Brief Review of VARs

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Vector Autoregressions

- Proposed by Chris Sims in 1970s, 1980s
- Major subsequent contributions by others (Bernanke, Blanchard-Watson, Blanchard-Quah)
- Useful Way to Organize Data
  - VARs serve as a ‘Battleground’ between alternative economic theories
  - VARs can be used to quantitatively construct a particular model
- Question that can (in principle) be addressed by VAR:
  - ‘How does the economy respond to a particular shock?’
  - Answer can be very useful:
    - for discriminating between models
    - For estimating the parameters of a given model
- VARs can’t *actually* address such a question
  - Identification problem
  - Need extra assumptions….Structural VAR (SVAR).

Outline of SVAR discussion

- What is a VAR?
- The Identification Problem
- Long run restrictions as a way to solve the problem
- Short Run Restrictions: Identification of Monetary Policy Shocks
- Results
- Historical Decompositions of Data

Estimating the Effects of Shocks to the Economy

- Vector Autoregression for a $N \times 1$ vector of observed variables:

$$
\begin{aligned}
Y_t &= B_1 Y_{t-1} + \ldots + B_p Y_{t-p} + u_t, \\
E u_t u_t' &= V
\end{aligned}
$$

- $B\prime s$, $u's$ and V are Easily Obtained by OLS.
- Problem: $u's$ are statistical innovations.
  - We want impulse response functions to fundamental economic shocks, e_t.

$$
\begin{aligned}
u_t &= C e_t, \\
E e_t e_t' &= I, \\
CC' &= V
\end{aligned}
$$

Estimating the Effects of a Shock to the Economy ...

VAR: $Y_t = B_1 Y_{t-1} + ... + B_p Y_{t-p} + Ce_t$

- Impulse Response to i^{th} Shock:

$$Y_t - E_{t-1}Y_t = C_i e_{it},$$

$$E_t Y_{t+1} - E_{t-1}Y_{t+1} = B_1 C_i e_{it}$$

$$\cdots$$

- To Compute Dynamic Response of Y_t to i^{th} Element of e_t We Need

$B_1, ..., B_p$ and C_i.

Identification Problem

$$Y_t = B_1 Y_{t-1} + ... + B_p Y_{t-p} + u_t$$

$$u_t = Ce_t, \; Eu_t u_t' = CC' = V$$

- We know $B's$ and V, we need C.
- Problem
  - N^2 Unknown Elements in C,
  - Only $N(N+1)/2$ Equations in

$$CC' = V$$

- Identification Problem: Not Enough Restrictions to Pin Down C
- Need More Identifying Restrictions!

Bivariate Blanchard and Quah Example

- Identification Assumption:

  Technology Shock is *Only* Shock that Has Long-Run Impact on (Forecast of) Level of Labor Productivity:

$$\text{(exclusion restriction)} \lim_{j\to\infty}\left[E_t y_{t+j} - E_{t-1} y_{t+j}\right] = f(\text{technology shock only})$$

$$\text{(sign restriction) } f' > 0$$

$$y_t = \frac{\text{output}}{\text{hour}}$$

- Blanchard-Quah/Jordi Gali:

  This Assumption Makes it Possible to Estimate Technology Shock, Even Without Direct Observations on Technology

Bivariate Blanchard and Quah Example ...

- Bivariate VAR:

$$Y_t = BY_{t-1} + u_t, \; Eu_tu_t' = V$$

$$u_t = Ce_t$$

$$Y_t = \begin{pmatrix} \Delta y_t \\ x_t \end{pmatrix}, \; C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, \; e_t = \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$$

e_{zt} ~ Technology Shock.

- From Applying OLS To Both Equations in VAR, We Know:

$$B, \; V$$

- Problem: $CC' = V$ Provides only **Three** Equations in **Four** Unknowns in C.
- Result: Assumption that e_{2t} Has No Long Run Impact on y_t Supplies the Extra Required Equation

Bivariate Blanchard and Quah Example ...

- Easy to Verify:

$$
\begin{aligned}
\overbrace{E_t[y_{t+1}] - E_{t-1}[y_{t+1}]}^{\overbrace{E_t[\Delta y_{t+1} + \Delta y_t] - E_{t-1}[\Delta y_{t+1} + \Delta y_t]}^{[E_t\Delta y_{t+1} - E_{t-1}\Delta y_{t+1}] + [E_t\Delta y_t - E_{t-1}\Delta y_t]}} &= (1,0)\,[B + I]\,Ce_t \\
E_t[y_{t+2}] - E_{t-1}[y_{t+2}] &= (1,0)\left[B^2 + B + I\right]Ce_t \\
E_t[y_{t+j}] - E_{t-1}[y_{t+j}] &= (1,0)\left[B^j + B^{j-1} + ... + B^2 + B + I\right]Ce_t
\end{aligned}
$$

as $j \to \infty$:

$$
\begin{aligned}
\lim_{j\to\infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] &= \\
\lim_{j\to\infty}(1,0)\left[... + B^j + B^{j-1} + ... + B^2 + B + I\right]Ce_t & \\
= (1,0)\,[I - B]^{-1}\,Ce_t &
\end{aligned}
$$

Bivariate Blanchard and Quah Example ...

- As $j \to \infty$:

$$\lim_{j\to\infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] \;=\; (1,0)\,[I-B]^{-1}\,Ce_t$$

- Identification Assumption About Technology:

$$[I-B]^{-1}\,C \;=\; \begin{bmatrix} \text{number} & 0 \\ \text{number} & \text{number} \end{bmatrix}$$

- Final Result: Solve for C Using

$$\text{(exclusion restriction) } 1,2 \text{ element of } [I-B]^{-1}\,C \text{ is } zero$$

$$\text{(sign restriction) } 1,1 \text{ element of } [I-B]^{-1}\,C \text{ is positive}$$

$$CC' = V$$

- Conclude: Long-Run Restriction Supplies Extra Equation Needed to Achieve Identification.

Arbitrary Variables, Arbitrary Lags

- More General Case of Arbitrary Number (N) of Variables and Lags:

$$X_t = B_1 X_{t-1} + B_2 X_{t-2} + \ldots + B_p X_{t-p} + u_t$$

- To Compute Impulse Response to Technology Shock,
  - Require: $B_1, \ldots, B_p$ and C_1, First Column of C in $CC' = V$
  - Can Obtain by OLS: $B_1, \ldots, B_p$ and V
  - Identification Problem: Find C_1
- Solution: Use Restriction, as $j \to \infty$:

$$\lim_{j \to \infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1, 0, \ldots, 0) \left[I - B(1)\right]^{-1} C e_t$$

$$B(1) \equiv B_1 + B_2 + \ldots + B_p.$$

Arbitrary Variables, Arbitrary Lags ...

- VAR:

$$X_t \;=\; B_1 X_{t-1} + B_2 X_{t-2} + ... + B_p X_{t-p} + u_t$$

- Long-Run Restriction:

$$\text{(exclusion restriction)}\ [I - B(1)]^{-1} C = \begin{bmatrix} \text{number} & 0, ..., 0 \\ \text{numbers} & \text{numbers} \end{bmatrix}$$

$$\text{(sign restriction)}\ (1,1)\ \text{element of}\ [I - B(1)]^{-1} C \text{ is } positive$$

$$CC' = V$$

- There Are Many C That Satisfy These Constraints. All Have the Same C_1.

Arbitrary Variables, Arbitrary Lags ...

- Using the Restrictions to Uniquely Pin Down C_1
- Let

$$\begin{aligned} D &\equiv [I - B(1)]^{-1} C \\ \text{so, } DD' &= [I - B(1)]^{-1} V [I - B(1)']^{-1} \equiv S_0 \text{ (Since } CC' = V) \end{aligned}$$

- Exclusion Restriction Requires:

$$D = \begin{bmatrix} d_{11} & 0, ..., 0 \\ D_{21} & D_{22} \end{bmatrix}$$

- So

$$DD' = \begin{bmatrix} d_{11}^2 & d_{11} D_{21}' \\ D_{21} d_{11} & D_{21} D_{21}' + D_{22} D_{22}' \end{bmatrix} = \begin{bmatrix} S_0^{11} & S_0^{21\prime} \\ S_0^{21} & S_0^{22} \end{bmatrix}.$$

- Sign Restriction:

$$d_{11} > 0.$$

- Then, First Column of D Uniquely Pinned Down:

$$d_{11} = \sqrt{S_0^{11}},\ D_{21} = S_0^{21}/d_{11}$$

- First Column of C Uniquely Pinned Down:

$$C_1 = [I - B(1)]\, D_1.$$

Shocks and Identification Assumptions

- Monetary Policy Shock
- Neutral Technology Shock
- Capital-Embodied Shock to Technology

Identifying Monetary Policy Shocks

- One strategy: estimate parameters of Fed's feedback rule

  – Rule that relates Fed's actions to state of the economy.

$$R_t = f(\Omega_t) + e_t^R$$

  – f is a linear function
  – Ω_t: set of variables that Fed looks at.
  – e_t^R: time t policy shock

What does this rule represent?

- Literal interpretation: structural policy rule of central bank.

- Combination of structural rule and other "stuff"

- Example: Clarida – Gertler
  - True policy rule

$$
\begin{aligned}
R_t &= \alpha E_t X_{t+1} + e_t^R \\
&= f(\text{all time } t \text{ data used in } E_t X_{t+1}) + e_t^R
\end{aligned}
$$

What is a Monetary Policy Shock?

- Shocks to preferences of monetary authority
- Strategic considerations can lead to exogenous variation in policy
  - Self-fulfilling expectation traps (Albanesi, Chari, Christiano)
- Technical factors like measurement error (Bernanke and Mihov)

Recursiveness Assumption

- Policy rule: $R_t = f(\Omega_t) + e_t^R$.

- Problem: not enough assumptions, yet, to identify e_t^R

- Assume:
  - Policy shocks, e_t^R are orthogonal to Ω_t.
  - Ω_t contains current prices and wages, aggregate quantities, lagged stuff

- Economic content of this assumption:
  - Fed sees prices and output when it makes its choice of R_t.
  - Prices and output don't respond at time *t* to e_t^R.

- Assumption implies e_t^R can be estimated by OLS

- Response of other variables can be obtained by regressing them on current and lagged e_t^R

Using VAR to Estimate Impulse Response Functions Under Recursiveness Assumption

- Vector autoregression:

$$
\begin{aligned}
Y_t &= B_1 Y_{t-1} + B_2 Y_{t-2} + \ldots + B_q Y_{t-q} + u_t \\
u_t &= C e_t.
\end{aligned}
$$

- To think about recursiveness assumption, it is convenient to work with

$$A_0 \equiv C^{-1}$$

so that:

$$
\begin{aligned}
A_0 Y_t &= A_0 B_1 Y_{t-1} + A_0 B_2 Y_{t-2} + \ldots + A_0 B_q Y_{t-q} + e_t, \\
A_0^{-1} \left(A_0^{-1} \right)' &= V
\end{aligned}
$$

- Consider:

$$Y_t = \begin{pmatrix} \underset{(k_1\times k_1)}{X_{1t}} \\ R_t \\ \underset{(k_2\times k_2)}{X_{2t}} \end{pmatrix}, \; A_0 = \begin{bmatrix} \underset{(k_1\times k_1)}{a_{11}} & \underset{(k_1\times 1)}{0} & \underset{(k_1\times k_2)}{0} \\ \underset{(1\times k_1)}{a_{21}} & \underset{(1\times 1)}{a_{22}} & \underset{(1\times k_2)}{0} \\ \underset{(k_2\times k_1)}{a_{31}} & \underset{(k_2\times 1)}{a_{32}} & \underset{(k_2\times k_2)}{a_{33}} \end{bmatrix}.$$

  where

  R_t interest rate (middle equation is policy rule)

  $X_{1t} \sim k_1$ variables whose current and lagged values do appear in policy rule

  $X_{2t} \sim k_2$ variables whose current values do no appear in the policy rule.

- Zero restrictions on A_0 are implied by recursiveness assumption:

  – Zero in middle row: current values of X_{2t} do not appear in policy rule

  – Zeros in first block of rows ensure that monetary policy shock does not affect X_{1t}

    ∗ First block of zeros: prevents direct effect, via R_t

    ∗ Second block of zeros: prevents indirect effect, via X_{2t}

Using VAR to Estimate Impulse Response Functions Under Recursiveness Assumption ...

- There are many A_0 matrices with given pattern of zeros, which satisfy

$$(*)\ A_0^{-1}\left(A_0^{-1}\right)' = V$$

  - One example: lower triangular A_0 with positive diagonal elements.
  - In this case, A_0^{-1} is lower triangular Choleski decomposition of V.

- Proposition:
  a. All A_0 matrices that satisfy (*) and zero restrictions imply same value for column of A_0^{-1} which corresponds to e_t^R.
     * So, we can work with lower triangular Choleski decomposition of V without loss of generality

  b. Suppose we change the ordering of the variables in X_{1t} and X_{2t}, but always pick lower triangular Choleski decomposition of V
     * dynamic response of impulse reponse of variables to e_t^R unaffected

- Proof: see Christiano, Eichenbaum and Evans (Handbook of Macro).

Long Run Identification of Technology Shocks (Blanchard-Quah, Fisher, JPE 2007)

- There are two types of technology shocks: neutral and capital embodied

$$X_t = Z_t F(K_t, L_t)$$

$$K_{t+1} = (1-\delta)K_t + V_t I_t$$

- These are only shocks that can affect the log level of labor productivity.

- The only shock which also has a long run effect on the relative price of capital is a capital embodied technology shock (V_t).

- These identification strategies require that the variables in the VAR be covariance stationary.

Technology Shocks…

- Advantage of this approach:
  - Don’t need to make all the usual assumptions required to construct Solow-residual based measures of technology shocks.
    - Functional form assumptions for production function, corrections for labor hoarding, capital utilization, and time-varying markups.
- Disadvantage: some models don’t satisfy our identifying assumption.
  - endogenous growth models where all shocks affect productivity in the long run.
  - Standard models when there are permanent shocks to the tax rate on capital income.

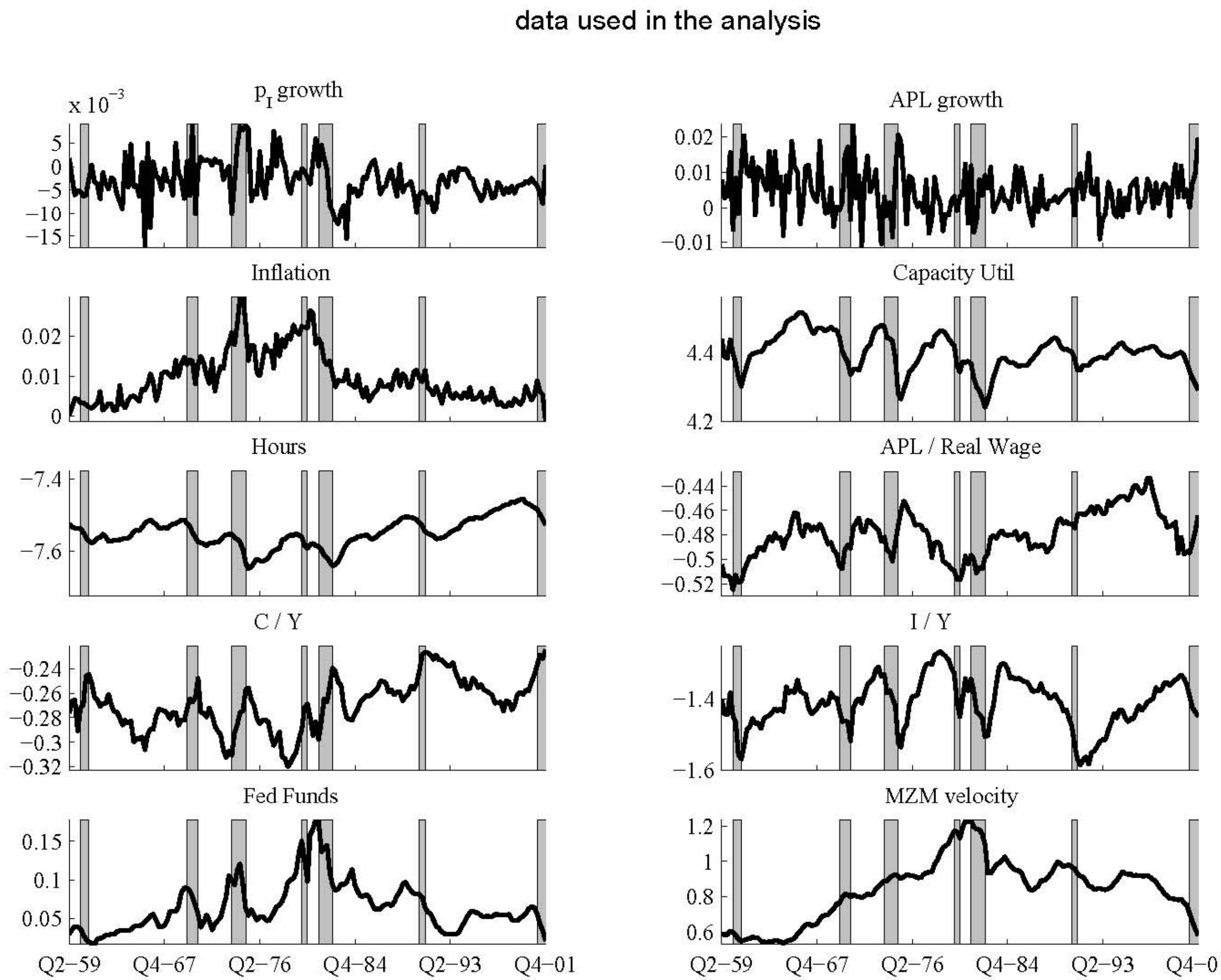
Identification

- When monetary policy shocks and technology shocks are identified separately, have exact identification
  - no restrictions on the estimated VAR parameters
- When shocks are identified simultaneously, there is over-identification.
- We test this and do not reject.
- See Altig, Christiano, Eichenbaum, Linde.

VAR estimation with the following data:

$$\underbrace{Y_t}_{10\times 1} = \begin{pmatrix} \Delta\ln(\text{relative price of investment}_t) \\ \Delta\ln(GDP_t/\text{Hours}_t) \\ \Delta\ln(GDP\ \text{deflator}_t) \\ \text{capacity utilization}_t \\ \ln(\text{Hours}_t) \\ \ln(GDP_t/\text{Hours}_t) - \ln(W_t/P_t) \\ \ln(C_t/GDP_t) \\ \ln(I_t/GDP_t) \\ \text{Federal Funds Rate}_t \\ \ln(GDP\ \text{deflator}_t) + \ln(GDP_t) - \ln(MZM_t) \end{pmatrix}$$

The data have been transformed to ensure stationarity
Sample period: 1959Q1-2007Q1


data used in the analysis
p_I growth
x 10^-3
APL growth
Inflation
Capacity Util
Hours
APL / Real Wage
C / Y
I / Y
Fed Funds
MZM velocity
Q2−59
Q4−67
Q2−76
Q4−84
Q2−93
Q4−01

- We will now estimate impulse responses using a VAR.
- First, however, we have to talk about the computation of standard errors.
- We'll discuss a standard bootstrap procedure.

Confidence Intervals and the Bootstrap

- Estimation Produces:

$$\begin{aligned} Y_t &= \hat{B}(L)Y_{t-1} + \hat{u}_t, \\ \hat{u}_t,\ t &= 1, ..., T. \end{aligned}$$

- Bootstrap
  - Generate $r = 1, ..., R$ artificial data sets, each of length T
    - For r^{th} dataset:

$$\lambda_t^r \in Uniform[0,1],\ t = 1, ..., T$$

    - Convert to integers $\in \{1, 2, ..., T\}$:

$$\tilde{\lambda}_t^r = \text{integer}(\lambda_t^r \times T),\ t = 1, ..., T$$

Confidence Intervals and the Bootstrap ...

* Draw shocks:

$$\hat{u}_{\tilde{\lambda}_1^r}, \ldots, \hat{u}_{\tilde{\lambda}_T^r}$$

* Generate artificial data:

$$Y_t^r = B(L)Y_{t-1}^r + \hat{u}_{\tilde{\lambda}_t^r},\ t = 1, ..., T.$$

– Suppose statistic of interest is ψ (could be vector of impulse response functions, serial correlation coefficients, etc.)

$$\psi^r = f\left(Y_1^r, ..., Y_T^r\right),\ r = 1, ..., R$$

* Compute

$$\sigma_\psi = \left\{ \frac{1}{R} \sum_{r=1}^{R} (\psi^r - \bar{\psi})^2 \right\}^{1/2}$$

* Report

$$\hat{\psi} \pm 2 \times \sigma_\psi.$$

VAR Diagnostics

- Whether or not to First Difference Hours Worked Important
- Choosing VAR Lag Length

$$\begin{aligned}
\text{Akaike} &: \quad s(p) = \log(\det \hat{V}_p) + (m + m^2 p)\frac{2}{T} \\
\text{Hannan-Quinn} &: \quad s(p) = \log(\det \hat{V}_p) + (m + m^2 p)\frac{2\log(\log(T))}{T} \\
\text{Schwarz} &: \quad s(p) = \log(\det \hat{V}_p) + (m + m^2 p)\frac{\log(T)}{Y}
\end{aligned}$$

T sample size, m; Number of Variables (10); p Number of Lags

– Choice:

$$\hat{p} = \min_p s(p).$$

VAR Diagnostics ...

– With $T = 170$:

$$\frac{2}{T} = 0.0118, \ \frac{2\log(\log(T))}{T} = 0.0192, \ \frac{\log(T)}{Y} = 0.0302$$

– Akaike Penalizes p the Least

  $*$ Known: In Population, Akaike Has Positive Probability of Overshooting True p

  $*$ Hannan-Quinn and Schwarz are Consistent.

- Results (see picklag.m): HQ and SC Choose $p = 1$, AIC Chooses $p = 2$:

Table: Standard VAR Lag Length Selection Criteria

h	AIC	HQ	SC
1	-101.24	-100.42	-99.21
2	-101.42	-99.84	-97.53
3	-101.28	-98.94	-95.52
4	-101.23	-98.13	-93.58
5	-101.02	-97.14	-91.46
6	-101.04	-96.37	-89.55
7	-101.02	-95.57	-87.60
8	-101.12	-94.88	-85.75

VAR Diagnostics ...

- Multivariate $Q(s)$ Statistic
  - Measure of Serial Correlation In Fitted Disturbances
  - Null Hypothesis: the First s Autocorrelations Are Zero:

$$Q(s) = T(T+2)\sum_{j=1}^{s} \frac{1}{T-j} trace\left[C_j C_0^{-1} C_j' C_0^{-1}\right],$$

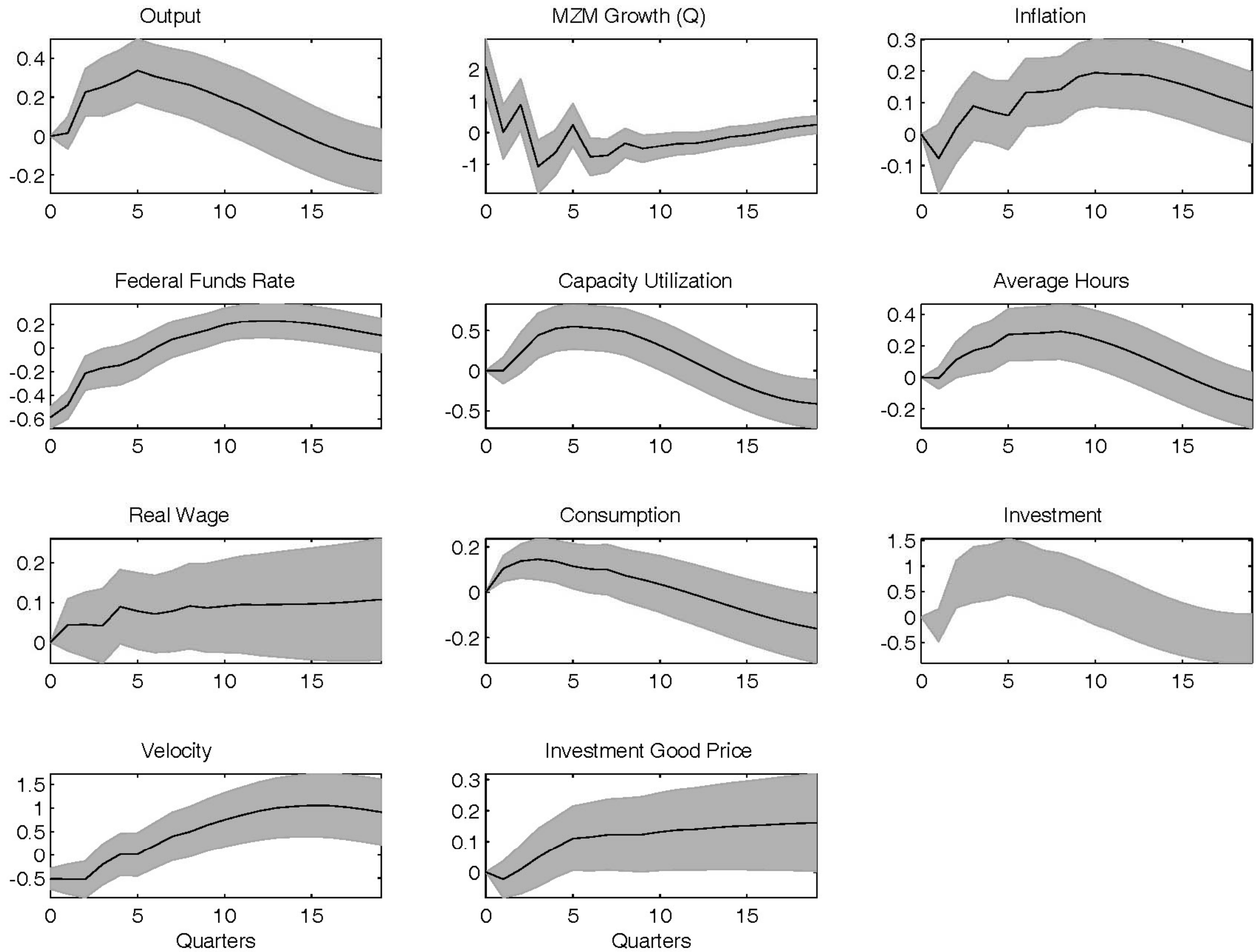
  where

$$C_j = \frac{1}{T}\sum_{t=j+1}^{T} \hat{u}_t \hat{u}_{t-j}'.$$

  - In the Scalar Case, It is the Weighted Sum of the Squares of the First s Correlations.
  - Null Distribution:

$$Q(s) \sim \chi^2_{m^2(s-p)}$$

Response to a monetary policy shock

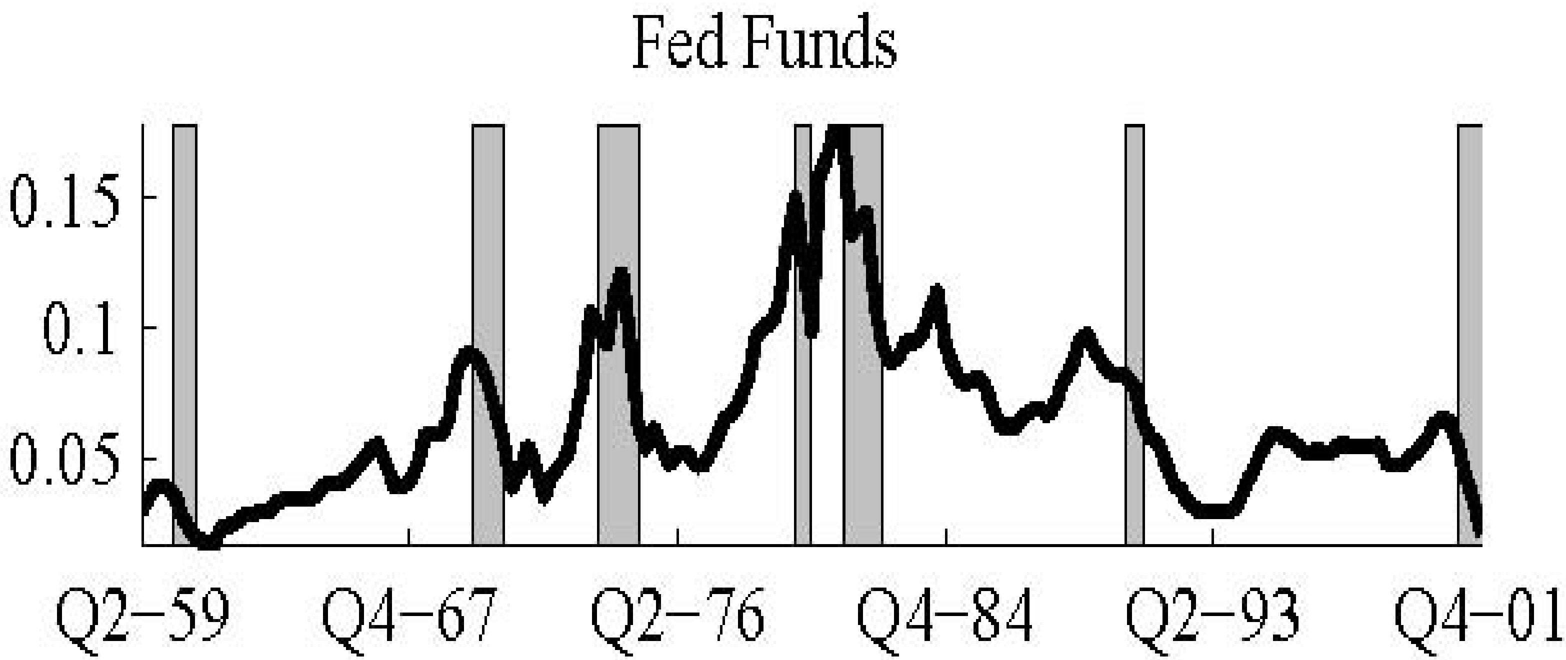


Interesting Properties of Monetary Policy Shocks

- Plenty of endogenous persistence:
  - money growth and interest rate over in 1 year, but other variables keep going….
- Inflation slow to get off the ground: peaks in roughly two years
  - It has been conjectured that explaining this is a major challenge for economics
  - Chari-Kehoe-McGrattan (*Econometrica*), Mankiw.
  - Kills models in which movements in *P* are key to monetary transmission mechanism (Lucas misperception model, pure sticky wage model)
  - Has been at the heart of the recent emphasis on sticky prices.
- Output, consumption, investment, hours worked and capacity utilization hump-shaped
- Velocity comoves with the interest rate

Timing Assumptions

- ‘Extreme’ Assumption:
  - Output Does Not Respond Instantly to Policy Shock
  - Policy Responds Instantly to Output
- Could Make a Continuum of Alternative Assumptions: Is Our Choice Arbitrary?
- Fact: Innovations in Output and Interest Rate are Positively Correlated


Fed Funds
0.15
0.1
0.05
Q2−59
Q4−67
Q2−76
Q4−84
Q2−93
Q4−01

Timing Assumptions…

- Identification Has to Come to Terms with Direction of Causation Underlying Positive Correlation

- Does it Reflect:

1. Output Responding to Policy?
2. Policy Responding to Output?
3. Something in Between?

- We adopt interpretation (2)

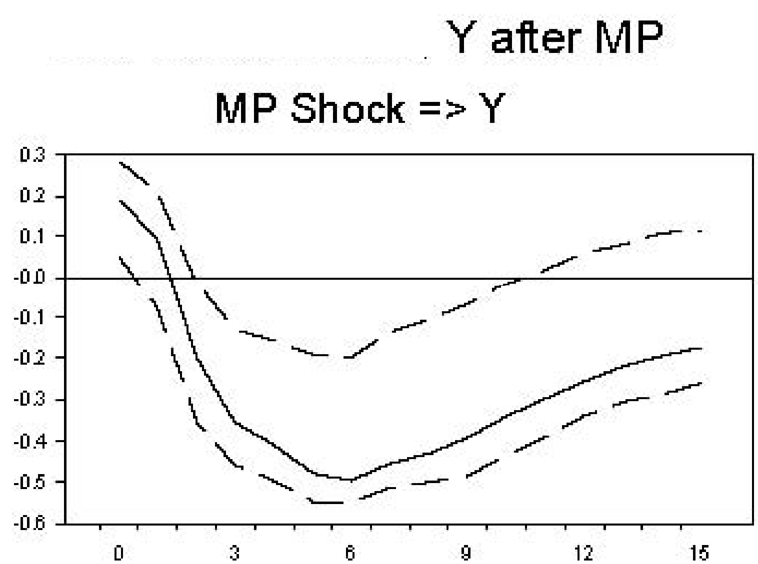
- Choices (1) or (3) Imply:

  - Monetary Policy Induced Rise in *R* Drives Output Up
  - Standard Monetary Models Inconsistent With This Implication

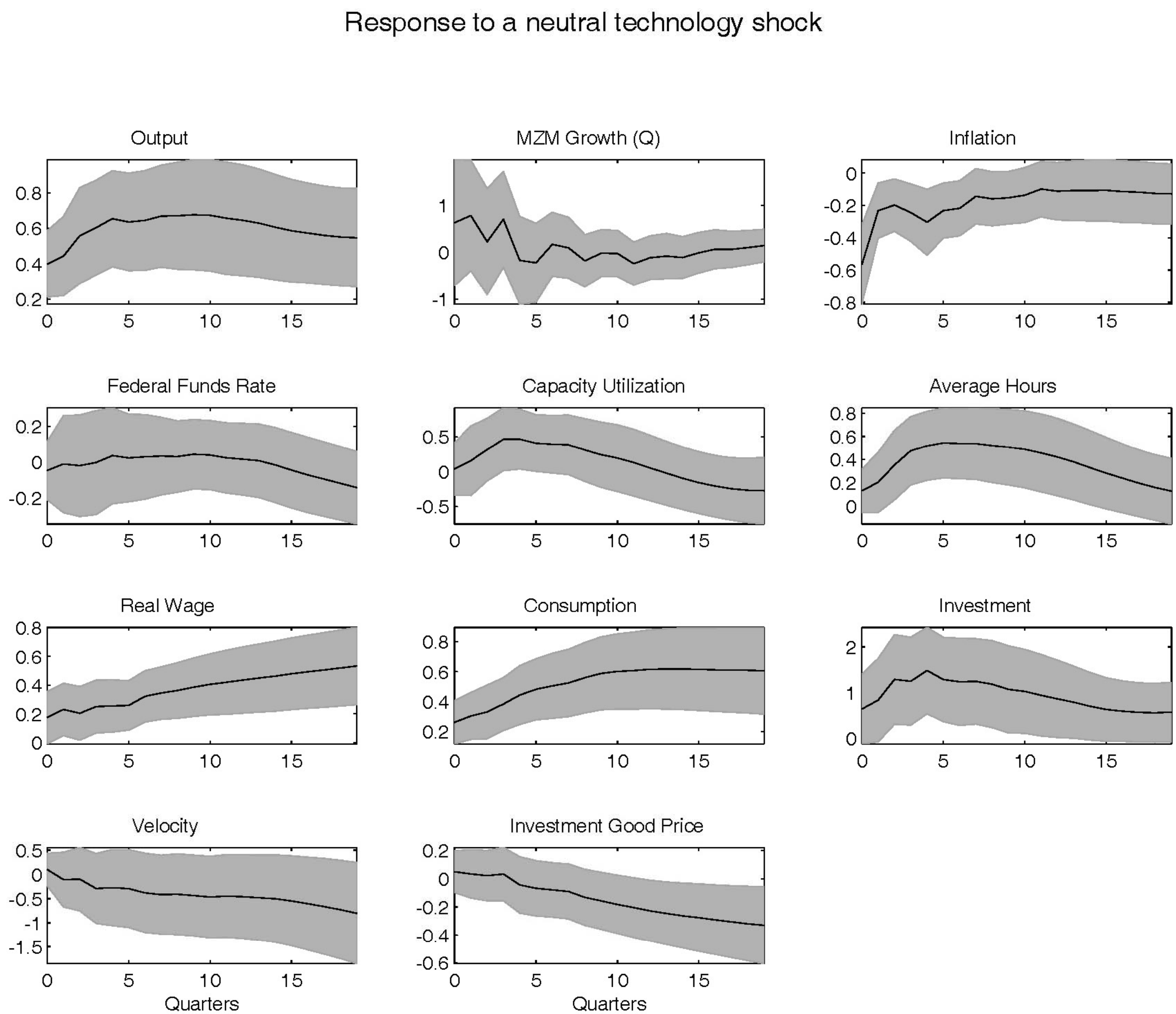
- Example: Presence of ambulances highly correlated with wounded people
  - Interpretation #1: ambulances cause people to be hurt
  - Interpretation #2: hurt people cause ambulances to come to them
  - Tough to go far with interpretation #1; prefer #2

Result of Allowing Output to Respond to Policy

- A rise in R induced by policy, e_t^R, produces a rise in *Y:*



- Seems difficult to build a theory around this


Response to a neutral technology shock
Output
MZM Growth (Q)
Inflation
Federal Funds Rate
Capacity Utilization
Average Hours
Real Wage
Consumption
Investment
Velocity
Investment Good Price
Quarters
Quarters

Observations on Neutral Shock

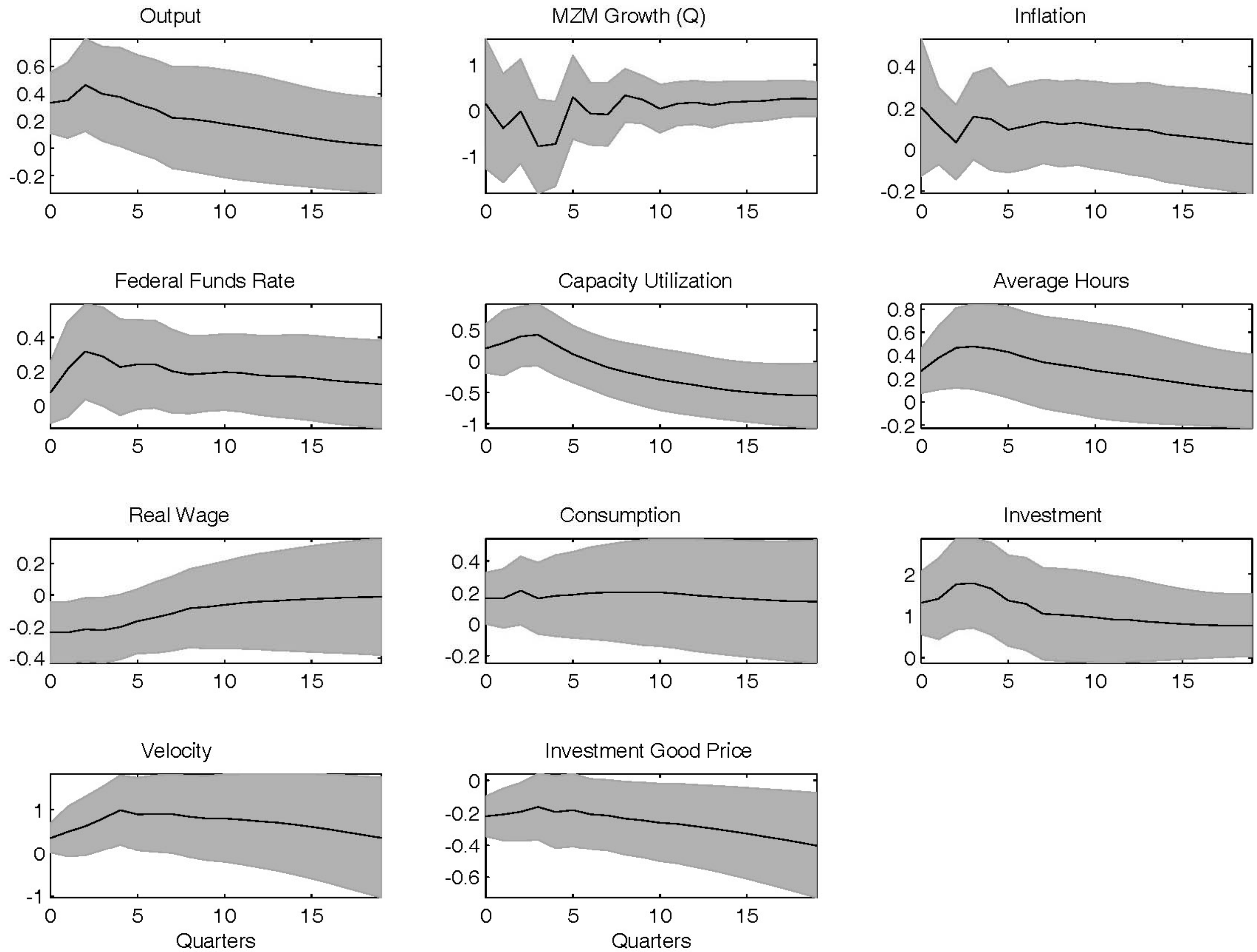
- Generally, results are ‘noisy’, as one expects.
  - Interest, money growth, velocity responses not pinned down.

- Interestingly, inflation response is immediate and *precisely* estimated.

- Does this raise a question about the conventional interpretation of the response of inflation to a monetary shock?

- Alternative possibility: information confusion stories.
  - A variant of recent work by Rhys Mendes that builds on Guido Lorenzoni’s work.

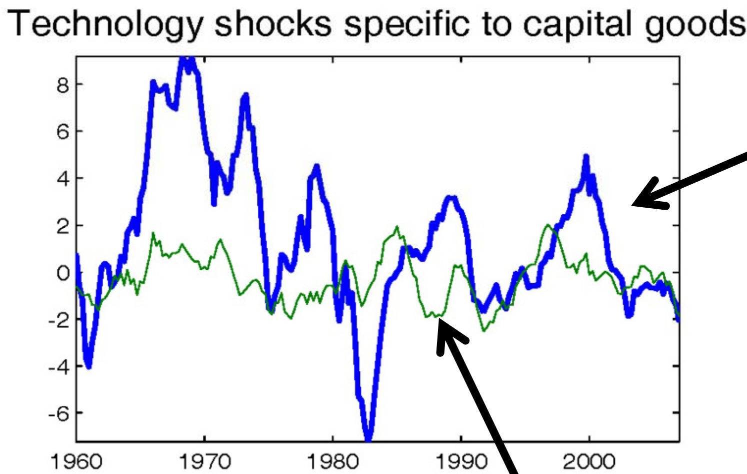
Response to an embodied technology shock



Historical Decomposition of Data into Shocks

- We can ask:
  - What would have happened if only monetary policy shocks had driven the data?
  - We can ask this about other identified shocks, or about combinations of shocks
  - We find that the three shocks together account for a large part of fluctuations

Historical decomposition of US GDP

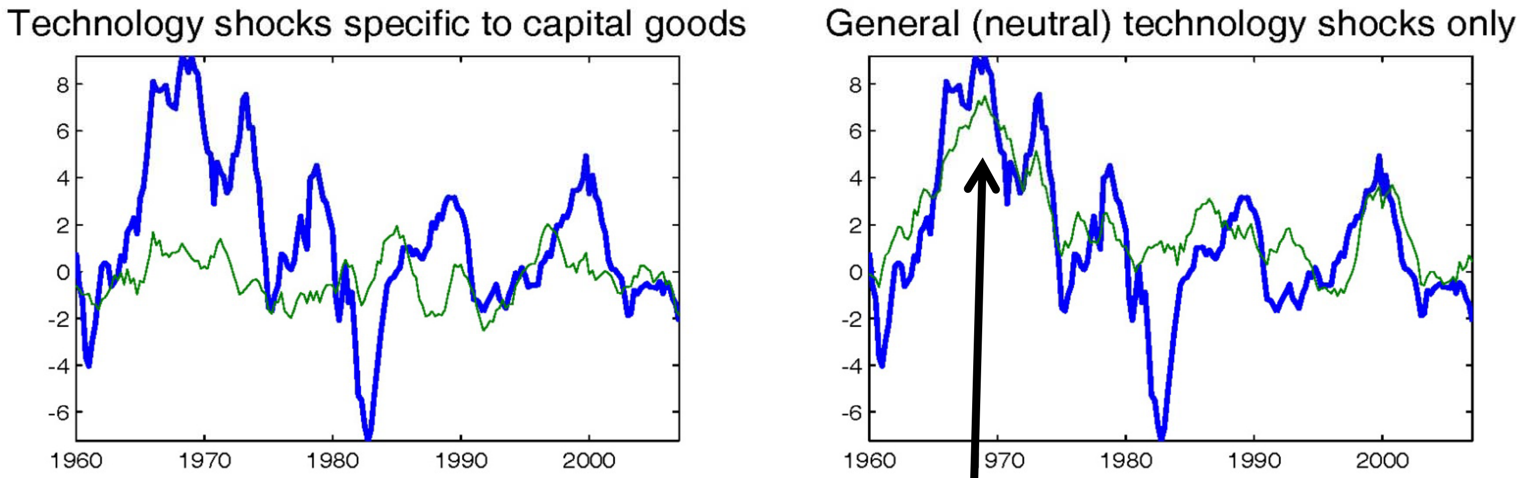


Dark line: detrended actual GDP

Thin line: what GDP would have been if there had only been one type of technology shock, the type that affects only the capital goods industry

These shocks have some effect, but not terribly important

Historical decomposition of US GDP



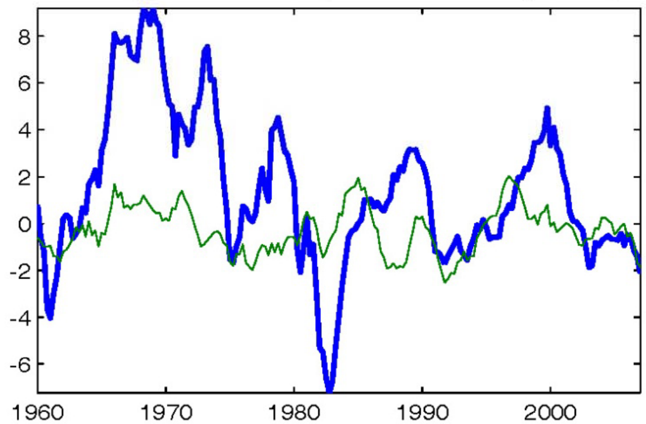
Type of technology shock that affects
all industries

This has very large impact on broad trends in the data, and a smaller impact on business cycles.

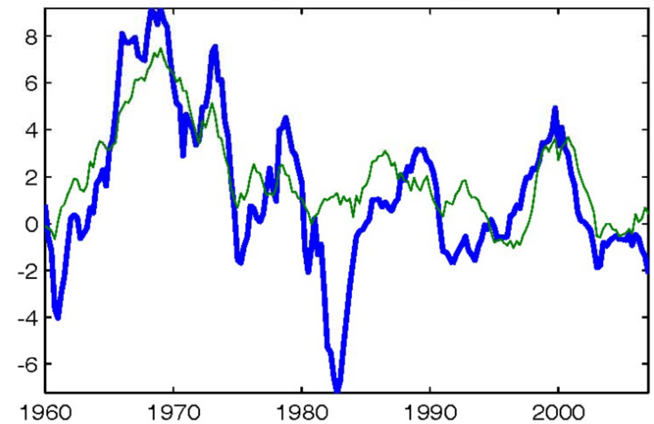
Has big impact on trend in data, and 2000 boom-bust

Historical decomposition of US GDP

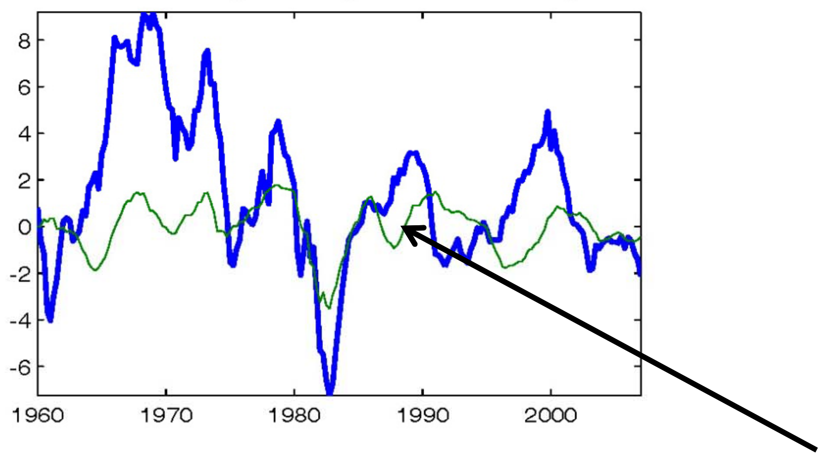
Technology shocks specific to capital goods



General (neutral) technology shocks only

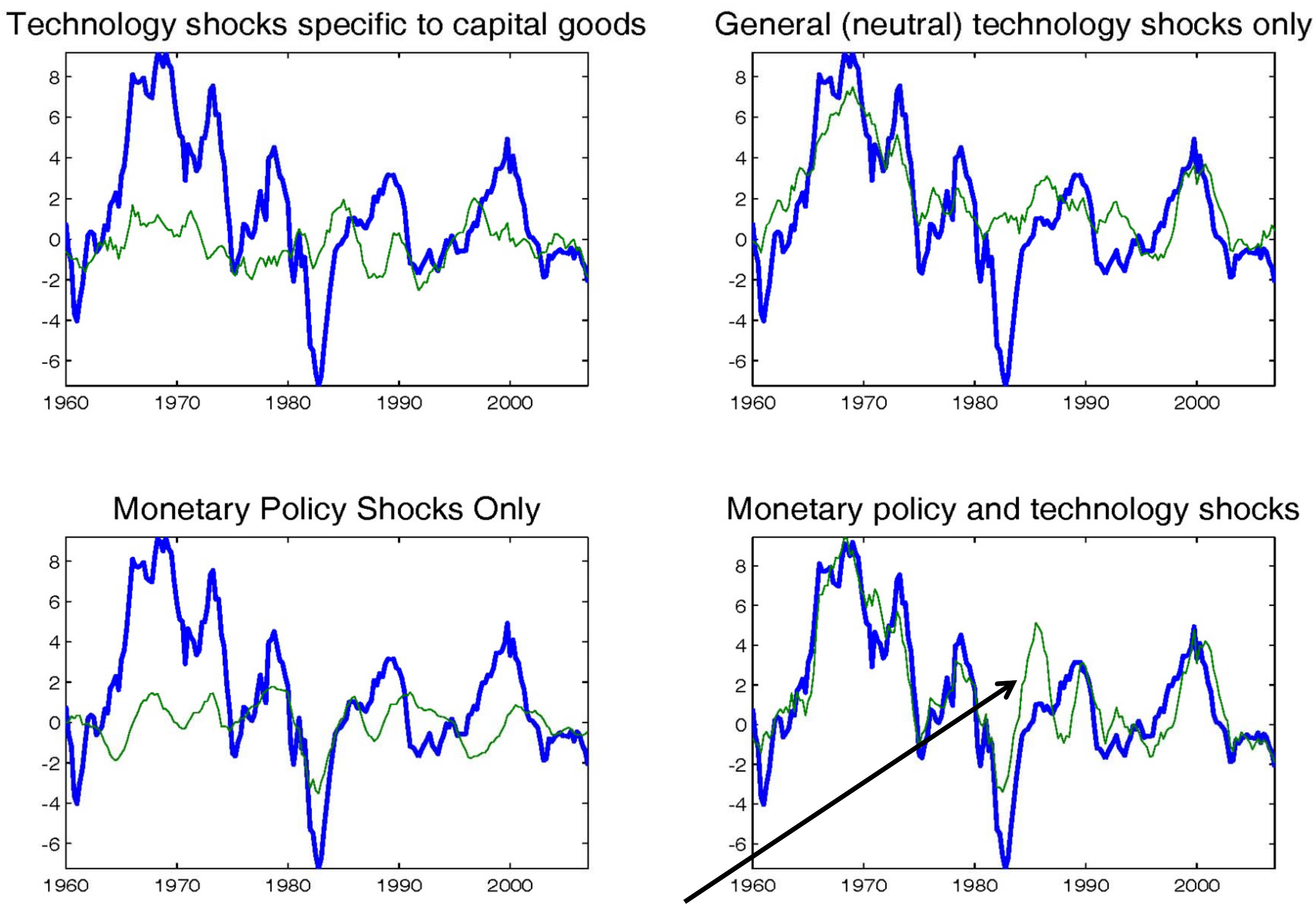


Monetary Policy Shocks Only



Monetary policy shocks have a big impact on 1980 'Volcker recession'

Historical decomposition of US GDP



All three shocks together account for
large part of business cycle

Variance Decomposition

Variable	BP(8,32)
Output	86 [18]
Money Growth	23 [11]
Inflation	33 [17]
Fed Funds	52 [16]
Capacity Util.	51 [16]
Avg. Hours	76 [17]
Real Wage	44 [16]
Consumption	89 [21]
Investment	69 [16]
Velocity	29 [16]
Price of investment goods	11 [16]

Figure 4: Historical decomposition - monetary policy and technology shocks

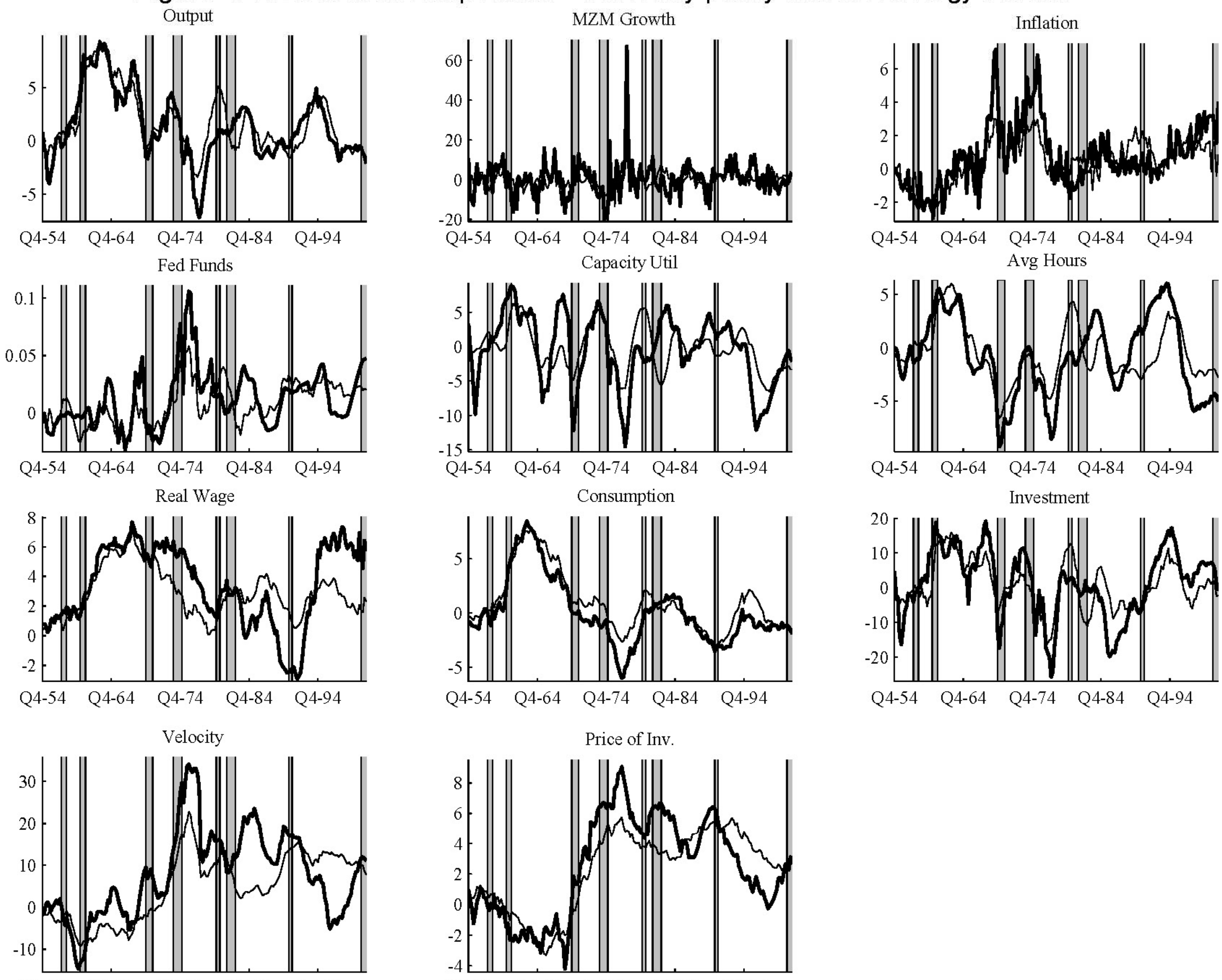


Table 1: Decomposition of Variance - In-sample Band Pass Filter and 30-Quarter Ahead Forecast Error								
Variable	**Embodied Technology**		**Neutral Technology**		**Monetary Policy**		**All Three Shocks**	
	BP(8,32)	Forec. Error	BP(8,32)	Forec. Error	BP(8,32)	Forec. Error	BP(8,32)	Forec. Error
Output	19 [10]	10 [8]	22 [13]	63 [15]	25 [9]	6 [3]	86 [18]	80 [12]
MZM Growth	2 [6]	3 [3]	3 [7]	3 [3]	17 [7]	13 [3]	23 [11]	18 [5]
Inflation	3 [10]	7 [11]	16 [12]	25 [9]	15 [7]	11 [5]	33 [17]	43 [11]
Fed Funds	6 [9]	14 [9]	2 [7]	1 [5]	45 [10]	20 [5]	52 [16]	36 [9]
Capacity Util.	7 [9]	13 [9]	7 [8]	7 [6]	25 [9]	11 [5]	51 [16]	31 [10]
Avg. Hours	19 [11]	19 [11]	18 [11]	34 [13]	22 [8]	7 [4]	76 [17]	60 [13]
Real Wage	28 [11]	5 [10]	7 [12]	49 [19]	2 [3]	2 [3]	44 [16]	57 [17]
Consumption	14 [10]	8 [11]	37 [17]	71 [20]	23 [8]	2 [3]	89 [21]	82 [17]
Investment	19 [10]	30 [12]	10 [10]	22 [12]	20 [8]	7 [4]	69 [16]	59 [12]
Velocity	7 [10]	11 [13]	1 [8]	5 [7]	24 [10]	12 [6]	29 [16]	27 [13]
Price of Inv.	9 [16]	26 [20]	4 [7]	13 [10]	3 [4]	5 [4]	11 [16]	44 [16]

Notes: Numbers are point estimates, number in square brackets are standard deviation of point estimates across bootstrap simulations. In the case of the forecast error decomposition row sums fail to add only because of rounding error. In the case of BP(8,32) row sums fail to add due to in-sample correlation between shocks.

Fiscal Shocks

- SVAR analysis of dynamic effects of fiscal shocks useful for discriminating between models
  - Neoclassical models suggest:
    - hours worked and aggregate output rise
    - real wages and consumption *fall* after an increase in gov't purchases.
  - Models with countercyclical markups suggest that hours worked and output rise, and real wages *rise* (Rotemberg and Woodford).
  - Blanchard and Perotti, (*QJE*, 2002), Gali, Lopez-Salido, Valles (2002), Ramey and Shapiro (1998, Carnegie-Rochester), Burnside, Eichenbaum and Fisher (1999)

Fiscal Shocks…

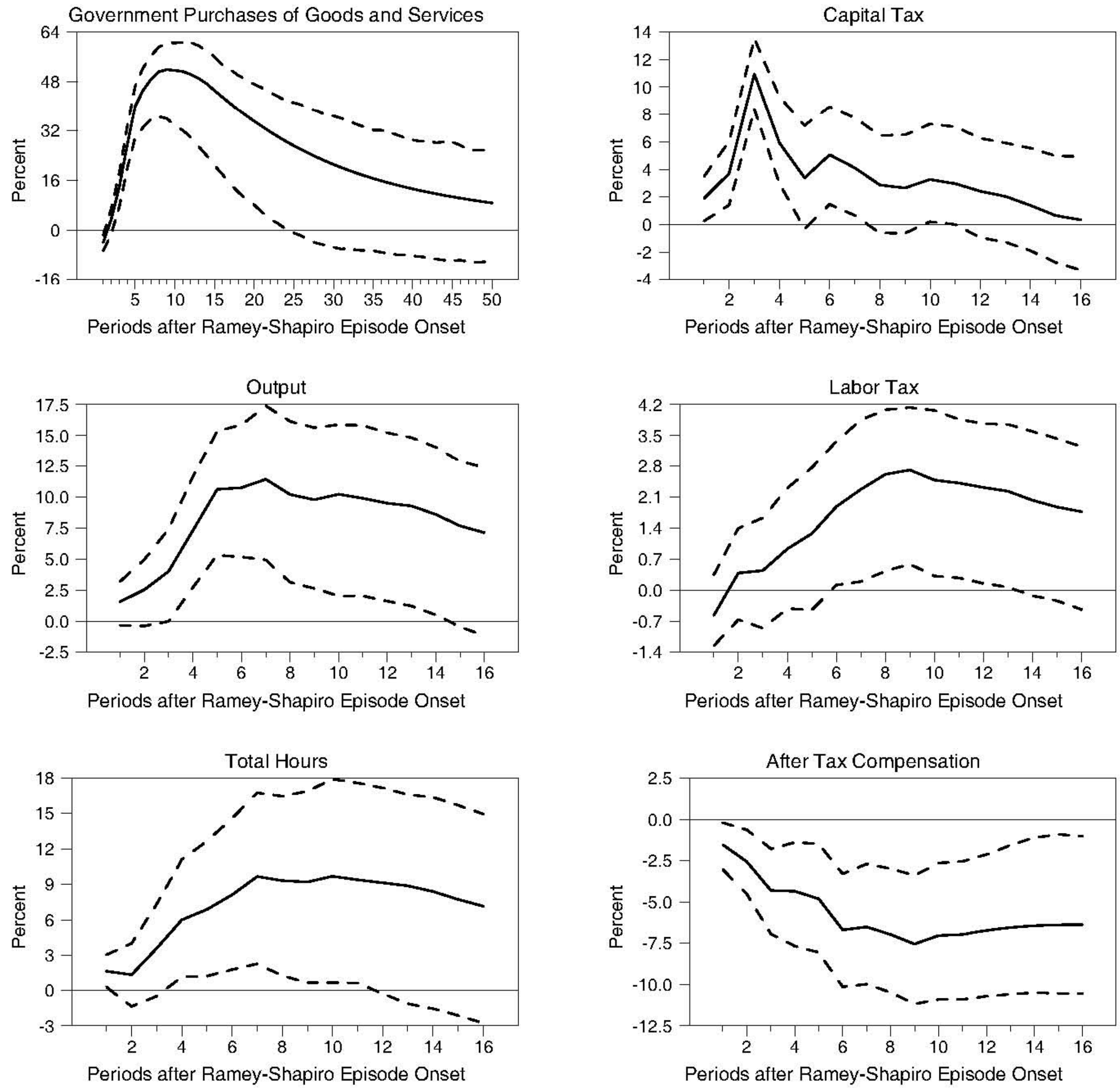
- Key empirical issue: identifying exogenous changes in fiscal policy.

- Hard to do standard VAR identification of fiscal shocks

  – In practice, people know that a fiscal shock is on the way, before it hits the data

Burnside, Eichenbaum and Fisher

- Ramey and Shapiro identify three political events that led to large military buildups:
    - Korean War -- 1950:3, Vietnam War -- 1965:1, Carter-Reagan build-up -- 1980:1
  - Weakness: only three episodes.
  - Advantage: assumption that war episodes are exogenous is compelling
- How does economy respond to these shocks?


Government Purchases of Goods and Services
Percent
Periods after Ramey-Shapiro Episode Onset
Capital Tax
Percent
Periods after Ramey-Shapiro Episode Onset
Output
Percent
Periods after Ramey-Shapiro Episode Onset
Labor Tax
Percent
Periods after Ramey-Shapiro Episode Onset
Total Hours
Percent
Periods after Ramey-Shapiro Episode Onset
After Tax Compensation
Percent
Periods after Ramey-Shapiro Episode Onset

- Evidence consistent with neoclassical model (real wage falls)

- When consumption is included, it turns out to rise, or remain unchanged.

- Problem: this contradicts neoclassical model.
  - Gali, Lopez-Salido, Valles have explored presence of liquidity constrained consumers
  - May be consistent with Sims-Woodford fiscal theory of the price level

Fiscal Theory

- Equation depicting payments to government bondholders:

$$\frac{B_0}{P_0} = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t [T_t - G_t]$$

- Standard Fiscal Theory
  – Price level flexible
  – When *G* or *T* changes, P_0 adjusts immediately

- Fiscal Theory with sticky prices
  – *G* jumps or *T* drops implies *r* falls
  – Low *r* stimulates consumption

- To explore Fiscal Theory story, must check what happens to *r* after fiscal shock.

- Fiscal policy shocks
  - Example of Sims' suggestion that VARs can be a 'battleground' for discriminating between different theories.

Conclusions of VAR discussion

- We have reviewed identification of shocks with VARs.

- We identified three shocks which together account for a large fraction of output fluctuations.

- We also identified their dynamic effects on the economy.

- We discussed their usefulness in debates.