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Assignment #7

Tutorial on Model Solution Using Dynare 4

The first set of two questions pertain to the RBC model ('example 3' in the handout on solution methods). The Dynare mod file, rbc.mod, and the steady state program, ssrbc.m, are in the zip file provided. As stated, the RBC model is inconsistent with the fact that consumption, investment and capital all grow over time. Question 3 shows one way to fix this counterfactual implication, by positing that the trend in the data reflects a stochastic drift in the growth rate of technology. The result is that the endogenous variables of the model follow a 'unit root' process, just like in the Clarida-Gali-Gerlter model of example 5.[1] The last question, question 4, pertains to the Clarida-Gali-Gertler model described in example 5 in the handout. The mod file for this model is also provided in the zip file. In addition to illustrating important technical points, these questions also illustrate some economic principles. You may want to look at the dynare4instructions.m file on the same website that contains this homework, for some notes on Dynare 4 syntax.

In the case of the RBC model, the following parameter values were used:

$$\alpha = 0.36,\ \beta = 0.97,\ \delta = 0.10,\ \psi = 2.5.$$

The magnitude of δ and β suggest this is an annual model (depreciation on capital is 10 percent per year, discount rate is 3 percent per year).

1. Consider the impulse response of the RBC model to a 1 percent innovation to technology, with $\rho = 0.5$. Preferences in this model are:

$$u(c,h) = \log(c) + \psi \log(1-h).$$

   (a) Note how the program displays the response of output as well as the inputs. Of course accuracy of the solution requires, among other things, that the relationship between output and inputs are consistent with the production function. Verify this.

[1] An alternative model of trend posits that the data are stationary about a deterministic trend. This case can be handled by a trivial perturbationon the unit root case.

(b) What is the percent response in the consumption to output ratio? To make this precise, let p_t denote the percent deviation of the consumption to output ratio from its steady state value:

$$p_t = \frac{\frac{c_t}{y_t} - \frac{c}{y}}{\frac{c}{y}}.$$

Then,

$$\begin{aligned} 1 + p_t &= \frac{\frac{c_t}{y_t}}{\frac{c}{y}} \\ p_t &\simeq \log(1 + p_t) = \log\left(\frac{c_t}{y_t}\right) - \log\left(\frac{c}{y}\right) \\ &= [\log(c_t) - \log(c)] - [\log(y_t) - \log(y)]. \end{aligned}$$

Thus, the percent response in the consumption to output ratio is just the difference between the impulse response of log consumption and log income.

(c) Note the national income identity in the model:

$$y_t = c_t + i_t,$$

so that

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{\imath}_t,$$

where

$$\hat{x}_t = \frac{dx_t}{x} = \frac{x_t - x}{x} \simeq \log\left(\frac{x_t}{x}\right),$$

where absence of the time subscript denotes steady state. The graph generated by rbc.mod indicates the impact effect of the shock on output and consumption, $\hat{y}_t$ and $\hat{c}_t$. The program also prints information about the model's steady state to the screen. Use this information to deduce the impact effect of the shock on $\hat{\imath}_t$.

(d) Modify the program so that it computes $\hat{\imath}_t$. Rerun the program and verify that the impact on investment of the shock is what you anticipated.

(e) Compute the share of the increase in output going to consumption and to investment in the period of the technology shock:

$$\frac{c_t - c}{y_t - y} = \frac{\hat{c}_t}{\hat{y}_t}\frac{c}{y}, \; \frac{i_t - i}{y_t - y} = \frac{\hat{\imath}_t}{\hat{y}_t}\frac{i}{y}. \tag{1}$$

(f) Redo (b)-(e) for each of $\rho = 0$ and 0.999. What is the economics underlying the different response sizes?

(g) Note that when $\rho = 0.999$, the technology shock is virtually a random walk. So, from a consumption smoothing perspective one might expect consumption to jump substantially in the period of the technology shock. In some respects, the rise in consumption is in fact surprisingly small in the $\rho = 0.999$ case. To explain this, add a new variable, the expected (as of period t) rate of return on capital:

$$R_t^k = \alpha \left(\frac{z_{t+1} h_{t+1}}{k_{t+1}} \right)^{1-\alpha} + 1 - \delta.$$

What is the value of this variable in steady state? What is its value in the period of a technology shock in the $\rho = 0.999$ case? Does this explain the surprisingly small jump in consumption?

2. Consider the following alternative utility function specification

$$u(c, h) = \frac{\left[c\,(1-h)^{\psi}\right]^{1-\sigma}}{1-\sigma}.$$

In the model that is coded up in rbc.mod, implicitly σ has been set to unity, because the utility function there is log-linear in consumption and leisure. Write a new mod and steady state file to cover this case.

(a) Make sure you reproduce the results you obtained when $\sigma = 1$.

(b) The 'speed of adjustment' is the amount of time it takes to close 95 percent of an initial gap between the initial stock of capital and the steady state stock of capital. It is a measure of the internal persistence in a model. It is something that is defined in the nonstochastic version of the model, and can be computed from the policy rule for capital that solves the model: $k_{t+1} - k^{ss} =$

$\lambda\left(k_t - k^{ss}\right)$. If, at date $t = 0$ the gap is $k_0 - k^{ss}$, then the model predicts that at date $t \geq 0$, the gap will be $k_t - k^{ss} = \lambda^t\left(k_0 - k^{ss}\right)$. The amount of time it takes to close 95 percent of the initial gap, $k_0 - k^{ss}$, is the value of t that solves:

$$\lambda^t = \frac{k_t - k^{ss}}{k_0 - k^{ss}} = 0.05,$$

or, $t = \log(.05) / \log(\lambda)$. You can find the value of λ in the policy rule printed by Dynare to the screen, or you can find it in the structure, oo_.dr.ghx.

i. Examine what happens to speed of adjustment when the value of σ is increased to 3. Explain the economics of your result.
ii. What happens to the speed of adjustment as the value of δ increases to 0.9. Explain the economics of your result.
iii. What happens to the speed of adjustment as the value of α increases to 0.6. Explain the economics of your result.

3. Now return to the rbc.mod Dynare file. Suppose that the resource constraint is as before:

$$C_t + K_t - (1-\delta) K_{t-1} \leq K_{t-1}^{\alpha} (z_t h_t)^{1-\alpha} = Y_t.$$

But now we suppose that the technology shock evolves according to the following unit root process:

$$\begin{aligned} z_t &= z_{t-1}\chi_t \\ \log \chi_t &= (1-\rho)\log\chi + \rho\log\chi_{t-1} + e_t. \end{aligned}$$

Rewrite the equilibrium conditions in terms of the scaled variables,

$$c_t \equiv \frac{C_t}{z_t},\ k_t = \frac{K_t}{z_t},\ y_t = \frac{Y_t}{z_t}.$$

Note that when you write the equilibrium conditions in terms of c_t, k_t and y_t, z_t disappears and only χ_t remains. The scaled system has a steady state and can be solved by the standard methods. Modify the mod file to cover this case. For convenience, you should set $\chi = 1$ (although a more sensible value would be 1.015, because this would imply an average per capita growth of output of 1.5 percent per year.)

(a) Set $\rho = 0$ and compare the contemporaneous response of hours worked, consumption and output in this specification with what the code you wrote for question 1 when $\rho = .9999$. Note that the two models are essentially the same in this case. This impression will be confirmed only if you take into account that output, consumption and investment in the model of this question are scaled, while these variables are not scaled in the previous question.

(b) Compare the response of hours worked for $\rho = 0$ and $\rho = 0.99$. Provide an economic explanation for the different results.

4. Now, consider the Clarida-Gali-Gertler model.

(a) How high do you need to drive α_π before the hours response to technology in the model corresponds to the efficient response? Explain the economics underlying this result.

(b) Introduce a working capital channel into the Clarida-Gali-Gertler model by introducing r_t with a coefficient of unity in the Phillips curve. Observe that Dynare cannot solve the model because of indeterminacy (in the language of the handout, there are more than one A matrix besides $A = 0$ that solve the relevant matrix polynomial.) Explain the economics underlying this result. Note that with a coefficient substantially less than unity there does exist a unique (local to steady state) equilibrium. Conclude that the working capital channel *may* turn the standard view that the Taylor principle rules out self-fulfilling inflation on its head. It will in fact do so only if the working capital channel is strong enough. In standard models with the working capital channel, this channel is in fact *not* strong enough (for example, the JPE paper by CEE has a working capital channel and yet there is a unique equilibrium in a neighborhood of steady state). It will be interesting to see whether this conclusion about the impact of the working capital channel survives as alternative models of financial frictions are explored in the next decade.

(c) Modify the CGG model by supposing that technology instead evolves as an AR(1) in its log level:

$$a_t = \rho a_{t-1} + \varepsilon_t.$$

i. Show that now, the reduced form representation for the natural rate of interest is:

$$rr_t^* = rr - (1-\rho)\, a_t + \frac{1}{1+\varphi}(1-\lambda)\,\tau_t.$$

Explain the intuition for the difference in the sign of the coefficient on a_t.

ii. Compute the impulse responses in rr_t^*, r_t, a_t, π_t, h_t and y_t to ε_t. How do they compare to when the technology shock has the time series representation, $\Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t$?

(d) Modify the Clarida-Gali-Gertler model by supposing that agents receive an advance signal on the log-level of the technology shock. In particular, suppose that technology has the following representation:

$$a_t = \rho a_{t-1} + \varepsilon_t + \xi_{t-1},$$

where ε_t and ξ_{t-1} are iid and uncorrelated with each other[2], and the time subscript indicates when the variable is observed. With this model, agents get 'advanced observation' on a_{t+1} at time t, although the final realized value of that variable also depends on the realization of ε_{t+1}. The reduced form representation for the natural rate of interest is now:

$$rr_t^* = rr - (1-\rho)\, a_t + \xi_t + \frac{1}{1+\varphi}(1-\lambda)\,\tau_t,$$

Note how a positive realization of the signal about a_{t+1}, ξ_t, immediately raises the natural rate of interest.

i. What is the intuition for the latter result?

ii. Compute the impulse responses to the signal shock, ξ_t. The current realization of the signal drives up consumption, employment and hours worked. What is the reason for this? Despite the rise in all these measures of demand in the first period, the first period inflation rate *falls*. What is the reason for this?

[2] Note that $\varepsilon_t + \xi_{t-1}$ is iid so that an econometrician who just observes a time series on a_t would infer that a_t is a realization of the same $AR(1)$ model that we used before we introduced the signal.