Simple New Keynesian Model without Capital

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Objective

- Review the foundations of the basic New Keynesian model without capital.
  - Clarify the role of money supply/demand.
- Derive the Equilibrium Conditions.
  - Small number of equations and a small number of variables, which summarize everything about the model (optimization, market clearing, gov't policy, etc.).
- Look at some data through the eyes of the model:
  - Money demand.
  - Cross-sectoral resource allocation cost of inflation.
- Some policy implications of the model will be examined.
  - Many policy implications will be 'discovered' in later computer exercises.

Outline

- The model:
  - Individual agents: their objectives, what they take as given, what they choose.
    - Households, final good firms, intermediate good firms, gov't.
  - Economy-wide restrictions:
    - Market clearing conditions.
    - Relationship between aggregate output and aggregate factors of production, aggregate price level and individual prices.

- Properties of Equilibrium:
  - *Classical Dichotomy* - when prices flexible monetary policy irrelevant for real variables.
  - Monetary policy *essential* to determination of all variables when prices sticky.

Households

- Households' problem.
- Concept of Consumption Smoothing.

Households

- There are many identical households.
- The problem of the typical ('representative') household:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \exp\left(\tau_t\right) \frac{N_t^{1+\varphi}}{1+\varphi} + \gamma log \left(\frac{M_{t+1}}{P_t} \right) \right),$$

$$\text{s.t. } P_t C_t + B_{t+1} + M_{t+1}$$
$$\leq W_t N_t + R_{t-1} B_t + M_t$$
$$+\text{Profits net of government transfers and taxes}_t.$$

- Here, B_t and M_t are the beginning-of-period t stock of bonds and money held by the household.
- Law of motion of the shock to preferences:

$$\tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}$$

the preference shock is in the model for pedagogic purposes only, it is not an interest shock from an empirical point of view.

Household First Order Conditions

- The household first order conditions:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} (5)$$

$$e^{\tau_t} C_t N_t^{\varphi} = \frac{W_t}{P_t}.$$

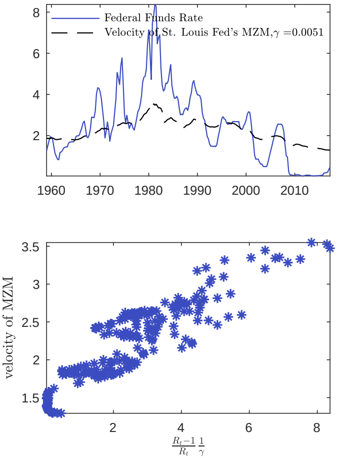
$$m_t = \left(\frac{R_t}{R_t - 1} \right) \gamma C_t \; (7),$$

where

$$m_t \equiv \frac{M_{t+1}}{P_t}.$$

- All equations are derived by expressing the household problem in Lagrangian form, substituting out the multiplier on budget constraint and rearranging.
- The last first order condition is real money demand, increasing in C_t and decreasing in $R_t \geq 1$.

Figure: Money Demand, Relative to Two Measures of Velocity



Notes: (i) velocity is GDP/M, (ii) With the MZM measure of money, the money demand equation does well qualitatively, but not quantitatively because the theory implies the scatters in the 2,1 and 2,2 graphs should be on the 45^0.

Consumption Smoothing

- Later, we'll see that *consumption smoothing* is an important principle for understanding the role of monetary policy in the New Keynesian model.
- Consumption smoothing is a characteristic of households' consumption decision when they expect a change in income and the interest rate is *not* expected to change.
  - Peoples' current period consumption increases by the amount that can, according to their budget constraint, be maintained indefinitely.

Consumption Smoothing: Example

- Problem:

$$\begin{aligned} & max_{c_1,c_2} log\left(c_1\right) + \beta log\left(c_2\right) \\ \text{subject to :} \quad & c_1 + B_1 \leq y_1 + rB_0 \\ & c_2 \leq rB_1 + y_{2.} \end{aligned}$$

- where y_1 and y_2 are (given) income and, after imposing equality (optimality) and substituting out for B_1,

$$c_1 + \frac{c_2}{r} = y_1 + \frac{y_2}{r} + rB_0,$$
$$\frac{1}{c_1} = \beta r \frac{1}{c_2},$$

second equation is fonc for B_1.

- Suppose $\beta r = 1$ (this happens in 'steady state', see later):

$$c_1 = \frac{y_1 + \frac{y_2}{r}}{1 + \frac{1}{r}} + \frac{r}{1 + \frac{1}{r}} B_0$$

Consumption Smoothing: Example, cnt'd

- Solution to the problem:

$$c_1 = \frac{y_1 + \frac{y_2}{r}}{1 + \frac{1}{r}} + \frac{r}{1 + \frac{1}{r}} B_0.$$

- Consider three polar cases:
  - *temporary change in income*: $\Delta y_1 > 0$ and $\Delta y_2 = 0 \implies \Delta c_1 = \Delta c_2 = \frac{\Delta y_1}{1+\frac{1}{r}}$
  - *permanent change in income*: $\Delta y_1 = \Delta y_2 > 0 \implies \Delta c_1 = \Delta c_2 = \Delta y_1$
  - *future change in income*: $\Delta y_1 = 0$ and $\Delta y_2 > 0 \implies \Delta c_1 = \Delta c_2 = \frac{\frac{\Delta y_2}{r}}{1+\frac{1}{r}}$
- Common feature of each example:
  - When income rises, then - assuming r does not change - c_1 increases by an amount that can be maintained into the second period: **consumption smoothing**.

Goods Production

- We turn now to the technology of production, and the problems of the firms.
- The technology requires allocating resources across sectors.
  - We describe the *efficient* cross-sectoral allocation of resources.
  - With price setting frictions, the market may not achieve efficiency.

Final Goods Production

- A homogeneous final good is produced using the following (Dixit-Stiglitz) production function:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

- Each intermediate good, $Y_{i,t}$, is produced by a monopolist using the following production function:

$$Y_{i,t} = e^{a_t} N_{i,t}, \qquad a_t \sim \text{exogenous shock to technology.}$$

- Before discussing the firms that operate these production functions, we briefly investigate the socially efficient allocation of resources across i.

Efficient Sectoral Allocation of Resources

- With Dixit-Stiglitz final good production function, there is a socially optimal allocation of resources to all the intermediate activities, $Y_{i,t}$.
- It is optimal to run them all at the same rate, *i.e.*, $Y_{i,t} = Y_{j,t}$ for all $i, j \in [0, 1]$.
- For given N_t, *allocative efficiency*: $N_{i,t} = N_{j,t} = N_t$, for all $i, j \in [0, 1]$.
  In this case, final output is given by

$$Y_t = \left[\int_0^1 \left(e^{a_t} N_{i,t} \right)^{\frac{\varepsilon - 1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon - 1}} = e^{a_t} N_t.$$

- One way to understand allocated efficiency result is to suppose that labor is *not* allocated equally to all activities.
- Explore one simple deviation from $N_{i,t} = N_{j,t}$ for all $i, j \in [0, 1]$.

Suppose Labor *Not* Allocated Equally

- Example:

$$N_{it} = \begin{cases} 2\alpha N_t & i \in \left[0, \frac{1}{2}\right] \\ 2(1-\alpha)N_t & i \in \left[\frac{1}{2}, 1\right] \end{cases}, \; 0 \leq \alpha \leq 1.$$

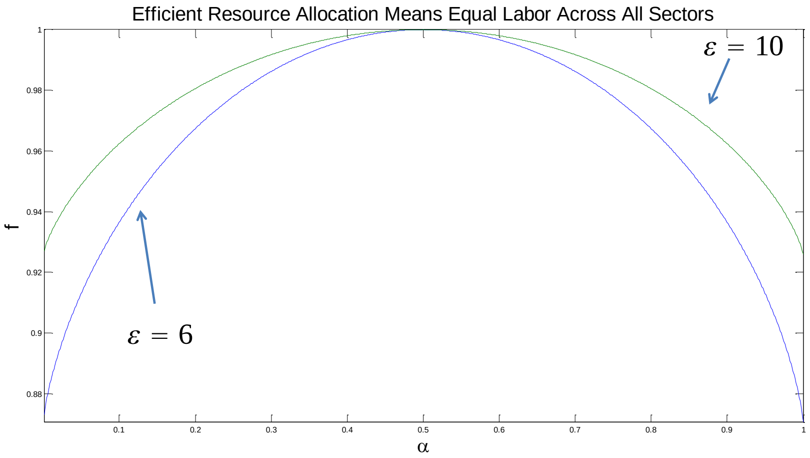
- Note that this is a particular distribution of labor across activities:

$$\int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1-\alpha) N_t = N_t$$

Labor *Not* Allocated Equally, cnt'd

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\int_0^{\frac{1}{2}} Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} (2\alpha N_t)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha)N_t)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\int_0^{\frac{1}{2}} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t f(\alpha)
\end{aligned}
$$

$$f(\alpha) = \left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$



Final Good Producers

- Competitive firms:
  - maximize profits

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,$$

subject to P_t, $P_{i,t}$ given, all $i \in [0,1]$, and the technology:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Foncs:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}}\right)^{\varepsilon} \to \overbrace{P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{1}{1-\varepsilon}}}^{\text{"cross price restrictions"}}$$

Intermediate Good Producers

- The i^{th} intermediate good is produced by a monopolist.
- Demand curve for i^{th} monopolist:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon}.$$

- Production function:

$$Y_{i,t} = e^{a_t} N_{i,t},\ a_t \text{ ~ exogenous shock to technology.}$$

- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}.$$

Marginal Cost of Production

- An important input into the monopolist's problem is its marginal cost:

$$s_t = \frac{dCost}{dOutput} = \frac{\frac{dCost}{dWorker}}{\frac{dOutput}{dWorker}} = \frac{(1-\nu)\,\frac{W_t}{P_t}}{e^{a_t}}$$
$$= \frac{(1-\nu)\, e^{\tau_t} C_t N_t^{\varphi}}{e^{a_t}}$$

  after substituting out for the real wage from the household intratemporal Euler equation.
- The tax rate, ν, represents a subsidy to hiring labor, financed by a lump-sum government tax on households.
- Firm's job is to set prices whenever it has the opportunity to do so.
  - It must always satisfy whatever demand materializes at its posted price.

Present Discounted Value of Intermediate Good Revenues

- i^{th} intermediate good firm's objective:

$$E_t^i \sum_{j=0}^{\infty} \beta^j \, \upsilon_{t+j} \overbrace{\left[\overbrace{P_{i,t+j} Y_{i,t+j}}^{\text{revenues}} - \overbrace{P_{t+j} s_{t+j} Y_{i,t+j}}^{\text{total cost}} \right]}^{\text{period } t+j \text{ profits sent to household}}$$

υ_{t+j} - Lagrange multiplier on household budget constraint

- Here, E_t^i denotes the firm's expectation over future variables, including the future probability that the firm gets to reset its price.

Firms that Can Change Price at t

- Let $\widetilde{P}_t$ denote the price set by the $1-\theta$ firms who optimize at time t.
- Expected value of future profits sum of two parts:
  - future states in which price is still $\widetilde{P}_t$, so $\widetilde{P}_t$ matters.
  - future states in which the price is not $\widetilde{P}_t$, so $\widetilde{P}_t$ is irrelevant.
- That is,

$$E_t^i \sum_{j=0}^{\infty} \beta^j v_{t+j} \left[P_{i,t+j} Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j}\right]$$

$$= \overbrace{E_t \sum_{j=0}^{\infty} (\beta\theta)^j v_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j}\right]}^{Z_t} + X_t,$$

where

  - Z_t is the present value of future profits over all future states in which the firm's price is $\tilde{P}_t$.
  - X_t is the present value over all other states, so $dX_t/d\widetilde{P}_t = 0$.

Decision By Firm that Can Change Its Price

- Substitute out demand curve:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right]$$

$$= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[\tilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon} \right].$$

- Differentiate with respect to $\tilde{P}_t$:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[(1-\varepsilon) \left(\tilde{P}_t \right)^{-\varepsilon} + \varepsilon P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon-1} \right] = 0,$$

or,

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0.$$

  - When $\theta = 0$, get standard result - price is fixed markup over marginal cost.

Decision By Firm that Can Change Its Price

- Substitute out the multiplier:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \overbrace{\frac{u'\left(C_{t+j}\right)}{P_{t+j}}}^{=v_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0.$$

  - Using assumed log-form of utility,

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \right] = 0,$$

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t},\ \bar{\pi}_t \equiv \frac{P_t}{P_{t-1}},\ X_{t,j} = \begin{cases} \frac{1}{\bar{\pi}_{t+j}\bar{\pi}_{t+j-1}\cdots\bar{\pi}_{t+1}},\ j \geq 1 \\ 1,\ j = 0. \end{cases},$$

'recursive property': $X_{t,j} = X_{t+1,j-1} \dfrac{1}{\bar{\pi}_{t+1}},\ j > 0$

Decision By Firm that Can Change Its Price

- Want $\tilde{p}_t$ in:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon - 1} s_{t+j}\right] = 0$$

- Solving for $\tilde{p}_t$, we conclude that prices are set as follows:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} \frac{Y_{t+j}}{C_{t+j}} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_t}{F_t}.$$

- Need convenient expressions for K_t, F_t.

Decision By Firm that Can Change Its Price

- Recall,

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_t}{F_t}$$

The numerator has the following simple representation:

$$K_t = E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+j}$$

$$= \frac{\varepsilon}{\varepsilon - 1} \frac{Y_t}{C_t} \frac{(1-\nu)\, e^{\tau_t} C_t N_t^{\varphi}}{e^{a_t}} + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1} \text{ (1)},$$

after using $s_t = (1-\nu)\, e^{\tau_t} C_t N_t^{\varphi} / e^{a_t}$.

- Similarly,

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{1-\varepsilon} F_{t+1} \text{ (2)}$$

Moving On to Aggregate Restrictions

- Link between aggregate price level, P_t, and $P_{i,t}$, $i \in [0,1]$.
  - Potentially complicated because there are MANY prices, $P_{i,t}$, $i \in [0,1]$.

- Link between aggregate output, Y_t, and N_t.
  - Potentially complicated because of earlier example with $f(\alpha)$.
  - Analog of $f(\alpha)$ will be a function of degree to which $P_{i,t} \neq P_{j,t}$.

- Market clearing conditions.
  - Money and bond market clearing.
  - Labor and goods market clearing.

Aggregate Price Index

- Important Calvo result:

$$P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}$$
$$= \left((1-\theta) \tilde{P}_t^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

- Divide by P_t :

$$1 = \left((1-\theta) \tilde{p}_t^{1-\varepsilon} + \theta \left(\frac{1}{\bar{\pi}_t} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

- Rearrange: $\tilde{p}_t = \left[\frac{1-\theta(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}}$

Aggregate Output vs Aggregate Labor and Tech (Tack Yun, JME1996)

- Define Y_t^*:

$$Y_t^* \equiv \int_0^1 Y_{i,t} di \qquad \left(= \int_0^1 e^{a_t} N_{i,t} di = e^{a_t} N_t \right)$$

$$\overbrace{=}^{\text{demand curve}} \quad Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} di = Y_t P_t^{\varepsilon} \int_0^1 \left(P_{i,t} \right)^{-\varepsilon} di$$

$$= Y_t P_t^{\varepsilon} \left(P_t^* \right)^{-\varepsilon}$$

where, using 'Calvo result':

$$P_t^* \equiv \left[\int_0^1 P_{i,t}^{-\varepsilon} di \right]^{\frac{-1}{\varepsilon}} = \left[(1-\theta) \tilde{P}_t^{-\varepsilon} + \theta \left(P_{t-1}^* \right)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}$$

- Then

$$Y_t = p_t^* Y_t^*, \; p_t^* = \left(\frac{P_t^*}{P_t} \right)^{\varepsilon}.$$

Gross Output vs Aggregate Labor

- Relationship between aggregate inputs and outputs:

$$Y_t = p_t^* Y_t^*$$

or,

$$Y_t = p_t^* e^{a_t} N_t.$$

- Note that p_t^* is a function of the ratio of two averages (with different weights) of $P_{i,t}$, $i \in (0,1)$

- So, when $P_{i,t} = P_{j,t}$ for all $i, j \in (0,1)$, then $p_t^* = 1$.

- But, what is p_t^* when $P_{i,t} \neq P_{j,t}$ for some (measure of) $i, j \in (0,1)$?

Tack Yun Distortion

- Consider the object,

$$p_t^* = \left(\frac{P_t^*}{P_t} \right)^{\varepsilon},$$

where

$$P_t^* = \left(\int_0^1 P_{i,t}^{-\varepsilon} di \right)^{\frac{-1}{\varepsilon}}, \; P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}$$

- In following slide, use Jensen's inequality to show:

$$p_t^* \leq 1.$$

Tack Yun Distortion

- Let $f(x) = x^4$, a convex function. Then,

$$\text{convexity: } \alpha x_1^4 + (1-\alpha) x_2^4 > (\alpha x_1 + (1-\alpha) x_2)^4$$

  for $x_1 \neq x_2$, $0 < \alpha < 1$.

- Applying this idea:

$$\text{convexity:} \quad \int_0^1 \left(P_{i,t}^{(1-\varepsilon)}\right)^{\frac{\varepsilon}{\varepsilon-1}} di \geq \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\iff \left(\int_0^1 P_{i,t}^{-\varepsilon} di\right) \geq \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\iff \overbrace{\left(\int_0^1 P_{i,t}^{-\varepsilon} di\right)^{\frac{-1}{\varepsilon}}}^{P_t^*} \leq \overbrace{\left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{1}{1-\varepsilon}}}^{P_t}$$

Law of Motion of Tack Yun Distortion

- We have

$$P_t^* = \left[(1-\theta) \tilde{P}_t^{-\varepsilon} + \theta \left(P_{t-1}^* \right)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}$$

- Dividing by P_t:

$$p_t^* \equiv \left(\frac{P_t^*}{P_t} \right)^{\varepsilon} = \left[(1-\theta) \tilde{p}_t^{-\varepsilon} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1}$$

$$= \left((1-\theta) \left[\frac{1-\theta (\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right]^{\frac{-\varepsilon}{1-\varepsilon}} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right)^{-1} \qquad (4)$$

using the restriction between $\tilde{p}_t$ and aggregate inflation developed earlier.

Government

- Government budget constraint: expenditures = receipts

$$\overbrace{P_t G_t}^{\text{purchases of final goods}} + \overbrace{\nu W_t N_t}^{\text{subsidy payments}} + \overbrace{B^g_{t+1}}^{\text{gov't bonds (lending, if positive)}} + \overbrace{T_t^{trans}}^{\text{transfer payments to households}} = \overbrace{M_t \mu_t}^{\text{money injection, if positive}} + \overbrace{T_t^{tax}}^{\text{tax revenues}} + R_{t-1} B^g_t$$

where μ_t denotes money growth rate.

- Then,

$$T_t^{tax} - T_t^{trans} = \nu W_t N_t + B^g_{t+1} + P_t G_t - M_t \mu_t - R_{t-1} B^g_t$$

- Government's choice of μ_t determines evolution of money supply:

$$M_{t+1} = (1 + \mu_t) M_t, \ \mu_t \sim \text{ money growth rate.}$$

Government

- The law of motion for money places restrictions on m_t:

$$m_t \equiv \frac{M_{t+1}}{P_t} = \frac{M_{t+1}}{M_t}\frac{M_t}{P_{t-1}}\frac{P_{t-1}}{P_t}$$

$$\rightarrow m_t = \left(\frac{1+\mu_t}{\bar{\pi}_t}\right) m_{t-1} \, (8),$$

for $t = 0, 1, ...$.

Market Clearing

- We now summarize the market clearing conditions of the model.
  - Money, labor, bond and goods markets.

Money Market Clearing

- We temporarily use the bold notation, $\mathbf{M}_t$, to denote the per capita supply of money at the start of time t, for $t = 0, 1, 2, ...$.
- The supply of money is determined by the actions, μ_t, of the government:

$$\mathbf{M}_{t+1} = \mathbf{M}_t + \mu_t \mathbf{M}_t,$$

  for t=0,1,2,...
- Households being identical means that in period $t = 0$,

$$\mathbf{M}_0 = M_0,$$

  where M_0 denotes beginning of time $t = 0$ money stock of the representative household.
- Money market clearing in each period, $t = 0, 1, ...$, requires

$$\mathbf{M}_{t+1} = M_{t+1},$$

  where M_{t+1} denotes the representative household's time t choice of money.
- From here on, we do not distinguish between $\mathbf{M}_t$ and M_t.

Other Market Clearing Conditions

- Bond market clearing:

$$B_{t+1} + B^g_{t+1} = 0,\ t = 0, 1, 2, ...$$

- Labor market clearing:

$$\overbrace{N_t}^{\text{supply of labor}} = \overbrace{\int_0^1 N_{i,t} di}^{\text{demand for labor}}$$

- Goods market clearing:

$$\overbrace{C_t + G_t}^{\text{demand for final goods}} = \overbrace{Y_t}^{\text{supply of final goods}},$$

and, using relation between Y_t and N_t:

$$C_t + G_t = p_t^* e^{a_t} N_t \ (6)$$

Next

- Collect the equilibrium conditions associated with private sector behavior.
- Comparison of NK model with RBC model (i.e., $\theta = 0$)
  - *Classical Dichotomy*: In flexible price version of model real variables determined independent of monetary policy.
  - Fiscal policy still matters, because equilibrium depends on how government deals with the monopoly power, i.e., selects value for subsidy, ν.
  - In NK model, markets don't necessarily work well and good monetary policy essential.
- To close model with $\theta > 0$ must take a stand on monetary policy.

Equilibrium Conditions

- 8 equations in 8 unknowns: $m_t, C_t, p_t^*, F_t, K_t, N_t, R_t, \bar{\pi}_t$, and 3 policy variables: ν, μ_t, G_t.

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{Y_t}{C_t} \frac{(1-\nu)\, e^{\tau_t} C_t N_t^{\varphi}}{A_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \text{ (1)}$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \text{ (2)}, \quad \frac{K_t}{F_t} = \left[\frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \text{ (3)}$$

$$p_t^* = \left[(1-\theta) \left(\frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \text{ (4)}$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \text{ (5)}, \quad C_t + G_t = p_t^* e^{a_t} N_t \text{ (6)}$$

$$m_t = \frac{\gamma C_t}{\left(1 - \frac{1}{R_t}\right)} \text{ (7)}, \quad m_t = \left(\frac{1+\mu_t}{\bar{\pi}_t} \right) m_{t-1} \text{ (8)}$$

Classical Dichotomy Under Flexible Prices

- *Classical Dichotomy*: when prices flexible, $\theta = 0$, then real variables determined regardless of the rule for μ_t (i.e., monetary policy).
  - Equations (2),(3) imply:

$$F_t = K_t = \frac{Y_t}{C_t},$$

which, combined with (1) implies

$$\frac{\varepsilon(1-\nu)}{\varepsilon - 1} \times \overbrace{e^{\tau_t} C_t N_t^{\varphi}}^{\text{Marginal Cost of work}} = \overbrace{e^{a_t}}^{\text{marginal benefit of work}}$$

  - Expression (6) with $p_t^* = 1$ (since $\theta = 0$) is

$$C_t + G_t = e^{a_t} N_t.$$

- Thus, we have two equations in two unknowns, N_t and C_t.

Classical Dichotomy: No Uncertainty

- Real interest rate, $R_t^* \equiv R_t/\bar{\pi}_{t+1}$, is determined:

$$R_t^* = \frac{\frac{1}{C_t}}{\beta\frac{1}{C_{t+1}}}.$$

- So, with $\theta = 0$, the following are determined:

$$R_t^*, C_t, N_t,\ t = 0, 1, 2, ...$$

- What about the nominal variables?
  - Suppose the monetary authority wants a given sequence of inflation rates, $\bar{\pi}_t$, $t = 0, 1, ...$.
  - Then,

$$R_t = \bar{\pi}_{t+1} R_t^*,\ t = 0, 1, 2, ...$$

  - What money growth sequence is required?
    - From (7), obtain m_t, $t = 0, 1, 2, ...$. Also, m_{-1} is given by initial M_0 and P_{-1}.
    - From (8)

$$1 + \mu_t = \frac{m_t}{m_{t-1}}\bar{\pi}_t,\ t = 0, 1, 2, ...$$

Classical Dichotomy versus New Keynesian Model

- When $\theta = 0$, then the Classical Dichotomy occurs.
- In this case, monetary policy (i.e., the setting of μ_t, $t = 0, 1, 2, ...$) cannot affect the real interest rate, consumption and employment.
  - Monetary policy simply affects the split in the real interest rate between nominal and real rates:

$$R_t^* = \frac{R_t}{\bar{\pi}_{t+1}}.$$

  - For a careful treatment when there is uncertainty, see.
- When $\theta > 0$ (NK model) then real variables are not determined independent of monetary policy.
  - In this case, monetary policy matters.

Monetary Policy in New Keynesian Model

- Suppose $\theta > 0$, so that we're in the NK model and monetary policy matters.
- The standard assumption is that the monetary authority sets μ_t to achieve an interest rate target, and that that target is a function of inflation:

$$R_t/R = (R_{t-1}/R)^{\alpha} \exp\left[(1-\alpha)\,\phi_{\pi}(\bar{\pi}_t - \bar{\pi}) + \phi_x x_t\right] \text{ (7)',}$$

  where x_t denotes the log deviation of actual output from target (more on this later).
- This is a *Taylor rule*, and it satisfies the *Taylor Principle* when $\phi_{\pi} > 1$.
- Smoothing parameter: α.
  - Bigger is α the more persistent are policy-induced changes in the interest rate.

Equilibrium Conditions of NK Model with Taylor Rule

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{Y_t}{C_t} \frac{(1-\nu)\, e^{\tau_t} C_t N_t^{\varphi}}{A_t} + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \text{ (1)}$$

$$F_t = \frac{Y_t}{C_t} + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon - 1} F_{t+1} \text{ (2)}, \quad \frac{K_t}{F_t} = \left[\frac{1 - \theta \bar{\pi}_t^{(\varepsilon - 1)}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} \quad \text{(3)}$$

$$p_t^* = \left[(1-\theta) \left(\frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon - 1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \quad \text{(4)}$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \text{ (5)}, \quad C_t + G_t = p_t^* e^{a_t} N_t \text{ (6)}$$

$$R_t / R = (R_{t-1}/R)^{\alpha} \exp\left[(1-\alpha)\, \phi_{\pi} (\bar{\pi}_t - \bar{\pi}) + \phi_x x_t \right] \text{ (7)'.}$$

Conditions (7) and (8) have been replaced by (7)'.

Equilibrium Conditions of NK Model

- The model represents 7 equations in 7 unknowns:

$$C, p_t^*, F_t, K_t, N_t, R_t, \bar{\pi}_t$$

- After this system has been solved for the 7 variables, equations (7) and (8) can be used to solve for μ_t and m_t.
  - This last step is rarely taken, because researchers are uncertain of the exact form of money demand and because m_t and μ_t are in practice not of direct interest.

Natural Equilibrium

- When $\theta = 0$, then

$$\frac{\varepsilon\left(1-\nu\right)}{\varepsilon-1} \times \overbrace{e^{\tau_t} C_t N_t^{\varphi}}^{\text{Marginal Cost of work}} = \overbrace{e^{a_t}}^{\text{marginal benefit of work}}$$

  so that we have a form of efficiency when ν is chosen to that $\varepsilon\left(1-\nu\right) / \left(\varepsilon-1\right) = 1.$

- In addition, recall that we have allocative efficiency in the flexible price equilibrium.

- So, the flexible price equilibrium with the efficient setting of ν represents a natural benchmark for the New Keynesian model, the version of the model in which $\theta > 0$.
  - We call this the *Natural Equilibrium*.

- To simplify the analysis, from here on we set $G_t = 0$.

Natural Equilibrium

- With $G_t = 0$, equilibrium conditions for C_t and N_t:

$$\overbrace{e^{\tau_t} C_t N_t^{\varphi}}^{\text{Marginal Cost of work}} = \overbrace{e^{a_t}}^{\text{marginal benefit of work}}$$

aggregate production relation: $C_t = e^{a_t} N_t$.

- Substituting,

$$e^{\tau_t} e^{a_t} N_t^{1+\varphi} = e^{a_t} \rightarrow N_t = \exp\left(\frac{-\tau_t}{1+\varphi}\right)$$

$$C_t = \exp\left(\mathrm{a}_t - \frac{\tau_t}{1+\varphi}\right)$$

$$R_t^* = \frac{\frac{1}{C_t}}{\beta E_t \frac{1}{C_{t+1}}} = \frac{1}{\beta E_t \frac{C_t}{C_{t+1}}} = \frac{1}{\beta E_t \exp\left(-\Delta a_{t+1} + \frac{\Delta \tau_{t+1}}{1+\varphi}\right)}$$

Natural Equilibrium, cnt'd

- Natural rate of interest:

$$R_t^* = \frac{\frac{1}{C_t}}{\beta E_t \frac{1}{C_{t+1}}} = \frac{1}{\beta E_t \exp\left(-\Delta a_{t+1} + \frac{\Delta \tau_{t+1}}{1+\varphi}\right)}$$

- Two models for a_t :

$$DS: \ \Delta a_{t+1} = \rho \Delta a_t + \varepsilon_{t+1}^a$$
$$TS: \ a_{t+1} = \rho a_t + \varepsilon_{t+1}^a$$

- Model for τ_t :

$$\tau_{t+1} = \lambda \tau_t + \varepsilon_{t+1}^{\tau}$$

Natural Equilibrium, cnt'd

- Suppose the ε_t's are Normal. Then,

$$E_t \exp\left(-\Delta a_{t+1} + \frac{\Delta \tau_{t+1}}{1+\varphi}\right) = \exp\left(-E_t \Delta a_{t+1} + E_t \frac{\Delta \tau_{t+1}}{1+\varphi} + \frac{1}{2}V\right)$$

  where

$$V = \sigma_a^2 + \frac{\sigma_\tau^2}{(1+\varphi)^2}$$

- Then, with $r_t^* \equiv \log R_t^*$

$$r_t^* = -\log \beta + E_t \Delta a_{t+1} - E_t \frac{\Delta \tau_{t+1}}{1+\varphi} - \frac{1}{2}V.$$

- Useful: consider how natural rate responds to ε_t^a shocks under DS and TS models for a_t and how it responds to ε_t^τ shocks.
  - To understand how r_t^* responds, consider implications of consumption smoothing in absence of change in r_t^*.
  - Hint: in natural equilibrium, r_t^* steers the economy so that natural equilibrium paths for C_t and N_t are realized.

Conclusion

- Described NK model and derived equilibrium conditions.
  - The usual version of model represents monetary policy by a Taylor rule.
- When $\theta = 0$, so that prices are flexible, then monetary policy is (essentially) neutral.
  - Changes in money growth move prices and wages in such a way that real wages do not change and employment and output don't change.
- When prices are sticky, then a policy-induced reduction in the interest rate encourages more nominal spending.
  - The increased spending raises W_t more than P_t because of the sticky prices, thereby inducing the increased labor supply that firms need to meet the extra demand.
  - Firms are willing to produce more goods because:
    - The model assumes they *must* meet all demand at posted prices.
    - Firms make positive profits, so as long as the expansion is not too big they still make positive profits, even if not optimal.