A Theory of the Non-Neutrality of Money with Banking Frictions and Bank Recapitalization*

Zhixiong Zeng†
Department of Economics
Monash University

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Abstract

Policy actions by the Federal Reserve during the recent financial crisis often involve recapitalization of banks. This paper offers a theory of the non-neutrality of money for policy actions taking the form of injecting capital into banks via nominal transfers, in an environment where banking frictions are present in the sense that there exists an agency cost problem between banks and their private-sector creditors. The analysis is conducted within a general equilibrium setting with two-sided financial contracting. We first show that even with perfect nominal flexibility, the recapitalization policy can have real effects on the economy. We then study the design of the optimal long-run recapitalization policy as well as the optimal short-run policy responses to banking riskiness shocks.

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†Department of Economics, Monash University, Caulfield East, VIC 3145, Australia. Phone: +61-3-99034045, Fax: +61-3-99031128, Email: zhixiong.zeng@buseco.monash.edu.au, Homepage: http://faculty.ccer.edu.cn/zxzeng/.

1 Introduction

The Federal Reserve took a variety of unconventional policy actions during the recent financial crisis that started in 2007. As traditional interest rate policy that adjusts the federal funds rate was perceived to be ineffective (Cecchetti, 2009), the Fed adopted various measures of what Reis (forthcoming) classifies as “quantitative policy”, i.e., policy that changes the size of the Fed’s balance sheet and the composition of its liabilities, as well as “credit policy”, policy that manages the composition of its asset holdings. In addition to injecting liquidity into the financial system (Brunnermeier, 2009), some of the Fed’s policy measures also have the flavor of providing capital subsidy to banks, a point forcefully made by Cecchetti (2009). During the crisis, lending by the Fed to banks almost always involved a subsidy. By accepting collaterals at prices that were almost surely above their actual market prices (Tett, 2008), lending by the Fed in effect recapitalized the borrowing banks through nominal transfers. In response to the crisis, the Fed attempted to stimulate discount borrowing, which is collateralized, by reducing substantially the premium charged on primary discount lending (relative to the federal funds rate target) and raising the term of lending from overnight to as long as three months. In addition, to remove the stigma attached to discount borrowing[1], the Fed created the Term Auction Facility (TAF) in December 2007 and enlarged it later on in order to better provide funds to banks that need them most. The rules of the TAF allow banks to pledge collaterals that might otherwise have little market value.[2]

In the light of the celebrated Modigliani-Miller theorem, such bank recapitalization efforts, as short-run measures to cope with the adverse situation in the economy, would be impotent

[1] Traditionally, banks that borrowed from the discount window might be seen by other banks and institutions as having financial stress.

[2] For details, see Cecchetti (2009). Similar actions were taken by the Fed to help out other financial institutions (e.g., investment banks) through programs such as the Term Securities Lending Facility, the Primary Dealer Credit Facility, and the Term Asset-Backed Securities Loan Facility, etc.

in stimulating employment and output in a world where banks can frictionlessly raise funds to finance the loans they make, as the capital structure of banks would be irrelevant for their lending activities and the real market value of their loan portfolios. In that kind of world the classical dichotomy holds and the recapitalization of banks by the monetary authority is neutral, despite that it does involve a real transfer that enlarges banks' net worth relative to debt (because other sectors of the economy are not getting the same nominal transfer). However, as will be demonstrated in this paper, once an agency cost problem is introduced to the relationship between banks and their private-sector creditors (henceforth "depositors" for ease of exposition), the Modigliani-Miller theorem fails for banks, the classical dichotomy breaks down, and money is no longer neutral when the central bank policy takes the form of injecting money to the banking system to increase bank capital. In particular, a bank recapitalization effort by the monetary authority triggers a redistribution of wealth (nominal and real) in favor of the banks, reduces the cost of banks' external finance, stimulates bank lending, and raises employment and output. Importantly, this non-neutrality of money obtains even without any kind of nominal rigidities.

Needless to say, understanding the mechanism through which policy works is crucial for assessing the effectiveness of central bank reactions to the crisis. Impotent policy is clearly not interesting. The main thrust of the paper is that to make sense of the bank recapitalization policy, one has to take seriously frictions on the liability side of the bank balance sheet, i.e., frictions in the relationship between banks and depositors. The reason is that it is precisely frictions on the liability side of the bank balance sheet, rather than frictions on the asset side, that are responsible for the real effects of the bank recapitalization policy. As is already well known, on the asset side of the bank balance sheet there might exist informational asymmetry regarding the ability of (nonfinancial) firms to repay their loans, giving rise to an agency cost problem between banks and firms as emphasized in the seminal work of Bernanke and Gertler

(1989) and a large literature that follows. Frictions of this kind are the literature's main focus thus far. We shall refer to them as "credit frictions", for the sake of distinguishing it from the informational asymmetry and agency cost problem on the liability side of the bank balance sheet, which we shall call "banking frictions". To introduce the latter kind of frictions we apply the costly-state-verification (CSV) framework of Towsend (1979), Gale and Hellwig (1985), and Williamson (1986) to the bank-depositor relationship. In our model banks face idiosyncratic risks and depositors have to expend monitoring costs in order to verify banks' capacities to repay. As is shown in the paper, bank recapitalization by the monetary authority is neutral when banking frictions are absent, even if the conventionally studied credit frictions are present. This implies that what credit frictions do is at best to amplify and propagate the policy's real effects which are brought forth solely by the existence of banking frictions. We are thus compelled to give special attention to the roles banking frictions play. Modeling banking frictions and studying their implications for the effects of bank recapitalization policy is precisely the goal of this paper.

In our model economy, banks receive both deposits and central bank money injections to finance their lending activities. It should be clarified here that we use the term "deposits" in the broadest sense, referring to all liabilities of banks that are held by the private sector. Meanwhile, we lump all the private-sector creditors of banks, including consumers, nonfinancial businesses, and nonbank financial firms, into a single category of agents called "depositors". At the heart of our story is that the rate of default by banks and the cost of their external finance are positively related to their debt-equity ratios. Recapitalization by the monetary authority induces a real transfer in favor of the banks, no matter how the price level changes. This real transfer is not inconsequential: It lowers the banks' debt-equity ratio, leading to a decline in their default rate and the external finance premium, which in turn stimulates real bank lending and thus employment and output.

To highlight the mechanism at work, our model has abstracted from several aspects of the actual economy that might be considered important in other contexts. First, our analysis is conducted within a framework that allows for perfect nominal flexibility (i.e., there is no price or wage stickiness or adjustment cost on nominal savings). This allows us to isolate the real effects of the recapitalization policy from the non-neutrality produced by nominal rigidities. Second, insurance of deposits is not considered. This does not invalidate our analysis since a large fraction of bank liabilities remain uninsured. Neither are capital adequacy requirements incorporated. Hence the mechanism in our model does not work through the relaxation of binding capital adequacy requirements. Instead, it works through changing the banks' default rate and their cost of external finance. Third, our model is constructed in such a way that the firms' financial leverage is unaffected by the bank recapitalization policy in equilibrium, which enables us to focus on the role played by the banks' debt-equity ratio. Such a construct is innocuous as neither the non-neutrality result with banking frictions nor the neutrality result without banking frictions (but still with credit frictions) relies on the fixity of the equilibrium debt-equity ratio of firms.

In a model that allows for perfect nominal flexibility, some other sort of frictions must be employed to generate the non-neutrality of money. In Lucas' (1972) misperceptions theory it is the imperfect information about the overall price level that temporarily misleads suppliers and generates real effects of money supply shocks. It seems that information on money supply and other policy instruments are available to the public with little delay so there is no serious signal extraction problem to solve. Hence the misperceptions story might not be particularly relevant in our context. In contrast, this paper assumes full information on all aggregate variables but uses a different kind of information problem to generate the non-neutrality of money. The problem here concerns costly revelation of banks' information to depositors, which leads to the breakdown of the Modigliani-Miller theorem and gives rise to a nontrivial role for banks' capital structure. Although the idea that the Modigliani-Miller theorem might not apply for banks have been put forth by Kashyap and Stein (1995) and Stein (1998), our non-neutrality result with perfect nominal flexibility is novel.[3]

Although the focus of our paper is on the effects of short-run recapitalization efforts, i.e., policy actions intended to counteract adverse shocks to the economy, a more general formulation of the bank recapitalization policy is adopted in our analysis. We envision the policy as comprising a long-run component and a short-run component. The crucial difference between them is that the long-run policy involves a tradeoff between financial frictions and monetary frictions. The former is a combination of banking frictions and credit frictions, while the latter arises from the constraint that purchases of factor inputs must use cash. An increase in the long-run component reduces the extent of financial frictions while raising the risk-free nominal interest rate and hence the extent of monetary frictions. Balancing the effects of these two frictions results in an optimal long-run recapitalization policy, which turns out to be positive under reasonable parameterization of the model economy. In contrast, the short-run policy only affects the extent of financial frictions and leaves monetary frictions intact. This property allows the short-run policy to be used as a stabilization tool when the economy is subject to shocks to the level of the riskiness of banking, which gives rise to a short-run policy reaction function.[4]

The rest of the paper is organized as follows. Section 2 presents a model of two-sided financial contracting with idiosyncratic banking risks. A general equilibrium model with consumption/saving and labor supply decisions on the part of households is then developed in Section

[3] To be concrete, our model differs from theirs in two major respects. First, we use the CSV framework to model banking frictions, while Stein (1995) uses an adverse selection model, and Kashyap and Stein (1995) use a reduced-form formulation. Second, they rely on exogenously imposed incomplete adjustment of the price level to generate the non-neutrality of money, while our model assumes away all sorts of nominal rigidities.

[4] The level of banking riskiness is represented by a dispersion parameter of the distribution of the idiosyncratic bank productivities and is assumed to stochastic.

3. Section 4 characterizes the equilibrium, presents the non-neutrality result, and discusses the optimal long-run policy and the optimal short-run reaction function. The last section concludes. All proofs are relegated to the Appendix.

2 Financial Contracting with Banking Risks

2.1 Production and Information Structure

Consider an environment with a unit-mass continuum of regions indexed by i, $i \in [0,1]$. In region i there is one bank, called bank i, and a unit-mass continuum of firms indexed by ij, $j \in [0,1]$. Each firm resides in a distinct location, and operates a stochastic production technology that transforms labor and capital services into a homogeneous final output. The technology of firm ij is represented by the production function

$$y_{ij} = \theta_i \omega_{ij} F(k_{ij}, l_{ij}), \tag{1}$$

where y_{ij}, k_{ij}, and l_{ij} denote final output, capital input, and labor input, respectively, for firm ij. The function $F(\cdot)$ is linearly homogeneous, increasing and concave in its two arguments, and satisfies the usual Inada conditions. All sources of idiosyncratic risks are captured in the productivity factor, with θ_i being the random productivity specific to region i, and ω_{ij} the random productivity specific to location ij. We assume that θ_i is identical and independently distributed across regions, with c.d.f. $\Phi^r(\cdot)$ and p.d.f. $\phi^r(\cdot)$, and that ω_{ij} is identical and independently distributed across all locations, with c.d.f. $\Phi^l(\cdot)$ and p.d.f. $\phi^l(\cdot)$. Both θ_i and ω_{ij} have non-negative support and unit mean. Furthermore, θ_i and $\omega_{\tau j}$, $i, \tau, j \in [0,1]$, are uncorrelated with each other. The distributions are known by all agents in the economy.

Firms hire labor and rent capital from competitive factor markets at nominal wage rate W and rental rate R^k. Assume that each firm owns the same amount of physical capital K^f, and that each bank owns K^b. Both K^f and K^b are fixed. To simplify matters even further we assume that physical capital is not traded so that capital gains or losses (from changes in the price of capital) are not potential sources of changes in the net worth of firms and banks. Moreover, it cannot be transferred across different firms and banks. There is, however, a rental market. And the rental income of capital constitutes the firms and banks' internal funds.[5] Since the firms' internal funds are generated entirely from the current rental value of the capital stock they own, in a market clearing equilibrium the firms must borrow additional funds to finance their purchase of labor inputs supplied by workers plus rental services provided by the stock of physical capital owned by the banks. Our model thus emphasizes working capital financing as in Christiano and Eichenbaum (1992). Once firms acquire factor inputs, production takes place, and the region and location specific productivities realize. The final output is sold at price P in a competitive goods market.

We use the CSV approach of Towsend (1979), which is later adopted by Gale and Hellwig (1985) and Williamson (1986), to model financial frictions and financial contracting. It is assumed that there is an informational asymmetry regarding borrowers' ex post revenues. In particular, only borrowers themselves can costlessly observe their realized revenues, while lenders have to expend a verification cost in order to observe the same object. In our environment only firm ij can observe at no cost $s_{ij}^f \equiv \theta_i \omega_{ij}$, and only bank i can observe θ_i costlessly. For a bank to observe s_{ij}^f (or ω_{ij}) and for a depositor to observe θ_i, verification costs have to be incurred. Note that by lending to a continuum of firms in a particular region each bank effectively diversifies away all the firm/location specific risks. But the region specific risk is not diversifiable, giving rise to the possibility that a bank becomes insolvent when an adverse regional shock occurs. Our model thus features potential bankruptcy of banks in addition to potential bankruptcy of

[5] Note that the assumption of fixed capital stock does not prevent it from generating variable internal funds, because in the general equilibrium the rental rate responds to aggregate shocks.

nonfinancial firms. Note that even if the working capital loans are perfectly safe for the banks (no default by the firms), the depositors still regard their claims on the banks as being risky due to the informational asymmetry about the idiosyncratic bank/region productivities.

The concept of "regions" should not be interpreted literally as reflecting geographic areas, albeit this is certainly one of the many possible interpretations. Rather, it is a device designed to generate risks idiosyncratic to individual banks. If banks are subject to risks that cannot be fully diversified, then the kind of agency problem between banks and firms applies equally well to the relationship between banks and depositors. In that case there are needs to "monitor the monitor", in the terminology of Krasa and Villamil (1992a). Bank-level risks might stem from geographic confinement of an individual bank's operation to specific areas, as in the U.S. when out-of-state branching was restricted (see Williamson, 1989). They might also be due to the concentration of a bank's lending activities in specific industries. Savings and loan associations in the U.S., which historically concentrated on mortgage loans, was a good example. It should be noted that even without branching restrictions or regulations on banks' lending and investment activities, an individual bank might optimally choose to limit its scale and/or scope of operation so that the risks associated with its lending activities are not fully diversified. An example appears in Krasa and Villamil (1992b), who consider the trade-off involved in increasing the size of a bank's portfolio (i.e., lending to additional borrowers). In their model balancing the gains from decreased default risk with the losses from increased monitoring costs leads to an optimal scale for banks. Another example is Cerasi and Daltung (2000), who introduce considerations on the internal organization of banks that render scale economies in the banking sector rapidly exhausted.[6] In this paper we follow Krasa and Villamil (1992a) and Zeng (2007) to assume that an individual bank cannot contract with a sufficient variety of borrowers so that the credit risks

[6]Specifically, loan officers, who are the ones actually making loans, have to be monitored by the banker.

are not perfectly diversifiable.

2.2 The Two-Sided Debt Contract

The three groups of players—firms, banks, and depositors—in the model are connected via a two-sided contract structure. Both sides of the contract, one between the firms and banks and the other between the banks and depositors—fit into a generic framework we now develop. Here attention is restricted to deterministic monitoring.[7] It is also assumed that all contracting parties are risk neutral. It then follows that the optimal contract between a generic borrower and a generic lender takes the form of a standard debt contract, in Gale and Hellwig (1985)'s term.

Suppose that the borrower's revenue is given by Vs, where V is a component freely observable to the lender, and $s \geq 0$ is a unit-mean risky component that is subject to informational asymmetry, whereby the borrower can costlessly observe s while the lender has to expend a verification cost in order to do so. The verification cost is assumed to be μ times the borrower's revenue, with $\mu \in (0, 1)$. The c.d.f. of s, given by $\Phi(\cdot)$, is also common knowledge. The contract specifies a set of realizations of s for which monitoring occurs, together with a payment schedule. An incentive compatible contract must specify a fixed payment for s in the non-monitoring set, otherwise the borrower will always report the value of s for which the payment is lowest among non-monitoring states. A standard debt contract with monitoring threshold $\bar{s}$ is an incentive compatible contract with the following features: (i) the monitoring set is $\{s|s < \bar{s}\}$, (ii) the fixed payment is $V\bar{s}$ for $s \in \{s|s \geq \bar{s}\}$, and (iii) the payment is Vs for $s \in \{s|s < \bar{s}\}$. The standard

[7] The assumption of deterministic monitoring is actually less restrictive than it appears. Krasa and Villamil (2000) articulates a costly enforcement model that justifies deterministic monitoring when commitment is limited and enforcement is costly and imperfect. See also Mookherjee and Png (1989) and Boyd and Smith (1994) on deterministic versus stochastic monitoring.

debt contract is particularly interesting because it resembles many financial contracts in the real world. It features fixed payment for non-default states and state-contingent payment when default occurs. Requiring the borrower to repay as much as possible in default states allows the fixed payment for non-default states to be minimized, thus minimizing the probability of verification and thus the expected monitoring cost.

Under the standard debt contract, the borrower and the lender each obtains a share of the expected revenue V. The borrower receives $V\Gamma(\bar{s};\Phi)$ where

$$\Gamma(\bar{s};\Phi) \equiv \int_{\bar{s}}^{\infty} (s-\bar{s})\, d\Phi(s)\,, \tag{2}$$

reflecting the fact that with s above $\bar{s}$, the borrower gives out the fixed payment $V\bar{s}$ and keeps the remaining, while with s below $\bar{s}$, all revenues are confiscated by the lender. The lender receives $V\Psi(\bar{s};\Phi)$ where

$$\Psi(\bar{s};\Phi) \equiv \bar{s}\left[1-\Phi(\bar{s})\right] + (1-\mu)\int_{0}^{\bar{s}} s d\Phi(s)\,. \tag{3}$$

When s is larger than or equal to $\bar{s}$, which occurs with probability $1-\Phi(\bar{s})$, the lender recoups the fixed proportion $\bar{s}$ of the expected revenue V. If s falls below $\bar{s}$, the lender takes all of the realized revenue while expending a verification cost which equals a fraction μ of the revenue. Note that

$$\Gamma(\bar{s};\Phi) + \Psi(\bar{s};\Phi) = 1 - \mu\int_{0}^{\bar{s}} s d\Phi(s) < 1,$$

indicating that there is a direct deadweight loss $\mu\int_{0}^{\bar{s}} s d\Phi(s)$ due to costly monitoring. The following assumption is imposed.

Assumption 1. (a) The p.d.f $\phi(\cdot)$ is positive, bounded, and continuously differentiable on $(0,\infty)$, and (b) $s\phi(s)/\left[1-\Phi(s)\right]$ is an increasing function of s.

Assumption 1(b), that $s\phi(s)/[1-\Phi(s)]$ is increasing in s, is weaker than the increasing hazard assumption commonly made in the incentive contract literature, which requires $\phi(s)/[1-\Phi(s)]$ to be monotonically increasing in s. Yet the latter property is already satisfied by a fairly large class of distributions. It can be shown that for $\bar{s}>0$,

$$\Gamma'(\bar{s};\Phi) = -[1-\Phi(\bar{s})] < 0,$$

$$\Psi'(\bar{s};\Phi) = 1-\Phi(\bar{s}) - \mu\bar{s}\phi(\bar{s}) > 0, \text{ if } \bar{s} < \hat{s},$$

and

$$\Gamma'(\bar{s};\Phi) + \Psi'(\bar{s};\Phi) = -\mu\bar{s}\phi(\bar{s}) < 0,$$

where the primes denote derivatives and $\hat{s}$ satisfies $1-\Phi(\hat{s})-\mu\hat{s}\phi(\hat{s})=0$. We rule out the possibility of credit rationing by requiring $V\Psi(\hat{s};\Phi)$ to be no less than the opportunity cost of funds for the lender (see Williamson, 1986). Thus the domain of $\bar{s}$ we are interested in is $[0,\hat{s})$ and $\Psi'(\bar{s};\Phi)>0$ on this interval. It is interesting to note that changes in the monitoring threshold (and hence the default probability) generate redistributions of the expected revenue between the borrower and the lender. An increase in $\bar{s}$ reduces the share Γ received by the borrower, while raising the share Ψ received by the lender. The total effect on the returns to the two parties, however, is negative since the marginal increase in the lender's share is less than the marginal increase in the borrower's share, reflecting the additional monitoring cost born by the lender at the margin. Furthermore,

$$\begin{aligned}
\lim_{\bar{s}\to 0}\Gamma(\bar{s};\Phi) &= 1, \quad \lim_{\bar{s}\to 0}\Psi(\bar{s};\Phi)=0, \quad \lim_{\bar{s}\to 0}[\Gamma(\bar{s};\Phi)+\Psi(\bar{s};\Phi)]=1,\\
\lim_{\bar{s}\to 0}\Gamma'(\bar{s};\Phi) &= -1, \quad \lim_{\bar{s}\to 0}\Psi'(\bar{s};\Phi)=1, \quad \lim_{\bar{s}\to 0}\left[\Gamma'(\bar{s};\Phi)+\Psi'(\bar{s};\Phi)\right]=0,
\end{aligned}$$

whenever the probability density $\phi(\bar{s})$ is bounded as in Assumption 1(a). These limits indicate that starting from a small default rate, where the borrower grabs virtually all of the revenues, an increase in the monitoring threshold generates a nearly one-for-one transfer of returns from the borrower to the lender *without* producing discernible effects on the sum of returns (that is, the marginal direct deadweight loss is practically zero).

We now apply this generic debt contract framework to the bank-firm relationship. The firm's revenue can be written as $V^f\omega$, where $V^f \equiv PF(k,l)\,\theta$ is freely observable to the bank, and ω is the risk that can be observed by the bank only with a cost.[8] The contract between the bank and the firm specifies a monitoring threshold, denoted by $\overline{\omega}$, for the firm/location specific productivity ω. Conditional on the region specific productivity θ, the expected return to the firm is then given by $PF(k,l)\,\theta\Gamma^f\left(\overline{\omega};\Phi^l\right)$ and the revenue of the bank from lending to the firms in its region is $PF(k,l)\,\theta\Psi^b\left(\overline{\omega};\Phi^l\right)$, where $\Gamma^f\left(\overline{\omega};\Phi^l\right)$ and $\Psi^b\left(\overline{\omega};\Phi^l\right)$ result from substituting $\left(\overline{\omega};\Phi^l\right)$ for $(\overline{s};\Phi)$ in (2) and (3).[9]

The contracting problem between the bank and its depositors specifies a monitoring threshold for the bank risk θ. To fit this into the generic setup, write the bank's revenue as $V^b\theta$, where $V^b \equiv PF(k,l)\,\Psi^b\left(\overline{\omega};\Phi^l\right)$. Here $\overline{\omega}$—the monitoring threshold in the bank-firm contract—is freely observable to both the bank and the depositors. Let $\overline{\theta}$ represent the monitoring threshold for θ in the bank-depositor contract. Then the expected return to the bank from the contract is $V^b\Gamma^b\left(\overline{\theta};\Phi^r\right)$ and the expected return to the depositors is $V^b\Psi^d\left(\overline{\theta};\Phi^r\right)$, where $\Gamma^b\left(\overline{\theta};\Phi^r\right)$ and $\Psi^d\left(\overline{\theta};\Phi^r\right)$ obtain from substituting $\left(\overline{\theta};\Phi^r\right)$ for $(\overline{s};\Phi)$ in (2) and (3).

2.3 Optimal Competitive Contract

To motivate competitive banking assume that in principle a bank is allowed to operate beyond its region. But that entails a fixed cost. It follows that the bank in region i must offer to

[8] From the bank's perspective, monitoring $x^f \equiv \theta\omega$ is equivalent to monitoring ω given its information in θ.

[9] By the law of large numbers, the revenue of the bank from lending to all of the firms in its region is the same as the expected revenue from lending to one firm, the expectation taken over the distribution of ω and conditional on θ.

the firms in that region financial contracts that maximize the firms' expected return such that if bank j, $j \neq i$ offers the same contracts to the same firms the expected return earned by bank j will equal the opportunity cost of its funds plus the cost of operating outside region j. Otherwise bank j would offer alternative contracts with terms that are preferable to the firms and make a profit itself. If the out-of-region operating cost goes to zero, then the limit case is perfect competition for the banking industry, where each bank offers contracts that maximize the expected return to the firms in its region such that the bank itself at least earns the riskless return on its funds. We focus on this limit situation and state formally the optimal competitive contract as solving the following problem.

Problem 1.

$$\max_{k,l,\overline{\omega},\overline{\theta},N^d} PF(k,l)\,\Gamma^f\left(\overline{\omega};\Phi^l\right)$$

subject to

$$PF(k,l)\,\Psi^b\left(\overline{\omega};\Phi^l\right)\Gamma^b\left(\overline{\theta};\Phi^r\right) \geq RN^b, \tag{4}$$

$$PF(k,l)\,\Psi^b\left(\overline{\omega};\Phi^l\right)\Psi^d\left(\overline{\theta};\Phi^r\right) \geq RN^d, \tag{5}$$

$$R^k k + Wl \leq N^f + N^b + N^d, \tag{6}$$

where R is the risk-free nominal rate of interest. Here $PF(k,l)\,\Gamma^f\left(\overline{\omega};\Phi^l\right)$ is the expected return to the firm, unconditional on θ. Inequality (4) is the individual rationality (IR) constraint for the bank, which says that the bank must obtain at least what it can earn by investing all of its capital (in the financial sense) in riskless securities. The amount of the bank's financial capital equals the rental value of the physical capital stock it owns plus the injection of capital from the central bank, Z. That is, $N^b \equiv R^k K^b + Z$. Inequality (5) is the IR constraint for the depositors, which says that the contract guarantees a riskless return R on their deposits. Finally, inequality (6) is the flow-of-funds constraint for the firms. The total bill for the firms' factor inputs is

$R^k k + Wl$, which has to be covered by the internal funds of the firms themselves, $N^f \equiv R^k K^f$, and bank loans that equal the sum of bank capital N^b and deposits N^d. In Problem 1 N^f and N^b are taken as given.

Define the "debt-equity ratios" for the bank and firms, denoted by ζ^b and ζ^f respectively, as

$$\zeta^b \equiv \frac{N^d}{N^b}, \quad \zeta^f \equiv \frac{N^b + N^d}{N^f}.$$

As shown in the Appendix, the solution to Problem 1 satisfies the following conditions:

$$F_k(k,l) = q\left(\overline{\omega}, \overline{\theta}\right) R \frac{R^k}{P}, \tag{7}$$

$$F_l(k,l) = q\left(\overline{\omega}, \overline{\theta}\right) R \frac{W}{P}, \tag{8}$$

$$\frac{\Psi^d\left(\overline{\theta}; \Phi^r\right)}{\Gamma^b\left(\overline{\theta}; \Phi^r\right)} = \zeta^b, \tag{9}$$

$$q\left(\overline{\omega}, \overline{\theta}\right) \Psi^b\left(\overline{\omega}; \Phi^l\right) \left[\Gamma^b\left(\overline{\theta}; \Phi^r\right) + \Psi^d\left(\overline{\theta}; \Phi^r\right)\right] = \frac{\zeta^f}{1+\zeta^f}, \tag{10}$$

where

$$q\left(\overline{\omega}, \overline{\theta}\right) \equiv \left\{\left[\Psi^b\left(\overline{\omega}; \Phi^l\right) - \Gamma^f\left(\overline{\omega}; \Phi^l\right) \frac{\Psi^{b\prime}\left(\overline{\omega}; \Phi^l\right)}{\Gamma^{f\prime}\left(\overline{\omega}; \Phi^l\right)}\right]\left[\Psi^d\left(\overline{\theta}; \Phi^r\right) - \Gamma^b\left(\overline{\theta}; \Phi^r\right) \frac{\Psi^{d\prime}\left(\overline{\theta}; \Phi^r\right)}{\Gamma^{b\prime}\left(\overline{\theta}; \Phi^r\right)}\right]\right\}^{-1}. \tag{11}$$

The factor $q\left(\overline{\omega}, \overline{\theta}\right) > 1$ whenever $\overline{\omega}, \overline{\theta} > 0$, and $\lim_{\overline{\omega}, \overline{\theta} \to 0} q\left(\overline{\omega}, \overline{\theta}\right) = 1$.

Conditions (7)-(10) capture the notion that monetary frictions *and* financial frictions lead to inefficient use of resources. Equations (7) and (8) are the first-order conditions for factor demand. They state that capital and labor inputs are employed up to the points where their marginal products equal real factor prices, times the gross nominal interest rate R, and times an object labeled q which is determined by the terms of the financial contract, with both R and q larger than or equal to one. In the first-best world productive efficiency requires equating the marginal product of factor inputs to their real prices. In our model, however, there are various sources of frictions that prevent the economy from achieving the first best.

The first friction arises from the requirement that factor market transactions must use cash, a friction we call monetary friction. A gross nominal interest rate that is strictly greater than one creates wedges between the marginal products of factor inputs and their real prices, leading to underemployment of factor inputs. The second and third sources of distortions, measured in combination by the factor $q\left(\overline{\omega},\overline{\theta}\right)$, which we shall call the *financial friction indicator*, lie in the agency cost problem between borrowers and lenders. If either $\overline{\omega}>0$ or $\overline{\theta}>0$ (or both) then $q\left(\overline{\omega},\overline{\theta}\right)$ is strictly greater than one. Here $\overline{\omega}>0$ indicates a positive default rate by the firms and reflects the agency cost in the bank-firm relationship. This is what the existing literature on credit market imperfections has typically focused on. On the other hand, $\overline{\theta}>0$ corresponds to a positive rate of default by the banks (to the depositors) and reflects the agency cost in the bank-depositor relationship. These financial frictions create additional wedges between the marginal products of factor inputs and their real prices. The variable $q\left(\overline{\omega},\overline{\theta}\right)$ measures the overall distortions caused by the conventionally studied credit frictions and the sort of banking frictions we introduce. Again, the presence of financial frictions leads to underemployment of resources. The distinction between monetary frictions and financial frictions is important. In the general equilibrium model to be presented in the next section, the "long-run" recapitalization policy will involve a tradeoff between these two kinds of frictions, represented by movements of R and q in opposite directions, while the "short-run" recapitalization policy impacts on the economy only through its effect on q.

Equations (9) and (10) reflect the fact that the optimal competitive contract entails binding IR constraints for both the bank and the depositors. Essentially, the terms of contract dictate a division of expected revenues between borrowers and lenders. Equation (9) says that in the bank-depositor contract the share of expected revenue received by the depositors, relative to the share received by the bank, is positively related to the bank's debt-equity ratio. Since

$\Psi^d\left(\overline{\theta};\Phi^r\right)/\Gamma^b\left(\overline{\theta};\Phi^r\right)$ is increasing in $\overline{\theta}$, the bank's default probability increases along with $\overline{\theta}$ when it has a larger debt-equity ratio ζ^b. Equation (10) says that the total share of expected revenue that goes to the bank *and* the depositors, adjusted for the factor $q\left(\overline{\omega},\overline{\theta}\right)$, is positively related to the firms' debt-equity ratio ζ^f.

3 General Equilibrium

We now embed the two-sided financial contract articulated in the previous section to a full-blown general equilibrium model. The goal is to analyze how a bank recapitalization policy, taking the form of central bank money injection into the banking system, will affect the economy.

3.1 The Environment

Time is discrete and there is a representative household. Following Lucas (1990), we model the household as a multi-member "family". The household is populated with a unit-mass continuum of members. Each member has the same utility function, defined over consumption and leisure streams. They work to earn wage income in the labor market, and are also engaged in financial transactions with the banks, thereby playing the roles of "depositors" as described in the previous section. We assume that each member has the same amount of deposits. At the end of each period all members reconvene and submit all of their income to the household. Note that different members might have different amounts of income to bring to the household, depending on the realizations of the idiosyncratic risks of the banks they contracted with. Since the household, through its members, contracts with all the banks in the economy, it effectively holds a perfectly diversified (with respect to θ) portfolio of deposits. Thus the household's total income is not exposed to idiosyncratic bank risks: the total return on all deposits always equals the expected return on each individual member's deposits by the law of large numbers.[10] This income pooling assumption enables us to envision a perfect risk-sharing allocation designed by the household that assigns equal amounts of consumption (and leisure) to its members, which effectively renders each member risk neutral with respect to the banking risk θ. This justifies our treatment of the depositors as being risk neutral in the financial contracting problem. We also assume that the firms and banks do not retain earnings in order to invest in consecutive periods, so that the financial contracting problem is of period-by-period nature and is as formulated in Problem 1.

Suppose that an individual household member has preferences represented by the following life-time utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) + \nu \log\left(1 - L_t\right)\right], \tag{12}$$

where C_t is consumption in period t, L_t is hours worked (the time endowment is normalized to be one), $\nu > 0$ is a constant that weighs leisure relative to consumption, $\beta \in (0,1)$ is the time discount factor, and E_0 is the expectation operator conditional on time-0 information.[11] The assumption of perfect risk sharing against bank risks implies that for the purpose of characterizing the behavior of aggregate variables it suffices to consider consumption, leisure, and saving (in the form of bank deposits) as being chosen by the household who maximizes (12), where the expectation is taken over the distribution of aggregate shocks conditional on time-0 aggregate information.[12]

Let M_t denote the quantity of money outstanding at the beginning of period t. In equilibrium this is all held by the household. In period t the central bank injects $Z_t \equiv M_{t+1} - M_t$ into the

[10] Note that the household, as the owner of all the banks and firms in the economy, also receives all the profit. Again, by the law of large numbers, the total profit it receives from all the banks (firms) always equals the expected profit from each individual bank (firm).

[11] The assumption that the period utility is logarithmic and separable in consumption and leisure allows us to arrive at an analytical characterization of the equilibrium of the model economy.

[12] The household members do not bear the consequences of bank risks but still have to bear the consequences of aggregate shocks, such as policy shocks or banking riskiness shocks, which are not diversifiable.

economy by means of nominal transfers to the banking system, which effectively recapitalizes the banks. Every bank receives the same amount of transfer. The quantity of money injection is public information so that the model assumes full information on aggregate variables. In the sequel we normalize all nominal quantities and prices by M_t, following the practice of Christiano (1991) and Christiano and Eichenbaum (1992). The resultant variables will be denoted by corresponding lowercase letters. Let $z_t \equiv Z_t/M_t$ be the recapitalization rate. We model z_t as consisting of two components—a long-run component, represented by the constant $\eta \geq 0$, and a short-run component, denoted by x_t. That is

$$z_t = \eta + x_t. \tag{13}$$

Here $x_t \geq -\eta$ so that z_t is always nonnegative. Furthermore, x_t is assumed to be a mean zero, i.i.d. stochastic process. The i.i.d. assumption prevents the "anticipated inflation effect" of a short-run increase in money growth from arising (see Christiano, 1991 and Williamson, 2005 for an exposition). The short-run component should be thought of as adjustment of the recapitalization policy around the long-run component. For the present we treat both η and x_t as exogenous. Later on we will study how they might react to variations in the extent of banking frictions.

After observing the value of z_t, the household chooses its portfolio by dividing the nominal balance m_t between savings n_t^d, to be deposited in the banks, and cash holdings $m_t - n_t^d$ (these quantities obtain after normalization by M_t). We assume that there is always a zero supply of risk-free government bonds, so that in equilibrium all of the household's savings are in the form of deposits in the banks. Nevertheless, the zero-supply risk-free bonds can still be priced (at $1/R_t$). The bank-depositor contracts ensure that the risk-free return R_t accrues to household deposits n_t^d. Contrary to the limited participation literature, we assume that there is no cost or other barrier for the household to adjust its nominal savings in response to realizations of x_t. Hence our model also abstracts away the "liquidity effect" of a short-run increase in money growth. Removal of both the anticipated inflation effect and the liquidity effect makes the risk-free nominal interest rate unresponsive to x_t, which greatly simplifies the analysis of the effects of the short-run recapitalization policy.

There is a cash-in-advance (CIA) constraint, standard in the literature, on the household's purchase of consumption:

$$p_t C_t \leq m_t - n_t^d + w_t L_t, \tag{14}$$

where $p_t \equiv P_t / M_t$ is the scaled price level. This formulation is consistent with our previous assumption that firms must acquire cash to purchase labor inputs (from workers). Implicit in (14) is the notion that the wage income can be used to purchase consumption, along with the cash balance the household set aside at the beginning of period t. Formulation like this allows us to derive a standard quantity equation of money (see the next section). The household's cash holdings evolve according to

$$m_{t+1}(1+z_t) = \Big(m_t - n_t^d + w_t L_t - p_t C_t\Big) + R_t n_t^d + \pi_t, \tag{15}$$

where the term in the parentheses on the right-hand side is the unspent cash in the goods market, $R_t n_t^d$ is the gross return on deposits, and π_t is the total profit of banks and firms, paid out to the household in accordance with its ownership.[13]

The household maximizes (12) subject to (14) and (15). Its optimal plan obeys the following conditions:

$$\frac{\nu C_t}{1-L_t} = \frac{w_t}{p_t}, \tag{16}$$

$$E_t \left\{ \frac{1}{p_t C_t} - \beta \frac{R_t}{p_{t+1} C_{t+1}(1+z_t)} \right\} = 0, \tag{17}$$

[13] The household takes π_t as given. But in equilibrium $\pi_t = p_t F(K_t, L_t) \left[\Gamma^f \left(\overline{\omega}_t, \Phi^l \right) + \Psi^b \left(\overline{\omega}_t, \Phi^l \right) \Gamma^b \left(\overline{\theta}_t, \Phi^r \right) \right]$.

where E_t is the expectation operator conditional on time-t aggregate information. Equation (16) is the first-order condition for labor supply, while equation (17) is the standard consumption/saving Euler equation, modified to the current monetary environment.

Finally, we assume that the production function takes the standard Cobb-Douglas form:

$$F(K, L) = K^{\alpha} L^{1-\alpha}, \quad \alpha \in (0, 1),$$

where we have used K and L to replace k and l in (1) in anticipation of factor-market clearing.

3.2 Competitive Equilibrium Defined

We now define a competitive equilibrium for our model economy with banking frictions and two-sided financial contracting.

Definition 1. A *competitive equilibrium* of the model economy is a policy $\{z_t\}_{t=0}^{\infty}$, an allocation $\{C_t, , m_{t+1}, n_t^d, K, L_t\}_{t=0}^{\infty}$, a price system $\{p_t, w_t, r_t^k, R_t\}_{t=0}^{\infty}$, and terms of financial contract $\{\overline{\omega}_t, \overline{\theta}_t\}_{t=0}^{\infty}$ such that

i. Given the policy and prices, $\{C_t, , m_{t+1}, n_t^d, L_t\}_{t=0}^{\infty}$ solves the household's problem and satisfies (16)-(17). The CIA constraint (14) holds with equality whenever $R_t > 1$.

ii. Given the policy and prices, $\{K, L_t, \overline{\omega}_t, \overline{\theta}_t\}_{t=0}^{\infty}$ solves the financial contracting problem (Problem 1) and satisfies (7)-(10).

iii. The money market, loan market, and goods market clear. That is, $m_t = 1$ in addition to

$$w_t L_t = n_t^d + z_t, \tag{18}$$

$$C_t = F(K, L_t)\, \varphi\left(\overline{\omega}_t, \overline{\theta}_t\right), \tag{19}$$

where

$$\varphi\left(\overline{\omega}_t, \overline{\theta}_t\right) \equiv \Gamma^f\left(\overline{\omega}_t; \Phi^l\right) + \Psi^b\left(\overline{\omega}_t; \Phi^l\right)\left[\Gamma^b\left(\overline{\theta}_t; \Phi^r\right) + \Psi^d\left(\overline{\theta}_t; \Phi^r\right)\right]. \tag{20}$$

iv. $R_t \geq 1$ for all time.

In the goods-market clearing condition (19), the factor $\varphi\left(\overline{\omega}_t, \overline{\theta}_t\right) < 1$ whenever $\overline{\omega}_t > 0$, or $\overline{\theta}_t > 0$, or both, reflecting the direct deadweight loss due to the agency cost problems.[14] We call φ the net output factor since it gives the proportion of the gross output that is not dissipated in the agency process. The loan market clearing condition takes the form of (18) because the firms' rental payment on capital is covered by the rental value of the stock of capital owned by the firms and banks. It remains that their wage bills are to be ultimately financed by household deposits and the monetary authority's transfers to the banks.[15]

For analytic purpose it will be especially convenient to look at the behavior of the model economy around a situation where no default by either the banks or the firms occurs. We define such a situation as follows.

Definition 2. A *zero-default equilibrium* is the competitive equilibrium of the model economy obtained when the distributions for θ and ω are degenerate.

Essentially, the asymmetric information problems disappear when θ and ω are non-stochastic, giving rise to zero default in equilibrium. Proof of the existence and uniqueness of the zero-default equilibrium is trivial. Our analysis will focus on the neighborhood of the zero-default equilibrium where the default rates are small. According to Fisher (1999), the historical average of bankruptcy rate is indeed quite small. Using the Dun & Bradstreet dataset, he finds an average quarterly bankruptcy rate of roughly one percent for nonfinancial firms. This does not, however, mean that the distortions caused by financial frictions are negligible. In fact, Bernanke, Gertler, and Gilchrist (1999) show that a similar magnitude of bankruptcy rate is

[14] Remember that there is also an indirect social loss due to the distortions on the marginal costs of production caused by $q > 1$.

[15] To write the loan market clearing condition in full, we have $r_t^k K + w_t L_t = n_t^f + n_t^b + n_t^d = r_t^k \left(K^f + K^b\right) + n_t^d + z_t$. This simplifies to (18) since $K = K^f + K^b$.

consistent with an average external finance premium, or risk spread, of about two hundred basis points per annum.[16] Therefore the focus of our analysis in the neighborhood of the zero-default equilibrium does not entail a large deviation from the reality.

4 The Effects of Bank Recapitalization

4.1 Characterization of Equilibrium

As the policy process z_t is assumed to be stationary, the equilibrium allocation, prices, and contract terms in period t are functions of z_t, the functions being invariant with respect to t. Hence the time subscripts will be dropped in the subsequent analysis whenever possible. To avoid confusion, denote the random policy variable by $\mathbf{z}$ and its realization by z. Similarly, denote the random short-run component of the policy by $\mathbf{x}$ and its realization by x. Given the constant long-run component η, we have $\mathbf{z} = \eta + \mathbf{x}$ and $z = \eta + x$. Below we develop an algorithm to solve for the equilibrium. In preparation we note the following.

First, the loan market clearing condition (18) together with the binding CIA constraint (14) imply the quantity equation:

$$pC = 1 + z. \tag{21}$$

Second, the risk-free nominal interest rate R is constant for given η. Substitution of the quantity equation (21) into the Euler equation (17) gives

$$R = \left[\beta E\left(\frac{1}{1+\mathbf{z}}\right)\right]^{-1}, \tag{22}$$

where $E(\cdot)$ denotes unconditional expectation. The i.i.d. assumption on x_t implies that R is independent of the realized value of the short-run policy..

[16] In Bernanke et al. (1999), the empirical measure of the risk spread is taken to be the difference between the prime lending rate and the six-month T-bill rate.

Third, the debt-equity ratio of firms is constant:

$$\zeta^f \equiv \frac{n^b + n^d}{n^f} = \frac{\left(r^k K^b + z\right) + (wL - z)}{r^k K^f} = \frac{K^b}{K^f} + \frac{wL}{r^k K}\frac{K}{K^f} = \frac{K^b}{K^f} + \frac{1-\alpha}{\alpha}\frac{K}{K^f}. \tag{23}$$

The last equality follows from the Cobb-Douglas form of technology.

Fourth, the debt-equity ratio of banks is given by

$$\zeta^b \equiv \frac{n^d}{n^b} = \frac{wL - z}{r^k K^b + z}. \tag{24}$$

Absent the term z, ζ^b is also a constant, given by $(1-\alpha)K/\left(\alpha K^b\right)$. Hence by construction our model features a debt-equity ratio of firms that is unaffected by the bank recapitalization policy, along with a debt-equity ratio of banks that can be perturbed by the policy. This feature allows us to highlight the bank capital/liability side of the story.

Fifth, the bank debt-equity ratio ζ^b is a *sufficient statistic* for the monitoring thresholds $\left(\overline{\omega}, \overline{\theta}\right)$ and hence the financial friction indicator q as well as the net output factor φ. The dependence of q on ζ^b is a central relationship in our analysis as it highlights the impact of changes in the banks' capital structure on the extent of financial frictions. The following lemma states that an increase in ζ^b leads to a larger value of q in the neighborhood of zero-default. The increase in q would reduce labor demand, ceteris paribus. However, the increase in ζ^b might also lead to an increase in the direct deadweight loss and hence a decrease in φ, which in turn would produce a positive impact on labor supply due to a wealth effect. It turns out that the positive effect on q dominates the potentially negative effect on φ as long as the default rates are sufficiently small. The reason is that under such situations changes in φ are only of second-order importance compared to changes in q. This justifies the focus of our analysis on q.

Lemma 1. $dq/d\zeta^b > 0$ and $d(q\varphi)/d\zeta^b > 0$ whenever $\left(\overline{\omega}, \overline{\theta}\right) \in (0, \hat{\omega}) \times \left(0, \hat{\theta}\right)$ for some $\hat{\omega}, \hat{\theta} > 0$.

A subtlety arises when the nominal capital transfer z is so large that it exceeds wL. Note that in this case ζ^b should be set to be zero, meaning that the banks have zero debt-equity ratio, as they do not have to take in any debt. We assume that any excess of z over wL is rebated to the household immediately. For all situations where $z > wL$ the monitoring threshold in the bank-depositor contract is kept at zero, i.e., $\overline{\theta} = 0$, and any amount of nominal capital transfer beyond what is necessary to maintain a zero debt-equity ratio for the banks will not mitigate banking frictions any further.[17]

Our strategy of solving for the equilibrium is to collapse all the equilibrium conditions into one single equation as follows:

$$(1-\alpha)\left(\frac{K}{L}\right)^{\alpha} = q\left(\overline{\omega},\overline{\theta}\right)\varphi\left(\overline{\omega},\overline{\theta}\right) R\nu\frac{K^{\alpha}L^{1-\alpha}}{1-L},$$

Essentially, this equation characterizes equilibrium in the labor market, taking into account all the relevant information from the rest of the economy: it is obtained by using the labor supply condition (16) to substitute $\nu C/(1-L)$ for w/p in the labor demand condition (8), and by further substituting $K^{\alpha}L^{1-\alpha}\varphi\left(\overline{\omega},\overline{\theta}\right)$ for C in accordance with the resource constraint (19). Obviously this condition can be further simplified to

$$\frac{1-L}{L} = \frac{\nu}{1-\alpha} Rq\left(\overline{\omega},\overline{\theta}\right)\varphi\left(\overline{\omega},\overline{\theta}\right). \tag{25}$$

The left-hand side of (25) is a decreasing function of L. In general the right-hand side is also a function of L (and the policy variable z as well), which we now derive in the following steps. First, by substituting the quantity equation (21) into the labor supply condition (16) we have

$$w = \nu\frac{1+z}{1-L}. \tag{26}$$

[17] To incorporate the case of $z > wL$, the following conditions should be modified. First, the loan market clearing condition (18) becomes $n^d = \max\{wL - z, 0\}$. The CIA constraint (14) should be modified to $pC \leq m - n^d + wL + \max\{0, z - wL\}$, reflecting the fact that any excess of z over wL is rebated to the household immediately. The evolution of household cash holdings (15) should be modified accordingly. Note that the modified loan market clearing condition and the equality version of the modified CIA constraint implies the same quantity equation as in (21).

Second, dividing (7) by (8) yields $r^k/w = (\alpha L) / [(1-\alpha) K]$, which implies

$$r^k = \frac{\alpha}{1-\alpha} \frac{\nu}{K} (1+z) \frac{L}{1-L}. \tag{27}$$

Third, substitution of (26) and (27) into (24) gives

$$\zeta^b = \frac{\frac{\nu L}{1-L} - \frac{z}{1+z}}{\frac{\alpha}{1-\alpha} \frac{K^b}{K} \frac{\nu L}{1-L} + \frac{z}{1+z}} \tag{28}$$

for $L \geq \frac{z}{z+\nu(1+z)}$ (i.e., $wL \geq z$). For $L < \frac{z}{z+\nu(1+z)}$ we set $\zeta^b = 0$. Finally, solve for $\overline{\omega}$ and $\overline{\theta}$ given ζ^b using (9)-(10). This also allows us to compute $q\left(\overline{\omega}, \overline{\theta}\right)$ and $\varphi\left(\overline{\omega}, \overline{\theta}\right)$ as functions of ζ^b, and hence as functions of L (and z). The following proposition concerns the existence and uniqueness of equilibrium.

Proposition 1. In the neighborhood of the zero-default equilibrium, a competitive equilibrium of the model economy with banking frictions and two-sided financial contracting exists and is unique.

A unique competitive equilibrium of the model economy exists if and only if a unique solution to condition (25) exists for all $z \geq 0$. Figure 1 illustrates the determination of L for given z. As shown in the figure the left-hand side (LHS) of condition (25) is a monotonically decreasing function of L, with $\lim_{L \to 0} (1-L)/L = \infty$ and $\lim_{L \to 1} (1-L)/L = 0$. For the right-hand side (RHS) both z and R, the latter solely determined by the distribution of $\mathbf{z}$ and independent of particular values of z, are taken as given. To see how RHS depends on L, we consider two intervals separately. First, for $L \in \left(\frac{z}{z+\nu(1+z)}, 1\right]$, the bank debt-equity ratio $\zeta^b > 0$ since $wL = \nu(1+z)\frac{L}{1-L} > z$. It can be shown that ζ^b is a monotonic function of L on this interval: it is increasing in L for $z > 0$ and constant for $z = 0$. We already know from Lemma 1 that the factor $q\varphi$ is monotonically increasing in ζ^b in the neighborhood of the zero-default equilibrium. Taken together, RHS is a monotonic, positive, finite-valued, continuous function of

L on $\left(\frac{z}{z+\nu(1+z)},1\right]$. Second, for $L \in \left[0,\frac{z}{z+\nu(1+z)}\right]$ we have $\zeta^b = 0$ since $wL = \nu\left(1+z\right)\frac{L}{1-L} \leq z$. This implies that RHS is constant with respect to L on this interval, its value being equal to $\frac{\nu}{1-\alpha}Rq\left(\overline{\omega},\overline{\theta}\right)\varphi\left(\overline{\omega},\overline{\theta}\right)\Big|_{\overline{\theta}=0}$, where $\overline{\omega}$ is given by (10) with $\overline{\theta} = 0$. This is also the limit of RHS as L tends to $\frac{z}{z+\nu(1+z)}$ (meaning that ζ^b and $\overline{\theta}$ both tend to 0) from the right. Hence RHS is a non-increasing, positive, finite-valued, continuous function of L on $[0,1]$.

For $z = 0$ RHS is a horizontal line, as shown in Figure 1. For $z > 0$ there is kink at $L = \frac{z}{z+\nu(1+z)} \in (0,1)$. To the left of the kink RHS is a horizontal segment. To the right it is upward-sloping. The kink tends to zero as z tends to zero, and tends to $1/\left(1+v\right) \in (0,1)$ as z tends to infinity. Several RHS curves are shown in Figure 1, corresponding to different values of z. Note that the curves with $z > 0$ differ from the one with $z = 0$ in that the value of RHS at $L = 0$ corresponds to $\zeta^b = \frac{1-\alpha}{\alpha}\frac{K}{K^b}$ when $z = 0$, but corresponds to $\zeta^b = 0$ when $z > 0$. This is because whenever $z > 0$, it exceeds wL at $L = 0$, no matter how small z is. Whether $z = 0$ or $z > 0$, the RHS curves cut LHS from below. Hence the solution to condition (25) exists, is unique, and is interior for all $z \geq 0$, implying that a competitive equilibrium with banking frictions exists and is unique in the neighborhood of the zero-default equilibrium.

[Insert Figure 1 about here.]

4.2 The Non-Neutrality of Money

We are now ready to analyze the effects of the bank recapitalization policy. We first present an analytical characterization of the effects of the short-run policy.

Proposition 2. Take the long-run policy η as given and consider the neighborhood of the zero-default equilibrium. There exists $\hat{x} \in (-\eta,\infty)$ such that a marginal increase in x raises employment, output, and consumption as long as $x < \hat{x}$. For $x \geq \hat{x}$ a marginal increase in x has no real effect. The cutoff $\hat{x}$ satisfies

$$\frac{1+\eta+\hat{x}}{\eta+\hat{x}} = \frac{Rq\left(\overline{\omega},0\right)\varphi\left(\overline{\omega},0\right)}{1-\alpha}, \tag{29}$$

where R is given by (22) and $\overline{\omega}$ is given by (10), with $\overline{\theta}=0$ and $\zeta^f = \frac{K^b}{K^f} + \frac{1-\alpha}{\alpha}\frac{K}{K^f}$.

To understand the result in Proposition 2, refer again to Figure 1. Consider first the starting situation where $z > 0$ $(x > -\eta)$. Recall that for given z, the kink of RHS of condition (25) occurs at $L = \frac{z}{z+\nu(1+z)}$. At this point the value of RHS equals $\frac{\nu}{1-\alpha}Rq\left(\overline{\omega},0\right)\varphi\left(\overline{\omega},0\right)$, where $\overline{\omega}$ is given by (10) with $\overline{\theta}=0$. At the same L the value of LHS equals $\nu\frac{1+z}{z}$. Given η, $\hat{x}$ is the value of x such that LHS and RHS of condition (25) intersect at exactly the kink. Such $\hat{x}$ exists, is unique and finite, and is such that $\hat{z} \equiv \eta+\hat{x}$ is positive. If $x < \hat{x}$ then the equilibrium L is to the right of the kink. In this case a marginal increase in x lowers the bank debt-equity ratio ζ^b and lowers RHS, resulting in a higher equilibrium value of L. Since in the neighborhood of the zero-default equilibrium changes in φ are only of second-order importance as compared to changes in q (and hence L), consumption $C = F\left(K,L\right)\varphi$ will also increase in response to the marginal increase in z. If on the other hand $x \geq \hat{x}$ then the equilibrium L is at the kink or to the left. In this case a marginal increase in x has no effect on ζ^b, which is already zero, and hence does not have any real effect at all. Analysis of the situation where we start from $z=0$ $(x=-\eta)$ is straightforward. Under this situation the equilibrium $L \in (0,1)$ and a marginal increase in z lowers ζ^b and RHS, and hence the equilibrium values of L and C. Obviously this falls into the case of $x < \hat{x}$. Proposition 2 thus implies that starting from the zero-recapitalization benchmark, a short-run nominal capital injection into the banks is non-neutral.

It is important to note that the neutrality result for $x \geq \hat{x}$ does not mean that the Modigliani-Miller theorem applies for the banks. Rather, the banks' capital structure still matters. It is just that large values of x allow the banks to be one hundred percent internally financed, which lead to a zero bank debt-equity ratio and a zero monitoring threshold in the bank-depositor contract. For $x < \hat{x}$, the non-neutrality result we state in Proposition 2 depends crucially on the presence of banking frictions, i.e., frictions on the liability side of the bank balance sheet due to the informational asymmetry in the bank-depositor relationship. Without such frictions, a neutrality result will obtain regardless of the value of x.[18] The result holds even with the presence of credit frictions, i.e., frictions on the asset side of the bank balance sheet due to the informational asymmetry in the bank-firm relationship. It is therefore precisely the presence of banking frictions (and the fact that banks are the institutions being recapitalized) that is responsible for the potency of the recapitalization policy.

Characterizing the effects of the long-run recapitalization policy is much more complicated. The intuition, however, is quite simple. Basically, two opposing forces are at work when the long-run policy η changes. First, holding the risk-free nominal interest rate R fixed, an increase in η has similar effects as an increase in x, including a drop in the financial friction indicator q. Second, holding the extent of financial frictions fixed, an increase in η raises R for any given distribution of the short-run policy $\mathbf{x}$. Reflected in Figure 1, the former would shift RHS of condition (25) down, while the latter would shift it up. Hence changes in the long-run recapitalization policy involve tradeoffs between financial frictions, as represented by q, and monetary frictions, as represented by R. Spelling out the exact conditions under which one of these two forces dominates is difficult, if possible at all. We therefore resort to numerical experiments in the next subsection.

[18] To see this we can take away banking frictions from the model simply by assuming that the distribution of the region specific productivity is degenerate. It is straightforward to show that in such an environment changes in the short-run recapitalization policy are irrelevant for employment, output, consumption, real factor prices, and the firms' default rate.

4.3 Banking Riskiness and the Optimal Recapitalization Policy

Our analysis thus far has treated the bank recapitalization policy as being exogenous. Conditions under which such policy is potent were established. In this section we investigate how the policy can be used in an optimal fashion when there are shocks to the "riskiness" of banking. To introduce the concept of banking riskiness, we assume that the bank/region specific productivity θ follows a unit-mean log-normal distribution on $(0, \infty)$, i.e., $\log(\theta) \sim \mathcal{N}\left(-\frac{1}{2}\sigma_\theta^2, \sigma_\theta^2\right)$, where $\mathcal{N}$ stands for the normal distribution. The distribution is completed by assigning a zero p.d.f. for $\theta = 0$. In our model, it is the costly verification of θ that gives rise to the bankruptcy of banks. The default rate of banks tends to zero as σ_θ tends to zero from the right. Therefore the dispersion parameter σ_θ captures the extent of the riskiness of banking. Here we allow σ_θ to be random. Specifically, its time-t value is

$$\sigma_{\theta,t} = \sigma_\theta^s + \varepsilon_t, \tag{30}$$

where σ_θ^s, representing the steady-state level of riskiness, is a positive constant, and ε_t is an i.i.d. disturbance, with $\varepsilon_t > -\sigma_\theta^s$ for all t. We interpret ε_t as the banking riskiness shock.[19]

In our view, shocks to banking riskiness are highly relevant in the light of the erratic behavior of the risk spreads for banks' external finance. The historical average of the spread between the 3-month CD rate and the 3-month T-bill rate is about 0.75 percent per annum, based on a sample period from 1973Q1 to 2009Q4. From 2001Q1 to 2007Q2, the spread averages only 0.27 percent per annum. In contrast, its average in the second half of 2007 and the year of 2008 rises to as high as 1.53 percent per annum, with a hike at 2.52 percent per annum in the fourth quarter of 2008. In our model, there is a direct linkage between the level of banking riskiness and the external finance premium faced by the banks. The gross interest rate at which the banks borrow from the

[19] Our formulation of riskiness and the riskiness shocks parallels Christiano, Motto, and Rostagno (2009), who consider the costly state verification problem between financial intermediaries and nonfinancial firms.

depositors is simply the non-default payment specified in the bank-depositor contract divided by the amount of deposits, i.e., $R^b = pF(K, L)\Psi^b(\overline{\omega}; \Phi^l)\overline{\theta}/n^d$. Using the binding IR constraint for the depositors (5), we obtain the model's bank risk spread: $R^b - R = R\left[\overline{\theta}/\Psi^d(\overline{\theta}; \Phi^r) - 1\right]$.[20] Other things equal, an increase in σ_θ raises $\overline{\theta}$ and hence the bank risk spread.[21] Fluctuations in the banking riskiness thus give rise to fluctuations in the bank risk spread.

It is easy to see that when the policy z_t and the banking riskiness $\sigma_{\theta,t}$ are as specified in (13) and (30), the existence and uniqueness results for the competitive equilibrium, as established in Proposition 1, remain valid.[22] Proposition 2, which establishes the effectiveness, to a certain extent, of the short-run recapitalization policy, applies as well. Importantly, the potency of the short-run policy allows it to become a stabilization tool in the face of banking riskiness shocks. Taking the long-run policy η and the steady-state riskiness σ_θ^s as given, we aim to analyze how the short-run policy x_t can be used to buffer the economy from the disturbance ε_t to the level of banking riskiness. We shall see that stabilization considerations give rise to a particular kind of short-run policy reaction function, or policy rule, which dictates how x_t should respond to ε_t in a systematic fashion. Endogenizing x_t to be a function of ε_t also retains the i.i.d. nature of x_t as in the specification of (13).

Ideally, the policy should completely insulate employment L_t and consumption C_t from the disturbances. But from (19) and (25), this would require the financial friction indicator q_t and the net output factor φ_t to be completely stabilized. This is impossible since we would have two targets and only one policy instrument. However, as we emphasized earlier, compared to q_t the variable φ_t is only of second-order importance in the neighborhood of the zero-default

[20] Similarly, the risk spread faced by the firms in the model is given by $R^f - R = R\left\{\overline{\omega}/\left[\Psi^b(\overline{\omega}; \Phi^l)\left(\Gamma^b(\overline{\theta}; \Phi^r) + \Psi^d(\overline{\theta}; \Phi^r)\right)\right] - 1\right\}$.

[21] Note that $R^b - R$ is increasing in $\overline{\theta}$ since $\Psi^{d\prime} < 1$.

[22] Essentially, the model's equilibrium is of period-by-period nature, aside from the intertemporal linkage as represented by R, which is solely determined by the time-invariant distribution of z_t.

equilibrium. Hence, an approximately optimal policy needs only seek to stabilize the financial friction indicator q_t. Our numerical result, to be presented momentarily, shows that targeting q_t alone actually achieves near-complete stabilization of both q_t and φ_t and hence near-complete stabilization of L_t and C_t.

Since complete stabilization can be approximately obtained, the recapitalization policy can be made to nearly fix employment and consumption over time. The question is what values of L and C and the associated q are optimal from the welfare point of view. This amounts to finding the optimal value of the long-run policy η, i.e., the value of η that maximizes steady-state household utility. As we argued previously, changes in the long-run policy entail tradeoffs between distortions caused by monetary frictions and distortions caused by financial frictions. Roughly speaking, the marginal effect of η on the risk-free nominal interest rate R, which captures the extent of monetary frictions, is given by $1/\beta$.[23] On the other hand, the steady-state marginal effect of η on the financial friction indicator q depends on the average level of banking riskiness, as represented by σ_θ^s. The optimal long-run policy thus balances the marginal effects of the two distortions, and is naturally a function of σ_θ^s. Denote the optimal long-run recapitalization policy by η^*, and the value of q associated with η^* by q^*. For the short-run recapitalization policy, q^* serves as the target.

In order to derive the approximately optimal reaction function for the short-run policy, denote the mapping of $\left(\varepsilon_t, \zeta_t^b\right)$ to q_t by $\mathbf{q}\left(\varepsilon_t, \zeta_t^b\right)$: the realization of ε_t gives the value of $\sigma_{\theta,t}$, which, together with the bank debt-equity ratio ζ_t^b, determines $\left(\overline{\omega}_t, \overline{\theta}_t\right)$ and hence q_t through (9) and (10). Given ε_t, targeting q_t at q^* amounts to targeting ζ_t^b at ζ_t^{b*}, where ζ_t^{b*} is such that $\mathbf{q}\left(\varepsilon_t, \zeta_t^{b*}\right) = q^*$. Let L^*, w^*, and r^* be the values of L, w, and r that correspond to the optimal long-run policy η^*. Then according to (24), the optimal time-t short-run policy, denoted by x_t^*,

[23] Strictly speaking, R depends not only on η but also on the distribution of x_t (see (22)). However, as long as the dispersion of x_t is not large, $R = (1+\eta)/\beta$ holds approximately.

satisfies

$$\zeta_t^{b*} = \frac{w^* L^* - (\eta^* + x_t^*)}{r^* K^b + (\eta^* + x_t^*)},$$

which leads to the optimal reaction function:

$$x_t^* = \frac{w^* L^*}{1 + \zeta_t^{b*}} - \frac{\zeta_t^{b*} r^* K^b}{1 + \zeta_t^{b*}} - \eta^*.$$

To target q^*, an increase in ε_t requires a lower value of ζ_t^{b*}. But a reduction in ζ_t^{b*} calls for an increase in x_t^*. Hence x_t^* varies positively with ε_t, with $x_t^* = 0$ when $\varepsilon_t = 0$. Such a reaction function entails recapitalization efforts that counteract banking riskiness: there is more (less) nominal capital transfer to the banks when banking becomes more (less) risky.[24]

To demonstrate numerically the optimal setting of the long-run policy and the short-run reaction function, we calibrate the model economy as follows. Let a time period correspond to a quarter, and consider a no-recapitalization benchmark. We set $\beta = 0.99$ to match an annual risk-free real interest rate of 4%. The weight on leisure in the household utility function, ν, is chosen to deliver $L = 1/3$ absent shocks. The elasticity parameter in the production function, α, is set to be 1/2, implying an asset-net worth ratio of about 2 for the firms (see Bernanke et al.).[25] With the aggregate capital stock K being normalized to one, the banks' share of capital, K^b, equals 0.082, which is consistent with the historical average of an asset-net worth ratio of 13.18 for U.S. commercial banks.[26] The monitoring cost parameter, μ, is set to be 0.3.[27] Similar to the bank/region specific productivity, we assume that the firm/location specific productivity

[24] Note that the above analysis implicitly assumes that the maximum value of x_t^* (corresponding to the maximum value of ε_t) does not exceed the value of $\hat{x}$ associated with η^* as defined in (29). This will be true if ε_t is not too large.

[25] If the variable K in the production function were interpreted literally as "physical capital", then 1/2 would be too large a value for α. Nevertheless, a broader interpretation can be adopted: the variable might be thought to include bank and firm managers' human capital, e.g., managerial skills, as well.

[26] This calculation is based on "Assets and Liabilities of Commercial Banks in the United States" of the Federal Reserve. The sample period is 1973Q1-2009Q4.

[27] Altman (1984) estimates the sum of direct and indirect bankruptcy costs to be about 20 percent of firms' total asset. By comparing the value of a firm as a going concern with its liquidation value, Alderson and Betker (1995) estimate that liquidation costs are equal to approximately 36 percent of firms assets. The value we adopt for the bankruptcy cost parameter lies in between these two estimates.

ω follows a unit-mean log-normal distribution on $(0, \infty)$, completed with the assignment of a zero p.d.f. for $\omega = 0$. For $\omega > 0$, $\log(\omega) \sim \mathcal{N}\left(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2\right)$. We assume that σ_ω is fixed, while $\sigma_{\theta,t}$ follows the specification in (30). The value of σ_ω and the steady-state value of $\sigma_{\theta,t}$, i.e., σ_θ^s, are chosen to match (1) a spread between the firms' borrowing rate and the risk-free rate of 2.93 percent per annum, and (2) a spread between the banks' borrowing rate and the risk-free rate of 0.75 percent per annum.[28]

Figure 2 depicts the optimal recapitalization policy in relation to the level of banking riskiness. Panel (a) pertains to the long-run policy. The left part illustrates the tradeoff between R and q as induced by changes in η, with σ_θ^s set to be 0.031, the calibrated steady-state value of $\sigma_{\theta,t}$. An increase in η raises the risk-free nominal interest rate R while lowering the financial friction indicator q. Under our parameterization, the utility maximizing value of the long-run policy, denoted by η^*, occurs at 1.14%, which corresponds to an annual money growth of 4.64%. The optimal long-run policy η^* varies as σ_θ^s changes. The right part of Panel (a) shows that η^* is an increasing, approximately linear function of σ_θ^s. An increase in the steady-state level of banking riskiness gives more weight to the mitigation of banking frictions and results in a higher value of η^*. As long as σ_θ^s is greater than 0.023, long-run considerations call for positive values of η^*.[29]

Systematic reductions in banking riskiness, i.e., reductions in σ_θ^s, can in principle be brought about by establishment and improvement of a bank safety net, whereby inter-bank transfers are undertaken in order to smooth the impacts of bank/region specific productivities. In a given time period, banks that have low realizations of θ are "subsidized" while those having high realizations are taxed, with the aggregate net transfer being equal to zero. Over time a bank

[28] The empirical measures of the risk-free rate, the banks' borrowing rate, and the firms' borrowing rate are the 3-month T-bill rate, the 3-month CD rate, and the prime lending rate, respectively. The data are from the Federal Reserve. The sample period is again 1973Q1-2009Q4.

[29] For σ_θ^s less than 0.023, all positive values of η are inferior to the zero value.

is subsidized in periods with low realizations of θ and taxed in periods with high realizations. The effect of improving the bank safety net is equivalent to a systematic reduction in banking riskiness, which allows the long-run recapitalization rate to be lowed.[30] Implementation of the bank safety net, however, relies on the implicit assumption that the central bank has superior information regarding realizations of the bank/region specific productivities. Such information advantage can only be obtained at costs. The better the quality of information, the higher the costs. Hence complete elimination of the impacts of banking riskiness by the bank safety net does not seem likely. The recapitalization policy still has a role to play.

[Insert Figure 2 about here.]

Panel (b) of the figure is concerned with the short-run policy in relation to shocks to banking riskiness. Here σ_θ^s equals 0.031 and η is set to be the corresponding optimal value, i.e., $\eta^* = 1.14\%$. The left part shows the employment effects (expressed in percent deviations of L from its steady-state value) of the shock, ε_t, to banking riskiness. The dashed line corresponds to the case where there is no reaction of the short-run policy to the shocks (x_t equals zero identically). The solid line corresponds to the case where the short-run policy reacts in the approximately optimal fashion described above. As can be seen from the no-reaction line, the effect of a positive (resp. negative) shock to banking riskiness is to lower (resp. raise) employment. The effects are asymmetric in that the effects of positive shocks are larger. This is because negative shocks drive the economy toward the situation without banking frictions, which provides the supremum for the employment effect. The asymmetry is also evident from the larger marginal employment effects of positive shocks (the dashed line is steeper to the right of $\varepsilon_t = 0$). By reacting to the banking riskiness shocks in the approximately optimal fashion, the short-run policy almost

[30] It should be noted that the sort of inter-bank transfers in the bank safety net are different from the bank recapitalization policy, which calls for government transfers to *all* banks in the economy at the same time in our model.

completely stabilizes employment, as shown by the solid line. The optimal reaction function is plotted on the right part of Panel (b), where x_t^* turns out be an increasing, approximately linear function of ε_t.[31] A final point to notice is that the computation of x_t^* in the figure ignores the restriction that $z_t^* \equiv \eta^* + x_t^*$ be nonnegative. This is innocuous as long as the banking riskiness shock is not too negative. In fact, $x_t^* > -\eta^*$ whenever $\varepsilon_t > -0.07$. For $\varepsilon_t < -0.07$, the unrestricted x_t^* renders z_t^* negative. However, truncating x_t^* at $-\eta^*$ or z_t^* at 0 will not produce much different outcome since the marginal employment effect of ε_t is already close to zero in this region. Nearly complete stabilization can still be maintained.

5 Conclusions

This paper develops a general equilibrium framework with banking frictions and two-sided financial contracting. The framework is used to analyze the effects of bank recapitalization, taking the form of nominal capital transfers to the banking system. The design of optimal recapitalization policy, in relation to the riskiness of banking, is also investigated. The paper contributes to understanding the transmission mechanisms of the unconventional monetary policy adopted in the recent financial crisis, and to understanding how policy should be designed to mitigate the adverse effects of financial frictions.

Although our study has mainly concerned the effects of bank recapitalization by the monetary authority and the analysis has been carried out in a highly stylized model, the theoretical framework we develop can be extended to study a wide spectrum of issues related to policy and regulation, as well as the monetary transmission mechanism, in perhaps more realistic ways. First, nominal rigidities and richer dynamics, such as capital accumulation, can be introduced to allow for a quantitative assessment of the effects of policy. Second, deposit insurance can

[31] The approximate linearity obtains since the marginal employment effect of x_t is also weaker when the marginal employment effect of ε_t is weaker, i.e., when banking is less risky.

be incorporated in order to study the effects of raising the limit of deposit insurance, as was implemented in the U.S. in 2008. Third, one can consider situations where some sort of capital adequacy requirements bind. In those situations, bank recapitalization policy may work through relaxing these constraints. Fourth, the model can be extended to allow changes in asset prices to affect the net worth of banks (and firms), as in Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2009). Finally, our analysis can be extended to include credit rationing as a possible equilibrium outcome as in Williamson (1986) so that another dimension in which policy exerts influence on the economy can be explored.[32] We conclude that thorough analysis of frictions in the banking sector should be an integral part of future research on the interaction of money, finance, and the macroeconomy.

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[32] In our setup one can imagine two possible types of credit rationing. The first is rationing on the banks' asset side, where firms are unable to obtain the bank loans they desire. This type of credit rationing has been extensively studied in the literature (e.g., Stiglitz and Weiss, 1981 and Williamson, 1986). The second type is rationing on the banks' liability side, where banks are unable to raise the loanable funds they desire. The latter type of credit rationing is an interesting aspect to explore in future research.

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Appendix

Derivation of the First-Order Conditions for Problem 1.

To avoid cluttering of notations we omit the arguments, such as $\left(\overline{\omega};\Phi^l\right)$, $\left(\overline{\theta};\Phi^r\right)$, and $\left(\overline{\omega},\overline{\theta}\right)$, of various functions. We first show that the first-order conditions (7)-(10) hold. Let λ^b and λ^d be the Lagrangian multipliers for (4) and (5), respectively. Then the first-order conditions with respect to $\overline{\omega}$ and $\overline{\theta}$ are

$$\Gamma^{f\prime} + \Psi^{b\prime}\left(\lambda^b\Gamma^b + \lambda^d\Psi^d\right) = 0, \tag{A.1}$$

$$\lambda^b\Gamma^{b\prime} + \lambda^d\Psi^{d\prime} = 0. \tag{A.2}$$

Equations (A.1) and (A.2) imply

$$\lambda^b = -\frac{\Gamma^{f\prime}}{\Psi^{b\prime}}\frac{\Psi^{d\prime}}{\Gamma^b\Psi^{d\prime} - \Gamma^{b\prime}\Psi^d},$$

$$\lambda^d = \frac{\Gamma^{f\prime}}{\Psi^{b\prime}}\frac{\Gamma^{b\prime}}{\Gamma^b\Psi^{d\prime} - \Gamma^{b\prime}\Psi^d}.$$

The first-order conditions with respect to k and l are given by (7) and (8), where

$$q \equiv \frac{\lambda^d}{\Gamma^f + \Psi^b\left(\lambda^b\Gamma^b + \lambda^d\Psi^d\right)} = \left[\left(\Psi^b - \Gamma^f\frac{\Psi^{b\prime}}{\Gamma^{f\prime}}\right)\left(\Psi^d - \Gamma^b\frac{\Psi^{d\prime}}{\Gamma^{b\prime}}\right)\right]^{-1},$$

as in (11). The linear homogeneity of $F(\cdot)$ together with (7) and (8) imply

$$PF(k,l) = qR\left(R^k k + Wl\right). \tag{A.3}$$

At the optimum constraints (4) and (5) both bind. Substituting (A.3) into the equality version of (4) and (5) yields

$$q\left(R^k k + Wl\right)\Psi^b\Gamma^b = N^b, \tag{A.4}$$

$$q\left(R^k k + Wl\right)\Psi^b\Psi^d = N^d. \tag{A.5}$$

Dividing (A.5) by (A.4) gives (9). Adding (A.4) and (A.5) and using the equality version of (6) gives (10).

We then show that $q > 1$ for all $\overline{\omega}, \overline{\theta} > 0$ and $\lim_{\overline{\omega},\overline{\theta}\to 0} q = 1$. Since $\left(-\Psi^{d\prime}/\Gamma^{b\prime}\right) < 1$ and $\left(-\Psi^{b\prime}/\Gamma^{f\prime}\right) < 1$ for all $\overline{\omega}, \overline{\theta} > 0$, we have

$$q^{-1} = \left(\Psi^b - \Gamma^f \frac{\Psi^{b\prime}}{\Gamma^{f\prime}}\right)\left(\Psi^d - \Gamma^b \frac{\Psi^{d\prime}}{\Gamma^{b\prime}}\right) < \left(\Gamma^f + \Psi^b\right)\left(\Gamma^b + \Psi^d\right) < 1,$$

and hence $q > 1$ for all $\overline{\omega}, \overline{\theta} > 0$. Since $\lim_{\overline{\theta}\to 0}\left(-\Psi^{d\prime}/\Gamma^{b\prime}\right) = 1$, $\lim_{\overline{\omega}\to 0}\left(-\Psi^{b\prime}/\Gamma^{f\prime}\right) = 1$, $\lim_{\overline{\theta}\to 0}\left(\Gamma^b + \Psi^d\right) = 1$, $\lim_{\overline{\omega}\to 0}\left(\Gamma^f + \Psi^b\right) = 1$, we have $\lim_{\overline{\omega},\overline{\theta}\to 0} q^{-1} = 1$.

Proof of Lemma 1.

To prove $dq/d\zeta^b > 0$ we first show that $q_{\overline{\omega}}\left(\overline{\omega}, \overline{\theta}\right) \equiv \partial q\left(\overline{\omega}, \overline{\theta}\right)/\partial\overline{\omega} > 0$ and $q_{\overline{\theta}}\left(\overline{\omega}, \overline{\theta}\right) \equiv \partial q\left(\overline{\omega}, \overline{\theta}\right)/\partial\overline{\theta} > 0$ in the neighborhood of $\overline{\omega}, \overline{\theta} = 0$. Let $q_{\overline{\omega}}^{-1} \equiv \partial q^{-1}/\partial\overline{\omega}$ and $q_{\overline{\theta}}^{-1} \equiv \partial q^{-1}/\partial\overline{\theta}$. We obtain from differentiating (11)

$$q_{\overline{\omega}}^{-1} = \frac{\Gamma^f}{\Gamma^{f\prime 2}}\left(\Psi^{b\prime}\Gamma^{f\prime\prime} - \Psi^{b\prime\prime}\Gamma^{f\prime}\right)\left(\Psi^d - \Gamma^b \frac{\Psi^{d\prime}}{\Gamma^{b\prime}}\right),$$

$$q_{\overline{\theta}}^{-1} = \left(\Psi^b - \Gamma^f \frac{\Psi^{b\prime}}{\Gamma^{f\prime}}\right)\frac{\Gamma^b}{\Gamma^{b\prime 2}}\left(\Psi^{d\prime}\Gamma^{b\prime\prime} - \Psi^{d\prime\prime}\Gamma^{b\prime}\right).$$

But $\Psi^d - \Gamma^b\Psi^{d\prime}/\Gamma^{b\prime} > 0$, $\Psi^b - \Gamma^f\Psi^{b\prime}/\Gamma^{f\prime} > 0$, and

$$\Psi^{d\prime}\Gamma^{b\prime\prime} - \Psi^{d\prime\prime}\Gamma^{b\prime} = -\mu\phi^r\left(\overline{\theta}\right)\left[1 - \Phi^r\left(\overline{\theta}\right)\right]\left[1 + \frac{\overline{\theta}\phi^r\left(\overline{\theta}\right)}{1 - \Phi^r\left(\overline{\theta}\right)} + \frac{\overline{\theta}\phi^{r\prime}\left(\overline{\theta}\right)}{\phi^r\left(\overline{\theta}\right)}\right],$$

$$\Psi^{b\prime}\Gamma^{f\prime\prime} - \Psi^{b\prime\prime}\Gamma^{f\prime} = -\mu\phi^l\left(\overline{\omega}\right)\left[1 - \Phi^l\left(\overline{\omega}\right)\right]\left[1 + \frac{\overline{\omega}\phi^l\left(\overline{\omega}\right)}{1 - \Phi^l\left(\overline{\omega}\right)} + \frac{\overline{\omega}\phi^{l\prime}\left(\overline{\omega}\right)}{\phi^l\left(\overline{\omega}\right)}\right].$$

To sign $\left(\Psi^{d\prime}\Gamma^{b\prime\prime} - \Psi^{d\prime\prime}\Gamma^{b\prime}\right)$ we consider two cases. Case 1: $\lim_{\overline{\theta}\to 0}\phi^r\left(\overline{\theta}\right) > 0$. In this case $\lim_{\overline{\theta}\to 0}\left(\Psi^{d\prime}\Gamma^{b\prime\prime} - \Psi^{d\prime\prime}\Gamma^{b\prime}\right) = -\mu\lim_{\overline{\theta}\to 0}\phi^r\left(\overline{\theta}\right) < 0$. Case 2: $\lim_{\overline{\theta}\to 0}\phi^r\left(\overline{\theta}\right) = 0$. But Assumption 1(a) requires $\phi^r(\cdot)$ to be positive, bounded, and continuously differentiable on $(0, +\infty)$. Hence in this case we must have $\lim_{\overline{\theta}\to 0}\phi^{r\prime}\left(\overline{\theta}\right) > 0$. This means that for $\overline{\theta}$ positive and sufficiently close to 0, we have $\phi^r\left(\overline{\theta}\right) > 0$ and $\phi^{r\prime}\left(\overline{\theta}\right) > 0$ and hence $\left(\Psi^{d\prime}\Gamma^{b\prime\prime} - \Psi^{d\prime\prime}\Gamma^{b\prime}\right) < 0$. In both cases when $\overline{\theta}$ is positive and sufficiently close to 0, we have $q_{\overline{\theta}}^{-1} < 0$ and hence $q_{\overline{\theta}} > 0$. The argument is similar for the signs of $\left(\Psi^{b\prime}\Gamma^{f\prime\prime} - \Psi^{b\prime\prime}\Gamma^{f\prime}\right)$ and $q_{\overline{\omega}}$. Hence there exist $\hat{\omega} > 0$ and $\hat{\theta} > 0$ such that $q_{\overline{\theta}} > 0$ and $q_{\overline{\omega}} > 0$ for all $\left(\overline{\omega}, \overline{\theta}\right) \in (0, \hat{\omega}) \times \left(0, \hat{\theta}\right)$.

Next, we have from (9)

$$\frac{d\overline{\theta}}{d\zeta^b} = \frac{\Gamma^{b2}}{\Psi^{d\prime}\Gamma^b - \Psi^d\Gamma^{b\prime}} > 0.$$

Given $\overline{\theta}$, condition (10), i.e., $q\Psi^b\left(\Gamma^b + \Psi^d\right) = \zeta^f / \left(1 + \zeta^f\right)$ determines $\overline{\omega}$. There can only be two cases for the change in $\overline{\omega}$ in response to an increase in ζ^b and hence $\overline{\theta}$. Case 1: $\overline{\omega}$ increases or stays unchanged. In this case $dq/d\zeta^b = q_{\overline{\theta}}\left(d\overline{\theta}/d\zeta^b\right) + q_{\overline{\omega}}\left(d\overline{\omega}/d\zeta^b\right) > 0$ for $\left(\overline{\omega}, \overline{\theta}\right) \in (0, \hat{\omega}) \times \left(0, \hat{\theta}\right)$. Case 2: $\overline{\omega}$ decreases. In this case Ψ^b decreases, too. In addition $\left(\Gamma^b + \Psi^d\right)$ decreases with the increase in $\overline{\theta}$. Hence condition (10) implies that q must increase, i.e., $dq/d\zeta^b > 0$.

We now prove $d(q\varphi)/d\zeta^b > 0$. Note that

$$\frac{d(q\varphi)}{d\zeta^b} = \frac{dq}{d\zeta^b}\varphi + q\frac{d\varphi}{d\zeta^b} = \frac{dq}{d\zeta^b}\varphi\left[1 + \frac{q}{\varphi}\frac{d\varphi\left(\overline{\omega}, \overline{\theta}\right)/d\zeta^b}{dq\left(\overline{\omega}, \overline{\theta}\right)/d\zeta^b}\right].$$

We need only show that $\left[d\varphi\left(\overline{\omega}, \overline{\theta}\right)/d\zeta^b\right] / \left[dq\left(\overline{\omega}, \overline{\theta}\right)/d\zeta^b\right]$ is sufficiently close to zero whenever $\left(\overline{\omega}, \overline{\theta}\right)$ is close to zero. Let $\varphi_{\overline{\theta}} \equiv \partial\varphi/\partial\overline{\theta}$ and $\varphi_{\overline{\omega}} \equiv \partial\varphi/\partial\overline{\omega}$. Since ζ^b is a sufficient statistic for $\left(\overline{\omega}, \overline{\theta}\right)$. It suffices to show $\lim_{\overline{\omega}, \overline{\theta} \to 0}\left(\varphi_{\overline{\theta}}/q_{\overline{\theta}}\right) = \lim_{\overline{\omega}, \overline{\theta} \to 0}\left(\varphi_{\overline{\omega}}/q_{\overline{\omega}}\right) = 0$.

To prove $\lim_{\overline{\omega}, \overline{\theta} \to 0}\left(\varphi_{\overline{\theta}}/q_{\overline{\theta}}\right) = 0$ note that $q_{\overline{\theta}} = -q^2 q_{\overline{\theta}}^{-1}$ and $\varphi_{\overline{\theta}} = \Psi^b\left(\Gamma^{b\prime} + \Psi^{d\prime}\right)$. We have $q_{\overline{\theta}} > 0$ and $\varphi_{\overline{\theta}} < 0$ for all $\overline{\omega}, \overline{\theta} > 0$. Taking limits, we obtain $\lim_{\overline{\omega}, \overline{\theta} \to 0}\varphi_{\overline{\theta}} = 0$, $\lim_{\overline{\omega}, \overline{\theta} \to 0} q_{\overline{\theta}} = -\lim_{\overline{\omega}, \overline{\theta} \to 0} q_{\overline{\theta}}^{-1}$, and

$$\lim_{\overline{\omega}, \overline{\theta} \to 0} q_{\overline{\theta}}^{-1} = \lim_{\overline{\theta} \to 0}\left(\Psi^{d\prime}\Gamma^{b\prime\prime} - \Psi^{d\prime\prime}\Gamma^{b\prime}\right)$$

since $\lim_{\overline{\theta} \to 0}\Gamma^b = \lim_{\overline{\theta} \to 0}\left(-\Gamma^{b\prime}\right) = \lim_{\overline{\omega} \to 0}\left(-\Psi^{b\prime}/\Gamma^{f\prime}\right) = \lim_{\overline{\omega} \to 0}\left(\Gamma^f + \Psi^b\right) = 1$. Again we discuss two cases. Case 1: $\lim_{\overline{\theta} \to 0}\phi^r\left(\overline{\theta}\right) > 0$. In this case $\lim_{\overline{\omega}, \overline{\theta} \to 0} q_{\overline{\theta}}^{-1} = -\mu\lim_{\overline{\theta} \to 0}\phi^r\left(\overline{\theta}\right) < 0$. Hence $\lim_{\overline{\omega}, \overline{\theta} \to 0} q_{\overline{\theta}} > 0$ and $\lim_{\overline{\omega}, \overline{\theta} \to 0}\left(\varphi_{\overline{\theta}}/q_{\overline{\theta}}\right) = 0$. Case 2. $\lim_{\overline{\theta} \to 0}\phi^r\left(\overline{\theta}\right) = 0$. In this case

$$\lim_{\overline{\omega}, \overline{\theta} \to 0}\frac{q_{\overline{\theta}}^{-1}}{\varphi_{\overline{\theta}}} = \lim_{\overline{\omega}, \overline{\theta} \to 0}\frac{\Psi^{d\prime}\Gamma^{b\prime\prime} - \Psi^{d\prime\prime}\Gamma^{b\prime}}{\Psi^b\left(\Gamma^{b\prime} + \Psi^{d\prime}\right)} = \infty.$$

This is because $\lim_{\overline{\omega}\to 0}\Psi^b = 0$ and

$$\begin{aligned}
& \lim_{\overline{\theta}\to 0}\frac{\Psi^{d\prime}\Gamma^{b\prime\prime}-\Psi^{d\prime\prime}\Gamma^{b\prime}}{\Gamma^{b\prime}+\Psi^{d\prime}} \\
= & \lim_{\overline{\theta}\to 0}\frac{\mu\phi^r\left(\overline{\theta}\right)\left[1-\Phi^r\left(\overline{\theta}\right)\right]\left[1+\frac{\overline{\theta}\phi^r\left(\overline{\theta}\right)}{1-\Phi^r\left(\overline{\theta}\right)}+\frac{\overline{\theta}\phi^{r\prime}\left(\overline{\theta}\right)}{\phi^r\left(\overline{\theta}\right)}\right]}{\mu\overline{\theta}\phi^r\left(\overline{\theta}\right)} \\
= & \lim_{\overline{\theta}\to 0}\left[\frac{1}{\overline{\theta}}+\frac{\phi^{r\prime}\left(\overline{\theta}\right)}{\phi^r\left(\overline{\theta}\right)}\right]=\infty,
\end{aligned}$$

which is so since $\lim_{\overline{\theta}\to 0}\phi^r\left(\overline{\theta}\right) = 0$ implies $\lim_{\overline{\theta}\to 0}\phi^{r\prime}\left(\overline{\theta}\right) > 0$ by Assumption 1(a). Hence $\lim_{\overline{\omega},\overline{\theta}\to 0}\left(\varphi_{\overline{\theta}}/q_{\overline{\theta}}\right) = 0$ in this case, too. The proof for $\lim_{\overline{\omega},\overline{\theta}\to 0}\left(\varphi_{\overline{\omega}}/q_{\overline{\omega}}\right) = 0$ is similar and is therefore omitted for brevity.

Q.E.D.

Proof of Proposition 1.

The left-hand side of condition (25) is a monotonically decreasing function of L, with $\lim_{L\to 0}(1-L)/L = \infty$ and $\lim_{L\to 1}(1-L)/L = 0$. For the right-hand side (RHS) both z and R, which is solely determined by the distribution of $\mathbf{z}$, are taken as given. To see how RHS depends on L, we consider two intervals separately. For $L \in \left(\frac{z}{z+\nu(1+z)}, 1\right]$, $\zeta^b > 0$ since $wL = \nu(1+z)\frac{L}{1-L} > z$. From (28) we have

$$\frac{\partial\zeta^b}{\partial L} = \frac{\nu\left(1+\frac{\alpha}{1-\alpha}\frac{K^b}{K}\right)z(1+z)}{\left\{\left[\frac{\alpha}{1-\alpha}\frac{K^b}{K}\nu(1+z)-z\right]L+z\right\}^2} \gtreqless 0 \ \text{ if } z \gtreqless 0.$$

Hence ζ^b is a monotonic function of L. According to Lemma 1, $d(q\varphi)/d\zeta^b > 0$ when default rates are sufficiently small. Hence in the neighborhood of the zero-default equilibrium, RHS is a monotonic, positive, finite-valued continuous function of L on $\left(\frac{z}{z+\nu(1+z)}, 1\right]$. Now consider those values of $L \in \left[0, \frac{z}{z+\nu(1+z)}\right]$. For all these values $wL \leq z$ and $\zeta^b = 0$, implying that RHS is constant with respect to L, its value being equal to $\frac{\nu}{1-\alpha}Rq\left(\overline{\omega},\overline{\theta}\right)\varphi\left(\overline{\omega},\overline{\theta}\right)\Big|_{\overline{\theta}=0}$, where $\overline{\omega}$ is given by (10) with $\overline{\theta} = 0$. This is also the limit of RHS as L tends to $\frac{z}{z+\nu(1+z)}$ (meaning that ζ^b

and $\overline{\theta}$ both tend to 0) from the right. Hence RHS is a non-increasing, positive, finite-valued, continuous function of L on $[0,1]$. Hence the solution to condition (25) exists, is unique, and is interior for all $z \geq 0$, implying that a competitive equilibrium with banking frictions exists and is unique in the neighborhood of the zero-default equilibrium.

Q.E.D.

Proof of Proposition 2.

Consider first the starting situation where $z > 0$ ($x > -\eta$). For $L \in \left(\frac{z}{z+\nu(1+z)}, 1\right]$, we have from differentiating (28)

$$\frac{\partial \zeta^b}{\partial z} = \frac{-\nu \frac{L}{1-L}\left(1 + \frac{\alpha}{1-\alpha}\frac{K^b}{K}\right)}{\left[\frac{\alpha}{1-\alpha}\frac{K^b}{K}\nu\left(1+z\right)\frac{L}{1-L} + z\right]^2} < 0$$

and hence $\partial \text{RHS}/\partial z < 0$. For $L \in \left[0, \frac{z}{z+\nu(1+z)}\right]$ we have $\partial \text{RHS}/\partial z = \partial \zeta^b/\partial z = 0$. Note that for all $z > 0$, $\frac{\nu}{1-\alpha} Rq\varphi\Big|_{L=\frac{z}{z+\nu(1+z)}} = \frac{\nu}{1-\alpha} Rq\left(\overline{\omega}, 0\right)\varphi\left(\overline{\omega}, 0\right)$,where $\overline{\omega}$ is given by (10), with $\overline{\theta} = 0$ and $\zeta^f = \frac{K^b}{K^f} + \frac{1-\alpha}{\alpha}\frac{K}{K^f}$. Let $\hat{z}$ be such that

$$\frac{1-L}{L}\Bigg|_{L=\frac{\hat{z}}{\hat{z}+\nu(1+\hat{z})}} = \frac{\nu}{1-\alpha} Rq\varphi\Bigg|_{L=\frac{\hat{z}}{\hat{z}+\nu(1+\hat{z})}, z=\hat{z}}$$

or equivalently

$$\frac{1+\hat{z}}{\hat{z}} = \frac{Rq\left(\overline{\omega}, 0\right)\varphi\left(\overline{\omega}, 0\right)}{1-\alpha},$$

where $\overline{\omega}$ is again given by (10), with $\overline{\theta} = 0$ and $\zeta^f = \frac{K^b}{K^f} + \frac{1-\alpha}{\alpha}\frac{K}{K^f}$. Obviously such $\hat{z}$ exists and is unique, positive, and finite. Taking η as given, define $\hat{x} \equiv \hat{z} - \eta$. If $-\eta < x < \hat{x}$ ($0 < z < \hat{z}$), then $\frac{1-L}{L}\Big|_{L=\frac{z}{z+\nu(1+z)}} = \nu\frac{1+z}{z} > \frac{\nu}{1-\alpha} Rq\varphi\Big|_{L=\frac{z}{z+\nu(1+z)}}$, hence the equilibrium $L \in \left(\frac{z}{z+\nu(1+z)}, 1\right)$. In this case $\partial \text{RHS}/\partial z < 0$, and a marginal increase in z raises equilibrium L. Since in the neighborhood of the zero-default equilibrium changes in φ are only of second-order importance as compared to changes in q (and hence L), consumption $C = F\left(K, L\right)\varphi$ will also increase in response to the marginal increase in z. If on the other hand $x \geq \hat{x}$ ($z \geq \hat{z}$), then the equilibrium

$L \in \left(0, \frac{z}{z+\nu(1+z)}\right]$. In this case $\partial \text{RHS}/\partial z = 0$, and a marginal increase in z has no effect on equilibrium L and C.

Now consider the starting situation of $z = 0$ ($x = -\eta < \hat{x}$). We have the equilibrium $L \in (0,1)$ and

$$\left.\frac{\partial \zeta^b}{\partial z}\right|_{z=0} = \frac{-\left(1 + \frac{\alpha}{1-\alpha}\frac{K^b}{K}\right)}{\left(\frac{\alpha}{1-\alpha}\frac{K^b}{K}\right)^2 \frac{\nu L}{1-L}} < 0,$$

implying $\partial RHS/\partial z|_{z=0} < 0$. Hence in this situation a marginal increase in z raises equilibrium L and C.

Q.E.D.

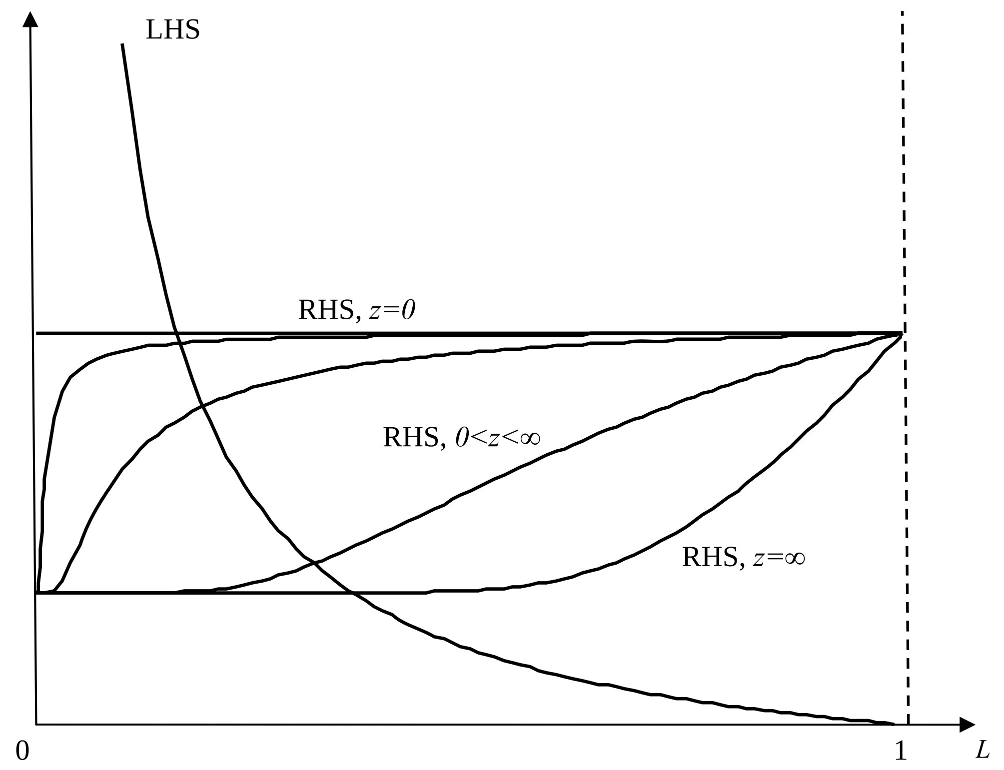
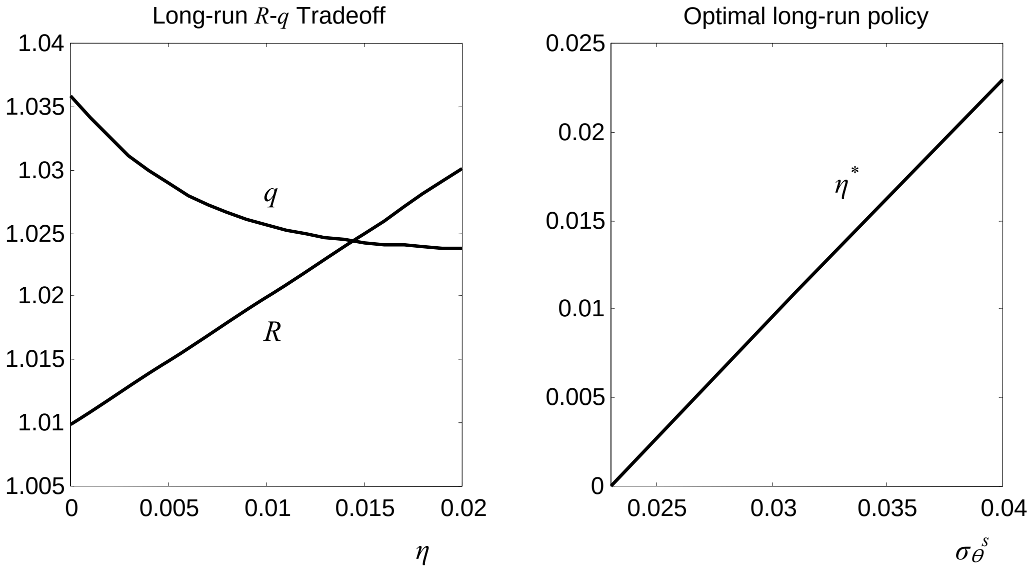
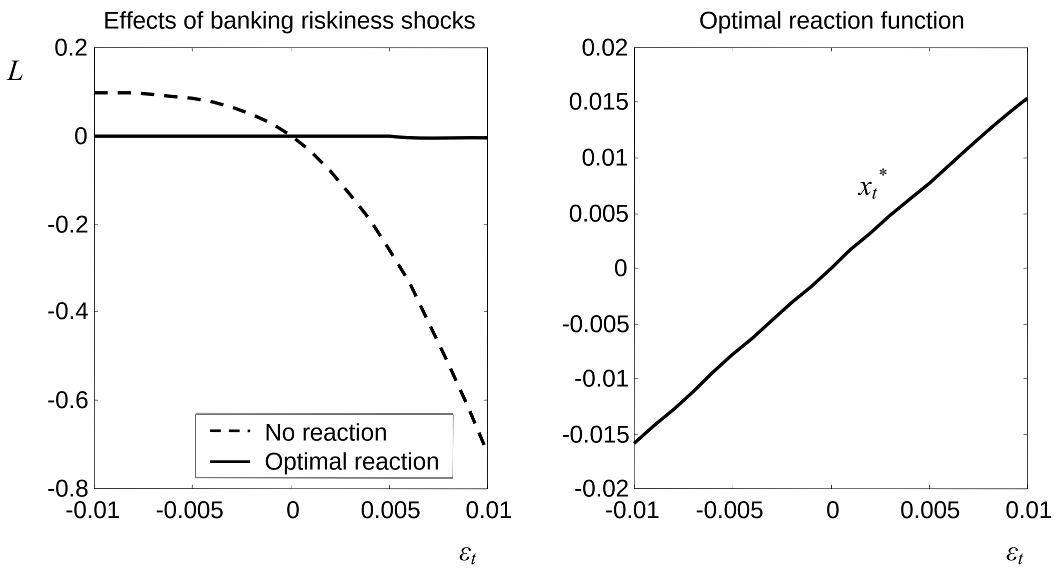


Figure 1. Determination of equilibrium

LHS and RHS refer to the left and right-hand sides, respectively, of condition (25). For RHS, R is taken as given. RHS is a horizontal line when $z=0$ and is kinked when $z>0$. For given z, the equilibrium L is determined by the intersection of LHS and RHS.



(a)



(b)

Figure 2. Banking Riskiness and the Optimal Recapitalization Policy

Panel (a) pertains to the long-run policy. The left part shows the tradeoff between R and q induced by changes in η, with σ_θ^s set to be 0.031. The right part shows the optimal long-run policy, η^*, as a function of σ_θ^s. Panel (b) pertains to the short-run policy. The left part shows the employment effects (percent deviations of L from the steady state) of shocks to banking riskiness, ε_t. The dashed line corresponds to the case where there is no reaction of the short-run policy to the shocks ($x_t=0$ identically). The solid line corresponds to the case where the short-run policy reacts in an approximately optimal fashion. The optimal reaction function is plotted on the right part.