phi}_t \sim \text{mean zero, iid.}\end{aligned}$$

- Discussion of $\tilde{\phi}_a$.
 - $\tilde{\phi}_a > 0$ implies (i) if $a_t^f > \bar{a}$, then return on foreign assets low and $a_t^f \downarrow$; (ii) if $a_t^f < \bar{a}$, then return on foreign assets high and $a_t^f \uparrow$
 - implication: $\tilde{\phi}_a > 0$ is a force that drives $a_t^f \rightarrow \bar{a}$ in steady state, independent of initial conditions.
 - logic is same as reason why steady state stock of capital in neoclassical growth model is unique, independent of initial conditions.
 - in practice, $\tilde{\phi}_a$ is tiny, so that its only effect is to pin down a_t^f in steady state and not affect dynamics (see Schmitt-Grohe and Uribe).

Extension to Small Open Economy ...

- Discussion of $\tilde{\phi}_t$
 - Captures, informally, the possibility that there is a shock to the required return on domestic assets. Perhaps this could be a crude stand-in for a ‘sub-prime mortgage crisis’, because it implies that people require a higher return on domestic assets if they are to hold them.
- Discussion of $\tilde{\phi}_s$.
 - $\tilde{\phi}_s$ is an important reduced form feature, designed to correct a flaw in models of international finance. It represents a quick fix for the problem, not a substitute for a longer-run solution.
 - to better explain this, it is convenient to first solve for the model’s steady state.

Extension to Small Open Economy ...

- Steady state

– household intertemporal efficiency conditions:

$$0 = E_t \left[\frac{1}{c_t} - \beta \frac{s_{t+1} R_t^f \Phi_t}{\pi_{t+1}^c c_{t+1} \exp(\Delta a_{t+1})} \right], \text{ steady state: } 1 = \beta \frac{s R^f \Phi}{\pi^c} \quad (17)$$

$$0 = E_t \left[\frac{1}{c_t} - \beta \frac{1}{c_{t+1} \pi_{t+1}^c} \frac{R_t}{\exp(\Delta a_{t+1})} \right], \text{ steady state: } 1 = \beta \frac{R}{\pi^c} \quad (18)$$

– assumption about foreign households:

$$1 = \beta \frac{R^f}{\pi^f} \quad (19)$$

$$\pi_t^f \equiv \frac{P_t^f}{P_{t-1}^f} \text{ (exogenous)}$$

Extension to Small Open Economy ...

– Taylor rule:

$$\pi^c = \pi^{-c} \text{ (central bank's inflation target).} \quad (20)$$

– from (10):

$$\pi^c = \bar{\pi} \equiv \frac{P_t}{P_{t-1}}. \quad (21)$$

– using price friction equilibrium conditions:

$$p^* = \frac{\frac{1-\theta\bar{\pi}^\varepsilon}{1-\theta}}{\left(\frac{1-\theta(\bar{\pi})^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}}, \text{ (no distortion if } \bar{\pi} = 1, \text{)} \quad (22)$$

$$F = \frac{1}{1 - \beta\theta\bar{\pi}^{\varepsilon-1}}, \text{ (don't allow } \bar{\pi}^{\varepsilon-1}\beta\theta < 1 \text{)} \quad (23)$$

$$K = \frac{\frac{\varepsilon}{\varepsilon-1} (1 - \nu) \exp(\tau) N^{\varphi+1} p^* (1 - \psi + \psi R)}{1 - \beta\theta\bar{\pi}^\varepsilon}, \text{ (} \beta\theta\bar{\pi}^\varepsilon < 1 \text{)} \quad (24)$$

$$K = F \left[\frac{1 - \theta\bar{\pi}^{\varepsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}}. \quad (25)$$

Extension to Small Open Economy ...

– other steady state conditions:

$$p^c = \left[(1 - \omega_c) + \omega_c (p^{m,c})^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \quad (26)$$

$$p^{m,c} = p^c q (1 - \psi^f + \psi^f R^f) \quad (27)$$

$$qp^x p^c = \nu^x R + 1 - \nu^x \quad (28)$$

$$p^* N = (1 - \omega_c) (p^c)^{\eta_c} c + x + g \quad (29)$$

$$a^f + p^c q (1 - \psi^f + \psi^f R^f) \omega_c \left(\frac{p^c}{p^{m,c}} \right)^{\eta_c} c = p^c qp^x x + \frac{sR^f \Phi a^f}{\bar{\pi}} \quad (30)$$

$$x = (p^x)^{-\eta_f} y^f \quad (31)$$

– 15 equations: (17)-(31), 15 unknowns:

$$p^c, p^{m,c}, q, p^x, c, x, a^f, R, \Phi, \bar{\pi}, K, F, p^*, \pi^c, s$$

– for convenience, set exogenous variables, g, \bar{a} ,

$$g = \eta_g y, \bar{a} = \eta_a y, \text{ where } y = p^* N.$$

Extension to Small Open Economy ...

– algorithm for solving for the steady state:

* p^* , F , K can be computed from (21), (22), (23) and (25).

* solve (24) for N .

* solve

$$\begin{aligned}g &= \eta_g p^* N \\ a^f &= \bar{a} = \eta_a p^* N.\end{aligned}$$

* (18), (19) imply

$$\frac{R^f}{\pi^f} = \frac{R}{\pi^c} \quad (32)$$

* steady state depreciation, s , can be computed from the inflation differential:

$$q_t \rightarrow q \text{ implies (see (13)) } s\pi^f = \pi^c.$$

* (17), (18) imply

$$sR^f\Phi = R, \quad (33)$$

or after multiplication by π^f and rearranging,

$$\frac{R^f}{\pi^f}\Phi = \frac{R}{\pi^c}, \text{ so (see (32)) } \Phi = 1 \text{ and } a_t = \bar{a} \text{ (see (16))}$$

Extension to Small Open Economy ...

- rest of the algorithm solves a single non-linear equation in a single unknown.
- set

$$\tilde{\varphi} = p^c q.$$

- use (27), (28), (26):

$$p^{m,c} = \tilde{\varphi} R^{\nu,*}$$

$$p^x = \frac{R^x}{\tilde{\varphi}},$$

$$p^c = \left[(1 - \omega_c) + \omega_c (p^{m,c})^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}$$

$$q = \frac{\tilde{\varphi}}{p^c}.$$

- solve the resource constraint, (29), for c in terms of x :

$$c = \frac{p^c q p^x x + \frac{sR^f a^f}{\pi} - a^f}{p^c q (1 - \psi^f + \psi^f R^f) \omega_c \left(\frac{p^c}{p^{m,c}} \right)^{\eta_c}}.$$

Extension to Small Open Economy ...

- use the latter to substitute out for c in the current account, (30):

$$\begin{aligned} & a^f + p^c q (1 - \psi^f + \psi^f R^f) \omega_c \left(\frac{p^c}{p^{m,c}} \right)^{\eta_c} \frac{p^c q p^x x + \frac{sR^f a^f}{\bar{\pi}} - a^f}{p^c q (1 - \psi^f + \psi^f R^f) \omega_c \left(\frac{p^c}{p^{m,c}} \right)^{\eta_c}} \\ &= p^c q p^x x + \frac{sR^f a^f}{\bar{\pi}}, \end{aligned}$$

which can be solved linearly for x .

- evaluate (31) and adjust φ^{\sim} until it is satisfied. In practice, we set $\varphi^{\sim} = 1$ and used (31) to determine y^f .

Extension to Small Open Economy ...

- Uncovered interest rate parity puzzle and Φ_t^b

– subtract equations (17) and (18):

$$E_t \left[\frac{R_t - s_{t+1} R_t^f \Phi_t}{c_{t+1} \pi_{t+1}^c \exp(\Delta a_{t+1})} \right] = 0. \quad (34)$$

– totally differentiate the object in square brackets, and evaluate in steady state

$$\begin{aligned} d \frac{R_t - s_{t+1} R_t^f \Phi_t}{c_{t+1} \pi_{t+1}^c \exp(\Delta a_{t+1})} &= \frac{dR_t}{c\pi^c} - \frac{1}{c\pi^c} \left[sR^f d\Phi_t + sdR_t^f + R^f ds_{t+1} \right] \\ &\quad - \frac{R - sR^f}{[c\pi^c]^2} d [c_{t+1} \pi_{t+1}^c \exp(\Delta a_{t+1})], \end{aligned}$$

so that, taking into account (33), (34) is, to a first approximation:

$$\hat{R}_t = E_t \hat{s}_{t+1} + \hat{R}_t^f + \hat{\Phi}_t, \quad \hat{x}_t = \log(x_t) - \log(x) = \frac{x_t - x}{x}.$$

Extension to Small Open Economy ...

– Note:

$$\hat{R}_t = \log R_t - \log (R) \simeq r_t - \log R, \quad \hat{R}_t^f = \log R_t^f - \log (R^f) \simeq r_t^f - \log R^f$$

$$R_t \equiv 1 + r_t, \quad R_t^f \equiv 1 + r_t^f,$$

so that:

$$r_t - \log (R) = \log S_{t+1} - \log S_t - \log s + r_t^f - \log R^f + \hat{\Phi}_t.$$

It follows from:

$$\log (R) - \log s - \log R^f = \log \left(\frac{R}{sR^f} \right) = 0,$$

that

$$\boxed{r_t = E_t \log S_{t+1} - \log S_t + r_t^f + \hat{\Phi}_t} \quad (35)$$

$\hat{\Phi}_t = \log \Phi_t = -\tilde{\phi}_a \left(a_t^f - \bar{a} \right) - \tilde{\phi}_s \left(r_t^f - r_t - (r^f - r) \right) + \tilde{\phi}_t$
which is our log-linear expansion of (34).

Extension to Small Open Economy ...

– Uncovered Interest Parity (UIP)

* Under UIP, $\hat{\Phi}_t \equiv 0$ and

$r_t > r_t^f \rightarrow$ there must be an anticipated depreciation (instantaneous appreciation) of the currency for people to be happy holding the existing stock of net foreign assets.

* Consider the standard ‘UIP regression’ ($\tilde{\phi}_a \simeq 0, \tilde{\phi}_t = 0$), involving risk free rate differentials:

$$\log S_{t+1} - \log S_t = \alpha + \beta \left(r_t - r_t^f \right) + u_t.$$

* Substitute out for $\log S_{t+1} - \log S_t$ from (35) and make use of the fact that a (rational expectations) forecast error is orthogonal to date t information, to obtain:

$$\hat{\beta} = \frac{\text{cov} \left(\log S_{t+1} - \log S_t, r_t - r_t^f \right)}{\text{var} \left(r_t - r_t^f \right)} = 1 - \tilde{\phi}_s,$$

Extension to Small Open Economy ...

* In data,

· $\hat{\beta} \simeq -0.75$, so UIP rejected (that's the *UIP puzzle*)

· $\tilde{\phi}_s = 1.75 \rightarrow \hat{\beta} = -0.75$.

* VAR impulse responses by Eichenbaum and Evans (QJE, 1992)

· data: $r_t \uparrow$ after monetary policy shock $\rightarrow \log S_{t+j}$ falls slowly for $j = 1, 2, 3, \dots$.

· UIP puzzle: $r_t \uparrow$ and expected appreciation of the currency represents a double-boost to the return on domestic assets. On the face of it, it appears that there is an irresistible profit opportunity. Why doesn't the double-boost to domestic returns launch an avalanche of pressure to buy the domestic currency? In standard models, this pressure produces a greater instantaneous appreciation in the exchange rate, until the familiar UIP overshooting result emerges -the pressure to buy the currency leads to such a large appreciation,

Extension to Small Open Economy ...

that expectations of depreciation emerge. In this way, UIP leads to the counterfactual prediction that a higher r_t will be followed (after an instantaneous appreciation) by a period of time during which the currency depreciates.

- model's resolution of the UIP puzzle: when $r_t \uparrow$ the return required for people to hold domestic bonds rises. This is why the double-boost to domestic returns does not create an appetite to buy large amounts of domestic assets. Possibly this is a reduced form way to capture the notion that increases in r_t make the domestic economy more risky. (However, the precise mechanism by which the domestic required return rises - earnings on *foreign* assets go up - may be difficult to interpret. An alternative specification was explored, with risk-premia affecting domestic bonds, but this resulted in indeterminacy problems.)

Extension to Small Open Economy ...

- Model dynamics

- 16 equations: price setting, (1), (2),(3) and (4); monetary policy rule, (5); household intertemporal Euler equations (6), (7); relative price equations (13), (8), (9), (10), (12); aggregate resource condition, (14); current account, (15); risk term, (16); demand for exports (11).
- 16 endogenous variables: $p_t^c, p_t^{m,c}, q_t, R_t, \bar{\pi}_t, \pi_t^c, p_t^x, N_t, p_t^*, a_t^f, \hat{\Phi}_t, s_t, x_t, c_t, K_t, F_t$.
- exogenous variables: $R_t^f, y_t^f, \tilde{\phi}_t, g_t, \varepsilon_{R,t}, \Delta a_t, \tau_t, \pi_t^f$.
for the purpose of numerical calculations, these were modeled as independent scalar AR(1) processes.
- the model was solved in the manner described above:
 - * compute the steady state using the formulas described above
 - * log-linearize the 16 equations about steady state
 - * solve the log-linearized system
 - * these calculations were made easy by implementing them in Dynare.

Extension to Small Open Economy ...

- Numerical examples
- Parameter values:

foreign and domestic inflation same $\overbrace{\bar{\pi}^c = \pi^f} = 1.005$, $\overbrace{\psi = \psi^f = \nu^x = 0}$, $\overbrace{\tilde{\phi}_a = 0.03}$, $\overbrace{\text{small weight on } a_t^f \text{ in } \Phi_t}$,

prices unchanged on average for 1 year $\beta = 1.03^{-1/4}$, $\overbrace{\theta = 3/4}$, $\overbrace{\varphi = 1}$, $\overbrace{1/\varphi \text{ Frisch elasticity}}$,

modest elasticity of demand for domestic intermediate goods $\overbrace{\varepsilon = 6}$, $\overbrace{1 - \nu_t = \frac{\varepsilon - 1}{\varepsilon}}$, $\overbrace{\text{subsidy extinguishes monopoly power in labor margin}}$,

elasticity of substitution between domestic and foreign inputs in producing domestic consumption $\overbrace{\eta_c = 5}$

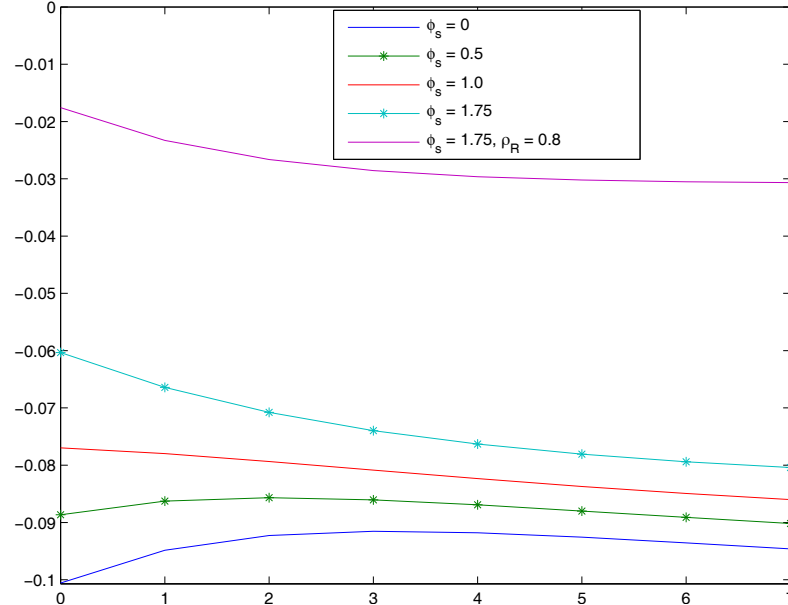
roughly 60% of domestic domestic consumption is composed of domestic content $\overbrace{\omega_c = 0.4}$, $\overbrace{\eta_g = 0.3}$, $\overbrace{\eta_a = 0}$, $\overbrace{\text{share of } g \text{ in } y \text{ net foreign assets/y}}$,

elasticity of demand for exports as function of relative price paid by foreigners $\overbrace{\eta_f = 1.5}$, $\overbrace{\rho_R = 0.9, r_\pi = 1.5, r_y = 0.15}$, $\overbrace{\text{monetary policy rule parameters}}$

Extension to Small Open Economy ...

- iid shock, 0.01, to $\varepsilon_{R,t}$.
 - $\tilde{\phi}_s = 0 \rightarrow$ after instantaneous appreciation, positive $\varepsilon_{R,t}$ shock followed by depreciation.
 - for higher $\tilde{\phi}_s$, shock followed by appreciation.
 - long run appreciation is increasing function of persistence of ρ_R .

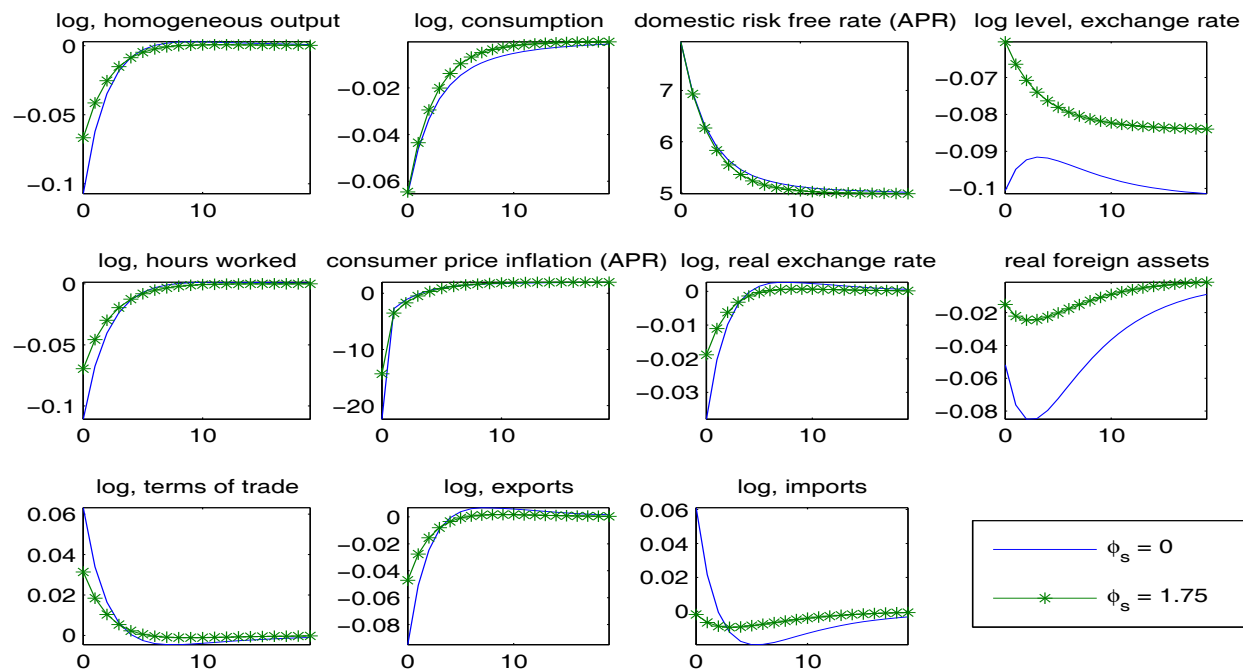
Response of log exchange rate to monetary policy shock



Extension to Small Open Economy ...

We now consider a monetary policy shock, $\varepsilon_{R,t} = 0.01$. According to (5), implies a four percentage point (at an annual rate) policy-induced jump in R_t . The dynamic effects are displayed in the following figure, for $\tilde{\phi}_s = 0$, $\tilde{\phi}_s = 1.75$.

response to monetary policy shock

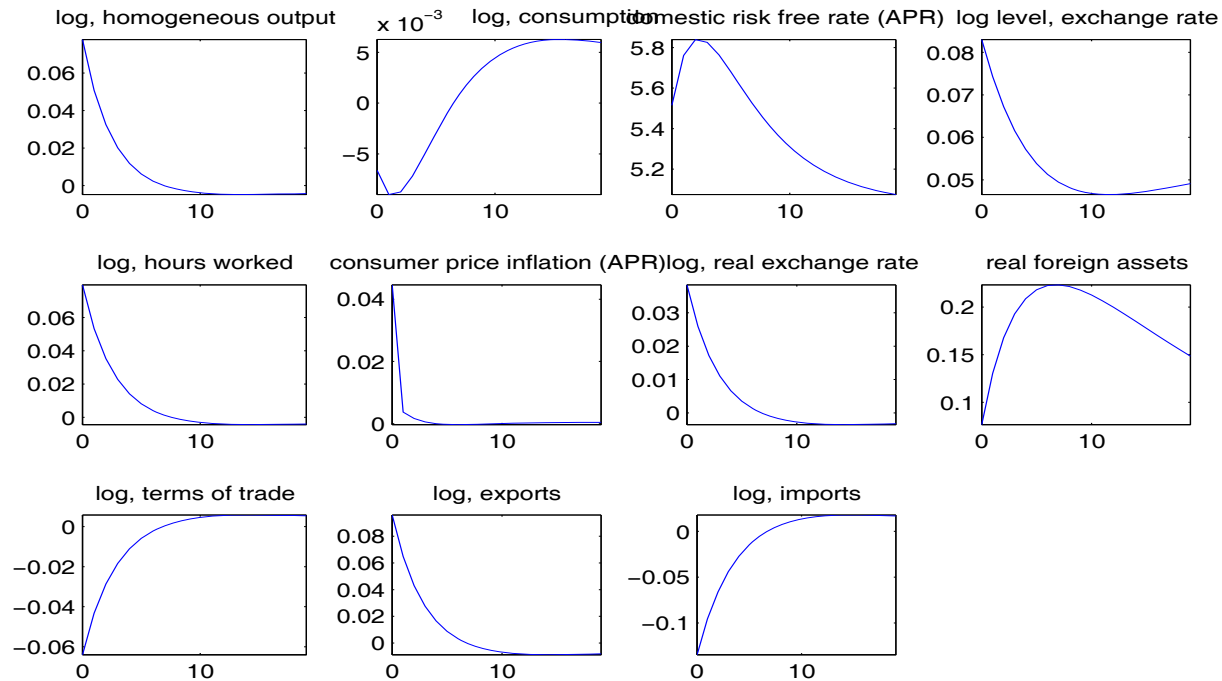


Note: (i) appreciation smaller, though a more drawn out, when $\tilde{\phi}_s$ is big; (ii) smaller appreciation results in smaller drop in net exports, so less of a drop in demand, so less fall in output and in inflation; (iii) smaller drop in net exports results in smaller drop in real foreign assets.

Extension to Small Open Economy ...

Consider now a domestic economy risk premium shock, a jump in the innovation to $\tilde{\phi}_t$ equal to 0.01.

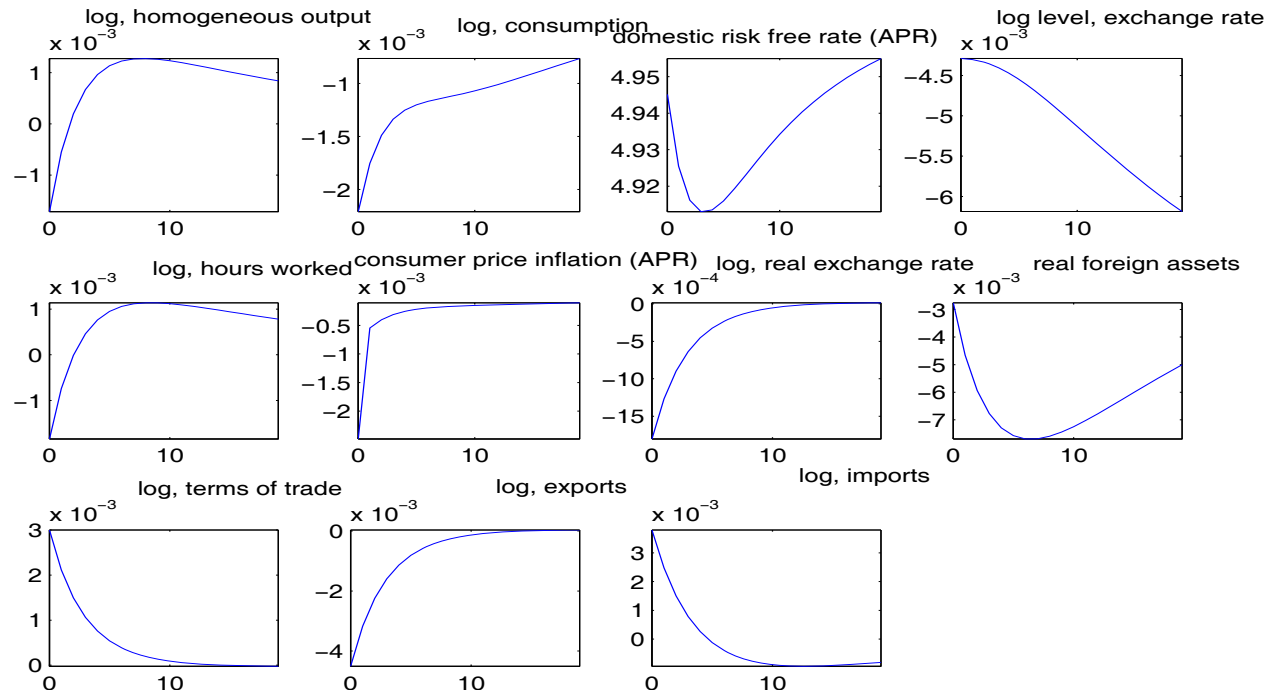
response to country risk premium shock



With the reduced interest in domestic assets, (i) the currency depreciates, (ii) net exports rise, (iii) hours and output rise, (iv) the upward pressure on costs associated with higher output leads to a rise in prices.

Extension to Small Open Economy ...

Next we consider a 0.01 innovation in log, government consumption, g_t .
response to government consumption shock



After a delay, the higher g_t leads to a rise in output. However, there is so much crowding out in the short run that output actually falls. There is crowding out of net exports and consumption because of the effects created by a higher interest rate. The higher interest rate directly reduces consumption, and by making the currency appreciate, it produces a fall in net exports. The initial drop in government spending in the wake of a rise in government spending is interesting.

Model with Capital, Richer Import Sector and Price Frictions that Slow Down Exchange Rate Pass-Through (CTW)

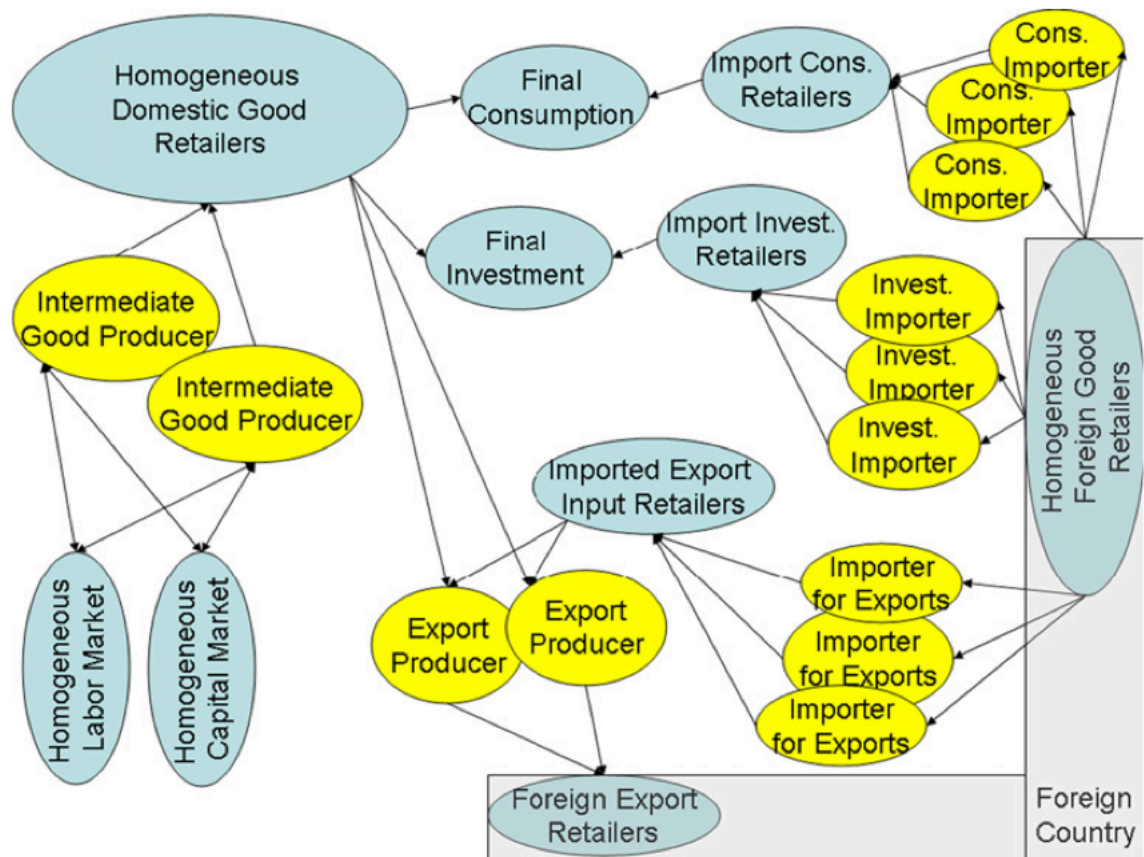
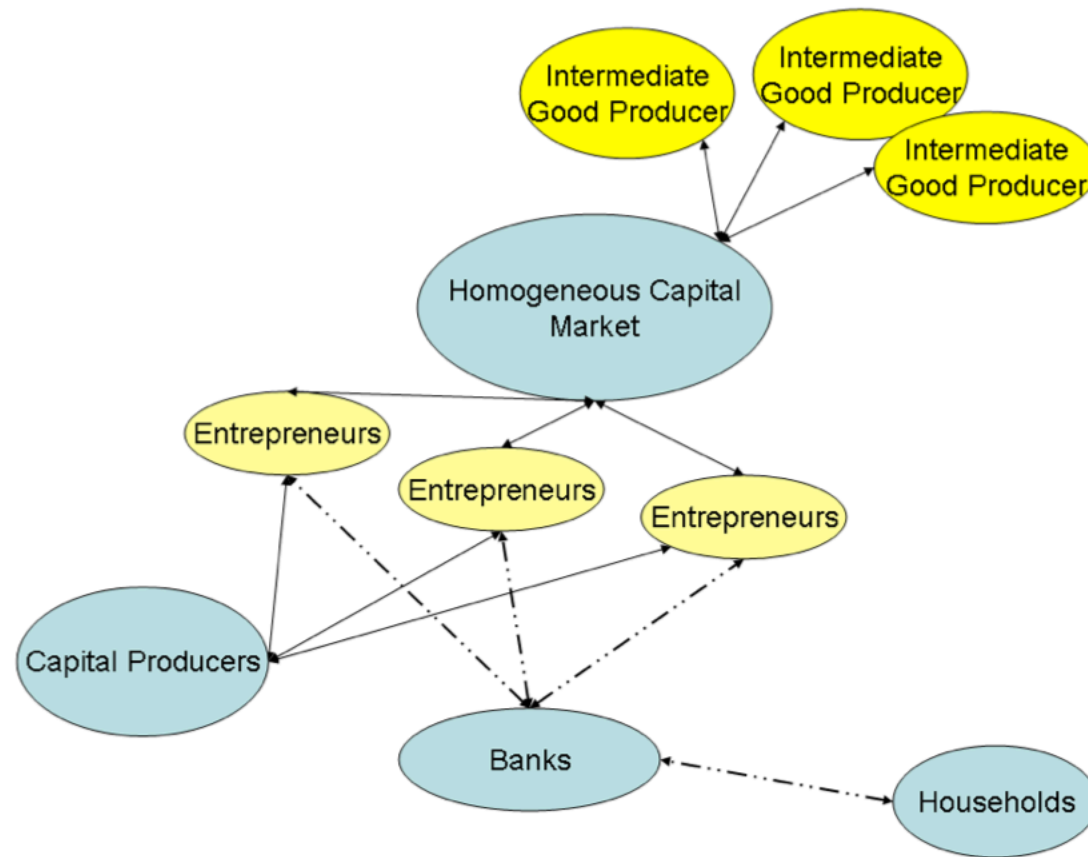


Fig. 1. Graphical illustration of the goods production part of the model.

Financial Frictions as the BGG or CMR in the Open Economy Model



Question Confronting Many Emerging Market Economies

- At some point, the Fed will implement its 'exit strategy' and raise US interest rates (300-350 basis points?).
- In the past, when Fed raised rates sharply (e.g., 1982 Volcker disinflation, 1994 run-up in interest rates), hit the rest of world like a brick:
 - Chilean crisis of 1982, Mexican default of August 1982. Mexican crisis of 1994.
- Will the US 'exit strategy' inflict financial crises around the world, especially in emerging market economies?
 - Last summer's 'taper episode' makes people worry about this possibility.
- I'll call the above possibility the BIS scenario.

BIS Scenario

- Low US interest rates since 2008 have encouraged 'excessive' accumulation of debt in the world.
- This has particularly affected emerging market economies in Asia and Latin America.

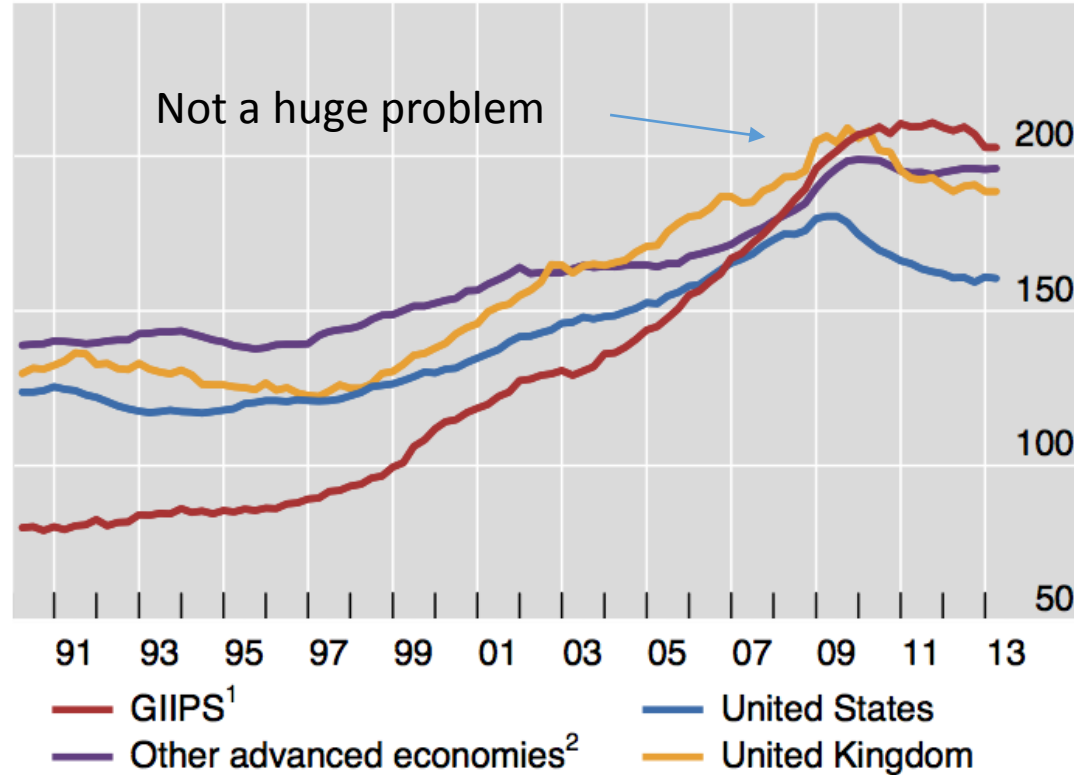
Credit to the private sector

Levels low relative to US, but still they grew to levels equal or greater than what they were in previous crises.

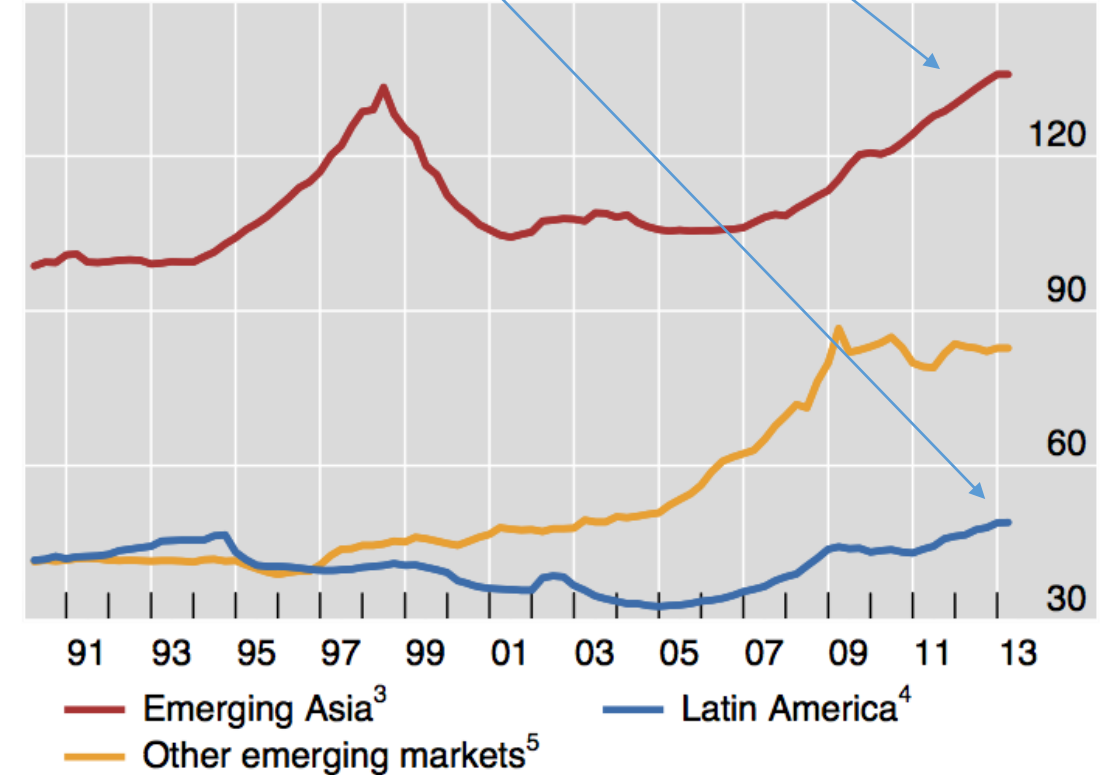
As a percentage of GDP; unweighted averages

Graph 2

Advanced economies



Emerging market economies



¹ Greece, Ireland, Italy, Portugal and Spain. ² Belgium, Canada, Denmark, France, Germany, Japan, the Netherlands, Norway and Sweden. ³ China, Hong Kong SAR, India, Indonesia, Korea, Malaysia, Singapore and Thailand. ⁴ Argentina, Brazil, Chile, Colombia, Mexico and Peru. ⁵ The Czech Republic, Hungary, Poland, Russia, South Africa and Turkey.

Sources: National data; BIS calculations.

Currency Mismatch Problem May Be Understated

(Hyun Shin, 'The Second Phase of Global Liquidity...', November, 2013)

- Many emerging market borrowers issue dollar-denominated debt through foreign subsidiaries (say in the UK).
- By the usual definition (based on the *residence* of the issuer), the bonds are a liability of the UK entity.
- But, it's the consolidated balance sheet that matters to the emerging market firm.
- So, the debt issued via a foreign subsidiary could make the emerging market firm vulnerable to currency mismatch problems.

- Hyun Shin argues:
 - Amount of dollar denominated debt from emerging market firms may be greatly understated.
 - This is suggested by evidence that foreign currency debt by *nationality* can be much larger than foreign debt by the usual *residency* definition.

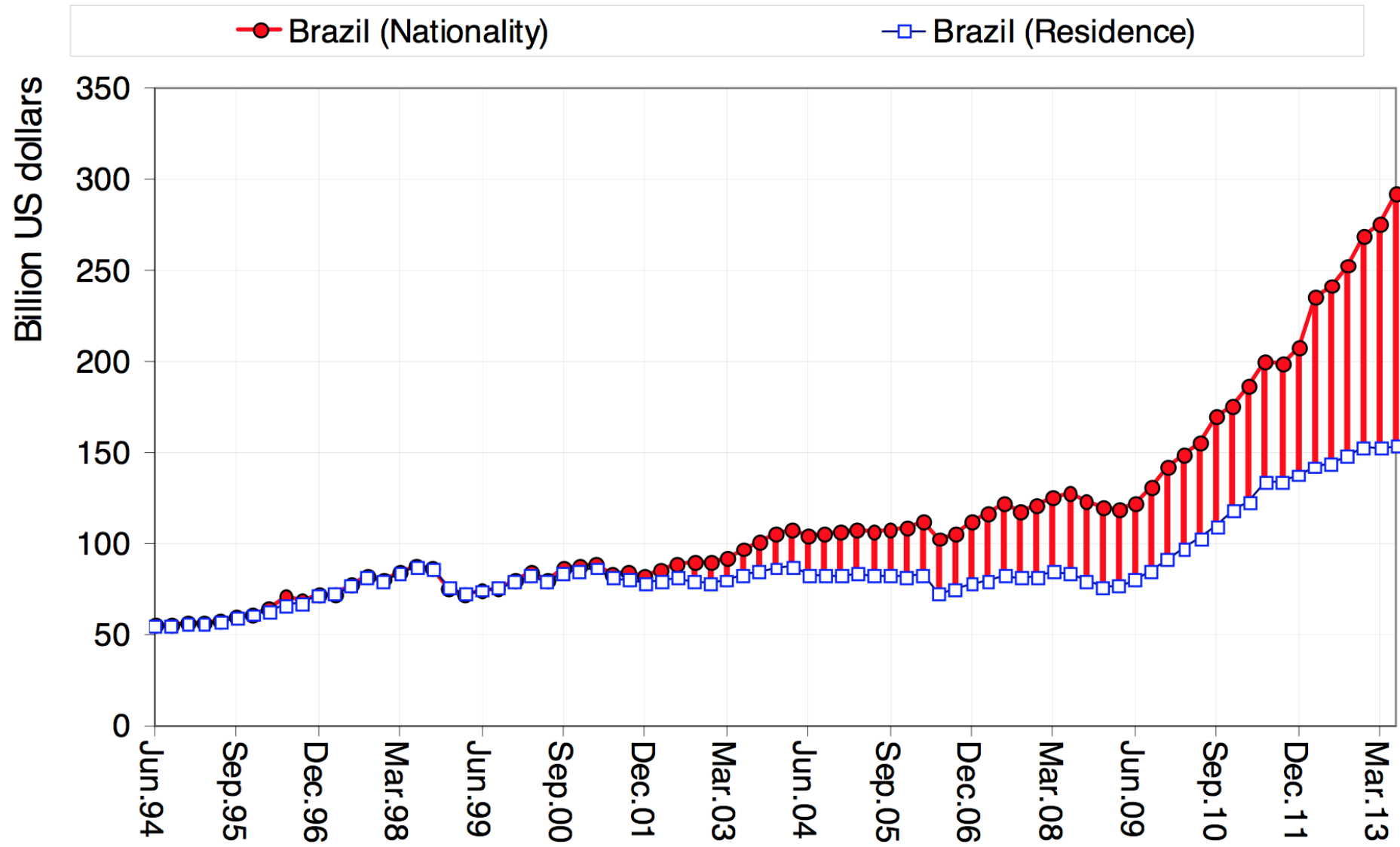


Figure 5. International debt securities outstanding (all borrowers) from Brazil by nationality and by residence (Source: BIS Debt Securities Statistics, Table 11A and 12A)

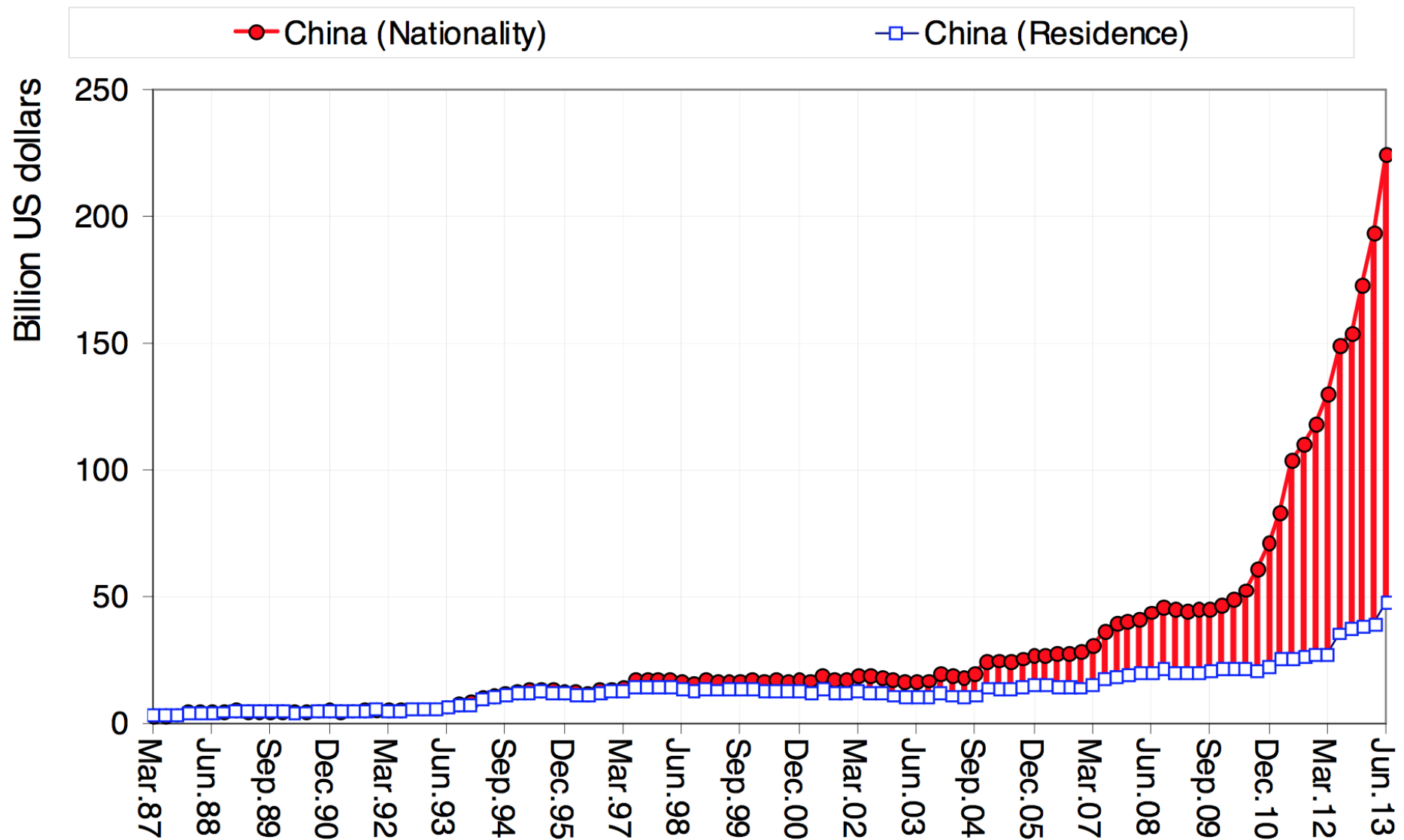


Figure 6. International debt securities outstanding (all borrowers) from China by nationality and by residence (Source: BIS Debt Securities Statistics, Table 11A and 12A)

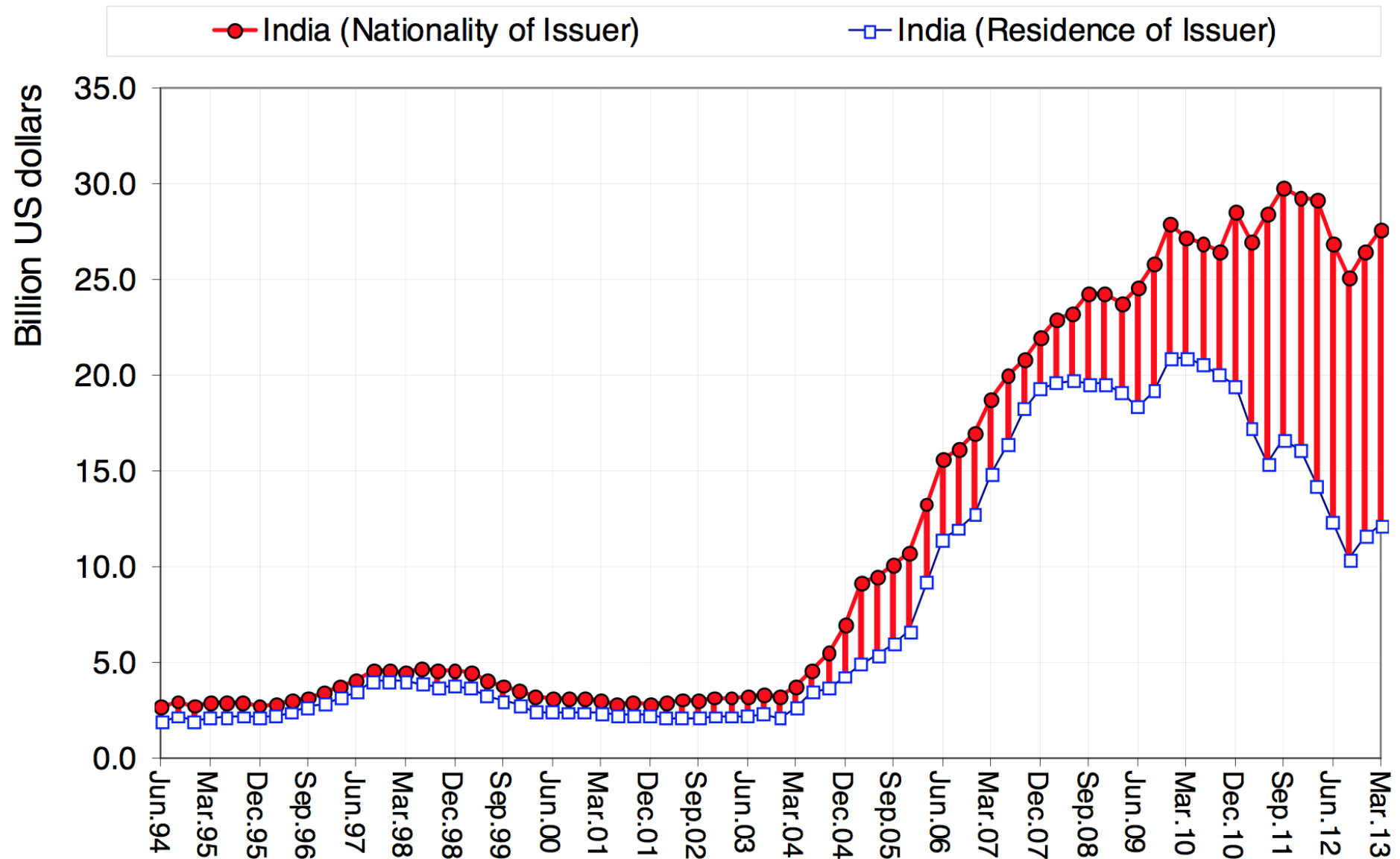


Figure 7. International debt securities outstanding for non-financial corporates from India by nationality and by residence (Source: BIS Debt Securities Statistics, Table 11D and 12D)

- Shin conjectures that distinction between external debt according to nationality and residence helps to resolve the ‘taper’ puzzle:
 - convulsions in emerging markets during taper episode last summer seem inconsistent with apparently small net external debt position (measured in residence terms) of firms in emerging markets.
 - Less surprising if external debt position is in fact much bigger.

BIS Scenario

- US raises interest rates.
- Emerging market exchange rates depreciate.
- Financial health of emerging market firms compromised.
- They cut back on investment activity...recessions start.
- Runs on emerging market banks known to have made loans to now-questionable emerging market non-financial firms.
- And so on...

Locomotive Scenario

- Previous episodes of US interest rate hikes may be playing too big a role in the pessimistic outlook.
- The circumstances in which the US raises interest rates may make a difference.
 - Some previous episodes in which US raised rates were designed to fight inflation, not a response to strong US economy.
- In present circumstances, Fed has (credibly, I think) committed to only raise rates until ***well after*** the US economy has returned to health.
- Under these circumstances, interest rate hikes occur when the US is firmly in the position of a locomotive, pulling the rest of the world economy forward in its wake.

Which Will it Be: BIS or Locomotive Scenario?

- Need a model to think about this question.
- Build in the BIS-type factors that raise concerns about the world economy.
 - Accurately capture the degree of foreign currency indebtedness of financial and non-financial firms (i.e., avoid the biases that Hyun is concerned about).
 - Build in the nature of the constraints that cause firms to pull back when their net worth contracts with exchange rate depreciation.
- Build in the 'locomotive' scenario:
 - Carefully model 'forward guidance' – Fed commitment to keep interest rates low even after the US economy has begun to strengthen.

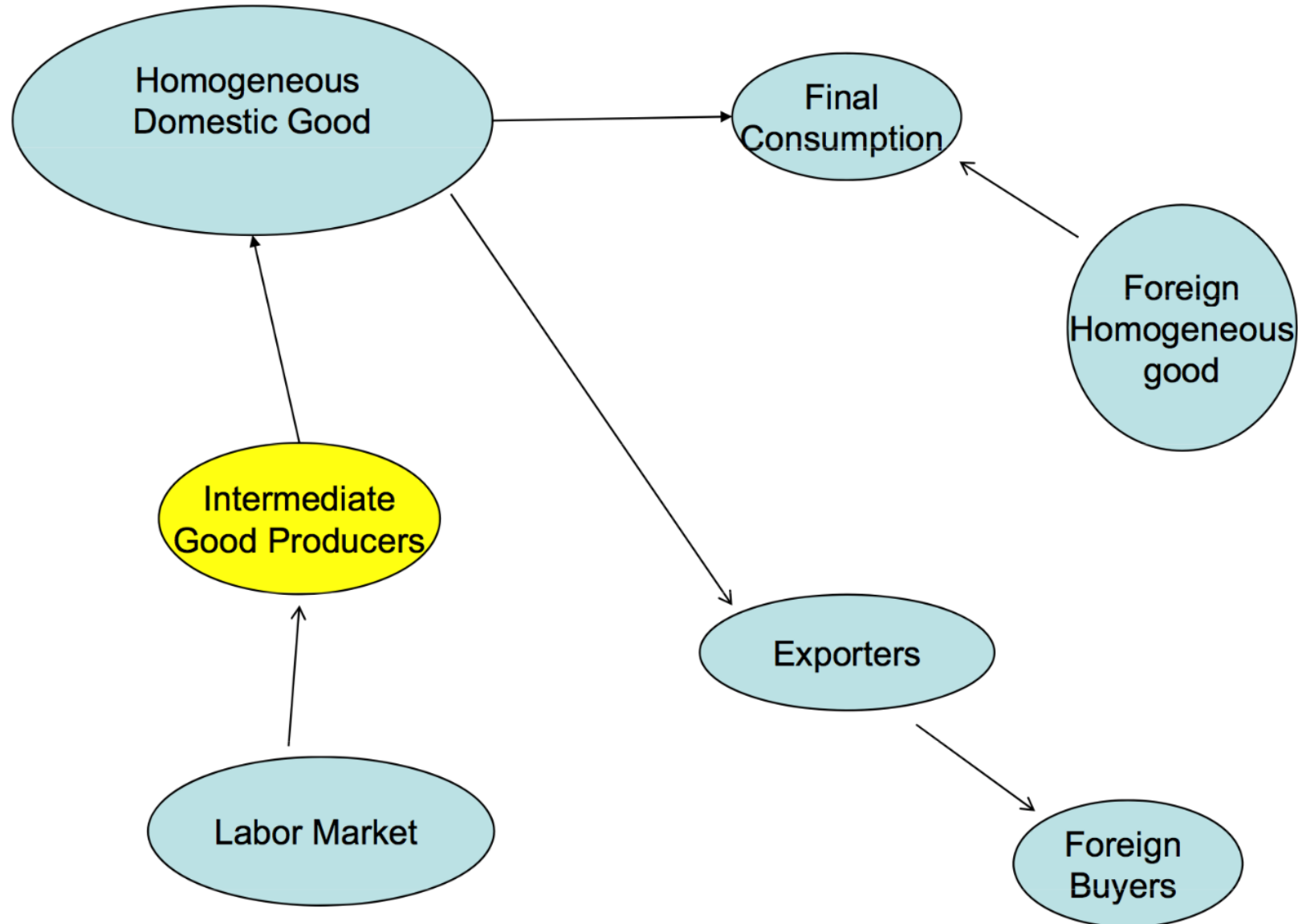
A Model

- Mihai Copaciu (Romanian Central Bank)
- Constructs small open economy models in which investment is sustained by purchases of entrepreneurs, who earn their revenues in domestic currency units.
 - Entrepreneurs need financing, and the amount of financing they can get is partially a function of their accumulated net worth.
 - Some of the financing is obtained from abroad.
 - When the currency depreciates, entrepreneurs that borrowed abroad make capital losses and their net worth suffers.
 - They are forced to cut back on expenditures, so that investment crashes, bringing down the economy.
- Same model also contains the usual features that imply an expansion in the US acts as a locomotive on the rest of the world.

Mihai Copaciu's Model of Currency Mismatch

(Banca Națională a României)

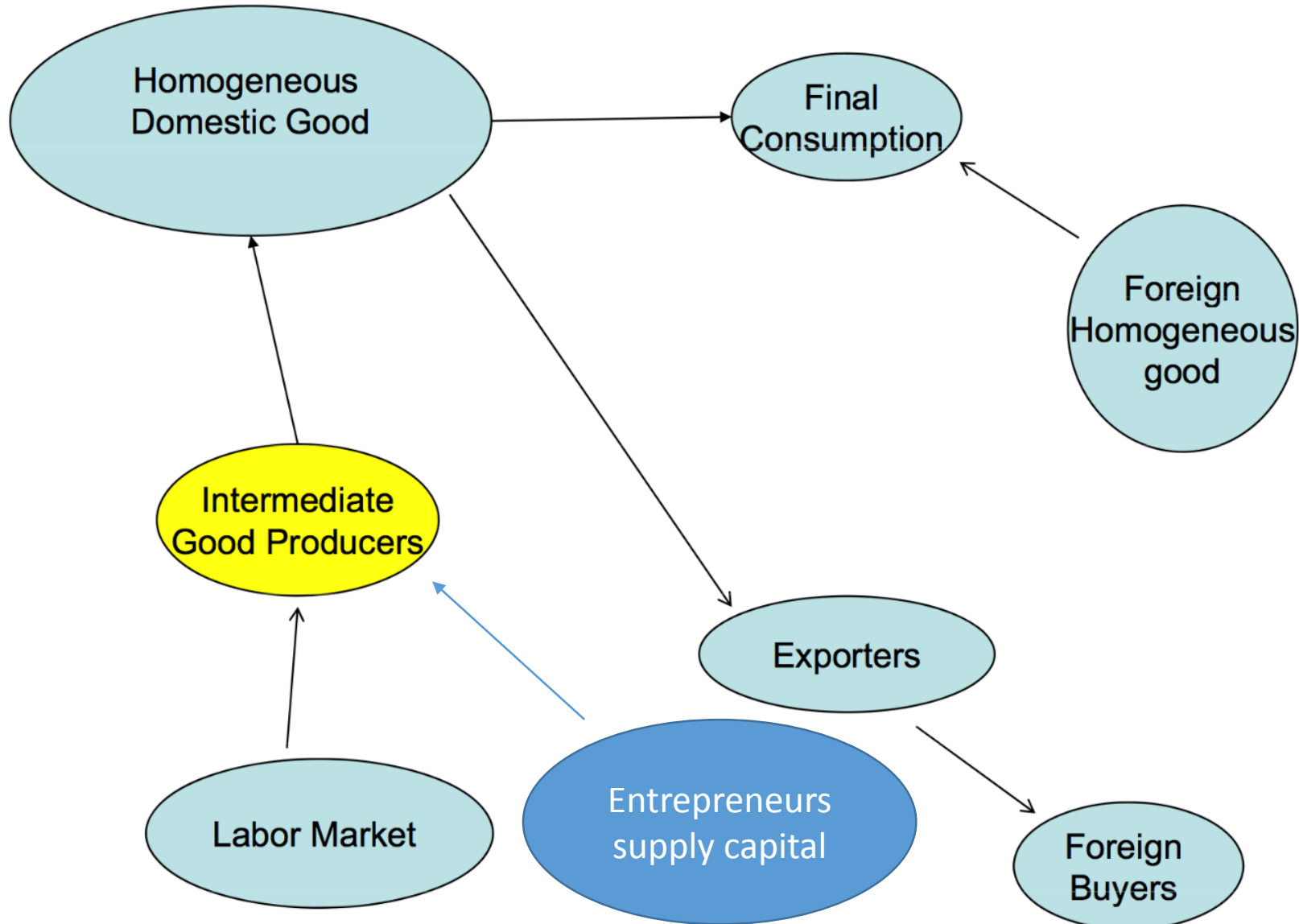
Typical small
open economy



Mihai Copaciu's Model of Currency Mismatch

(Banca Națională a României)

Typical small
open economy



DSGE Models Can Play a Useful Role in Discussions about Fed `Exit Strategy`

- New Keynesian Open Economy Models can Capture Various Sides:
 - Potential for US take-off to:
 - be a locomotive.
 - cause a loss of net worth by foreign firms/financial institutions and force a cut-back in foreign investment ('BIS scenario').
- Two aspects of the BIS scenario –
 - Hyun Shin has drawn attention to measurement problems.
 - Modeling: there are straightforward ways to get collateral constraints into models.