Fiscal Policy and the Zero Bound

Based on work by:
Christiano, Eichenbaum, Rebelo, ‘When is the Government Spending Multiplier Big?’
Background

• There is anxiety about the prospects for economic growth. Pessimists focus on:
  – Jump in private savings rate (2% before 2008, 6% now).
  – Downward pressure on firm marginal costs stemming from excess capacity and unemployment raises risk of deflation.
• The pessimists are especially alarmed because policy rates are close to zero.
• Monetary policy has done about all it can do, and so focus has shifted to fiscal policy.
Systematic Rise in Savings Rate that has Pessimists Worried
Issues and Questions

• Why are deflation, high saving such fearful things when the policy interest rate is at its lower bound?

• What can we hope to get from fiscal policy?
  – Empirical data tends to be ambiguous because
    • movements in G tend to be accompanied by other shocks.
    • a priori considerations suggest effects of G depends on state of economy.
  – Approach taken here: investigate what the equilibrium models which fit the data well have to say.

• What about tax policy?

• What role, quantitatively, does the fact that the policy rate is at the zero bound, play in the dynamics of the data?
Findings on the Fiscal Multiplier

• We will see that a standard equilibrium model implies:

  – in ‘normal times’ multiplier may be bigger than unity, but depends on the nature of monetary policy.

  – When lower bound on nominal interest rate is binding, multiplier may be quite large.
Outline of Discussion of Fiscal Multiplier

• Fiscal multiplier in normal times.

• Fiscal multiplier when non-negativity constraint on nominal rate of interest is binding.
Derivation of Model Equilibrium Conditions

• Households
  – First order conditions

• Firms:
  – final goods and intermediate goods
  – marginal cost of intermediate good firms

• Aggregate resources

• Monetary policy

• Three linearized equilibrium conditions:
  – Intertemporal, Pricing, Monetary policy

• Results
Model

• Household preferences and constraints:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^\gamma (1-N_t)^{1-\gamma}}{1-\sigma} - 1 \right] + v(G_t) 
\]

\[P_t C_t + B_{t+1} \leq W_t N_t + (1 + R_t)B_t + T_t, \quad T_t \sim \text{lump sum taxes and profits}\]

• Optimality conditions

\[
\bar{u}_{c,t} = E_t \beta u_{c,t+1} \quad \frac{1 + R_{t+1}}{1 + \pi_{t+1}}
\]

\[
\frac{\bar{u}_{N,t}}{u_{c,t}} = \frac{W_t}{P_t}
\]
Linearized Intertemporal Equation

• Inter-temporal Euler equation

\[ E_t \left[ u_{c,t} - \beta u_{c,t+1} \frac{1+R_{t+1}}{1+\pi_{t+1}} \right] = 0 \]

• In zero inflation no growth steady state:

\[ 1 = \beta(1 + R) \]

• Totally differentiate:

\[ du_{c,t} - \left[ \beta(1 + R)du_{c,t+1} + \beta u_c dR_{t+1} - \beta u_c (1 + R)d\pi_{t+1} \right] = 0 \]

  – Log-differentiation:

\[ u_c \hat{u}_{c,t} - \beta(1 + R)u_c \left[ \hat{u}_{c,t+1} + \frac{1}{1+R}dR_{t+1} - d\pi_{t+1} \right] = 0 \]

  – Finally:

\[ \hat{u}_{c,t} - \left[ \hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1} \right] = 0 \]
Linearized intertemporal, cnt’d

• Repeat:

\[ \hat{u}_{c,t} - \left[ \hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1} \right] = 0 \]

\[ u = \frac{\left[ C_t^{\gamma(1-N_t)^{1-\gamma}} \right]^{1-\sigma} - 1}{1-\sigma} \rightarrow u_{c,t} = \gamma C_t^{\gamma(1-\sigma)-1} (1 - N_t)^{(1-\gamma)(1-\sigma)} \]

\[ \hat{u}_{c,t} = [\gamma (1 - \sigma) - 1] \hat{C}_t - \frac{(1-\gamma)(1-\sigma)N}{1-N} \hat{N}_t \]
Firms

• Final, homogeneous good

\[ Y_t = \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \varepsilon > 1 \]

– Efficiency condition:

\[ P_t(i) = P_t \left( \frac{Y_t}{Y_t(i)} \right)^{\frac{1}{\varepsilon}} \]

• i-th intermediate good

\[ Y_t(i) = N_t(i) \]

– Optimize price with probability 1-\( \theta \), otherwise

\[ P_t(i) = P_{t-1}(i) \]
Intermediate Good Firm Marginal Cost

• Marginal cost:

\[
MC_t = \frac{dCost_t}{dOutput_t - dWorker_t} = \frac{W_t}{MP_{L,t}} \frac{(1-\nu)}{\frac{-u_{N,t}}{u_{c,t}}} \\
\text{subsidy to undo effects of monopoly power } = (\epsilon-1)/\epsilon
\]

household first order condition

\[
= W_t (1 - \nu) = P_t \frac{-u_{N,t}}{u_{c,t}} (1 - \nu)
\]

• Real marginal cost

\[
S_t \equiv \frac{MC_t}{P_t} = \frac{-u_{N,t}}{u_{c,t}} (1 - \nu) \quad \text{in steady state}
\]

marginal cost to household of providing one more unit of labor

\[
\frac{-u_{N,t}}{u_{c,t}} \quad \text{in steady state}
\]

marginal benefit of one extra unit of labor

\[
\frac{-u_{N,t}}{u_{c,t}} \equiv \frac{\epsilon-1}{\epsilon} \equiv 1
\]
Aggregate Resources

- Resource relation:
  \[ C_t + G_t = Y_t = p_t^* N_t \]
  - \( p_t^* \) is ‘Tak Yun’ distortion
  - recall, distortion = 1 to first order:
    \[ \hat{Y}_t = \hat{N}_t \]

- Log-linear expansion:
  \[ (1 - g)\hat{C}_t + g\hat{G}_t = \hat{Y}_t, \quad g \equiv \frac{G}{Y} \]

- Consumption:
  \[ \hat{C}_t = \frac{1}{1-g} \hat{Y}_t - \frac{g}{1-g} \hat{G}_t \]
Simplifying Marginal Utility of $C$

In steady state

\[
\frac{-u_{N,t}}{u_{c,t}} \approx 1 \rightarrow \frac{1 - \gamma}{1 - N} = \frac{\gamma}{C}
\]

\[
\hat{u}_{c,t} = [\gamma (1 - \sigma) - 1] \hat{C}_t - \frac{(1 - \gamma)(1 - \sigma)N}{1 - N} \hat{N}_t
\]

\[
= [\gamma (1 - \sigma) - 1] \hat{C}_t - \frac{\gamma (1 - \sigma) N}{C} \hat{N}_t
\]

\[
= [\gamma (1 - \sigma) - 1] \hat{C}_t - \frac{\gamma (1 - \sigma)}{1 - g} \hat{N}_t
\]

\[
= [\gamma (1 - \sigma) - 1] \left[ \frac{1}{1 - g} \hat{Y}_t - \frac{g}{1 - g} \hat{G}_t \right] - \frac{\gamma (1 - \sigma)}{1 - g} \hat{Y}_t
\]

\[
= - \frac{1}{1 - g} \hat{Y}_t - [\gamma (1 - \sigma) - 1] \frac{g}{1 - g} \hat{G}_t
\]
Simplify Intertemporal Equation

• Intertemporal Euler equation:

\[ \hat{u}_{c,t} = \hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1} \]

• Substitute out marginal utility of consumption:

\[
- \frac{1}{1-g} \hat{Y}_t - [\gamma(1-\sigma) - 1] \frac{g}{1-g} \hat{G}_t \\
= -\frac{1}{1-g} \hat{Y}_{t+1} - [\gamma(1-\sigma) - 1] \frac{g}{1-g} \hat{G}_{t+1} + \beta dR_{t+1} - d\pi_{t+1}
\]

• Rearranging:

\[
\hat{Y}_t + [\gamma(1-\sigma) - 1]g\hat{G}_t \\
= \hat{Y}_{t+1} + [\gamma(1-\sigma) - 1]g\hat{G}_{t+1} - (1-g)[\beta dR_{t+1} - d\pi_{t+1}]
\]
Phillips Curve

• Equilibrium condition associated with price setting just like before:

\[ \pi_t = \beta \pi_{t+1} + \kappa \hat{S}_t, \]

\[ \kappa = \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \]

• Marginal cost:

\[ \hat{S}_t = \frac{(1 - \gamma)C_t}{\gamma(1 - N_t)} = \hat{C}_t - \left(1 - N_t\right) = \hat{C}_t + \frac{N}{1 - N} \hat{N}_t \]

\[ \left( \hat{C}_t = \frac{1}{1 - g} \hat{Y}_t - \frac{g}{1 - g} \hat{G}_t, \hat{N}_t = \hat{Y}_t \right) \]

\[ \Rightarrow \quad \left[ \frac{1}{1 - g} + \frac{N}{1 - N} \right] \hat{Y}_t - \frac{g}{1 - g} \hat{G}_t \]
Monetary Policy

- Monetary policy rule (after linearization)

\[ dR_{t+1} = \rho_R dR_t + (1 - \rho_R) \left[ \frac{\phi_1}{\beta} \pi_{t+k} + \frac{\phi_2}{\beta} \hat{Y}_{t+l} \right] \]

\[ dR_{t+1} \equiv R_{t+1} - R, \ R = \frac{1}{\beta} - 1 \]

\[ \hat{Y}_t \equiv \frac{Y_t - Y}{Y} \]

\[ k, l = 0, 1. \]
Pulling All the Equations Together

• IS equation:

\[ \hat{Y}_t + [\gamma(1 - \sigma) - 1]g\hat{G}_t \]
\[ = \hat{Y}_{t+1} + [\gamma(1 - \sigma) - 1]g\hat{G}_{t+1} - (1 - g)[\beta dR_{t+1} - d\pi_{t+1}] \]

• Phillips curve:

\[ \pi_t = \beta \pi_{t+1} + \kappa \left[ \left( \frac{1}{1-g} + \frac{N}{1-N} \right)\hat{Y}_t - \frac{g}{1-g}\hat{G}_t \right] \]

• Monetary policy rule:

\[ dR_{t+1} = \rho_R dR_t + (1 - \rho_R) \left[ \frac{\phi_1}{\beta} \pi_{t+k} + \frac{\phi_2}{\beta} \hat{Y}_{t+l} \right] \]
The Equations in Matrix Form

\[
\begin{bmatrix}
-\frac{1}{1-g} & -1 & 0 \\
0 & \beta & 0 \\
l(1 - \rho_R) \frac{\phi_2}{\beta} & k(1 - \rho_R) \frac{\phi_1}{\beta} & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{Y}_{t+1} \\
\pi_{t+1} \\
dR_{t+2}
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
\frac{1}{1-g} & 0 & \beta \\
0 & -1 & 0 \\
(1 - l)(1 - \rho_R) \frac{\phi_2}{\beta} & (1 - k)(1 - \rho_R) \frac{\phi_1}{\beta} & -1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{Y}_t \\
\pi_t \\
dR_{t+1}
\end{bmatrix}
+ \begin{bmatrix}
g[\gamma(\sigma-1)+1] \\
0 \\
0 \\
\end{bmatrix}
\hat{G}_{t+1}
+ \begin{bmatrix}
-g[\gamma(\sigma-1)+1] \\
0 \\
0
\end{bmatrix}
\hat{G}_t,
\]

\[
\\text{• or,} \quad \alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t = 0.
\]

\[
s_t = Ps_{t-1} + \varepsilon_t, \quad s_t \equiv \hat{G}_t, \quad P = \rho
\]
Solution:

• Undetermined coefficients, $A$ and $B$:

\[ z_t = Az_{t-1} + Bs_t \]

• $A$ and $B$ must satisfy:

\[
\begin{align*}
\alpha_0 A^2 + \alpha_1 A + \alpha_2 &= 0 \\
\alpha_0 (AB + BP) + \alpha_1 B + \beta_0 P + \beta_1 &= 0.
\end{align*}
\]

• When $\rho_R = 0$, $\alpha_2 = 0 \rightarrow A = 0$ works.
Results

• Fiscal spending multiplier small, but can easily be bigger than unity (i.e., $C$ rises in response to $G$ shock)

• Contrasts with standard results in which multiplier is less than unity
  – Typical preferences in estimated models:
    $$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\gamma}}{1+\gamma} + v(G_t) \right], \psi, \gamma, \sigma > 0.$$  
  – Marginal utility of $C$ independent of $N$ for CGG
  – Marginal utility of $C$ increases in $N$ for KPR.
Simulation Results

- Benchmark parameter values:

\[ \kappa = 0.035, \, \beta = 0.99, \, \phi_1 = 1.5, \, \phi_2 = 0, \, N = 0.23, \, g = 0.2, \, \sigma = 2, \, \rho = 0.8, \, \rho_R = 0 \]

Multiplier = 1.05, constant.

\[ \frac{G_I - G}{Y} = \frac{G_I - G}{G} \frac{G}{Y} = \hat{G}_t g \]
Multiplier for Alternative Parameter Values

\[ \phi_1 = 1.5, \phi_2 = 0, \rho_R = 0, \rho = 0.8, \kappa = 0.03, \]
\[ \beta = 0.99, \gamma = 0.28571, N = 0.33333, g = 0.2, \quad \sigma = 2 \]

- Results: multiplier bigger
  - the less monetary policy allows \( R \) to rise.
  - the more complementary are consumption and labor (i.e., the bigger is \( \sigma \)).
  - the smaller the negative income effect on consumption (i.e., the smaller is \( \rho \)).
  - smaller values of \( \kappa \) (i.e., more sticky prices)
Multiplier for Alternative Parameter Values

\[ \phi_1 = 1.5, \phi_2 = 0, \rho_R = 0, \rho = 0.8, \kappa = 0.03, \]
\[ \beta = 0.99, \gamma = 0.28571, N = 0.33333, g = 0.2, \quad \sigma = 2 \]

\[ dR_{t+1} = \rho_R dR_t + (1 - \rho_R) \left[ \frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \right] \]

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Multiplier for Alternative Parameter Values

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\[ \beta = 0.99, \gamma = 0.28571, N = 0.33333, g = 0.2, \quad \sigma = 2 \]

\[ u(C_t, N_t) = \frac{\left[C_t(1 - N_t)^{1-\gamma}\right]^{1-\sigma} - 1}{1 - \sigma} \]

\[ u_{c,t} = C_t^{\gamma(1-\sigma)-1}(1 - N_t)^{(1-\gamma)(1-\sigma)} \]

- Results: multiplier bigger
  - the more complementary are consumption and labor (i.e., the bigger is \( \sigma \)).
Multiplier for Alternative Parameter Values

\[ \hat{G}_t = \rho \hat{G}_{t-1} + \varepsilon_t \]

- Results: multiplier bigger
  - the smaller the negative income effect on consumption (i.e., the smaller is \( \rho \)).
Multiplier for Alternative Parameter Values

\[ \begin{align*}
\phi_1 &= 1.5, \quad \phi_2 = 0, \quad \rho_R = 0, \quad \rho = 0.8, \quad \kappa = 0.03, \\
\beta &= 0.99, \quad \gamma = 0.28571, \quad N = 0.33333, \quad g = 0.2, \quad \sigma = 2
\end{align*} \]

- Results: multiplier bigger
  - smaller values of \( \kappa \) (i.e., more sticky prices)
Analysis of Case when the Non-negativity Constraint on the Nominal Interest Rate is Binding

• Begin with intuition....
Zero Lower Bound (ZLB)

• Arguably, zero lower bound is now binding.
G Multiplier and Welfare

- Exploiting the big $G$ multiplier while in the zero bound may be welfare-improving

  - Rise in output associated with bigger $G$ may help correct gross inefficiency in lower bound crisis.

  - This is so, even though high $G$ in normal times might be inefficient.
Real Interest Rate

• In New Keynesian model, what matters is real rate. Zero lower bound on nominal rate places no inflexibility on real interest rate.

• Inflation expectations slow to rise implies lower bound on real rate.

• Reasons why expected inflation may not rise soon:
  – Fed officials frequently repeat the credibility of their ‘exit strategy’ from the recent monetary expansion.
  – In the previous ‘zero bound scare’, policy of committing to keeping interest rate low extra long (to raise expected inflation) is held responsible for the recent housing bubble and subsequent world financial crisis.
  – Policy makers have learned from the experience of the 1970s that a rise in inflation expectations can lead to a loss of control over inflation.
Consequence of Increase in Saving When there is Lower Bound on Real Interest Rate

Real Rate, \( \frac{1+R}{1+\pi^e} \) vs. Investment

Lower bound vs. Saving, Investment
Consequence of Increase in Saving When there is Lower Bound on Real Interest Rate

Real Rate, \( \frac{1+R}{1+\pi^e} \)

Investment

Saving

Lower bound

Saving, Investment
Consequence of Increase in Saving When there is Lower Bound on Real Interest Rate

\[ \frac{1 + R}{1 + \pi^e} \]

Equilibrium requires that saving equal investment.

Saving

Investment

Real Rate

Lower bound

Saving, Investment
Consequence of Increase in Saving When there is Lower Bound on Real Interest Rate

- Push back in saving could be accomplished
  - indirectly, by (possibly big) fall in output
  - directly, by rise in \( G \)
- taxes have no effect, because have Ricardian equivalence.
Drop in Output May Be Very Large

• So far, analysis resembles classic Keynesian analysis of ‘Paradox of Thrift’
  – Rationalizes a relatively modest drop in output.

• In NK model, drop in output may be much bigger.
  
  – A vicious deflationary cycle may trigger a perverse rise in the real interest rate.

  – The rise in the real rate of interest makes the excess saving problem worse, increasing the fall in income needed to achieve equilibrium in the loan market.
Deflation Cycle in Zero Bound

- Low spending
- High real interest rate
- Low marginal cost
- Low expected inflation
G Multiplier

• May be big in zero bound, by preventing the zero bound collapse.

• Another way to see the potential for G multiplier.....
Government Spending Multiplier with Constant $R$

• Normal times

$$G \uparrow \implies Y \uparrow \pi^e \uparrow \implies R \uparrow \; R - \pi^e \uparrow$$

  – Effect of $G$ dampened by monetary policy

• Zero bound

$$G \uparrow \implies Y \uparrow \pi^e \uparrow \implies R \text{ fixed}, \; R - \pi^e \downarrow$$

  – Effect of $G$ is now amplified
Turning to the formal analysis....

• Need a shock that puts us into the lower bound.

• One possibility: increased desire to save.
  – Seems particularly relevant at the current time.
  – Other shocks will do it too......

• Discount rate shock.
Monetary Policy

• Monetary policy rule (after linearization)

\[
Z_{t+1} = R + \rho_R (R_t - R) + (1 - \rho_R) \left[ \frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \right]
\]

\[
\hat{Y}_t = \frac{Y_t - Y}{Y}, \quad R = \frac{1}{\beta} - 1
\]

\[
R_{t+1} = \begin{cases} 
Z_{t+1} & \text{if } Z_{t+1} > 0 \\
0 & \text{if } Z_{t+1} \leq 0
\end{cases}
\]
Eggertsson-Woodford Saving Shock

• Preferences:

\[ u(C_0, N_0, G_0) + \frac{1}{1+r_1} E_0 \{ u(C_1, N_1, G_1) + \frac{1}{1+r_2} u(C_2, N_2, G_1) + \frac{1}{1+r_2} \frac{1}{1+r_3} u(C_3, N_3, G_3) \ldots \} \]

• Before \( t<0 \)
  
  – System was in non-stochastic, zero inflation steady state,

\[ r_{t+1} = R = \frac{1}{\beta} - 1 \]

\[ R_{t+1} = R \]

\[ \hat{G}_t = 0, \text{ for all } t \]
Saving Shock, cnt’d

• At time $t=0$,

\[ r_1 = r^l < 0 \]

\[ \text{Prob}[r_{t+1} = r | r_t = r^l] = 1 - p \]

\[ \text{Prob}[r_{t+1} = r^l | r_t = r^l] = p \]

\[ \text{Prob}[r_{t+1} = r^l | r_t = r] = 0 \]

• “Discount rate drops in $t=0$ and is expected to return permanently to its ‘normal’ level with constant probability, 1-$p$.”
Zero Bound Equilibrium

• simple characterization:

\[ \pi^l, \hat{\gamma}^l, R = 0, Z^l \leq 0 \quad \text{while discount rate is low} \]

\[ \pi_t = \hat{\gamma}_t = 0, R = r \quad \text{as soon as discount rate snaps back up} \]
Fiscal Policy

• Government spending is set to a constant deviation from steady state, during the zero bound.

• That is,

\[ \hat{G}_t \text{ may be nonzero while } r_{t+1} = r^l, \quad \hat{G}_t = 0 \text{ when } r_{t+1} = r \]
Equations With Discount Shock

- **IS equation:**
\[
\hat{Y}_t - g[\gamma(\sigma - 1) + 1]\hat{G}_t = -(1 - g)[\beta(\hat{R}_{t+1} - \hat{r}_{t+1}) - E_t\pi_{t+1}] + E_t\hat{Y}_{t+1} - g[\gamma(\sigma - 1) + 1]E_t\hat{G}_{t+1}
\]

- **Phillips curve:**
\[
\hat{Y}^l - g[\gamma(\sigma - 1) + 1]\hat{G}^l = -(1 - g)[\beta(0 - \hat{r}^l) - p\pi^l] + p\hat{Y}^l - g[\gamma(\sigma - 1) + 1]p\hat{G}
\]

- **Phillips curve:**
\[
\pi_t = \beta E_t\pi_{t+1} + \kappa\left[\left(\frac{1}{1-g} + \frac{N}{1-N}\right)\hat{Y}_t - \frac{g}{1-g}\hat{G}_t\right]
\]

- **Monetary Policy:**
\[
R_{t+1} = 0
\]
\[
Z_{t+1} = R + \rho_R(R_t - R) + (1 - \rho_R)\left[\frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t\right] \leq 0
\]
As $p$ increases, zero-bound becomes more severe...this is because with higher $p$, fall in output is more persistent, and reductions in output have smaller effect on saving.
Fiscal Expansion in Zero Bound Highly Effective, But is it *Desirable*?

- **Intuition:**
  - Yes....

  - the vicious cycle produces a huge, inefficient fall in output

  - in the first-best equilibrium, output, consumption and employment are invariant to discount rate shocks

  - If $G$ helps to partially undo this inefficiency, then surely it’s a good thing
Fiscal Expansion in Zero Bound Highly Effective, But is it Desirable?

• Preferences

\[
\sum_{t=0}^{\infty} \left( \frac{p}{1 + r^l} \right)^t \left[ \frac{(C^l)^\gamma (1 - N^l)^{1-\gamma}}{1 - \sigma} \right]^{1-\sigma} - 1 + v(G^l)
\]

\[
= \frac{1}{1 - \frac{p}{1 + r^l}} \left[ \frac{(C^l)^\gamma (1 - N^l)^{1-\gamma}}{1 - \sigma} \right]^{1-\sigma} - 1 + v(G^l)
\]

\[
= \frac{1}{1 - \frac{p}{1 + r^l}} \left[ \frac{(N(\hat{y}^l + 1) - Ng(\hat{G}^l + 1))^{\gamma} (1 - N(\hat{y}^l + 1))^{1-\gamma}}{1 - \sigma} \right]^{1-\sigma} - 1 + v(Ng(\hat{G}^l + 1))
\]

• Compute optimal $\hat{G}^l$

- (i) $v(G^l) = 0$,

- (ii) $v(G) = \psi_g \frac{G^{1-\sigma}}{1 - \sigma}$, $\psi_g$ chosen to rationalize $g = 0.2$ as optimal in steady state
Case Where G is not Valued

\[ \phi_1 = 1.5, \phi_2 = 0, \rho_R = 0, \rho = 0.8, \kappa = 0.03, \beta = 0.99, \]
\[ \gamma = 0.28571, N = 0.33333, g = 0.2, k = 0, l = 0, \hat{G} = 0, \sigma = 2, \rho \]

Optimal \( G \) is substantial, around 5%.
Case Where Gov’t Spending is Desirable

\[
\begin{align*}
\phi_1 &= 1.5, \quad \phi_2 = 0, \quad \rho_R = 0, \quad \rho = 0.8, \quad \kappa = 0.03, \quad \beta = 0.99 \\
gamma &= 0.28571, \quad N = 0.33333, \quad g = 0.2, \quad k = 0, \quad l = 0, \quad G_{\hat{t}} = 0, \quad \sigma = 2, \quad \psi \\
psig &= 0.015226
\end{align*}
\]

Optimal $Y$ higher than before crisis

The high level of output is necessary to get partial recovery in consumption
Introducing Investment

• Two findings:

  – With investment, likelihood of lower bound reduced (as real rate falls with rise in saving, investment expands to absorb rise in saving).

  – When the lower bound binds, multiplier could be larger because vicious cycle more severe.
Conclusion of G Multiplier Analysis

- Government spending multiplier in a neighborhood of unity in ‘normal times’.

- Multiplier can be large when the zero bound is binding (because $R$ constant then).

- Increase in $G$ is welfare improving during lower bound crisis.

- Caveat: focused exclusively on multiplier
  - Increasing $G$ may be bad idea because hard to reverse.
  - May be other ways of accomplishing similar thing (e.g., transition to VAT tax over time).
Cut in Labor Tax Rate (Eggertsson)

• Eggertsson: a cut in labor income tax would only make recession worse, if zero lower bound is binding.

• Seems to conflict with core Keynesian orthodoxy: cutting taxes gets you out of a recession.
  – Contradiction not actually there.
  – NK model drops standard Keynesian mechanism, operating through disposable income. NK model assumes consumers are Ricardian.
• Eggertsson’s finding is nevertheless still surprising.
  – Labor income tax cut increases labor supply. Should lead to *more* employment!
  – Actually leads to *less* employment.
  – Key is the deflation spiral
    • Increase labor supply implies lower wage
    • Lower wage means lower marginal cost
    • Drop in marginal cost implies deflation
    • Deflation necessarily produces higher real interest rate when zero bound is binding.
    • Higher real rate produces a cutback in spending, and output drops.

• In NK models with sticky wages, Eggertsson’s result is not quantitatively so important.
Figure 3: Sticky Wages and Labor Supply
Labor Income Tax (Eggertsson)

- Dixit-Stiglitz Production of final goods:
  \[ Y_t = \left[ \int_0^1 Y_{t,i}^{\lambda_f} di \right]^\lambda_f, \ 1 \leq \lambda_f < \infty. \]

- Production of intermediate goods:
  \[ Y_{t,i} = H_{t,i}, \]

- Calvo sticky prices:
  \[ P_{t,i} = \begin{cases} 
  P_{t-1,i} & \text{with probability } \xi_p \\
  \text{chosen optimally} & \text{with probability } 1 - \xi_p 
  \end{cases}, \]
EHL Sticky Wages

• Labor aggregator:

\[ H_t = \left[ \int_0^1 (h_{t,j})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, 1 \leq \lambda_w < \infty. \]

• \textit{j-th} household’s objective

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log C_t - A \frac{h_{t,j}^{1+\phi}}{1 + \phi} \right], \beta \in (0, 1), A, \phi > 0.
\]

\[ P_tC_t + B_{t+1} \leq R_{t-1}B_t + (1 - \tau_t)W_{t,j}h_{t,j} + \Pi_{t,j}, \]

• Calvo sticky wages:

\[
W_{t,j} = \begin{cases} 
W_{t-1,j} & \text{with probability } \xi_w \\
\text{chosen optimally} & \text{with probability } 1 - \xi_w
\end{cases}
\]
• Resource constraint and monetary policy:

\[ C_t = Y_t. \]

\[ Z_t = \frac{1}{\beta} - 1 + 1.5\pi_t, \beta \in (0, 1) \]

\[ R_t = \begin{cases} 
Z_t & Z_t \geq 0 \text{ ‘zero bound not binding’} \\
0 & \text{otherwise ‘zero bound binding’} 
\end{cases} \]

• Benchmark parameter values:

\[ \xi_p = \xi_w = 0.75, \beta = \frac{1}{1 + r} = 0.99, r = 0.01, \lambda_w = \lambda_f = 1.20, \phi = 1, \]

• Experiment:

− discount rate drops from 0.01 in steady state to -0.01, for 15 periods.
− then, back to normal.
Figure 4: Baseline Simulation
Figure 5: Different Degrees of Wage Stickiness

Time between wage reoptimization = 4 quarters

Circles indicate date when zero bound ceases to bind

1.25 quarters

Starred line: no policy response, solid line: tax rate increased from 0.30 to 0.40 while zero bound binds
Tax Policies More Generally

• Think of their impact on marginal cost or on willingness to buy goods.

• Marginal cost:
  – employment subsidy hugely counterproductive

• Spending:
  – tax levied on household capital income helps by shifting saving supply function left.
  – tax levied on the earnings of capital before it reaches the household hurts by shifting investment demand function left.
  – Investment tax credit helps.
Can Zero Bound, in Conjunction with Other Shocks Account for Recent Data?

• Suppose that something (thing, $x$) happened in 2008Q3.
• Identify impulse response of economy to $x$ by comparing what actually happened with forecast as of 2008Q2.
• Assume $x$ is a shock to households’ desire to save (the saving rate did go up), and wedge in the rate of return on capital (spreads did go up).
• First, what happened?.....
Actual and 2008Q2 Forecasts

- **per capita GDP**
- **Consumption**
- **investment**
- **St&loc purchases**
- **Gov C and I**
- **cpi**
- **Funds rate**
- **priv saving rate**

[Graphs showing actual and univariate forecast values for each category from 1998 to 2010]
Simulate ACEL, CEE Model

• Features:
  – Habit persistence in preferences
  – Adjustment costs in change of investment
  – Capital utilization costs small (ACEL).

• Shock to discount rate and to wedge in rate of return on capital

\[
R^k_{t+1} = (1 - \tau^k_{t+1}) \left[ \frac{r^k_{t+1} + P^k_{t+1}(1 - \delta)}{P^k_t} \right]
\]

• Wedge designed to capture increased financial frictions.
Computations

• Selected parameters values:

Calvo wage stickiness parameter: 0.72, habit parameter: 0.70, coefficient on marginal cost in Phillips curve: 0.0026
steady state price markup: 1.01, steady state wage markup: 1.05

• Experiment:

Calvo wage stickiness parameter: 0.72, price stickiness: 0.77, habit parameter: 0.70,
coefficient on marginal cost in Phillips curve: 0.0026
steady state price markup: 1.01, steady state wage markup: 1.05
Government consumption up 1/2 percent while zero bound is binding
Results: Model Replicates Impulse Responses Reasonably Well

- **net inflation**
- **net nominal rate of interest**
- **output**
- **consumption**
- **investment**

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**Z**

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**APR**

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**% dev from ss**
• Government consumption multiplier:

\[ \frac{dY_t}{dG} = 0.49, 2.0, 2.2, 2.3, 2.3, 2.3, 2.2, 2.0, 1.8, 1.7, 1.5, 1.3, 0.02, \quad t = 1, 2, \ldots, 13. \]

• Denominator: change in G operative while G is up (i.e., periods 2 to 12).
Conclusion of Simulation

- Can account for dynamics of recent data as reflecting the operation of the zero bound and two particular shocks.

- Many people would expect this not to be possible. Mindful of the deflation spiral, they would anticipate that the drop in inflation would be too great.

- In fact, had to assume a very small slope to Phillips curve.