

1. Introduction

In the 1990's it was often suggested that the economy had entered an 'overinvestment boom'. During such a boom, investment and the stock market are high, until a day of reckoning occurs, when it is realized that the hoped-for returns in fact will never be realized. On the day of reckoning, the stock market and investment collapse, and the economy slips into recession. On a small scale, examples are fiber optic cable that was over-produced in the 1990s and now lays around unused. On a larger scale, many observers interpret the 1920s in the US and the 1980s in Japan as examples when entire economies were caught up in overinvestment booms that were terminated in a stock market crash and recession. In recent decades, mainstream economists have been skeptical that the notion of an overinvestment boom could be placed on sound economic foundations. However, journalists and other observers often interpret the 1990s and the stock market crash that occurred in 2000 as another example of an overinvestment boom. This has encouraged economists to revisit the possibility that overinvestment booms may be a real possibility after all.¹ Of particular interest is the possibility that these events pose novel questions about the appropriate conduct of monetary and fiscal policy.

In this tutorial, we will explore one attempt to model an overinvestment boom, based on research I am currently pursuing with Robert Motto and Massimo Rostagno of the European Central Bank. The boom is modelled as being triggered by a signal which indicates to all that productivity will be higher in the future. When the future actually occurs, however, the signal turns out to be false. In effect the signal turns out to be a mirage. We describe a particular stochastic process for technology that allows us to formalize the mirage idea.

The model economy is a standard real business cycle model, with two additional twists. First, there is curvature in the technology for converting investment goods into installed capital. This makes it possible for the price of capital to move around, something that is essential if we are to develop a theory in which equity prices can. In addition, the curvature is of a particular kind. The adjustment costs in investment are on the difference of investment. That is, if investment is

¹For an important contribution, see Beaudry and Portier (2000).

to be increased, doing so quickly is very costly, and less waste is incurred if the increase is done slowly. This is intended to capture, in a reduced form way, the notion that learning must occur as investment increases from one level to another. Second, there is habit persistence in preferences: if consumption has been high recently, then the marginal utility of current consumption is higher. The particular adjustment costs we introduce, as well as the assumption of habit persistence, have been shown to be important ingredients in models of aggregate economic activity. In this tutorial, we will see that the adjustment costs and habit persistence are a critical ingredient in any theory of overinvestment booms that is built on the notion that they are driven by a mirage.

A surprising outcome of this analysis, is that the theory of overinvestment booms that has been outlined in fact does not generate a rise in asset prices during the boom. In fact, the theory implies - despite the presence of adjustment costs in investment and strong investment - that asset prices will be *low* during the run up to the boom.

In this tutorial, you will discover these observations for yourself, by simulating a model under alternative settings of the parameters. You will, in effect, see first-hand how equilibrium modelling can be used to sharpen up one's ideas about an economic phenomenon. You will also find that the mirage theory, as developed in this tutorial, is incomplete as a theory of overinvestment booms. Of course, one possibility is that the theory is fundamentally misspecified. But, my work with Motto and Rostagno raises another possibility, that a critical thing that is missing from the theory is a monetary sector. When we incorporate money into the model, and model monetary policy as following a Taylor rule, we find that the mirage theory offers reasonable theory of overinvestment booms, complete with stock market boom and collapse. The reason is simple. The Taylor rule specifies that when anticipated inflation rises, then the authorities should sharply raise nominal rates of interest. During the overinvestment boom, inflation actually drifts down. The monetary authority responds by driving the nominal rate of interest down. To do this requires increasing the money supply. When the money supply is increased in this way, this adds extra fuel to investment and raises stock prices as well. So, it may well be that the mirage theory that we develop here is a reasonable basis for a theory of overinvestment after all.

Below, we explore the non-monetary theory of overinvestment booms. Clearly, the theory as developed is missing some crucial elements (probably, monetary factors). Still, it suits our purposes. Basic components of the theory are revealed without the additional complications that monetary factors can bring. In addition,

the exercise provides a way for the student to get their ‘feet wet’ in the analysis and simulation of a dynamic, general equilibrium model. Accompanying this tutorial, there is software that can be used to simulate the overinvestment boom in MATLAB, written by Etienne Gagnon.

2. Model

Suppose the preferences of households are

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{[(C_t - bC_{t-1})(1 - h_t)^\psi]^{1-\gamma}}{1 - \gamma}.$$

Here, h_t is hours worked, C_t is consumption and the amount of time that is available is unity. When $b > 0$ then there is habit persistence in preferences. The resource constraint is

$$I_t + C_t \leq Y_t, \quad (2.1)$$

where K_t denotes the beginning of period t stock of physical capital, I_t is investment, C_t is consumption and Y_t is output of goods.

Output Y_t is produced using the technology

$$Y_t = K_t^\alpha (\exp(z_t) h_t)^{1-\alpha}, \quad (2.2)$$

where z_t represents a stochastic shock to technology. Its law of motion will be described shortly.

Physical capital is accumulated with the following technology

$$K_{t+1} = (1 - \delta)K_t + (1 - S\left(\frac{I_t}{I_{t-1}}\right))I_t, \quad (2.3)$$

where the function S captures the notion that there are adjustment costs in changing the level of investment. For convenience, $S(x_t)$ has the functional form

$$S(x_t) = \frac{\chi}{2} [\exp\{x_t - 1\} + \exp\{-(x_t - 1)\} - 2]$$

with $\chi > 0$. Notice that $S(1) = S'(1) = 0$ and $S''(1) = \chi$. The first two zeros guarantee that the adjustment costs have no impact on the steady state.

We model an over investment boom as follows. Up until period 1, the economy is in a steady state. In period 1, a signal arrives that suggests z_t will be high in

period z_{1+p} . Then, in period $1 + p$, the expected rise in technology in fact does not occur. A time series representation for z_t which captures this possibility is:

$$z_t = \rho z_{t-1} + \varepsilon_{t-p} + \xi_t, \quad (2.4)$$

where ε_t and ξ_t are uncorrelated over time and with each other. To see that this setup can capture the mirage idea, suppose $p = 1$. Then, if ε_1 is seen to have a high value, this shifts up the expected value of z_2 . But, if (by accident!) $\xi_2 = -\varepsilon_1$, the high expected value of z_2 does not materialize. In effect, ε_1 turns out to be a mirage. Of course this is not the only possible outcome. The variable, ε_1 , is not a mirage if $\xi_t = 0$ for $t = 2, 3, \dots$. In this case, the high value of z_2 signalled by ε_1 actually happens and we get $z_{2+t} = \rho^t z_2$, for $t = 1, 2, 3, \dots$. In this framework, an overinvestment boom is something that occurs in the event that ε_1 is high, but its effects are later cancelled by ξ_t .

For purposes of solving and simulating the model, it is useful to formulate this model of z_t in the canonical form discussed in class. To this end, consider the following formulation:

$$\begin{bmatrix} z_t \\ \varepsilon_t \\ \varepsilon_{t-1} \\ \vdots \\ \varepsilon_{t-p+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} z_{t-1} \\ \varepsilon_{t-1} \\ \varepsilon_{t-2} \\ \vdots \\ \varepsilon_{t-p} \end{bmatrix} + \begin{bmatrix} \xi_t \\ \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

The first equation here corresponds to (2.4). The second equation says that $\varepsilon_t = \varepsilon_t$ (who can argue with that!). The third says $\varepsilon_{t-1} = \varepsilon_{t-1}$, and so on. Let

$$s_t = \begin{bmatrix} z_t \\ \varepsilon_t \\ \varepsilon_{t-1} \\ \vdots \\ \varepsilon_{t-p+1} \end{bmatrix}, \quad \epsilon_t = \begin{bmatrix} \xi_t \\ \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad P = \begin{bmatrix} \rho & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}.$$

Then, we have

$$s_t = P s_{t-1} + \epsilon_t. \quad (2.5)$$

We suppose that at the beginning of period t , s_t is observed. Then, future values of s_t are forecasted using $E_t s_{t+j} = P^j s_t$, for $j > 0$.

To see how this ‘works’, consider the following example. Suppose $p = 2$. Then

$$s_t = \begin{bmatrix} z_t \\ \varepsilon_t \\ \varepsilon_{t-1} \end{bmatrix}, P = \begin{bmatrix} \rho & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \epsilon_t = \begin{bmatrix} \xi_t \\ \varepsilon_t \\ 0 \end{bmatrix}$$

For additional concreteness, suppose $\rho = 0.90$. Then,

$$P^2 = \begin{bmatrix} 0.81 & 1.0 & 0.9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Now, let’s see the impact on $E_1 z_3$ of a shock to $\varepsilon_t, \varepsilon_1$. In order to isolate the effect of ε_1 , suppose $\varepsilon_t = 0$ for $t < 1$ and $\xi_t = 0, t \leq 0$. These assumptions imply that z_1 is at its unconditional mean of zero. Note that

$$\begin{aligned} E_1 z_3 &= 0.81z_1 + \varepsilon_1 + 0.9\varepsilon_0 \\ &= \varepsilon_1, \end{aligned}$$

by our assumption that only ε_1 has moved and that $E_1 \xi_t = 0$ for $t > 1$. Note in particular that ε_1 leads to an upward revision in the expectation of z_3 . In period 1, agents will act on that expectation. In period 3, z_3 is actually realized, but its realization is determined in part by the realization of ξ_3 . It happens that $\xi_3 = -\varepsilon_1$, then the expected positive move in z_3 does not occur. Any investment that occurs based on the expectation of higher z_3 constitutes ‘overinvestment’.

This event (it is a combination of two events, a positive value of ε_1 and $\xi_3 = -\varepsilon_1$) can be simulated very simply. First, solve the model to get a representation of the endogenous variables of the model. Let the endogenous variables of the model, whose values are determined at time t be:

$$Z_t = \begin{bmatrix} \hat{K}_{t+1} \\ \hat{C}_t \\ \hat{I}_t \\ \hat{h}_t \\ \hat{Y}_t \\ \hat{\lambda}_t \\ \hat{P}_{K',t} \end{bmatrix}, \tag{2.6}$$

where λ_t is the multiplier on the resource constraint in the Lagrangian representation of the problem (see the next section for a formal discussion). We then (i)

compute the steady state of the model economy; (ii) log-linearize the equilibrium conditions around this steady state (the equilibrium conditions are summarized below); and (iii) solve the resulting linear system to obtain:

$$Z_t = AZ_{t-1} + Bs_t. \quad (2.7)$$

The full system is then composed of (2.5) and (2.7).

We simulate an ‘overinvestment boom’ as the economy’s response to

$$\epsilon_1 = \begin{bmatrix} 0 \\ \epsilon_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \epsilon_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \dots, \quad \epsilon_{t+p} = \begin{bmatrix} -\epsilon_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \epsilon_{t+j} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad j > p,$$

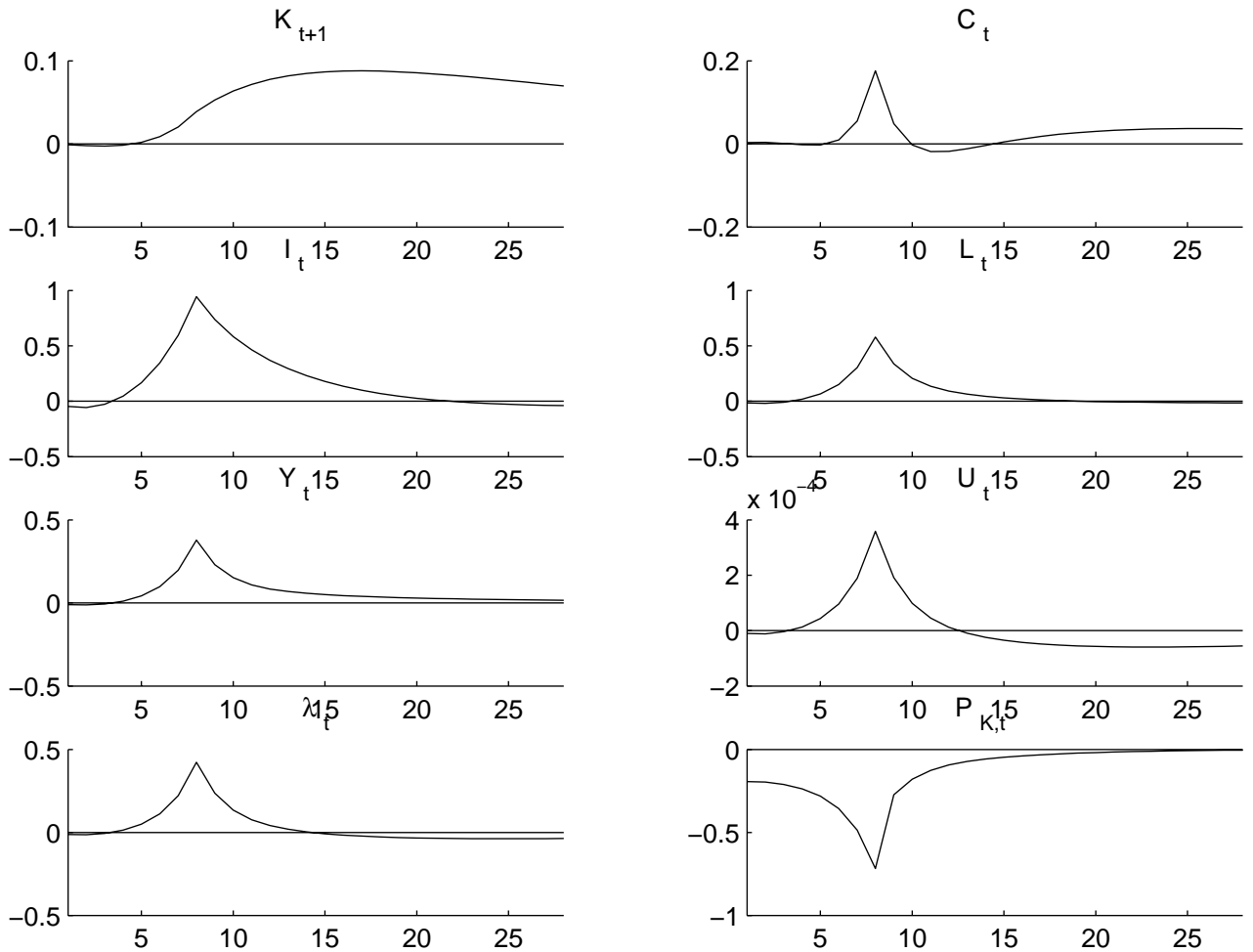
with initial condition:

$$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

To proceed with the exercise, you will need the software that accompanies it. Consider a benchmark parameterization, with $b = 0.6$, $\chi = 5$, $\delta = 0.02$, $\gamma = 1$, $\psi = 2.3$, $\rho = 0.95$, $p = 8$. To simulate the economy with this parameterization, run the MATLAB program, assignment4.m. This will produce the following figure:

Dynamic Response to a Signal About a Future Productivity
That Does Not Materialize

IRFs: Anticipated shock to technology is not realized (Logs)



Note how investment, consumption, output and employment respond positively to the signal about future technology. (Disregard U_t , capital utilization, which in any case does not move because utilization costs are set very high in the program.) Now, redo the same run, changing the value of χ to zero (this is chi in line 11 of the code). Note how dramatically different the findings are! Employment and output drop, while consumption is high, during the period leading up to $p = 8$.

Evidently, the rise in employment and investment in the benchmark specification is due to the investment adjustment costs. This is because, with this specification of adjustment costs, to be in a position to exploit the expected technology shock in the future it is efficient to slowly increase investment as soon as the signal comes in. In this way, the capital stock rises gradually to the point where it is at an efficient level to take advantage of the expected high level of technology. The response when $\chi = 0$ - raise the capital stock quickly at the last moment - generates too many adjustment costs.

Now go back to the benchmark parameter values, only set $b = 0$. In this case, the employment, output and investment booms occur, but consumption drops. With habit persistence, they don't like this fall in consumption preceding the rise in consumption that is anticipated to occur after the favorable shock is realized.

Now consider a version of the model with the following, different, specification of investment adjustment costs:

$$K_{t+1} = (1 - \delta)K_t + (1 - S(\frac{I_t}{K_t}))I_t,$$

where

$$S\left(\frac{I_t}{K_t}\right) = \frac{\chi}{2} \left[\exp\left\{\frac{I_t}{K_t} - \delta\right\} + \exp\left\{-\left(\frac{I_t}{K_t} - \delta\right)\right\} - 2 \right].$$

This is a more conventional specification of adjustment costs. Go back to the benchmark parameter values, and run the program with this specification of adjustment costs (set the parameter, `icf_lev` to 1). Note how investment fails to rise during the boom.

Now, return again to the benchmark parameter values. Consider the case in which the expected technology shock actually materializes (comment out line 48, and un-comment line 47). Notice how there is a sudden drop in employment in the period that the anticipated shock is realized. This was first observed in the paper by Jaimovic and Rebelo. Apparently, on the date that the technology shock is realized, for people to continue working at the same rate would imply a rise in consumption that is not worth it to the household.

3. The Price of Capital During the Overinvestment Boom

To understand how the price of capital evolves during the overinvestment boom, it is useful to study the first order condition with respect to investment, in the

model. For convenience, we work with a version of the model in which there is no uncertainty. Let the Lagrangian representation of the planning problem be:

$$\begin{aligned} & \sum \beta^t \left\{ \frac{[(C_t - bC_{t-1})(1 - h_t)^\psi]^{1-\gamma}}{1 - \gamma} + \lambda_t [(K_t)^\alpha (z_t h_t)^{1-\alpha} - C_t - I_t] \right. \\ & \left. + \mu_t \left[(1 - \delta)K_t + (1 - S \left(\frac{I_t}{I_{t-1}} \right))I_t - K_{t+1} \right] \right\}, \\ & \psi(C_t - bC_{t-1})^{1-\gamma} (1 - h_t)^{\psi(1-\gamma)-1} = \lambda_t (1 - \alpha) (K_t)^\alpha (z_t h_t)^{-\alpha} \end{aligned} \quad (3.1)$$

where λ_t and μ_t are non-negative multipliers. The first order condition with respect to C_t is:

$$\lambda_t = (C_t - bC_{t-1})^{-\gamma} (1 - h_t)^{\psi(1-\gamma)} - \beta b (C_{t+1} - bC_t)^{-\gamma} (1 - h_{t+1})^{\psi(1-\gamma)}. \quad (3.2)$$

Note that the right side of this expression is the marginal utility of consumption, taking into account habit persistence. So, λ_t is the marginal utility of consumption. The first order condition with respect to K_{t+1} is:

$$\mu_t = \beta \left[\lambda_{t+1} \alpha (K_{t+1})^{\alpha-1} (z_{t+1} h_{t+1})^{1-\alpha} + \mu_{t+1} (1 - \delta) \right].$$

Note that the object on the right side of the equality is the marginal utility of an extra unit of K_{t+1} . It is tomorrow's marginal physical product of capital, converted to marginal utility terms by multiplying by λ_{t+1} plus the value of the undepreciated part of K_{t+1} that is left over for use in subsequent periods, which is converted into marginal utility terms by μ_{t+1} . Divide both sides of the first order condition for K_{t+1} with respect to λ_t and rearrange:

$$\frac{\mu_t}{\lambda_t} = \beta \frac{\lambda_{t+1}}{\lambda_t} \left[\alpha (K_{t+1})^{\alpha-1} (z_{t+1} h_{t+1})^{1-\alpha} + \frac{\mu_{t+1}}{\lambda_{t+1}} (1 - \delta) \right].$$

Now, recall that μ_t is the marginal utility of K_{t+1} , loosely, dU/dK_{t+1} . Similarly, λ_t is the marginal utility of C_t , loosely dU/dC_t . Thus, the ratio is the consumption cost of a unit of K_{t+1} , or Tobin's q :

$$\frac{\mu_t}{\lambda_t} = \frac{\frac{dU_t}{dK_{t+1}}}{\frac{dU_t}{dC_t}} = \frac{dC_t}{dK_{t+1}}.$$

In light of this, it is natural to call μ_t/λ_t the ‘price of capital’, which we denote by $P_{k',t}$. Substituting this into the first order condition for K_{t+1} , we obtain:

$$P_{k',t} = \beta \frac{\lambda_{t+1}}{\lambda_t} \left[\alpha (K_{t+1})^{\alpha-1} (z_{t+1} h_{t+1})^{1-\alpha} + P_{k',t+1} (1 - \delta) \right].$$

This is the sort of first order condition we would expect if the households purchased capital in a competitive market at price, $P_{k',t}$. Thus, the left side has the cost of a unit of new capital in period t , $P_{k',t}$, and the right side has the payoff, discounted properly to the future. If the household had a real bond which paid a net return, r_{t+1} , from t to $t + 1$, it would turn out that the household’s first order condition would be:

$$\beta \frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{1 + r_{t+1}}.$$

Thus, the first order condition for K_{t+1} becomes:

$$P_{k',t} = \frac{1}{1 + r_{t+1}} \left[\alpha (K_{t+1})^{\alpha-1} (z_{t+1} h_{t+1})^{1-\alpha} + P_{k',t+1} (1 - \delta) \right]. \quad (3.3)$$

Finally, if there were a rental market for capital, with rental rate, R_t^k , that rental rate would be equal to the marginal product of capital. So, we obtain:

$$P_{k',t} = \frac{1}{1 + r_{t+1}} \left[R_{t+1}^k + P_{k',t+1} (1 - \delta) \right].$$

Note that by recursive substitution, this implies the standard formula for the price of capital:

$$\begin{aligned} P_{k',t} &= \frac{1}{1 + r_{t+1}} R_{t+1}^k + \frac{(1 - \delta)}{1 + r_{t+1}} P_{k',t+1} \\ &= \frac{1}{1 + r_{t+1}} R_{t+1}^k + \frac{(1 - \delta)}{1 + r_{t+1}} \left[\frac{1}{1 + r_{t+2}} R_{t+2}^k + \frac{(1 - \delta)}{1 + r_{t+2}} P_{k',t+2} \right] \\ &= \frac{1}{1 + r_{t+1}} R_{t+1}^k + \frac{(1 - \delta)}{(1 + r_{t+1})(1 + r_{t+2})} R_{t+2}^k + \frac{(1 - \delta)}{1 + r_{t+1}} \frac{(1 - \delta)}{1 + r_{t+2}} P_{k',t+2} \\ &= \dots \\ &= \sum_{i=1}^{\infty} \left(\prod_{j=1}^i \frac{1}{1 + r_{t+j}} \right) (1 - \delta)^{i-1} R_{t+i}^k. \end{aligned} \quad (3.4)$$

According to this, the price of capital is the present discounted value of its future earnings, where the discounting is done using the real rate of interest, and the fact that capital depreciates over time is taken into account. For our purposes, this formula is of limited use for thinking about $P_{k',t}$, because it has so many endogenous variables. However, since capital is reproducible in this model, there is first order condition corresponding to the construction of capital, and this first order condition (a variant of the ‘Tobin’s q’ condition), is useful for thinking about the price of capital.

The first order condition with respect to I_t is:

$$-\lambda_t + \mu_t(1 - S\left(\frac{I_t}{I_{t-1}}\right)) - \mu_t S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} + \beta \mu_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 = 0.$$

Rewriting this, taking into account the definition of the price of capital,

$$P_{K',t} = \frac{1}{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}} - \left(\frac{1}{1 + r_{t+1}}\right) \frac{P_{K',t+1} S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2}{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}}. \quad (3.5)$$

The right side of (3.5) is the marginal cost of an extra unit of capital. This marginal cost is the sum of two pieces. The first term is the usual marginal cost term that occurs in a static environment. It is the ratio of the consumption cost of a unit of investment goods, dC_t/dI_t (which is unity), divided by the marginal productivity (in producing new capital) of an extra investment good, dK_{t+1}/dI_t . To see that this is indeed marginal cost, note that this corresponds to

$$\frac{\frac{dC_t}{dI_t}}{\frac{dK_{t+1}}{dI_t}} = \frac{dC_t}{dK_{t+1}},$$

i.e., the consumption cost of capital. It is easy to verify, by differentiating (2.3) with respect to I_t , that the denominator in the first expression after the equality in (3.5) is indeed the marginal product of investment goods in producing K_{t+1} . If we just focus on this first term to the right of the equality, the puzzle about why $P_{K',t}$ drops during an overinvestment boom deepens. This is because, with the growth rate of investment high (see the above figure, which shows that I_t/I_{t-1} is high for several periods), the first term after the equality should unambiguously be high during the boom. Both S and S' rise, and this by itself makes $P_{K',t}$ rise.

Now it is time to look at the other term to the right of the equality in (3.5). This depresses the price of capital. Since $P_{K',t}$ falls, this must be the term that is dominating. What is it? When a technology shock is expected to rise in the future, then it is also expected that high investment will be desirable in the future. But, in this case, the value of investment today rises, because this helps reduce the adjustment costs associated with high future investment. The future terms shows the extra capital that can be produced from increased investment today, due to the reduction in future adjustment costs. This converted into future consumption units by $P_{K',t+1}$, and discounted to the present by $1 + r_{t+1}$. In a sense, although the cost of investment today is actually only one consumption good, the *net* cost, after counting the future benefits from current investment, is much smaller. Not only is it smaller because of the reduction in future adjustment costs, but it is also smaller because the real interest rate, $1 + r_{t+1}$, is lower. This can be seen from the above figure, which shows a sharp rise in λ_t during the boom.

The anticipated future technology shock in effect gives the planner who acquires investment goods today a ‘kickback’, in the form of a service in the future. This means the marginal cost an investment good is actually lower, making the marginal cost of capital lower. It is this reduction in the marginal cost of capital that gives the planner the incentive to increase capital and investment. The fall in $P_{K',t}$ is actually at the heart of the overinvestment boom. It is hard to see how one might contemplate an equilibrium with the property that $P_{K',t}$ rises in this type of environment.

As noted before, a modification of this environment pursued by Motto, Rostagno and me does produce a rise in $P_{K',t}$ during an overinvestment boom. A very natural monetary policy causes this to happen, by leading the central bank to increase the money supply in the wake of a signal that future technology will be high. This extra money adds additional vigor to the boom in investment, output, employment and consumption. The monetary policy is a Taylor rule in which the interest rate is increased vigorously when anticipated inflation rises.

4. Equilibrium Conditions

To complete the discussion of the equations that characterize equilibrium, we derive the first order condition with respect to labor. Differentiate (3.1) with respect to h_t to obtain:

$$\psi(C_t - bC_{t-1})^{1-\gamma} (1 - h_t)^{\psi(1-\gamma)-1} = \lambda_t (1 - \alpha) (K_t)^\alpha (z_t h_t)^{-\alpha}$$

This equation, together with (2.1), (2.2), (2.3), (3.2), (3.3), and (3.5), allow us to solve for the equilibrium. These seven equations, in effect, are sufficient to determine the evolution of the seven variables in Z_t , in (2.6). The steps in obtaining the linearized solution, A and B in (2.7), involve first computing the steady state values of the variables in Z_t . Then take a log-linear expansion of our six equations (the differentiation is done numerically in line 10 of getAB2.m). This gives rise to log-linearized system of Euler equations, and A and B are chosen to solve those (see lines 30 and 31 in getAB2.m).

References

- [1] Beaudry, Paul and Franck Portier, 2000, 'An Exploration into Pigou's Theory of Cycles,' manuscript, University of British Columbia.