The Zero Bound and Fiscal Policy

Based on work by:

Eggertsson and Woodford, 2003, 'The Zero Interest-Rate Bound and Optimal Monetary Policy,' Brookings Panel on Economic Activity.

Christiano, Eichenbaum, Rebelo, 'When is the Government Spending Multiplier Big?' (JPE, 2011)

Introduction

 The New Keynesian model suggests that an economy may be vulnerable to deep recession when the zero lower bound on the nominal interest rate is binding.

 Fiscal policy could be very effective and desirable in the zero lower bound, though it is relatively less effective in 'normal' times.

The ZLB Analysis (Over) Simplified

• Identity:

• If one group reduces spending, then GDP must fall unless another group increases.

Another group increases if real rate drops:

$$\frac{R}{\pi^e}$$

• If R is at lower bound and π^e cannot rise, have a problem.

The ZLB Analysis, cnt'd

• Two reasons people may be reluctant to raise π^e

 Ex post, monetary authority would not deliver high inflation (Eggertsson).

— Real-world monetary authorities spent years persuading people they would not use inflation to stabilize economy. Fears consequences of loss of credibility in case they now raise π^e for stabilization purposes.

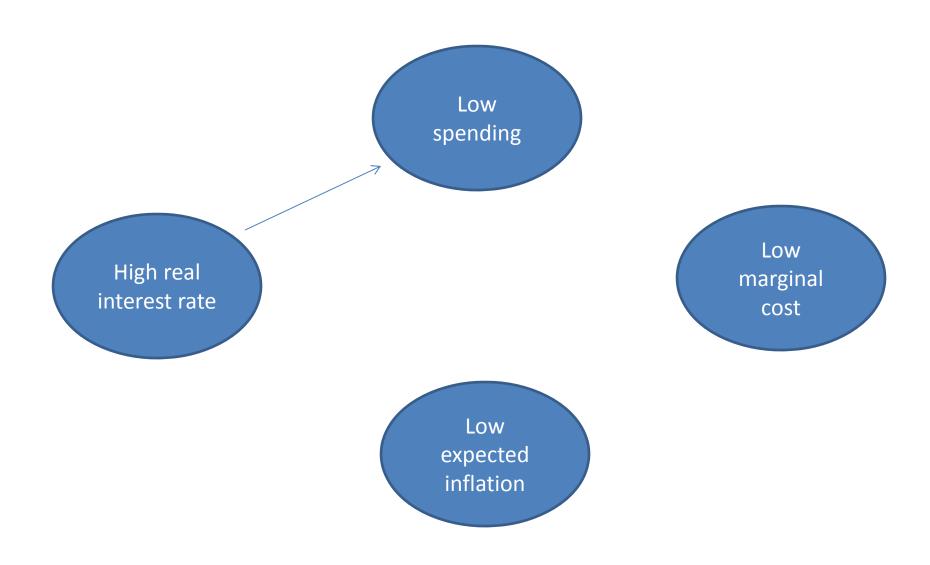
The ZLB Analysis (Over) Simplified

- Recession likely to follow, as real rate fails to drop.
- The recession could be very severe if a deflation spiral occurs.

$$\frac{R}{\pi^e}$$

- The decrease in spending leads to a fall in marginal cost, which makes firms cut prices.
- When there are price frictions, downward pressure on prices is manifest as a reduction in inflation.

Deflation Cycle in Zero Bound



The Whole Analysis, cnt'd

- The preceding indicates that the drop in output might be substantial.
- Options for solving zlb problem
 - Direct: by interrupting destructive deflation spiral, increase government spending may have a very large effect on output.
 - Tax credits
 - Investment tax credit
 - 'cash for clunkers'
 - Increase anticipated inflation
 - Convert to a VAT tax in the future (Feldstein, Correia-Fahri-Nicolini-Teles).
 - Don't: cut labor tax rate or subsidize employment (Eggertsson)

Outline

- Analysis in 'normal times' when zlb constraint on interest rate can be ignored.
 - Show that the government spending multiplier is fairly small.
- Analysis when zlb is binding.
 - Government spending can have a big, welfareimproving impact on output.

Derivation of Model Equilibrium Conditions

- Households
 - First order conditions
- Firms:
 - final goods and intermediate goods
 - marginal cost of intermediate good firms
- Aggregate resources
- Monetary policy
- Three linearized equilibrium conditions:
 - Intertemporal, Pricing, Monetary policy
- Results

Model
King-Plosser-Rebelo (KPR) preferences.

• Household preferences and constraints:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left[C_t^{\gamma} (1-N_t)^{1-\gamma} \right]^{1-\sigma} - 1}{1-\sigma} + v(G_t) \right]$$

 $P_tC_t + B_{t+1} \leq W_tN_t + (1+R_t)B_t + T_t$, T_t ~lump sum taxes and profits

Optimality conditions

marginal benefit tomorrow from saving more today extra goods tomorrow from saving more today

$$\overbrace{u_{c,t}} = E_t \beta u_{c,t+1}$$

$$\underbrace{\overline{-u_{N,t}}}_{u_{c,t}} = \underbrace{\overline{W_t}}_{P_t}$$

Linearized Intertemporal Equation

Inter-temporal Euler equation

$$E_t \left[u_{c,t} - \beta u_{c,t+1} \frac{1 + R_{t+1}}{1 + \pi_{t+1}} \right] = 0$$

In zero inflation no growth steady state:

$$1 = \beta(1+R)$$

Totally differentiate:

$$du_{c,t} - [\beta(1+R)du_{c,t+1} + \beta u_c dR_{t+1} - \beta u_c(1+R)d\pi_{t+1}] = 0$$

– Log-differentiation:

$$u_c \hat{u}_{c,t} - \beta (1+R) u_c \left[\hat{u}_{c,t+1} + \frac{1}{1+R} dR_{t+1} - d\pi_{t+1} \right] = 0$$

– Finally:

$$\hat{u}_{c,t} - [\hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1}] = 0$$

Linearized intertemporal, cnt'd

Repeat:

$$\hat{u}_{c,t} - \left[\hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1}\right] = 0$$

$$u = \frac{\left[C_t^{\gamma}(1-N_t)^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma} \rightarrow u_{c,t} = \gamma C_t^{\gamma(1-\sigma)-1} (1-N_t)^{(1-\gamma)(1-\sigma)}$$

$$\hat{u}_{c,t} = [\gamma(1-\sigma)-1]\hat{C}_t - \frac{(1-\gamma)(1-\sigma)N}{1-N}\hat{N}_t$$

Firms

Final, homogeneous good

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 1$$

– Efficiency condition:

$$P_t(i) = P_t\left(\frac{Y_t}{Y_t(i)}\right)^{\frac{1}{\varepsilon}}$$

i-th intermediate good

$$Y_t(i) = N_t(i)$$

– Optimize price with probability 1- θ , otherwise

$$P_t(i) = P_{t-1}(i)$$

Intermediate Good Firm Marginal Cost

Marginal cost:

subsidy to undo effects of monopoly power = $(\varepsilon-1)/\varepsilon$

in steady state

$$MC_t = \frac{\frac{dCost_t}{dWorker_t}}{\frac{dOutput_t}{dWorker}} = \frac{W_t}{MP_{L,t}}$$

household first order condition

$$= W_t(1-v) = P_t \qquad \underbrace{\overline{-u_{N,t}}}_{u_{c,t}} \qquad (1-v)$$

Real marginal cost

$$s_t \equiv \frac{MC_t}{P_t} = \frac{-u_{N,t}}{u_{c,t}} (1 - v)$$

marginal cost to household of providing one more unit of labor

$$\underbrace{\frac{-u_{N,t}}{u_{c,t}}}$$

in steady state

marginal benefit of one extra unit of labor



Aggregate Resources

Resource relation:

$$C_t + G_t = Y_t = p_t^* N_t$$

- $-p_t^*$ is 'Tak Yun' distortion
- recall, distortion = 1 to first order:

$$\hat{Y}_t = \hat{N}_t$$

Log-linear expansion:

$$(1-g)\hat{C}_t + g\hat{G}_t = \hat{Y}_t, g \equiv \frac{G}{Y}$$

• Consumption:

$$\hat{C}_t = \frac{1}{1-g} \hat{Y}_t - \frac{g}{1-g} \hat{G}_t$$

Simplifying Marginal Utility of C

in steady state
$$\frac{-u_{N,t}}{u_{c,t}} \stackrel{\text{in steady state}}{=} 1 \rightarrow \frac{1-\gamma}{1-N} = \frac{\gamma}{C}$$

$$\hat{u}_{c,t} = \left[\gamma(1-\sigma) - 1\right] \hat{C}_t - \frac{(1-\gamma)(1-\sigma)N}{1-N} \hat{N}_t$$

$$= \left[\gamma(1-\sigma) - 1\right] \hat{C}_t - \frac{\gamma(1-\sigma)N}{C} \hat{N}_t$$

$$= \left[\gamma(1-\sigma) - 1\right] \hat{C}_t - \frac{\gamma(1-\sigma)}{1-g} \hat{N}_t$$

$$= \left[\gamma(1-\sigma) - 1\right] \left[\frac{1}{1-g} \hat{Y}_t - \frac{g}{1-g} \hat{G}_t\right] - \frac{\gamma(1-\sigma)}{1-g} \hat{Y}_t$$

$$= -\frac{1}{1-g} \hat{Y}_t - \left[\gamma(1-\sigma) - 1\right] \frac{g}{1-g} \hat{G}_t$$

Simplify Intertemporal Equation

Intertemporal Euler equation:

$$\hat{u}_{c,t} = \hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1}$$

 Substitute out marginal utility of consumption:

$$-\frac{1}{1-g}\hat{Y}_{t} - [\gamma(1-\sigma) - 1]\frac{g}{1-g}\hat{G}_{t}$$

$$= -\frac{1}{1-g}\hat{Y}_{t+1} - [\gamma(1-\sigma) - 1]\frac{g}{1-g}\hat{G}_{t+1} + \beta dR_{t+1} - d\pi_{t+1}$$

Rearranging:

$$\hat{Y}_{t} + [\gamma(1-\sigma) - 1]g\hat{G}_{t}
= \hat{Y}_{t+1} + [\gamma(1-\sigma) - 1]g\hat{G}_{t+1} - (1-g)[\beta dR_{t+1} - d\pi_{t+1}]$$

Phillips Curve

 Equilibrium condition associated with price setting just like before:

$$\pi_t = \beta \pi_{t+1} + \kappa \widehat{s}_t,$$

$$\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

Marginal cost:

$$\widehat{S}_{t} = \frac{\widehat{(1-\gamma)C_{t}}}{\gamma(1-N_{t})} = \widehat{C}_{t} - \widehat{(1-N_{t})} = \widehat{C}_{t} + \frac{N}{1-N}\widehat{N}_{t}$$

$$\left(\widehat{C}_{t} = \frac{1}{1-g}\widehat{Y}_{t} - \frac{g}{1-g}\widehat{G}_{t}, \widehat{N}_{t} = \widehat{Y}_{t}\right)$$

$$= \left[\frac{1}{1-g} + \frac{N}{1-N}\right]\widehat{Y}_{t} - \frac{g}{1-g}\widehat{G}_{t}$$

Monetary Policy

Monetary policy rule (after linearization)

$$dR_{t+1} = \rho_R dR_t + (1 - \rho_R) \left[\frac{\phi_1}{\beta} \pi_{t+k} + \frac{\phi_2}{\beta} \hat{Y}_{t+l} \right]$$

$$dR_{t+1} \equiv R_{t+1} - R, R = \frac{1}{\beta} - 1$$

$$\hat{Y}_t \equiv \frac{Y_t - Y}{Y}$$

$$k, l = 0, 1.$$

Pulling All the Equations Together

• IS equation:

$$\hat{Y}_{t} + [\gamma(1-\sigma) - 1]g\hat{G}_{t}
= \hat{Y}_{t+1} + [\gamma(1-\sigma) - 1]g\hat{G}_{t+1} - (1-g)[\beta dR_{t+1} - d\pi_{t+1}]$$

Phillips curve:

$$\pi_t = \beta \pi_{t+1} + \kappa \left[\left(\frac{1}{1-g} + \frac{N}{1-N} \right) \hat{Y}_t - \frac{g}{1-g} \hat{G}_t \right]$$

Monetary policy rule:

$$dR_{t+1} = \rho_R dR_t + (1 - \rho_R) \left[\frac{\phi_1}{\beta} \pi_{t+k} + \frac{\phi_2}{\beta} \hat{Y}_{t+l} \right]$$

The Equations in Matrix Form

$$\begin{bmatrix} -\frac{1}{1-g} & -1 & 0 \\ 0 & \beta & 0 \\ l(1-\rho_R)\frac{\phi_2}{\beta} & k(1-\rho_R)\frac{\phi_1}{\beta} & 0 \end{bmatrix} \begin{pmatrix} \hat{Y}_{t+1} \\ \pi_{t+1} \\ dR_{t+2} \end{pmatrix} + \begin{bmatrix} \frac{1}{1-g} & 0 & \beta \\ \kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right) & -1 & 0 \\ (1-l)(1-\rho_R)\frac{\phi_2}{\beta} & (1-k)(1-\rho_R)\frac{\phi_1}{\beta} & -1 \end{bmatrix} \begin{pmatrix} \hat{Y}_t \\ \pi_t \\ dR_{t+1} \end{pmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \rho_R \end{bmatrix} \begin{pmatrix} \hat{Y}_{t-1} \\ \pi_{t-1} \\ dR_t \end{pmatrix} + \begin{pmatrix} \frac{g[\gamma(\sigma-1)+1]}{1-g} \\ 0 \\ 0 \end{pmatrix} \hat{G}_{t+1} + \begin{pmatrix} -\frac{g[\gamma(\sigma-1)+1]}{1-g} \\ -\frac{\kappa g}{1-g} \\ 0 \end{pmatrix} \hat{G}_t,$$

• or,
$$\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t = 0.$$
 $s_t = P s_{t-1} + \varepsilon_t, \ s_t \equiv \hat{G}_t, \ P = \rho$

Solution:

Undetermined coefficients, A and B:

$$z_t = A z_{t-1} + B s_t$$

A and B must satisfy:

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 = 0$$

$$\alpha_0 (AB + BP) + \alpha_1 B + \beta_0 P + \beta_1 = 0.$$

• When $\rho_R = 0$, $\alpha_2 = 0 \rightarrow A = 0$ works.

Results

- Fiscal spending multiplier small, but can easily be bigger than unity (i.e., C rises in response to G shock)
- Contrasts with standard results in which multiplier is less than unity
 - Typical preferences in estimated models:

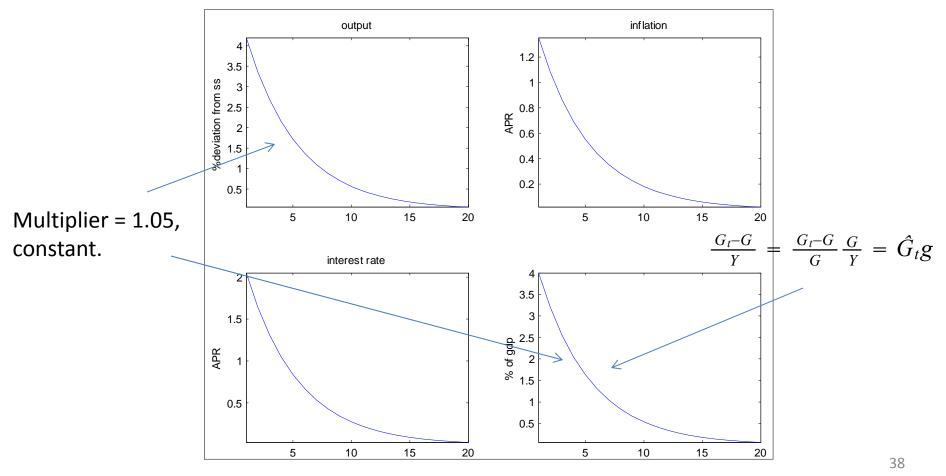
$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\gamma}}{1+\gamma} + \nu(G_t) \right], \ \psi, \gamma, \sigma > 0.$$

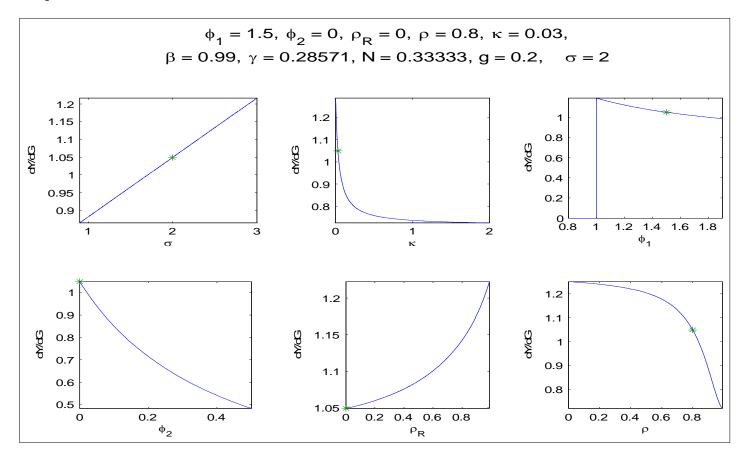
- Marginal utility of C independent of N for CGG
- Marginal utility of C increases in N for KPR.

Simulation Results

Benchmark parameter values:

$$\kappa = 0.035, \ \beta = 0.99, \ \phi_1 = 1.5, \ \phi_2 = 0, \ N = 0.23, \ g = 0.2, \ \sigma = 2, \ \rho = 0.8, \ \rho_R = 0$$

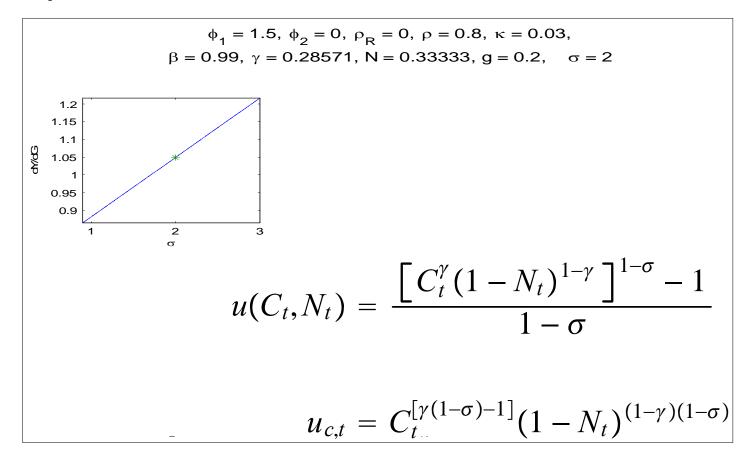




- Results: multiplier bigger
 - the less monetary policy allows R to rise.
 - the more complementary are consumption and labor (i.e., the bigger is $\,\sigma\,$).
 - the smaller the negative income effect on consumption (i.e., the smaller is ρ).
 - smaller values of κ (i.e., more sticky prices)

$$dR_{t+1} = \rho_R dR_t + (1 - \rho_R) \begin{bmatrix} \frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \end{bmatrix} \begin{bmatrix} \frac{\phi_1}{\delta} \sigma_{0.8} \\ \frac{\phi_2}{\delta} \sigma_{0.8} \\ \frac{\phi_1}{\delta} \sigma_{$$

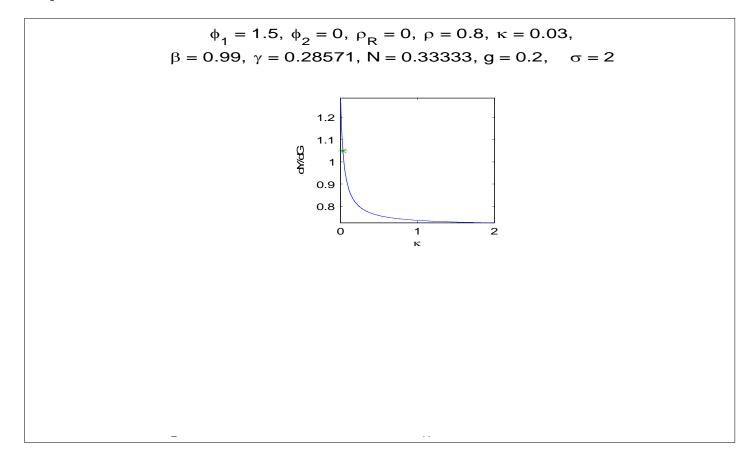
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- Results: multiplier bigger
 - the more complementary are consumption and labor (i.e., the bigger is σ).

$$\hat{G}_t = \rho \hat{G}_{t-1} + \epsilon_t$$

- Results: multiplier bigger
 - the smaller the negative income effect on consumption (i.e., the smaller is ρ).



Results: multiplier bigger

smaller values of κ (i.e., more sticky prices)

Analysis of Case when the Nonnegativity Constraint on the Nominal Interest Rate is Binding

- Need a shock that puts us into the lower bound.
- One possibility: increased desire to save.
 - Seems particularly relevant at the current time.
 - Other shocks will do it too.....
- Discount rate shock.

Monetary Policy

Monetary policy rule (after linearization)

$$Z_{t+1} = R + \rho_R(R_t - R) + (1 - \rho_R) \left[\frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \right]$$

$$\hat{Y}_t = \frac{Y_t - Y}{Y}, R = \frac{1}{\beta} - 1$$

$$R_{t+1} = \left\{ egin{array}{ll} Z_{t+1} & ext{if } Z_{t+1} > 0 \ 0 & ext{if } Z_{t+1} \leq 0 \end{array}
ight.$$
 nonlinearity

Eggertsson-Woodford Saving Shock

Preferences:

$$u(C_0, N_0, G_0) + \frac{1}{1+r_1} E_0 \left\{ u(C_1, N_1, G_1) + \frac{1}{1+r_2} u(C_2, N_2, G_1) + \frac{1}{1+r_2} \frac{1}{1+r_3} u(C_3, N_3, G_3) \dots \right\}$$

- Before *t=0*
 - System was in non-stochastic, zero inflation steady state,

$$r_{t+1} = R = \frac{1}{\beta} - 1$$

$$R_{t+1} = R$$

$$\hat{G}_t = 0$$
, for all t

Saving Shock, cnt'd

• At time *t=0*,

$$r_1 = r^l < 0$$
 $\Pr{ob[r_{t+1} = r | r_t = r^l]} = 1 - p$
 $\Pr{ob[r_{t+1} = r^l | r_t = r^l]} = p$
 $\Pr{ob[r_{t+1} = r^l | r_t = r]} = 0$

 "Discount rate drops in t=0 and is expected to return permanently to its 'normal' level with constant probability, 1-p."

Zero Bound Equilibrium

simple characterization:

$$\pi^l, \hat{Y}^l, R = 0, Z^l \leq 0$$
 while discount rate is low

 $\pi_t = \hat{Y}_t = 0$, R = r as soon as discount rate snaps back up

Fiscal Policy

 Government spending is set to a constant deviation from steady state, during the zero bound.

That is,

 \hat{G}_t may be nonzero while $r_{t+1} = r^l$, $\hat{G}_t = 0$ when $r_{t+1} = r$

Equations With Discount Shock

• IS equation:

$$\hat{Y}_{t} - g[\gamma(\sigma - 1) + 1]\hat{G}_{t} = -(1 - g)[\beta(R_{t+1} - r_{t+1}) - E_{t}\pi_{t+1}] + E_{t}\hat{Y}_{t+1} - g[\gamma(\sigma - 1) + 1]E_{t}\hat{G}_{t+1}$$

$$\hat{Y}^{l} - g[\gamma(\sigma - 1) + 1]\hat{G}^{l} = -(1 - g)[\beta(0 - r^{l}) - p\pi^{l}] + p\hat{Y}^{l} - g[\gamma(\sigma - 1) + 1]p\hat{G}$$

Phillips curve:

$$\pi_{t} = \beta E_{t} \pi_{t+1} + \kappa \left[\left(\frac{1}{1-g} + \frac{N}{1-N} \right) \hat{Y}_{t} - \frac{g}{1-g} \hat{G}_{t} \right]$$

$$\pi^{l} = \beta p \pi^{l} + \kappa \left(\frac{1}{1-g} + \frac{N}{1-N} \right) \hat{Y}^{l} - \frac{g}{1-g} \kappa \hat{G}^{l}$$

Monetary Policy:

$$R_{t+1} = 0$$

$$Z_{t+1} = R + \rho_R(R_t - R) + (1 - \rho_R) \left[\frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \right] \le 0$$

Solving for the Zero Bound Allocations

• Is equation:

$$\hat{Y}^l - g[\gamma(\sigma - 1) + 1]\hat{G}^l = -(1 - g)[\beta(0 - r^l) - p\pi^l] + p\hat{Y}^l - g[\gamma(\sigma - 1) + 1]p\hat{G}^l$$

Phillips curve:

$$\pi^{l} = \beta p \pi^{l} + \kappa \left(\frac{1}{1-g} + \frac{N}{1-N} \right) \hat{Y}^{l} - \frac{g}{1-g} \kappa \hat{G}^{l}$$

- Two equations in two unknowns!
 - Solve for \hat{Y}^l, π^l and verify that $Z^l \leq 0$

Solution

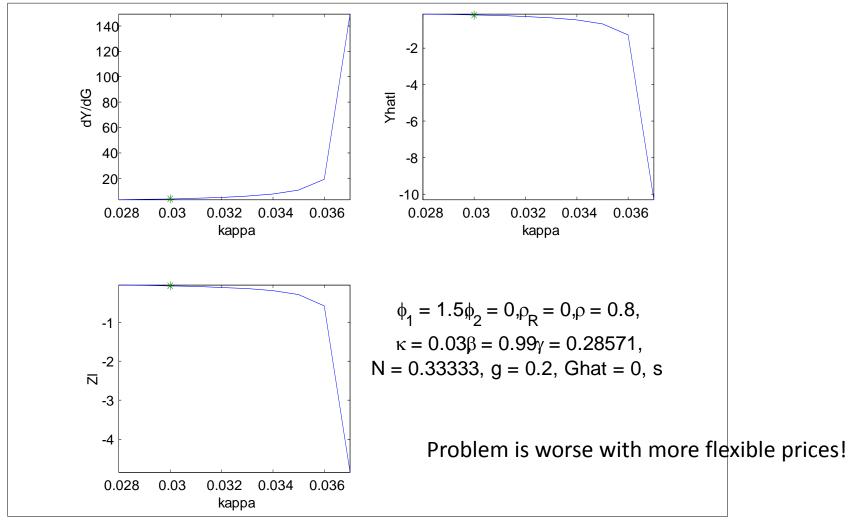
• Inflation:

$$\pi^{l} = \frac{\kappa \left(\frac{1}{1-g} + \frac{N}{1-N}\right) \left[g[\gamma(\sigma-1)+1]\hat{G}^{l} + \frac{1-g}{1-p}\beta r^{l}\right] - \frac{g}{1-g}\kappa \hat{G}^{l}}{1-\beta p - \kappa \left(\frac{1}{1-g} + \frac{N}{1-N}\right) p \frac{1-g}{1-p}}$$

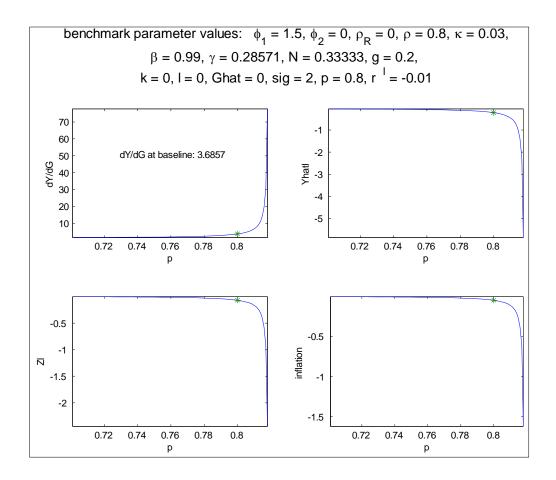
• Output:

$$\hat{Y}^l = g[\gamma(\sigma - 1) + 1]\hat{G}^l + \frac{1-g}{1-p}[\beta r^l + p\pi^l]$$

Numerical Simulations



 Results: multiplier 3.7 at benchmark parameter values and may be gigantic.



 As p increases, zero-bound becomes more severe...this is because with higher p, fall in output is more persistent and resulting negative wealth effect further depresses consumption.

Fiscal Expansion in Zero Bound Highly Effective, But is it *Desirable*?

- Intuition:
 - *Yes....*
 - the vicious cycle produces a huge, inefficient fall in output
 - in the first-best equilibrium, output, consumption and employment are invariant to discount rate shocks
 - If G helps to partially undo this inefficiency, then surely it's a good thing

Fiscal Expansion in Zero Bound Highly Effective, But is it *Desirable*?

Preferences

$$\sum_{t=0}^{\infty} \left(\frac{p}{1+r^{l}}\right)^{t} \left[\frac{\left[(C^{l})^{\gamma}(1-N^{l})^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma}+v(G^{l})\right]$$

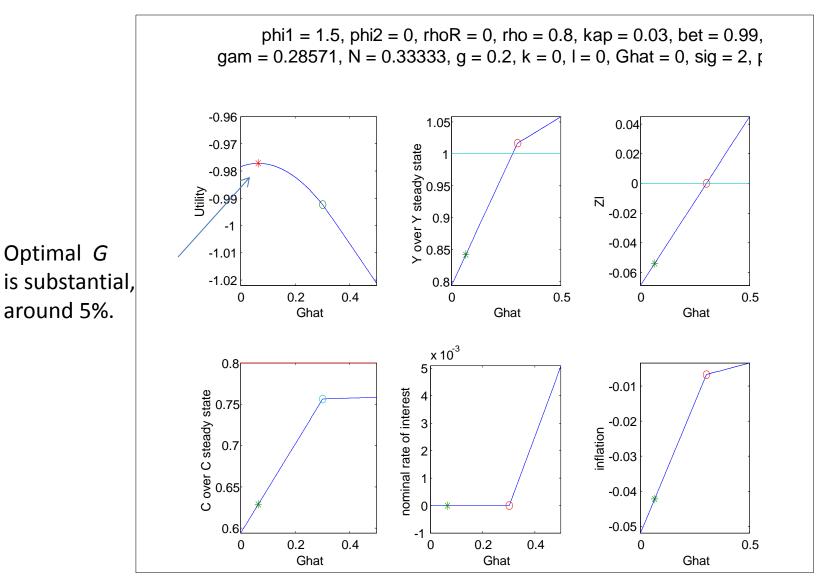
$$=\frac{1}{1-\frac{p}{1+r^{l}}} \left[\frac{\left[(C^{l})^{\gamma}(1-N^{l})^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma}+v(G^{l})\right]$$

$$=\frac{1}{1-\frac{p}{1+r^{l}}} \left[\frac{\left[(N(\hat{Y}^{l}+1)-Ng(\hat{G}^{l}+1))^{\gamma}(1-N(\hat{Y}^{l}+1))^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma}+v(Ng(\hat{G}^{l}+1))\right]$$

• Compute optimal \hat{G}^l

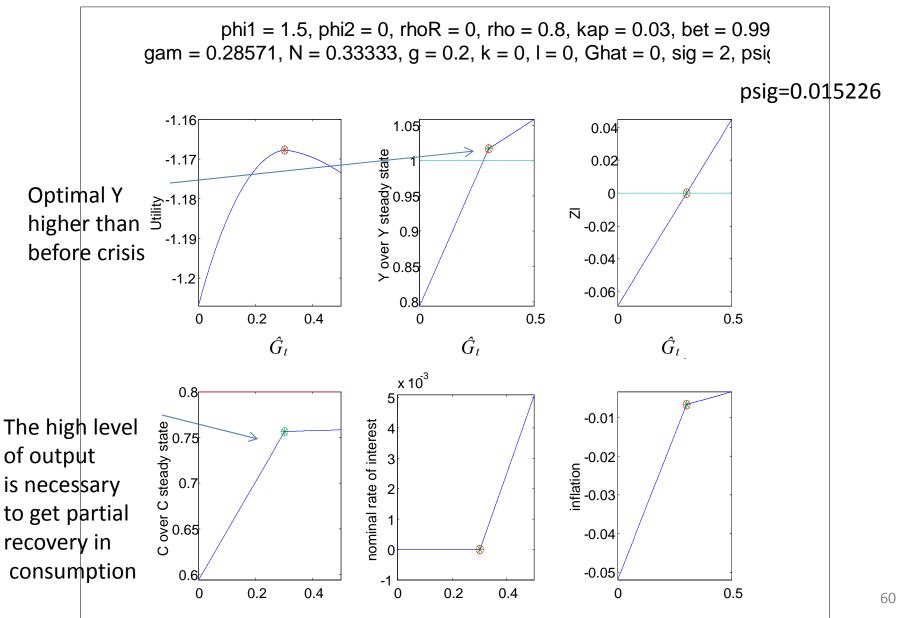
- (i)
$$v(G^l) = 0$$
,
- (ii) $v(G) = \psi_g \frac{G^{1-\sigma}}{1-\sigma}$, ψ_g chosen to rationalize $g = 0.2$ as optimal in steady state

Case Where G is not Valued



Optimal G

Case Where Gov't Spending is Desirable



Introducing Investment

 Inclusion of investment does not have a large, qualitative effect.

- Financial frictions could make things much worse.
 - Deflation hurts net worth of investors with nominal debt, and this forces those agents to cut spending by more.

Conclusion of G Multiplier Analysis

- Government spending multiplier in a neighborhood of unity in 'normal times'.
- Multiplier can be large when the zero bound is binding (because R constant then).
- Increase in G is welfare improving during lower bound crisis.
- Caveat: focused exclusively on multiplier
 - Increasing G may be bad idea because hard to reverse.
 - May be other ways of accomplishing similar thing (e.g., transition to VAT tax over time).