Using VARs to Estimate a DSGE Model

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Objectives

• Describe and motivate key features of standard monetary DSGE models.

• Estimate a DSGE model using VAR impulse responses reported in Eichenbaum’s lecture.

• Describe extensions of the model:
  – Small open economy (very rough sketch only, Rebelo will discuss more carefully)

  – Labor market search and matching

  – Financial frictions
• Very brief review of Marty Eichenbaum’s discussion of SVARs.
Identifying Monetary Policy Shocks

- Rule that relates Fed’s actions to state of the economy.

\[ R_t = f(\Omega_t) + e_t^R \]

- \( f \) is a linear function

- \( \Omega_t \): set of variables that Fed looks at.

- \( e_t^R \): time t policy shock, orthogonal to \( \Omega_t \)
Response to a monetary policy shock
Interesting Properties of Monetary Policy Shocks

• Plenty of endogenous persistence:
  – money growth and interest rate over in 1 year, but other variables keep going….

• Inflation slow to get off the ground: peaks in roughly two years
  – It has been conjectured that explaining this is a major challenge for economics
  – Kills models in which movements in $P$ are key to monetary transmission mechanism (Lucas misperception model, pure sticky wage model)
  – Has been at the heart of the recent emphasis on sticky prices.

• Output, consumption, investment, hours worked and capacity utilization hump-shaped

• Velocity comoves with the interest rate
Identification of Technology Shocks

- Two technology shocks:
  - One perturbs price of investment goods
  - One perturbs total factor productivity

- They are the only two shocks that affect labor productivity in the long run

- Only the shock to investment good prices have an impact on investment good prices in the long run.
Response to a neutral technology shock

Output

M2M Growth (Q)

Inflation

Federal Funds Rate

Capacity Utilization

Average Hours

Real Wage

Consumption

Investment

Velocity

Investment Good Price

Quarters

Quarters
Observations on Neutral Shock

• Generally, results are ‘noisy’, as one expects.
  – Interest, money growth, velocity responses not pinned down.

• Interestingly, inflation response is immediate and precisely estimated.

• Does this raise a question about the conventional interpretation of the response of inflation to a monetary shock?

• Alternative possibility: information confusion stories.
  – A variant of recent work by Rhys Mendes that builds on Guido Lorenzoni’s work.
Importance of Three Shocks

• According to VAR analysis, they account for a large part of economic fluctuations.
Historical decomposition of US GDP

Technology shocks specific to capital goods

Dark line: detrended actual GDP

Thin line: what GDP would have been if there had only been one type of technology shock, the type that affects only the capital goods industry

These shocks have some effect, but not terribly important
Type of technology shock that affects all industries

This has very large impact on broad trends in the data, and a smaller impact on business cycles.

Has big impact on trend in data, and 2000 boom-bust
Monetary policy shocks have a big impact on the 1980 'Volcker recession'.

**Historical decomposition of US GDP**

- Technology shocks specific to capital goods
- General (neutral) technology shocks only
- Monetary Policy Shocks Only

Monetary policy shocks have a big impact on the 1980 'Volcker recession'.
Historical decomposition of US GDP

All three shocks together account for large part of business cycle
## Variance Decomposition

<table>
<thead>
<tr>
<th>Variable</th>
<th>BP(8,32)</th>
</tr>
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<tbody>
<tr>
<td>Output</td>
<td>86</td>
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<td></td>
<td>[18]</td>
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<td>Money Growth</td>
<td>23</td>
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<td></td>
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<td>Inflation</td>
<td>33</td>
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<td></td>
<td>[17]</td>
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<td>Fed Funds</td>
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<td>[16]</td>
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<td>Capacity Util.</td>
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<tr>
<td>Avg. Hours</td>
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<td>[17]</td>
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<td>Real Wage</td>
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<td>[16]</td>
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<td>Consumption</td>
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<td>[21]</td>
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<td>Investment</td>
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<td>Velocity</td>
<td>29</td>
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<tr>
<td></td>
<td>[16]</td>
</tr>
<tr>
<td>Price of investment goods</td>
<td>11</td>
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<tr>
<td></td>
<td>[16]</td>
</tr>
</tbody>
</table>
Next

- Use Impulse Responses to Estimate a DSGE Model
  - Motivate the Basic Model Features.
  - Model Estimation.

- Determine if there is a conflict regarding price behavior between micro and macro data.
  - Macro Evidence:
    - Inflation responds slowly to monetary shock
    - Single equation estimates of slope of Phillips curve produce small slope coefficients.
  - Micro Evidence:
    - Bils-Klenow, Nakamura-Steinsson report evidence on frequency of price change at micro level: 5-11 months.
Single equation estimates of slope of Phillips curve

- Phillips curve: \[ \pi_t = \beta E_t \pi_{t+1} + \gamma s_t \]

\[ \gamma = \frac{(1 - \xi_p)(1 - \beta \xi p)}{\xi p} \]

- Rewrite:

\[ \pi_t - \beta \pi_{t+1} = \gamma s_t - \beta \eta_{t+1} \]

\[ \eta_{t+1} = \pi_{t+1} - E_t \pi_{t+1} \perp \text{date } t \text{ variables} \]

- Regression:

\[ \hat{\gamma} = \frac{\text{cov}(\pi_t - \beta \pi_{t+1}, \gamma s_t)}{\text{var}(s_t)} \]

\[ = \frac{\text{cov}(\gamma s_t - \beta \eta_{t+1}, \gamma s_t)}{\text{var}(s_t)} \]

\[ = \frac{\text{cov}(\gamma s_t, \gamma s_t)}{\text{var}(s_t)} = \gamma. \]
• Procedures like this tend to imply stickiness in prices (Gali-Gertler, Eichenbaum-Fisher):
\[
\frac{1}{1-\xi_p} \approx 6
\]

• At the same time, DSGE literature finds (see Smets-Wouters, Primiceri, others) highly serially correlated shock in Phillips curve:

\[
\pi_t = \beta E_t \pi_{t+1} + \gamma s_t + u_t,
\]

• Then,

\[
\hat{\gamma} = \frac{\text{cov} (\pi_t - \beta \pi_{t+1}, \gamma s_t)}{\text{var}(s_t)} = \frac{\text{cov} (\gamma s_t + u_t, \gamma s_t)}{\text{var}(s_t)} = \gamma \left[ 1 + \frac{\text{cov} (u_t, s_t)}{\text{var}(s_t)} \right] \overset{?}{<} \gamma
\]
• Could apply instrumental variables/GLS methods to estimate slope of Phillips curve, but these tend to produce noisy results.

• Alternative: impulse-response approach will in principle allow us to estimate slope of Phillips curve without making any detailed assumptions on the Phillips curve shock.

• If the slope of the Phillips curve is small, could in principle reconcile with micro evidence on frequency of price adjustment (Kimball aggregator, firm-specific capital). However, these approaches entail other questionable empirical implications.
Outline

• Model (Describe extensions that are subject of current research)

• Econometric Estimation of Model
  – Fitting Model to Impulse Response Functions

• Model Estimation Results

• Implications for Micro Data on Prices

• Evaluate the Reliability of VAR Analysis
Description of Model

• Timing Assumptions

• Firms

• Households

• Monetary Authority

• Goods Market Clearing and Equilibrium
Timing

- Technology Shocks Realized.
- Monetary Policy Shock Realized.
- Household Money Demand Decision Made.
- Production, Employment, Purchases Occur, and Markets Clear.
- Note: Wages, Prices and Output Predetermined Relative to Policy Shock.
Firm Sector

Final Good, Competitive Firms

Intermediate Good Producer 1

Intermediate Good Producer 2

Intermediate Good Producer infinity

Competitive Market for Homogeneous Labor Input

Household 1

Household 2

Household infinity

Competitive Market For Homogeneous Capital
Extension to open economy (Christiano, Trabandt, Walentin (2008))

- Domestic homogeneous good
  - Final consumption goods
  - Final investment goods
  - Final export goods
  - Imported consumption goods
  - Imported investment goods
  - Imported goods for re-export
Firms

Final Good Firms

• Technology:

\[ Y_t = \left[ \int_0^1 Y_{it}^{\frac{1}{\lambda_f}} \, di \right]^{\lambda_f}, \quad 1 \leq \lambda_f < \infty \]

• Objective:

\[ \max P_t Y_t - \int_0^1 P_{it} Y_{it} \, di \]

• Firms and Prices:

\[ \left( \frac{P_t}{P_{it}} \right)^{\frac{1}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}, \quad P_t = \left[ \int_0^1 P_{it}^{\frac{i}{1 - \lambda_f}} \, di \right]^{(1 - \lambda_f)} \]
Intermediate Good Firms -

- Each $Y_{it}$ Produced by a Monopolist, With Demand Curve:

$$\left( \frac{P_t}{P_{it}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}.$$  

- Technology:

$$Y_{it} = K_{it}^{\alpha} \left( z_t L_{it}^{1-\alpha} \right), \ 0 < \alpha < 1,$$

- Here, $z_t$ is a technology shock:

$$\mu_{z,t} = \log z_t - \log z_{t-1}, \ \hat{\mu}_{z,t} = \rho_{\mu_z} \hat{\mu}_{z,t-1} + \varepsilon_{\mu_z,t}$$
- Calvo Price Setting:
  
  - With Probability $1 - \xi_p$, $i^{th}$ Firm Sets Price, $P_{it}$, Optimally, to $\tilde{P}_t$.
  
  - With Probability $\xi_p$,
    
    $$P_{it} = \pi_{t-1} P_{i,t-1}, \quad \pi_t = \frac{P_t}{P_{t-1}}.$$  
  
  - Standard Approach in Literature:
    
    $$P_{it} = \bar{\pi} P_{i,t-1}, \text{ or } P_{it} = P_{i,t-1}.$$  
  
  - Stand on Indexing Matters
    
    Determines Extent of ‘Front-Loading’
Evidence from Midrigan, ‘Menu Costs, Multi-Product Firms, and Aggregate Fluctuations’

Figure 1: Distribution of price changes conditional on adjustment

Histograms of $\log(P_t/P_{t-1})$, conditional on price adjustment, for two data sets pooled across all goods/stores/months in sample.
• Combining Optimal Price and Aggregate Price Relation:

\[ \Delta \hat{\pi}_t = \beta E_t \Delta \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} E_t \hat{s}_t, \]

• Under Standard Price-Updating Scheme:

\[ P_{it} = \bar{\pi} P_{i,t-1}. \]

Associated Reduced Form:

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t \mid 1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} E_t \hat{s}_t. \]
Households: Sequence of Events

- Technology shock realized.

- Decisions: Consumption, Capital accumulation, Capital Utilization.

- Insurance markets on wage-setting open.

- Wage rate set.

- Monetary policy shock realized.

- Household allocates beginning of period cash between deposits at financial intermediary and cash to be used in consumption transactions.
Households...

- Monopoly supplier of differentiated labor
  - Sets wage subject to Calvo style frictions like firms
- Preferences of $j^{th}$ household

\[ E_t^j \sum_{l=0}^{\infty} \beta^{l-1} \left[ \log (C_{t+l} - bC_{t+l-1}) - \psi_L \frac{h_{j,t}^2}{2} \right] \]

- $E_t^j$: expectation operator, conditional on aggregate and household $j$ idiosyncratic information.
- $C_t$: consumption
- $h_{j,t}$: hours worked.
Dynamic Response of Consumption to Monetary Policy Shock

- In Estimated Impulse Responses:
  - Real Interest Rate Falls
    \[ R_t / \pi_{t+1} \]
  - Consumption Rises in Hump-Shape Pattern:
Consumption ‘Puzzle’

• Intertemporal First Order Condition:

\[ \frac{c_{t+1}}{\beta c_t} = \frac{MU_{c,t}}{\beta MU_{c,t+1}} \approx \frac{R_t}{\pi_{t+1}} \]

• With Standard Preferences:

\[ c_t \rightarrow \frac{c_{t+1}}{\beta c_t} = \frac{MU_{c,t}}{\beta MU_{c,t+1}} \approx \frac{R_t}{\pi_{t+1}} \]
One Resolution to Consumption Puzzle

• Concave Consumption Response Displays:
  – Rising Consumption (problem)
  – Falling Slope of Consumption

• Habit Persistence in Consumption

\[ U(c) = \log(c - b \times c_{-1}) \]

  – Marginal Utility Function of Slope of Consumption
  – Hump-Shape Consumption Response Not a Puzzle

• Econometric Estimation Strategy Given the Option, \( b > 0 \)
Households...

- Asset Evolution Equation:

\[ M_{t+1} = R_t \left[ M_t - Q_t + (x_t - 1)M_t^g \right] + A_{j,t} + Q_t + W_{j,t}h_{j,t} + P_t r^K_t u_t \tilde{K}_t + D_t - P_t \left[ (1 + \eta(V_t)) C_t + \gamma_t^{-1} (I_t + a(u_t)\bar{K}_t) \right] \]

- \( M_t \): Beginning of Period Base Money; \( Q_t \): Transactions Balances
- \( x_t \): Growth Rate of Base; \( u_t \): Utilization Rate of Capital
  * \( u_t = 1 \) in steady state, \( a(1) = 0, a'(1) > 0, \sigma_a = a''(1)/a'(1) \).
- \( \gamma_t^{-1} \): (Real) Price of investment goods, \( \mu_{\gamma,t} = \gamma_t/\gamma_{t-1} \),

\[ \hat{\mu}_{\gamma,t} = \rho_{\mu_{\gamma}} \hat{\mu}_{\gamma,t-1} + \varepsilon_{\mu_{\gamma},t} \]

- Velocity:

\[ V_t = \frac{P_tC_t}{Q_t}, \]
Money Demand

- Asset Evolution Equation:

\[ M_{t+1} = R_t [M_t - Q_t + (x_t - 1)M_t^a] + A_{j,t} + Q_t + W_{j,t}h_{j,t} + P_t r_t^{k} u_t \bar{K}_t + D_t - P_t \left[ (1 + \eta (V_t)) C_t + \gamma_t^{-1} (I_t + a(u_t)\bar{K}_t) \right] \]

- Increase in \( Q_t \):
  - Marginal Cost of Interest Foregone: \( R_t \)
  - Marginal Benefit:

\[ 1 - P_t \eta' (V_t) C_t \frac{dV_t}{dQ_t} \]

\[ = 1 + \eta' \left( \frac{P_t C_t}{Q_t} \right) \left( \frac{P_t C_t}{Q_t} \right)^2 \]

additional cash available at end of period

redemption in transactions costs due to extra cash
Money Demand ...

- Money Demand: Equate Marginal Benefits and Costs of $Q_t$ —

$$R_t = 1 + \eta' \left( \frac{P_t C_t}{Q_t} \right) \left( \frac{P_t C_t}{Q_t} \right)^2.$$

- Properties of Money Demand:
  - Unit Consumption Elasticity of Money Demand
    * Increase $C_t$ 1 percent and Hold $R_t$, $P_t$ Fixed $\Rightarrow$ Desired $Q_t$ increases 1 percent
  - $R_t \uparrow$ Implies $Q_t \downarrow$
    * To Induce Households to Hold Additional $Q$, Must Have Lower $R$
    * Money Demand Elasticity is Bigger, the Bigger is $\eta''$
Money Demand ...

- Quantitative Analysis of Money Demand
  - Consider the Following Parametric Function for $\eta$

$$\eta = AV_i + \frac{B}{V_i} - 2\sqrt{AB}$$

$$\Rightarrow$$

$$R = 1 + \eta'(V) \times V^2 = 1 + [A - BV^{-2}] V^2 = 1 - B + AV^2$$

- Data:
  * Money - St. Louis Fed’s MZM, 1974-2004
  * Consumption - NIPA Consumption of Services and Nondurables
  * Interest Rate - One Year T-Bills.
  * OLS Regression of $V^2$ on $R \Rightarrow A = 0.0174$ and $B = 0.0187$
Money Demand ...

- Top Graph: Velocity of Money
- Bottom Graph: Actual and Predicted Interest Rate

- Findings: Static Money Demand Equation Fits the Data Well!
Dynamic Response of Investment to Monetary Policy Shock

- In Estimated Impulse Responses:
  - Investment Rises in Hump-Shaped Pattern:
Investment ‘Puzzle’

• Rate of Return on Capital

\[ R_t^k = \frac{MP_{t+1}^k + P_{k',t+1}(1 - \delta)}{P_{k',t}}, \]

\[ P_{k',t} \sim \text{consumption price of installed capital} \]

\[ MP_t^k \sim \text{marginal product of capital} \]

\[ \delta \in (0, 1) \sim \text{depreciation rate.} \]

• Rough ‘Arbitrage’ Condition:

\[ \frac{R_t}{\pi_{t+1}} \approx R_t^k. \]

• Positive Money Shock Drives Real Rate:

\[ R_t^k \downarrow \]

• Problem: Burst of Investment!
One Solution to Investment Puzzle

• Adjustment Costs in Investment
  – Standard Model (Lucas-Prescott)
    \[ k' = (1 - \delta)k + F\left(\frac{I}{k}\right)I. \]
  – Problem:
    • Hump-Shape Response Creates Anticipated Capital Gains
      \[ \frac{P_{k',t+1}}{P_{k',t}} > 1 \]

- Optimal Under Standard Specification
- Data!
One Solution to Investment Puzzle…

• Cost-of-Change Adjustment Costs:

\[ k' = (1 - \delta)k + F \left( \frac{I}{I_{-1}} \right)I \]

• This Does Produce a Hump-Shape Investment Response
  – Other Evidence Favors This Specification
  – Theoretical: Matsuyama, David Lucca
Wage Decisions

• Each household is a monopoly supplier of a specialized, differentiated labor service.
  – Sets wages subject to Calvo frictions.
  – Given specified wage, household must supply whatever quantity of labor is demanded.

• Household differentiated labor service is aggregated into homogeneous labor by a competitive labor ‘contractor’.

\[ l_t = \left[ \int_0^1 (h_{t,j}) \frac{1}{\lambda_w} \, dj \right]^{\lambda_w}, \quad 1 \leq \lambda_w < \infty. \]
Nominal wage, $W$

Firms use a lot of Labor because it’s ‘cheap’.

Households must supply that labor.

Labor demand

Labor supply

Shock

Quantity of labor
'Barro critique'

• Most worker-firm relationships are long-term, and unlikely to be strongly affected by details of the timing of wage-setting.

• Standard sticky wage model implausible.

• Recent results in search-matching literature:
  – Must distinguish between intensive (hours) and extensive (employment) margin.
  – Barro critique applies to idea that wage frictions matter in the intensive margin.
  – Does not apply to idea that wage frictions matter for extensive margin.
Modification of labor market

• Mortensen-Pissarides search and matching frictions recently introduced into DSGE models (Gertler-Sala-Trigari, Blanchard-Gali, Christiano-Illut-Motto-Rostagno)

• Draw a distinction between hours (‘intensive margin’) and number of workers (‘extensive margin’)

• Intensive and extensive margins exhibit very different dynamics over business cycle

• Wage frictions thought to matter for extensive margin, not intensive margin.

• Extension to open economy (Christiano, Trabandt, Walentin)
Firms

Employment

Homogeneous Labor

unemployment

Employment Agency

Employment Agency

Employment Agency
Each period, employment agencies post vacancies to attract workers.
Efficient determination of hours worked in employment agency marginal benefit of one hour to agency = marginal cost to worker of one hour
Taylor wage contracting
Employment agencies equally divided between N cohorts. Each period one cohort negotiates an N-period wage with its workers.
Monetary and Fiscal Policy

\[ x_t = \frac{M_t}{M_{t-1}} \]

\[ \hat{x}_{M,t} = \rho_M \hat{x}_{M,t-1} + \varepsilon_{M,t} \]
\[ \hat{x}_{z,t} = \rho_{xz} \hat{x}_{z,t-1} + c_z \varepsilon_{z,t} + c_{z}^{p}\varepsilon_{z,t-1} \]
\[ \hat{x}_{\gamma,t} = \rho_{x\gamma} \hat{x}_{\gamma,t-1} + c_{\gamma} \varepsilon_{\gamma,t} + c_{\gamma}^{p}\varepsilon_{\gamma,t-1} \]

- \( \hat{x}_{M,t} \): response of monetary policy to a monetary policy shock, \( \varepsilon_{M,t}/ \)
- \( \hat{x}_{z,t} \): response of monetary policy to an innovation in neutral technology, \( \varepsilon_{z,t}. \)
- \( \hat{x}_{\gamma,t} \): response of monetary policy to an innovation in capital embodied technology, \( \varepsilon_{\gamma,t}. \)
- Government has access to lump sum taxes, pursues a Ricardian fiscal policy.
Loan Market and Final Good Market Clearing Conditions, Equilibrium

- Financial intermediaries receive $M_t - Q_t + (x_t - 1) M_t$ from the household.
  - Lend all of their money to intermediate good firms, which use the funds to pay for $H_t$.
- Loan market clearing
  \[ W_t H_t = x_t M_t - Q_t. \]
- The aggregate resource constraint is
  \[ (1 + \eta(V_t))C_t + \gamma_t^{-1} \left[ I_t + a(u_t) K_t \right] \leq Y_t. \]
- We adopt a standard sequence-of-markets equilibrium concept.
Econometric Methodology

• Variant of limited information strategy used in CEE (2004).
  – Impose a subset of assumptions made in equilibrium model to estimate impulse response functions of ten key macroeconomic variables to the three shocks in our model.
  – Neutral technology shocks, capital embodied technology shocks and monetary policy shocks.

• Choose values for key parameters of structural model to minimize difference between estimated impulse response functions and analogous objects in model.
Estimating Parameters in the Model

- Partition Parameters into Three Groups.
  - Parameters set a priori (e.g., $\beta$, $\delta$, ...)
  - $\zeta_1$: remaining parameters pertaining to the nonstochastic part of model

$$\zeta_2 = [\xi_w, \gamma, \sigma_a, b, S'', \epsilon]$$

- $\zeta_2$: parameters pertaining to stochastic part of the model

- Number of parameters, $\zeta = (\zeta_1, \zeta_2)$, to be estimated - 18

- Estimation Criterion
  - $\Psi(\zeta)$: mapping from $\zeta$ to model impulse responses
  - $\hat{\Psi}$: 592 impulse responses estimated using VAR
  - Estimation Strategy:

$$\hat{\zeta} = \arg \min_\zeta \left( \hat{\Psi} - \Psi(\zeta) \right)' V^{-1} \left( \hat{\Psi} - \Psi(\zeta) \right).$$

- $V$: diagonal matrix with sample variances of $\hat{\Psi}$ along the diagonal.
Parameter estimates

### TABLE 2: ESTIMATED PARAMETER VALUES $\zeta_1$

<table>
<thead>
<tr>
<th>Model</th>
<th>$\lambda_f$</th>
<th>$\xi_w$</th>
<th>$\gamma$</th>
<th>$\sigma_a$</th>
<th>$b$</th>
<th>$S''$</th>
<th>$\epsilon$</th>
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<tr>
<td>Benchmark</td>
<td>1.35</td>
<td>0.75</td>
<td>0.32</td>
<td>0.06</td>
<td>0.80</td>
<td>4.85</td>
<td>0.77</td>
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<tr>
<td></td>
<td>(0.17)</td>
<td>(0.06)</td>
<td>(0.32)</td>
<td>(0.18)</td>
<td>(0.04)</td>
<td>(2.15)</td>
<td>(0.27)</td>
</tr>
</tbody>
</table>

Parameters are surprisingly consistent with estimates reported in JPE (2005) based on studying only monetary policy shocks.

Note slope of Phillips curve is fairly large, but standard error is large too!

- At point estimates: $\xi_p = 0.58$, $\frac{1}{1 - \xi_p} = 2.38$ quarters

Other parameters ‘reasonable’: estimation results really want sticky wages!
• Parameters of exogenous shocks:

| TABLE 3: ESTIMATED PARAMETER VALUES $\zeta_2$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\rho_M$       | $\sigma_M$     | $\rho_{\mu_z}$ | $\sigma_{\mu_z}$ | $\rho_{xz}$     | $c_z$       | $c^p_z$       | $\rho_{\mu_Y}$ | $\sigma_{\mu_Y}$ | $\rho_{xY}$     | $c_Y$       | $c^p_Y$       |
| Benchmark Model |                 |                 |                 |                 |             |               |                 |                 |                 |             |               |
| $-0.10$        | $0.31$          | $0.91$          | $0.05$          | $0.36$          | $3.68$      | $2.49$       | $0.24$         | $0.17$          | $0.91$          | $-0.10$      | $0.63$        |
| $(0.12)$       | $(0.10)$        | $(0.03)$        | $(0.02)$        | $(0.22)$        | $(1.55)$    | $(1.22)$     | $(0.52)$       | $(0.06)$        | $(0.07)$        | $(0.57)$      | $(0.65)$      |

• Neutral technology shock, $\rho_{\mu_z}$, is highly persistent.
Figure 1: Response to a monetary policy shock (○ - Model, - VAR, grey area - 95% Confidence Interval)
Figure 2: Response to a neutral technology shock (o - Model, - VAR, grey area - 95% Confidence Interval)
Figure 3: Response to an embodied technology shock (o - Model, - VAR, grey area - 95 % Confidence Interval)
Monetary Policy and Technology Shocks

• Policy Issue:

  – How would the economy have responded to technology shocks if monetary policy had not been accommodative?
Benchmark model (•) and alternative model (○) - dynamic response to a neutral technology shock

- Output
- MZM Growth
- Inflation
- Fed Funds
- Capacity Util
- Avg Hours
- Real Wage
- Consumption
- Investment
- Velocity
- p_t
- Total money growth
Benchmark model (•) and alternative model (○) - dynamic response to an embodied technology shock
The Experiment

- Begin at steady state and assume there is an expansionary monetary policy shock in period 1.
- Period 1
  - Prices and output is the same for all firms.
- Period 2
  - \((1 - \xi_p)^2\) firms re-optimize and implement new price, \(\xi_p\) do not.
- Period 3: there are 4 types of firms.
  - \((1 - \xi_p)^2\) re-optimize in period 2 and 3,
  - \(\xi_p^2\) don’t re-optimize in either period 2 or period 3.
  - \((1 - \xi_p)\xi_p\) re-optimized in period 2 but not in period 3.
  - \(\xi_p(1 - \xi_p)\) did not re-optimize in period 2 but did re-optimize in period 3
- In period \(s\) there are \(2^{s-1}\) different firms.
- For each period \(s\) we calculated the distribution of output and prices across firms.
Implications of the Estimated Model for the Distribution of Production Across Firms

Features of the Distribution of Output and Prices Across Firms 12 quarters after a monetary shock

Output per firm category in Period 12

Distribution of firms in Period 12

Average relative price in Period 12
Extension to Incorporate Financial Frictions

• General idea:
  – Standard model assumes borrowers and lenders are the same people...no conflict of interest
  – Financial friction models suppose borrowers and lenders are different people, with conflicting interests
  – Financial frictions: features of the relationship between borrowers and lenders adopted to mitigate conflict of interest.
Standard Model

Households

Backyard capital accumulation: \( K_{t+1} = (1 - \delta)K_t + G(K_t, I_t) \).

Savers and investors are the same: NO FRICTIONS!

\[
 u_{c,t} = \beta E_t u_{c,t+1} \frac{R^k_{t+1}}{\Pi_{t+1}} \quad R^k_{t+1} = \frac{r^k_{t+1} + (1-\delta)P^k_{k',t+1}}{P^k_{k',t}}
\]
Townsend, Gale-Hellwig, Bernanke-Gertler-Gilchrist Model

- Those who supply funds and those who put funds to work are different people. They work through banks.

- When funds are put to work, idiosyncratic things happen that are known only to the borrower. Lender can see the shock, but only at a cost.

- Savers and borrowers can’t just share the output, because borrowers have an incentive to misreport earnings.

- **Standard debt contract** works well in this setting: (i) borrowers pay a fixed interest rate if they can and (ii) those who can’t declare ‘bankruptcy’ and give everything to the bank after being monitored.

- Shocks that affect the distribution of wealth between savers and investors have an aggregate impact because investors have special abilities.
Financial Friction Model

- **Firms**
  - Produce capital

- **Households**
  - Consumption goods
  - Investment goods

- **Banks**
  - Deposits with fixed nominal return
  - Loans to entrepreneurs

- **Entrepreneurs**
  - Own and manage capital

- **Capital Rental**
  - From firms to households

- **Investment Goods**
  - From firms to households
Entrepreneur of Type $\omega$, Where $E_\omega = 1$.

Households Lend Funds to Banks

1. Entrepreneurs receive standard debt contract
2. If productivity high enough, pay fixed rate
3. If bankrupt, turn over all assets and monitoring occurs
Modification to standard model, to introduce financial frictions

• Household intertemporal equation for capital replaced by three equations:
  
  – Zero profit condition for banks (competition in lending)
  – Law of motion for entrepreneurial net worth
  – Efficiency condition on entrepreneurial debt contracts.
• Key properties of the lending contract:
  – Interest paid by entrepreneurs fixed in nominal terms (Christiano-Motto-Rostagno)
  – Entrepreneur with more real net worth can borrow more.

• Law of motion of net worth

\[
\text{real net worth}_t = \text{real earnings on capital (rent plus capital gains)}_t - \frac{\text{nominal interest rate}_t}{\pi_t} \text{ real debt to banks}_t
\]
Prediction of financial friction model:

• Shocks that drive output and price in the same direction (‘demand’) accelerated by financial frictions.
  – Fisher and earnings effects reinforce each other.

• Shocks that drive output and price in opposite directions (‘supply’) not much affected by financial frictions.
  – Fisher and earnings effects cancel each other.
Big drop in investment and net worth

Response of system with and without financial frictions to 50 basis point monetary policy shock.
Note: the software for computing these charts may be found at http://faculty.wcas.northwestern.edu/~lchrist/course/financial.htm
• Prediction of financial friction model appears to be consistent with empirical evidence.

• Chari-Christiano-Kehoe (2008) show:
  
  – Financially constrained firms seem to be more affected by monetary shock than unconstrained (Gertler-Gilchrist)

  – Financially constrained and unconstrained firms roughly equally affected over the business cycle.
• Delivers new variables such as credit, risk spread

• Can ask interesting questions:
  – when risk in the economy increases, how should monetary policy react.
  – What role should data on credit and on the stock market (the price of capital) play in monetary policy?
Summary

• We constructed a dynamic GE model of cyclical fluctuations.

• Given assumptions satisfied by our model, we identified dynamic response of key US economic aggregates to 3 shocks
  – Monetary Policy Shocks
  – Neutral Technology Shocks
  – Capital Embodied Technology Shocks

• These shocks account for substantial cyclical variation in output.

• Estimated GE model does a good job of accounting for response functions (However, Misses on Inflation Response to Neutral Shock)

• Our point estimates suggest slope of Phillips curve steep, so there is no micro-macro price puzzle. However, large standard error.

• Described extensions of the model.
Summary…

- Calvo Sticky Prices and Wages Seems Like Good Reduced Form
  - What is the Underlying Structure?
  - Is it information frictions?