Examples in Which the Taylor Responds too Weakly to Shocks

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The result is presented in the form of a proposition at the end of this note. Here is the simple NK model, after linearization about a zero inflation steady state:

$$\begin{aligned}
x_t &= x_{t+1} - [r_t - \pi_{t+1} - r_t^*] \\
r_t &= \phi_\pi \pi_t \\
\pi_t &= \beta \pi_{t+1} + \kappa x_t \\
r_t^* &= E_t (a_{t+1} - a_t),
\end{aligned}$$

where

$$\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}(1+\varphi).$$

1 DS Representation

Suppose that the law of motion of a_t is as follows:

$$\Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t,$$

so that

$$r_t^* = E_t \Delta a_{t+1} = \gamma_0 \Delta a_t,$$

where

$$\gamma_0 = \rho. \tag{1}$$

Conjecture the following solution:[1]

$$\pi_t = \gamma_1 \Delta a_t, x_t = \gamma_2 \Delta a_t, r_t = \gamma_3 \Delta a_t$$

Substituting,

$$\gamma_2 = \rho\gamma_2 - \gamma_3 + \rho\gamma_1 + \rho \tag{2}$$

$$\gamma_3 = \phi_\pi \gamma_1 \tag{3}$$

$$\gamma_1 = \beta\gamma_1\rho + \kappa\gamma_2 \tag{4}$$

[1] When $\phi_\pi > 1$, this delivers the unique, non-explosive solution in a neighborhood of steady state. See this for details.

We also examine the response of the real interest rate, $\tilde{r}_t$:

$$\tilde{r}_t = r_t - E_t \pi_{t+1} = \gamma_4 \Delta a_t,$$

where

$$\gamma_4 = \gamma_3 - \gamma_1 \rho.$$

Rewriting (3):

$$\gamma_1 = \frac{\gamma_3}{\phi_\pi} \tag{5}$$

Solving (4) for γ_2 and using the previous expression:

$$\gamma_2 = \frac{(1-\beta\rho)}{\kappa}\gamma_1 = \frac{(1-\beta\rho)}{\kappa}\frac{\gamma_3}{\phi_\pi} \tag{6}$$

Rewriting (2):

$$(1-\rho)\,\gamma_2 + \gamma_3 - \rho\gamma_1 = \rho$$

Substituting (5) and (6) into the latter:

$$\left[\frac{(1-\beta\rho)}{\kappa}\frac{(1-\rho)}{\phi_\pi} + 1 - \frac{\rho}{\phi_\pi}\right]\gamma_3 = \rho. \tag{7}$$

Using (8) we obtain

$$\gamma_4 = \gamma_3 - \gamma_1\rho = \gamma_4 = \left(1 - \frac{\rho}{\phi_\pi}\right)\gamma_3 \tag{8}$$

so,

$$\begin{aligned}\gamma_4 &= \left(1-\frac{\rho}{\phi_\pi}\right)\frac{\rho}{\left(\frac{(1-\beta\rho)(1-\rho)}{(1-\beta\theta)(1-\theta)}\frac{\theta}{(1+\varphi)} - \rho\right)\frac{1}{\phi_\pi} + 1} \\ &= \frac{(\phi_\pi - \rho)\,\rho}{\frac{(1-\beta\rho)(1-\rho)}{(1-\beta\theta)(1-\theta)}\frac{\theta}{(1+\varphi)} + \phi_\pi - \rho},\end{aligned}$$

so that

$$\frac{\gamma_4}{\gamma_0} = \psi,$$

using (1), where

$$\psi = \frac{\phi_\pi - \rho}{\frac{(1-\beta\rho)(1-\rho)}{(1-\beta\theta)(1-\theta)}\frac{\theta}{(1+\varphi)} + \phi_\pi - \rho}. \tag{9}$$

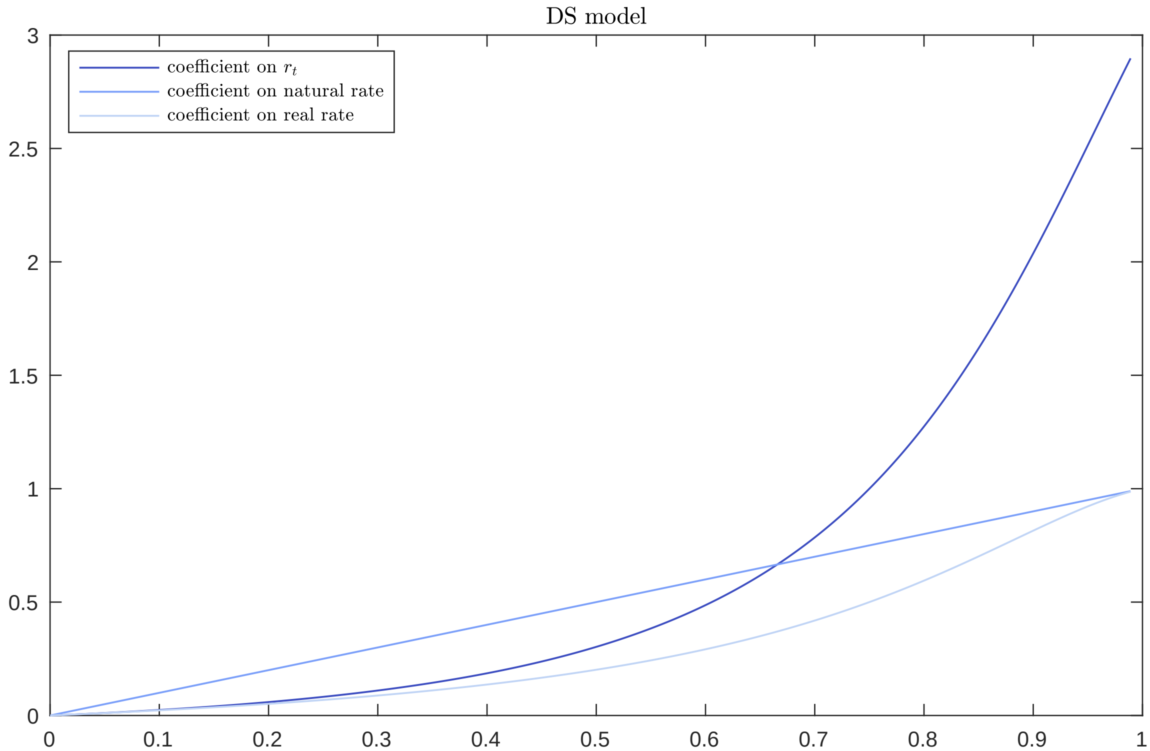
From this we see that, for $0 \leq \rho \leq 1$,

$$0 < \psi \leq 1, \text{ with equality only if } \rho = 1. \tag{10}$$

Figure 1 reports $\gamma_3, \gamma_0, \gamma_4$ for $\rho \in (0,1)$. We set

$$\theta = 3/4, \phi_\pi = 1.2, \beta = 1.03^{-1/4}, \varphi = 1,$$

Figure 1: DS Model Results



so that $\kappa = 0.17$, after rounding. From Figure 1, we see that the response of the real rate, γ_4, is with one exception, less than the response of the natural rate, γ_0, to a technology shock. The response of the nominal rate of interest, γ_3, to a technology shock could be bigger than the response of the natural rate, but that can only happen if the response of inflation is even stronger. Thus, in terms of moving the real rate of interest, the Taylor rule is almost always weaker than the natural rate in responding to a shock in technology. The exception is when the two responses are the same, when $\rho = 1$. Of course, what matters in the model is the real rate.

2 TS Model

Suppose that the law of motion of a_t is as follows:

$$a_t = \rho a_{t-1} + \varepsilon_t,$$

so that

$$r_t^* = E_t \Delta a_{t+1} = \gamma_0 a_t,$$

where

$$\gamma_0 = \rho - 1. \tag{11}$$

Note that now the response of the natural rate to a technology shock, γ_0, is related to ρ in a different way.

Conjecture the following solution:

$$\pi_t = \gamma_1 a_t, x_t = \gamma_2 a_t, r_t = \gamma_3 a_t$$

When these are substituted into the model, of equations (2)-(4), only (2) changes:

$$\gamma_2 = \gamma_2 \rho - \gamma_3 + \gamma_1 \rho + (\rho - 1).$$

Rewriting the first of these equations:

$$(1-\rho)\gamma_2 + \gamma_3 - \gamma_1 \rho = \rho - 1$$

As a result, we have, from (7),

$$\left[\frac{(1-\beta\rho)}{\kappa}\frac{(1-\rho)}{\phi_\pi} + 1 - \frac{\rho}{\phi_\pi}\right]\gamma_3 = \rho - 1 \tag{12}$$

The response of the real rate of interest, $\tilde{r}_t$, to a_t is γ_4 as defined in equation (8). The response of the natural rate of interest to a_t is γ_0, so by (8) and (12), we have we are now interested in

$$\frac{\gamma_4}{\gamma_0} = \left(1 - \frac{\rho}{\phi_\pi}\right)\frac{1}{\frac{(1-\beta\rho)}{\kappa}\frac{(1-\rho)}{\phi_\pi} + 1 - \frac{\rho}{\phi_\pi}} = \psi,$$

by (9) and (11). As before, ψ has the property, (10), even though the parameter ρ is now the parameter in the TS representation for a_t.

The values of $\gamma_4, \gamma_0, \gamma_3$ for $\rho \in (0,1)$ in the TS model are displayed in Figure 2. Note that, once again, the Taylor rule is too weak, in terms of moving the real rate of interest. That is, $\gamma_4 = \psi\gamma_0$, where ψ is given in (10).

3 Result

The result is stated in the form of a proposition:

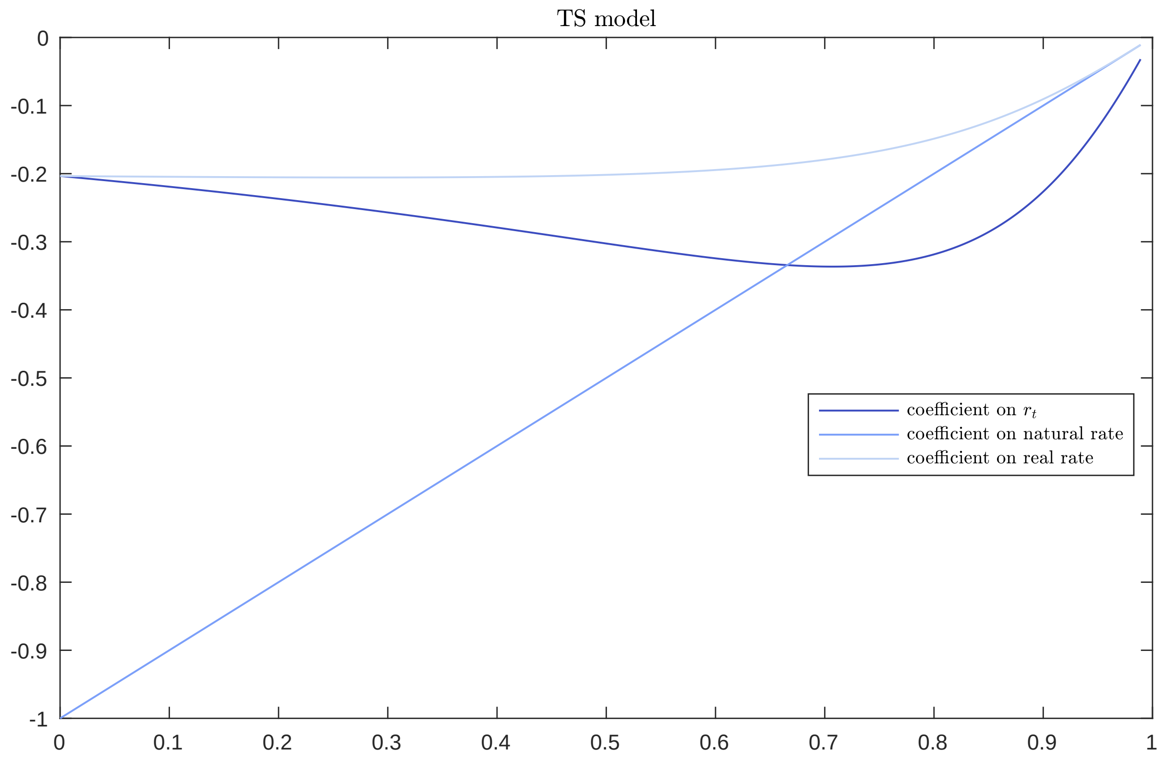
Proposition 1. *Suppose that the technology shock is driven by the DS or the TS model, with* $0 \leq \rho \leq 1$. *Also,* $\phi_\pi > 1$, $\beta, \theta \in (0,1)$, *and* $\varphi \geq 0$. *Then, real interest rate responds more weakly to a technology shock than the natural rate of interest does. That is,* $\gamma_4 = \psi\gamma_0$, *where* γ_0 *denotes the response of the natural rate of interest to a technology shock and* ψ *satisfies the restrictions in (10).*

4 What About Sunspots?

Suppose we allow for an additional shock that is non-fundamental in the sense that it does not enter the equilibrium conditions:

$$\nu_t = \xi \nu_{t-1} + u_t$$

Figure 2: TS model



The variable, ν_t, is also often referred to as a sunspot.

$$\pi_t = \gamma_1 \Delta a_t + \chi_1 \nu_t, x_t = \gamma_2 \Delta a_t + \chi_2 \nu_t, r_t = \gamma_3 \Delta a_t + \chi_3 \nu_t$$

Using the previous analysis, it is immediate that $\chi_i = 0$, $i = 1, 2, 3$, solves the model. The logic works like this. We can write (2), (3) and (4) in matrix form as follows:

$$A\gamma = b,$$

where A is a 3×3 matrix of model parameters, γ, is a 3×1 vector of the γ_i's and b is a 3×1 column vector in which the first element is not zero. The matrix A is non-singular. The solution for γ that we derived earlier is simply $A^{-1}b$. In the case of the χ_i's, they solve

$$A\chi = 0,$$

where 0 is a 3×1 column vector of zeros. Since A is non-singular, it follows that the unique solution for χ is zero. Thus, the non-fundamental shocks do not enter the solution to the model.

Suppose now that we change the monetary policy rule:

$$r_t = \phi_\pi E_t \pi_{t+1}.$$

The solution for γ is an obvious adjustment to is relevant to (2), (3) and (4):

$$\gamma_2 = \rho\gamma_2 - \gamma_3 + \rho\gamma_1 + \rho \tag{13}$$
$$\gamma_3 = \phi_\pi \rho \gamma_1 \tag{14}$$
$$\gamma_1 = \beta\gamma_1\rho + \kappa\gamma_2 \tag{15}$$

In matrix form, this is

$$A(\rho) = \begin{bmatrix} -\rho & 1-\rho & 1 \\ \phi_\pi \rho & 0 & 1 \\ 1-\beta\rho & \kappa & 0 \end{bmatrix}, b = \begin{pmatrix} \rho \\ 0 \\ 0 \end{pmatrix}$$

Then,

$$\gamma = A(\rho)^{-1} b.$$

At the same time, χ solves

$$\chi = A(\xi)^{-1} 0 = 0.$$

So, sunspots do not enter in this case.