Estimation of a DSGE Model

Lawrence J. Christiano

Based on:

Christiano, Eichenbaum, Evans, `Nominal Rigidities and the Dynamic Effects of a Shocks to Monetary Policy', JPE, 2005

Altig, Christiano, Eichenbaum and Linde, 'Firm-Specific Capital, Nominal Rigidities, and the Business Cycle', manuscript

Objectives

- Constructing a standard DSGE Model
  - Model Features.
  - Estimation of Model using VAR’s.
  - Indicate departures from the standard models.

- Determine if there is a conflict regarding price behavior between micro and macro data.

  - Macro Evidence:
    - Inflation responds slowly to monetary shock
    - Single equation estimates of slope of Phillips curve produce small slope coefficients.

  - Micro Evidence:
    - Bils-Klenow, Nakamura-Steinsson report evidence on frequency of price change at micro level: 5-11 months.

Single equation estimates of slope of Phillips curve

- Phillips curve: $\pi_t = \beta E_t \pi_{t+1} + \gamma s_t$

$$\gamma = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}$$

- Rewrite:

$$\pi_t - \beta\pi_{t+1} = \gamma s_t - \beta\eta_{t+1}$$

$$\eta_{t+1} = \pi_{t+1} - E_t\pi_{t+1} \perp \text{ date } t \text{ variables}$$

- Regression:

$$\hat{\gamma} = \frac{cov(\pi_t - \beta\pi_{t+1}, s_t)}{var(s_t)}$$

$$= \frac{cov(\gamma s_t - \beta\eta_{t+1}, s_t)}{var(s_t)}$$

$$= \frac{cov(\gamma s_t, s_t)}{var(s_t)} = \gamma.$$

- Procedures like this tend to imply stickiness in prices (Gali-Gertler, Eichenbaum-Fisher):

$$\frac{1}{1-\xi_p} \approx 6$$

- At the same time, DSGE literature finds (see Smets-Wouters, Primiceri, others) highly serially correlated shock in Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma s_t + u_t,$$

- Then,

$$\hat{\gamma} = \frac{cov(\pi_t - \beta\pi_{t+1}, s_t)}{var(s_t)} = \frac{cov(\gamma s_t + u_t, s_t)}{var(s_t)} = \gamma\left[1 + \frac{\overbrace{cov(u_t, s_t)}^{\text{expected to be negative}}}{var(s_t)}\right] \overbrace{<}^{?} \gamma.$$

- Could apply instrumental variables/GLS methods to estimate slope of Phillips curve, but these tend to produce noisy results.

- Alternative: impulse-response approach will in principle allow us to estimate slope of Phillips curve without making any detailed assumptions on the Phillips curve shock.

- If the slope of the Phillips curve is small, could in principle reconcile with micro evidence on frequency of price adjustment (Kimball aggregator, firm-specific capital). However, these approaches entail other questionable empirical implications.

Outline

- Model (Describe extensions that are subject of current research)
- Econometric Estimation of Model
  - Fitting Model to Impulse Response Functions
- Model Estimation Results
- Implications for Micro Data on Prices

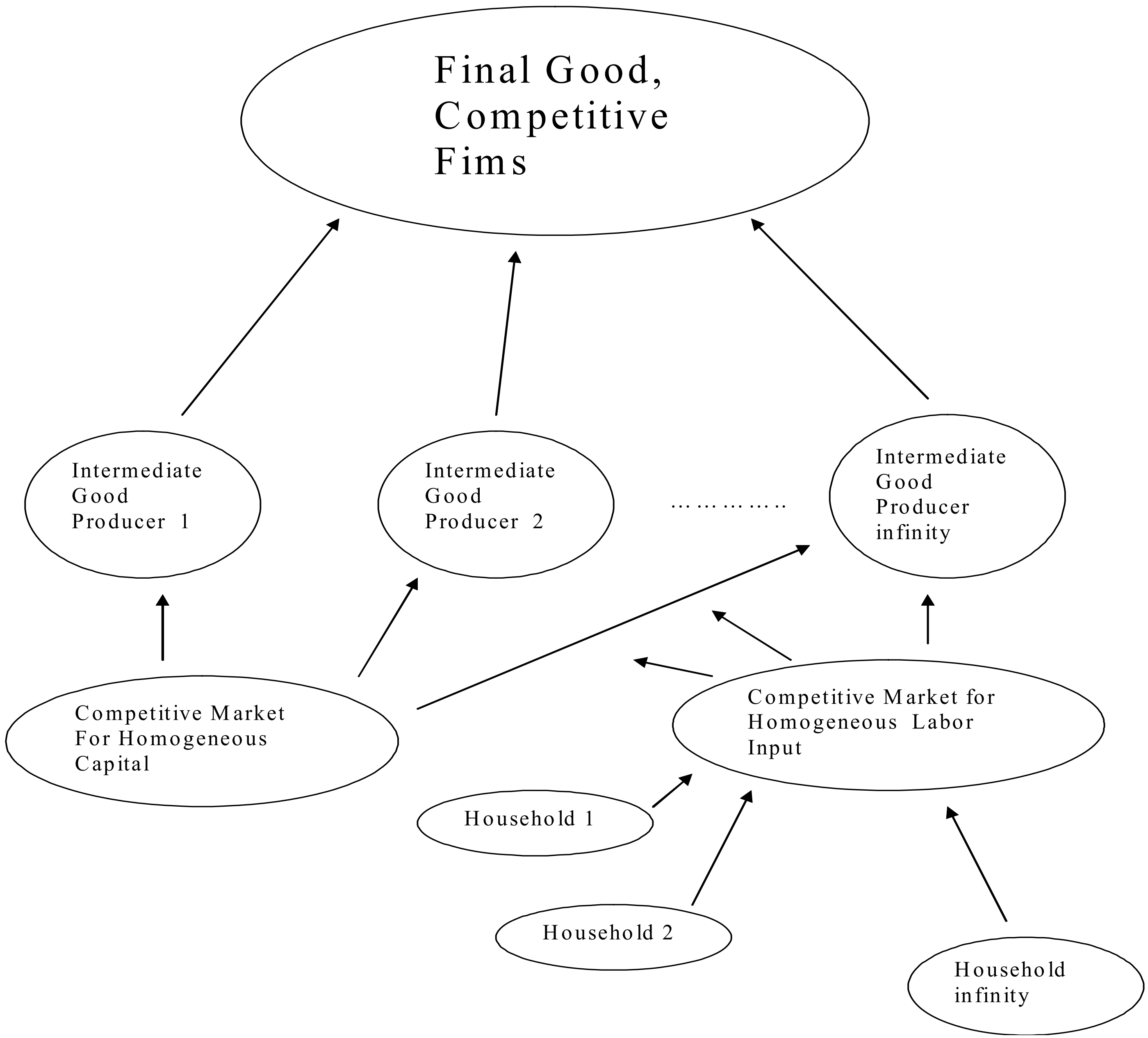
Description of Model

- Timing Assumptions
- Firms
- Households
- Monetary Authority
- Goods Market Clearing and Equilibrium

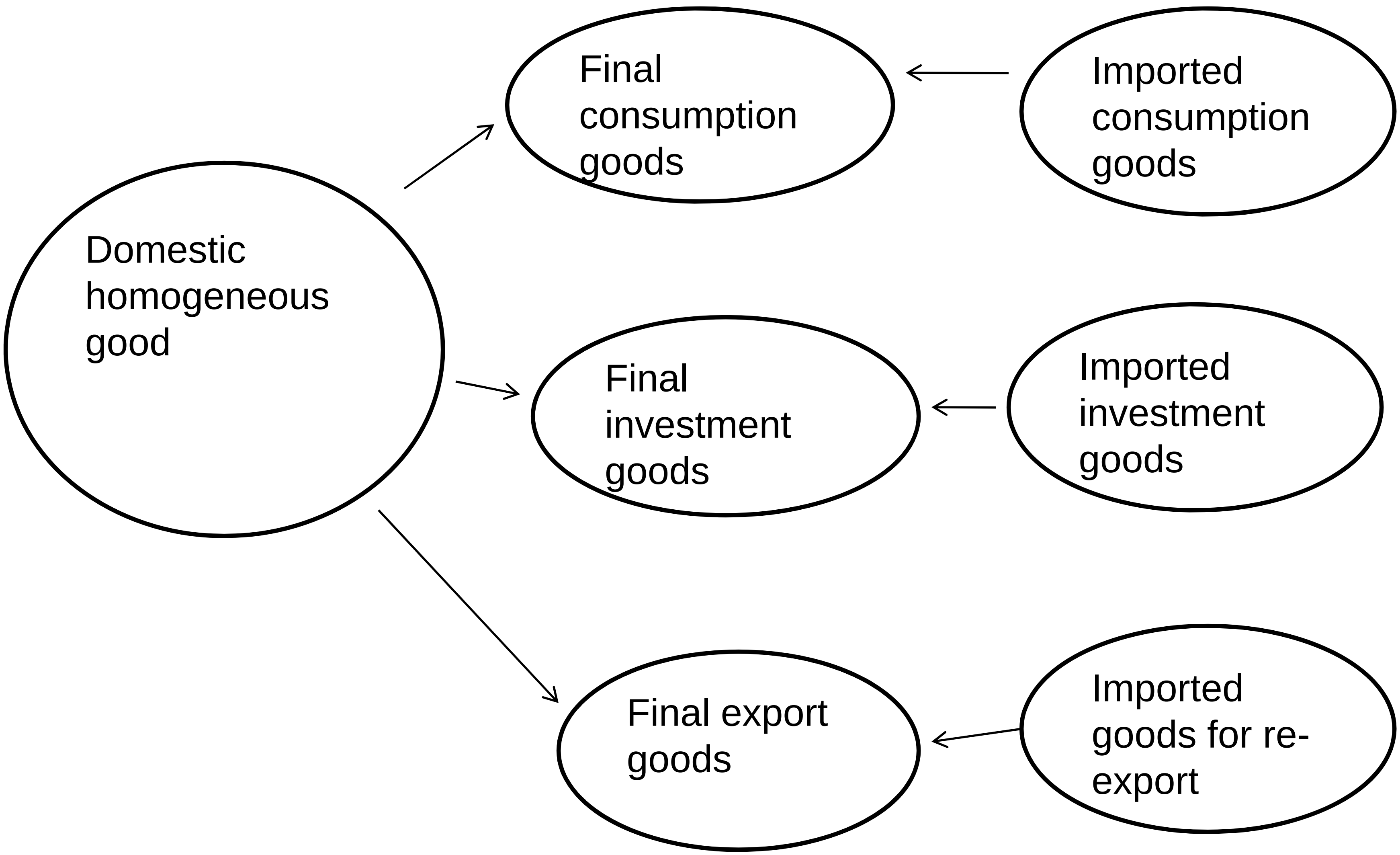
Timing

- Technology Shocks Realized.
- Agents Make Price/Wage Setting, Consumption, Investment, Capital Utilization Decisions.
- Monetary Policy Shock Realized.
- Household Money Demand Decision Made.
- Production, Employment, Purchases Occur, and Markets Clear.
- Note: Wages, Prices and Output Predetermined Relative to Policy Shock.

Firm Sector

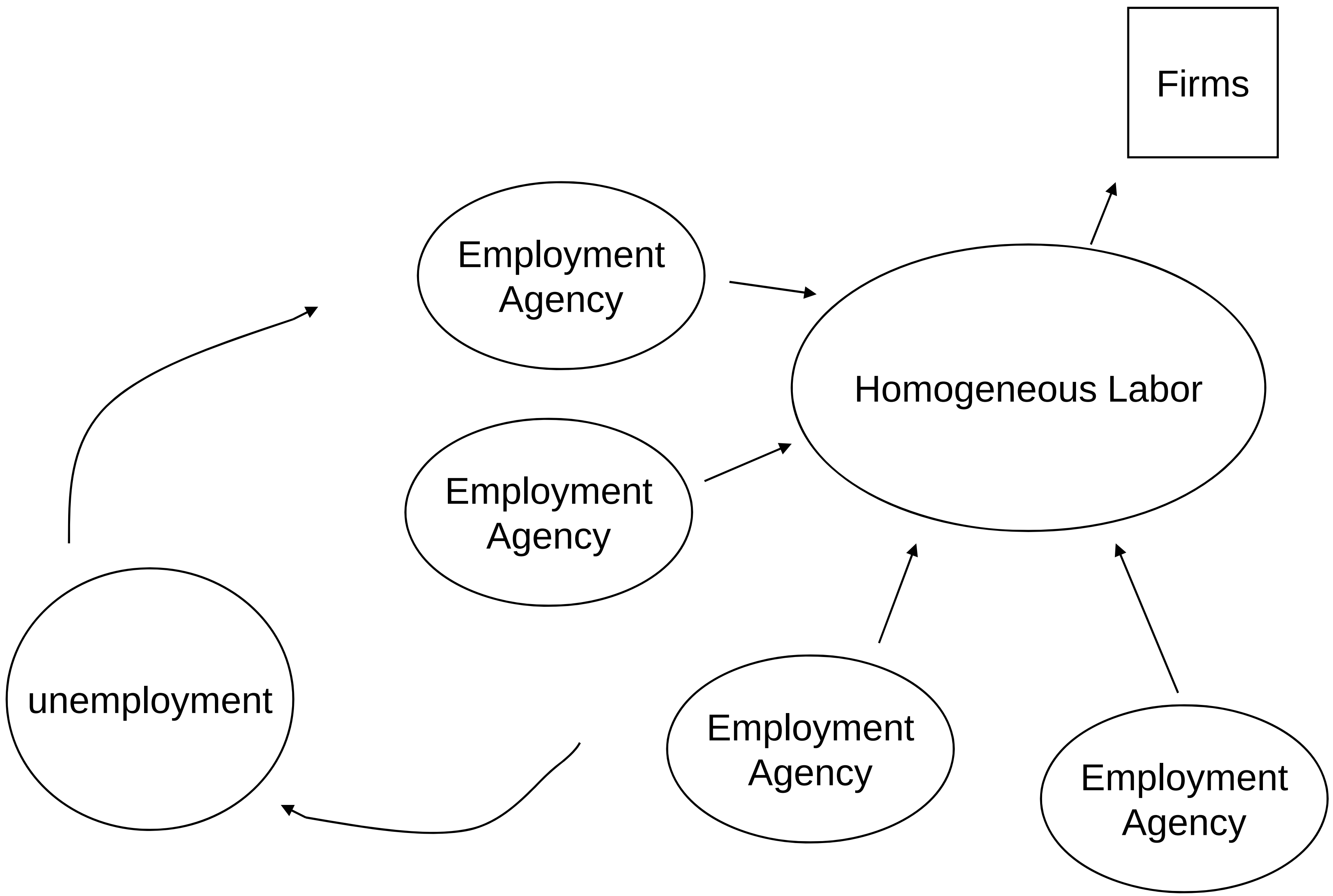

Final Good, Competitive Fims
Intermediate Good Producer 1
Intermediate Good Producer 2
……………..
Intermediate Good Producer infinity
Competitive Market For Homogeneous Capital
Competitive Market for Homogeneous Labor Input
Household 1
Household 2
Household infinity

Extension to open economy
(Christiano, Trabandt, Walentin (2007))



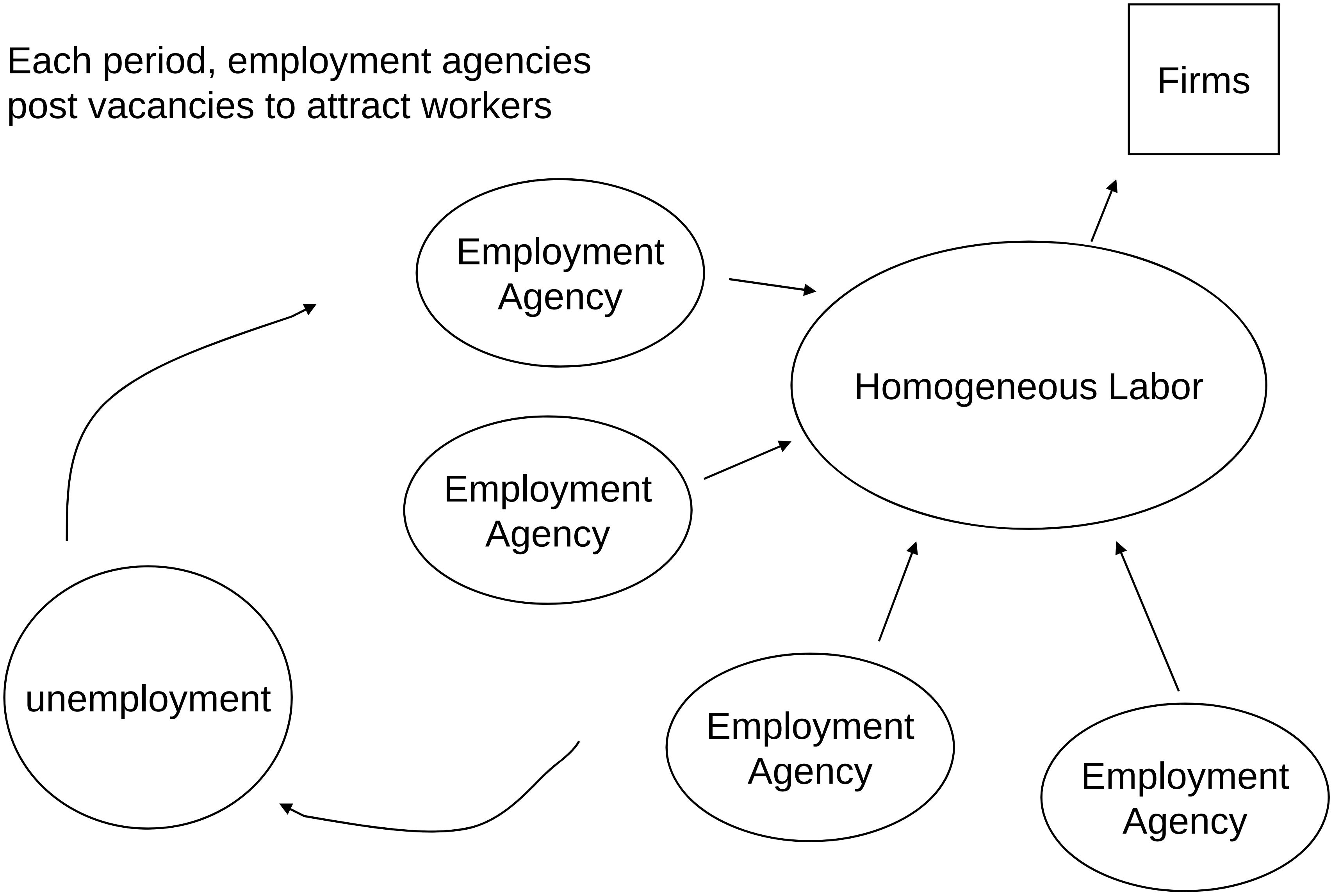
Modification of labor market

- In standard monetary DSGE model no distinction between hours (‘intensive margin’) and number of workers (‘extensive margin’)
- Intensive and extensive margins exhibit very different dynamics over business cycle
- Wage frictions thought to matter for extensive margin, not intensive margin.
- Labor market part of standard DSGE model has wage frictions affecting both margins
- Mortensen-Pissarides search and matching frictions recently introduced into DSGE models (Gertler-Sala-Trigari, Blanchard-Gali, Christiano-Ilut-Motto-Rostagno)
- Extension to open economy (Christiano, Trabandt, Walentin)

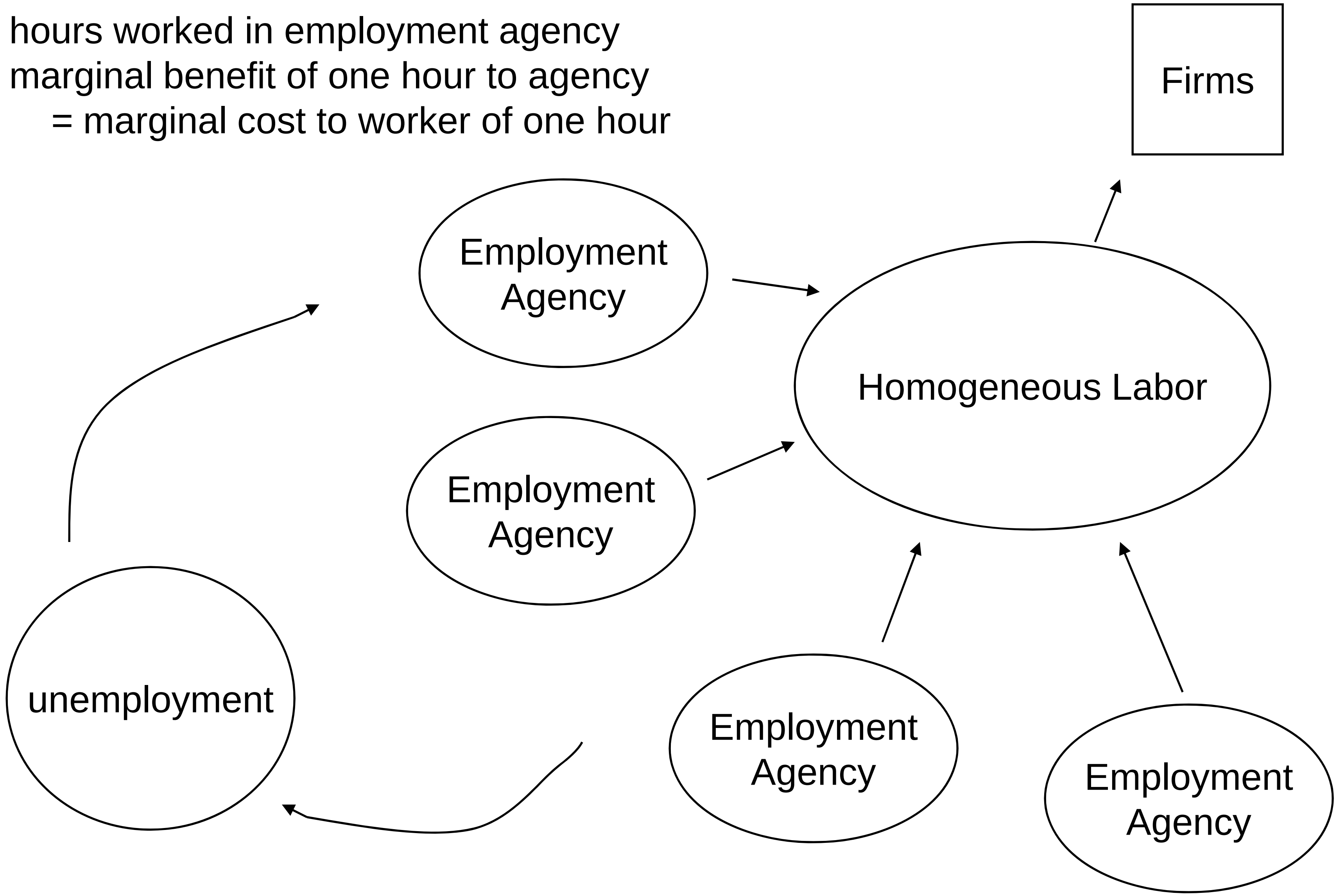


Firms
Employment Agency
Homogeneous Labor
Employment Agency
unemployment
Employment Agency
Employment Agency

Each period, employment agencies post vacancies to attract workers

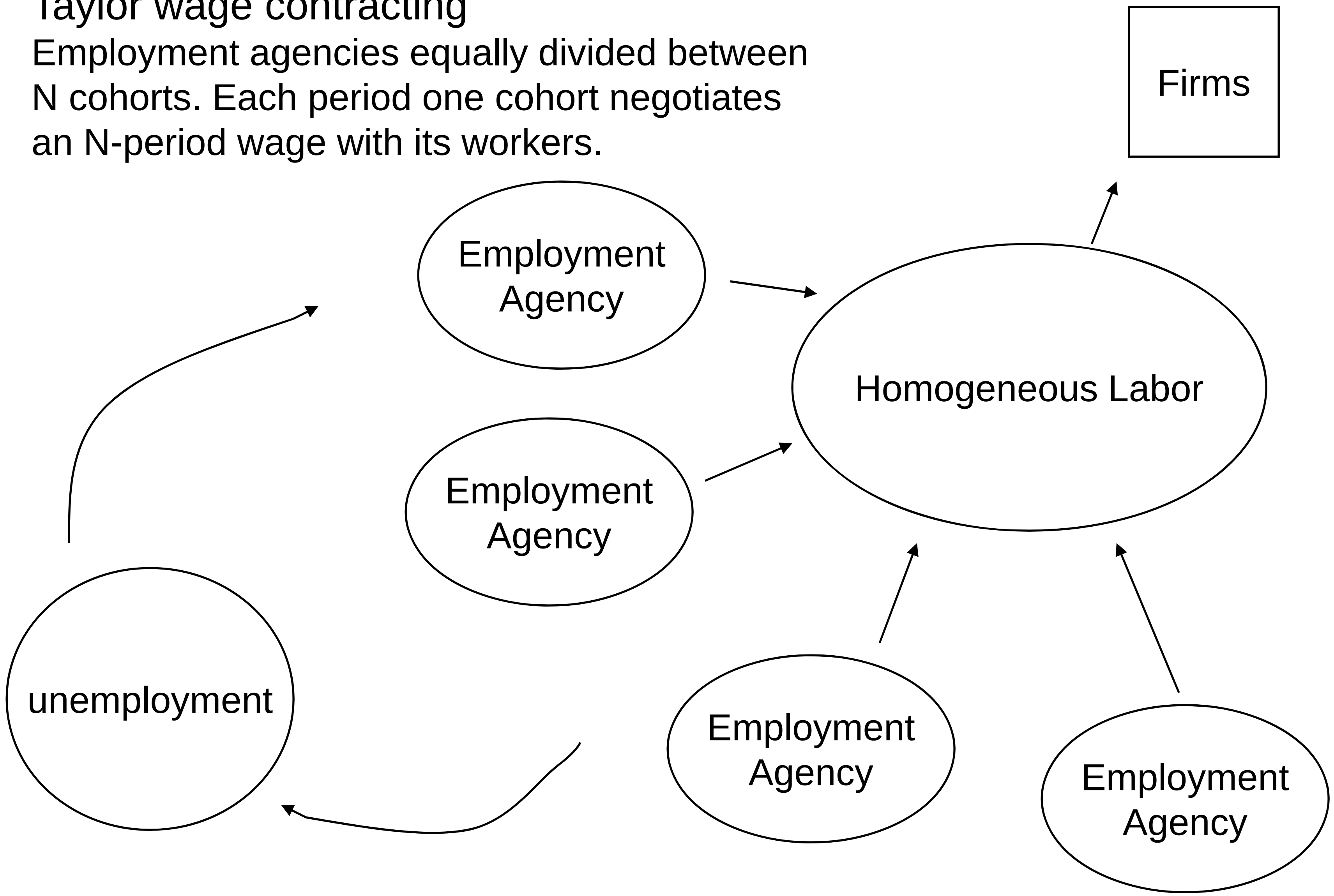


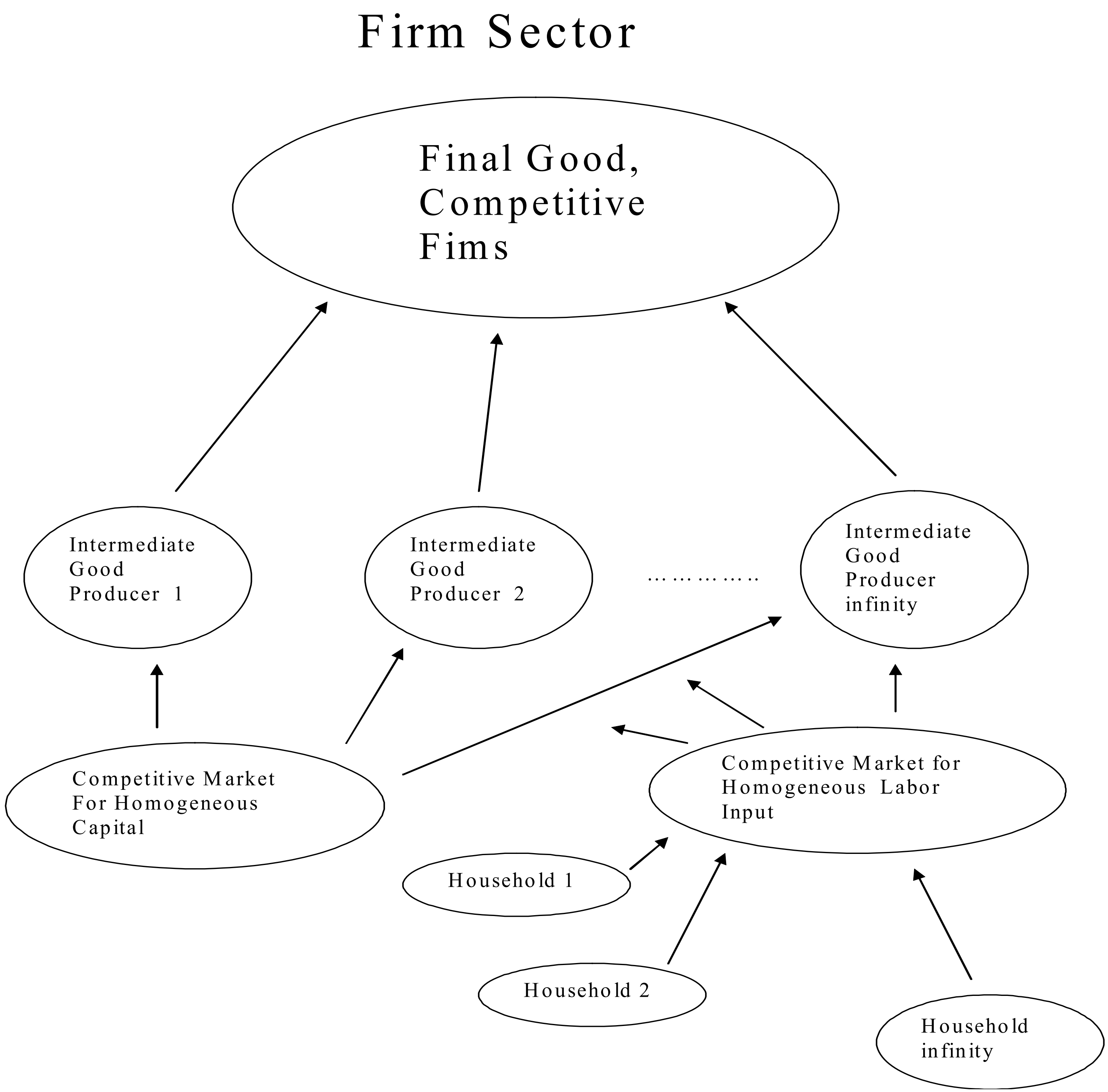
Efficient determination of
hours worked in employment agency
marginal benefit of one hour to agency
= marginal cost to worker of one hour



Taylor wage contracting

Employment agencies equally divided between N cohorts. Each period one cohort negotiates an N-period wage with its workers.

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Firms

Final Good Firms

- Technology:

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{1}{\lambda_f}} di\right]^{\lambda_f}, \; 1 \leq \lambda_f < \infty$$

- Objective:

$$\max P_t Y_t - \int_0^1 P_{it} Y_{it} di$$

- Foncs and Prices:

$$\left(\frac{P_t}{P_{it}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}, \; P_t = \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} di\right]^{(1-\lambda_f)}.$$

Intermediate Good Firms -

- Each Y_{it} Produced by a Monopolist, With Demand Curve:

$$\left(\frac{P_t}{P_{it}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}.$$

- Technology:

$$Y_{it} = K_{it}^{\alpha}\left(z_t L_{it}^{1-\alpha}\right),\ 0 < \alpha < 1,$$

- Here, z_t is a technology shock:

$$\mu_{z,t} = \log z_t - \log z_{t-1},\ \hat{\mu}_{z,t} = \rho_{\mu_z}\hat{\mu}_{z,t-1} + \varepsilon_{\mu_z,t}$$

- Calvo Price Setting:
  - With Probability $1-\xi_p$, i^{th} Firm Sets Price, P_{it}, Optimally, to $\tilde{P}_t$.
  - With Probability ξ_p,

$$P_{it} = \pi_{t-1} P_{i,t-1},\ \pi_t = \frac{P_t}{P_{t-1}}.$$

  - Standard Approach in Literature:

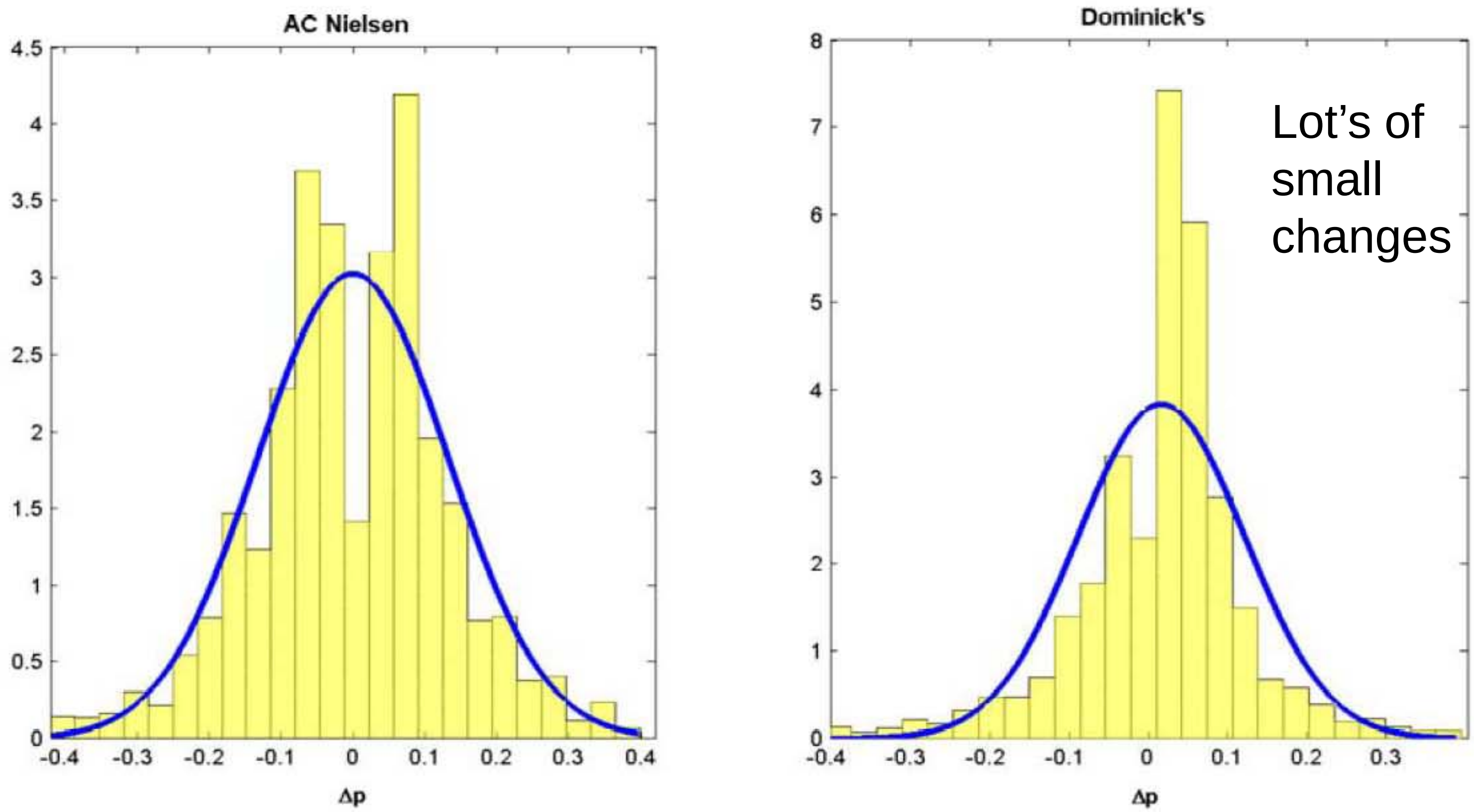
$$\begin{aligned} P_{it} &= \bar{\pi} P_{i,t-1}, \text{ or} \\ P_{it} &= P_{i,t-1}. \end{aligned}$$

  - Stand on Indexing Matters

    Determines Extent of ‘Front-Loading’

Evidence from Midrigan, ‘Menu Costs, Multi-Product Firms, and Aggregate Fluctuations’

Figure 1: Distribution of price changes conditional on adjustment



Note: superimposed is the pdf of a Gaussian distribution with equal mean and variance

Histograms of $\log(P_t/P_{t-1})$, conditional on price adjustment, for two data sets pooled across all goods/stores/months in sample.

- Combining Optimal Price and Aggregate Price Relation:

$$\Delta\hat{\pi}_t = \beta E_t \Delta\hat{\pi}_{t+1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p} E_t \hat{s}_t,$$

- Under Standard Price-Updating Scheme:

$$P_{it} = \bar{\pi} P_{i,t-1}.$$

Associated Reduced Form:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p} E_t \hat{s}_t.$$

Households: Sequence of Events

- Technology shock realized.
- Decisions: Consumption, Capital accumulation, Capital Utilization.
- Insurance markets on wage-setting open.
- Wage rate set.
- Monetary policy shock realized.
- Household allocates beginning of period cash between deposits at financial intermediary and cash to be used in consumption transactions.

Households...

- Monopoly supplier of differentiated labor
  - Sets wage subject to Calvo style frictions like firms
- Preferences of j^{th} household

$$E_t^j \sum_{l=0}^{\infty} \beta^{l-t} \left[\log\left(C_{t+l} - bC_{t+l-1}\right) - \psi_L \frac{h_{j,t+l}^2}{2} \right]$$

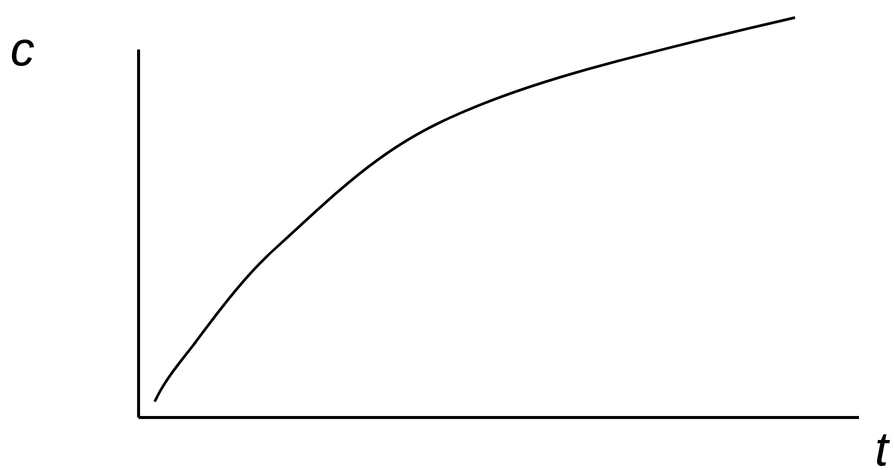
- E_t^j : expectation operator, conditional on aggregate and household j idiosyncratic information.
- C_t : consumption
- h_{jt} : hours worked.

Dynamic Response of Consumption to Monetary Policy Shock

- In Estimated Impulse Responses:
  - Real Interest Rate Falls

$$R_t/\pi_{t+1}$$

  - Consumption Rises in Hump-Shape Pattern:



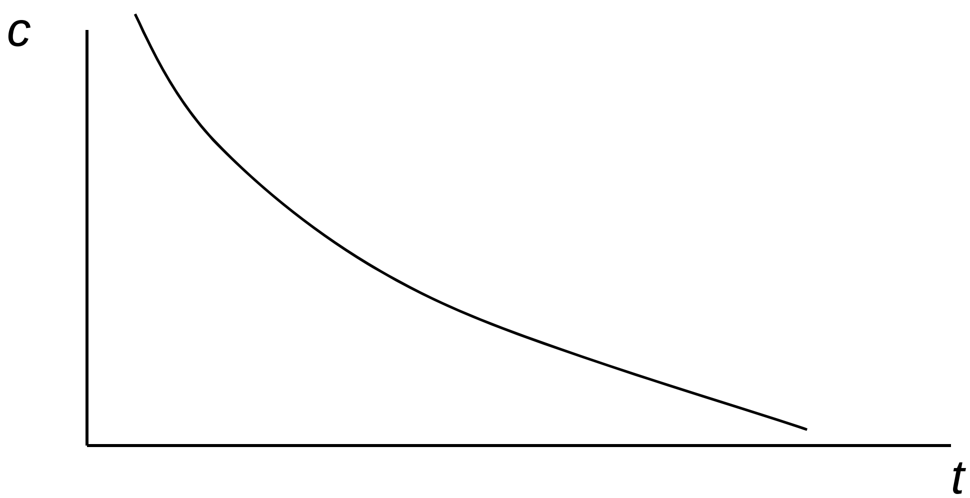
Consumption ‘Puzzle’

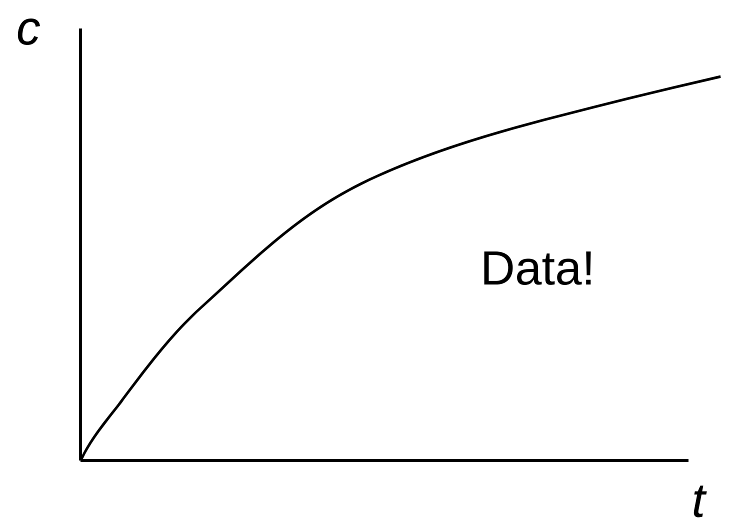
- Intertemporal First Order Condition:

‘Standard’ Preferences →

$$\frac{c_{t+1}}{\beta c_t} = \frac{MU_{c,t}}{\beta MU_{c,t+1}} \approx R_t/\pi_{t+1}$$

- With Standard Preferences:





One Resolution to Consumption Puzzle

- Concave Consumption Response Displays:
  - Rising Consumption (problem)
  - Falling Slope of Consumption

Habit parameter

- Habit Persistence in Consumption

$$U(c) = \log(c - b \times c_{-1})$$

  - Marginal Utility Function of *Slope* of Consumption
  - Hump-Shape Consumption Response Not a Puzzle

- Econometric Estimation Strategy Given the Option, *b>0*

Households...

- Asset Evolution Equation:

$$\begin{aligned} M_{t+1} &= R_t \left[M_t - Q_t + (x_t - 1) M_t^a \right] + A_{j,t} + Q_t + W_{j,t} h_{j,t} \\ &\quad + P_t r_t^k u_t \bar{K}_t + D_t - P_t \left[(1 + \eta (V_t)) C_t + \Upsilon_t^{-1} \left(I_t + a(u_t) \bar{K}_t \right) \right] \end{aligned}$$

  - M_t : Beginning of Period Base Money; Q_t : Transactions Balances
  - x_t : Growth Rate of Base; u_t : Utilization Rate of Capital
    - $u_t = 1$ in steady state, $a(1) = 0$, $a'(1) > 0$, $\sigma_a = a''(1)/a'(1)$.
  - Υ_t^{-1}: (Real) Price of investment goods, $\mu_{\Upsilon,t} = \Upsilon_t / \Upsilon_{t-1}$,

$$\hat{\mu}_{\Upsilon,t} = \rho_{\mu_\Upsilon} \hat{\mu}_{\Upsilon,t-1} + \varepsilon_{\mu_\Upsilon,t}$$

- Velocity:

$$V_t = \frac{P_t C_t}{Q_t},$$

Money Demand

- Asset Evolution Equation:

$$\begin{aligned} M_{t+1} \;=\; & R_t\left[M_t - Q_t + (x_t - 1)M_t^a\right] + A_{j,t} + Q_t + W_{j,t}h_{j,t} \\ & + P_t r_t^k u_t \bar{K}_t + D_t - P_t\left[(1+\eta(V_t))C_t + \Upsilon_t^{-1}\left(I_t + a(u_t)\bar{K}_t\right)\right] \end{aligned}$$

- Increase in Q_t :
  - Marginal Cost of Interest Foregone: R_t
  - Marginal Benefit:

$$1 - P_t\eta'(V_t)\,C_t\frac{dV_t}{dQ_t}$$

$$= \overbrace{1}^{\text{addititional cash available at end of period}} + \overbrace{\eta'\left(\frac{P_tC_t}{Q_t}\right)\left(\frac{P_tC_t}{Q_t}\right)^2}^{\text{reduction in transactions costs due to extra cash}}$$

Money Demand ...

- Money Demand: Equate Marginal Benefits and Costs of Q_t–

$$R_t \;=\; 1+\eta'\left(\frac{P_tC_t}{Q_t}\right)\left(\frac{P_tC_t}{Q_t}\right)^2.$$

- Properties of Money Demand:
  - Unit Consumption Elasticity of Money Demand
    * Increase C_t 1 percent and Hold R_t, P_t Fixed $\Rightarrow$ Desired Q_t increases 1 percent
  - $R_t \uparrow$ Implies $Q_t \downarrow$
    * To Induce Households to Hold Additional Q, Must Have Lower R
    * Money Demand Elasticity is Bigger, the Bigger is η''

- Quantitative Analysis of Money Demand
  - Consider the Following Parametric Function for η

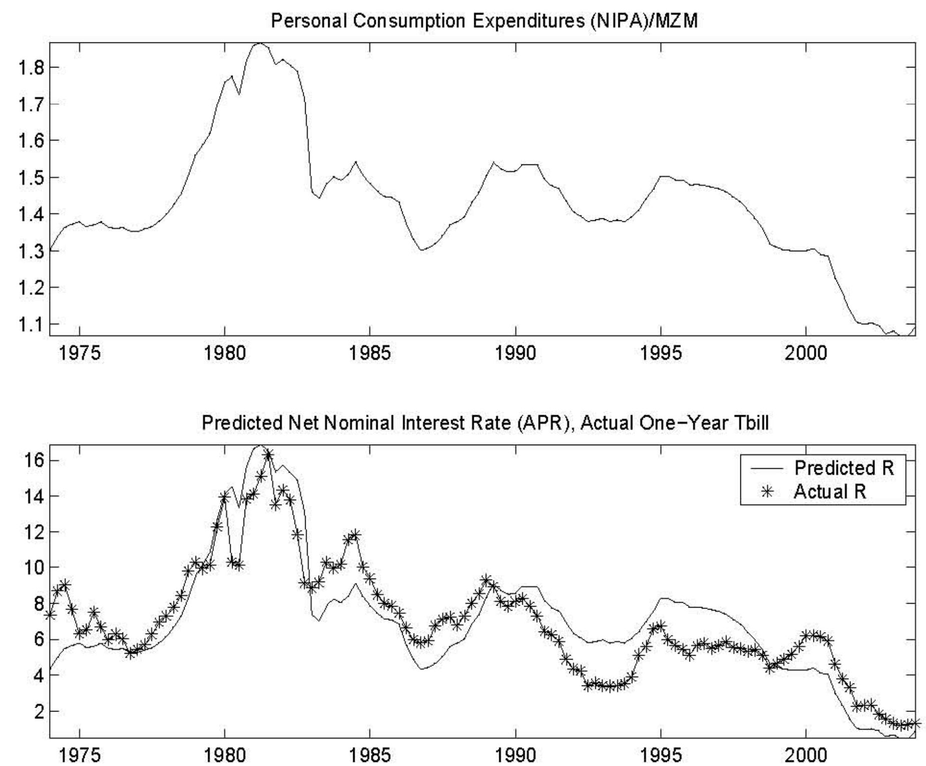
$$\eta = AV_t + \frac{B}{V_t} - 2\sqrt{AB}$$

$$\Rightarrow$$

$$R = 1 + \eta'(V) \times V^2 = 1 + \left[A - BV^{-2}\right] V^2 = 1 - B + AV^2$$

  - Data:
    * Money - St. Louis Fed's MZM, 1974-2004
    * Consumption - NIPA Consumption of Services and Nondurables
    * Interest Rate - One Year T-Bills.
    * OLS Regression of V^2 on $R \Rightarrow A = 0.0174$ and $B = 0.0187$

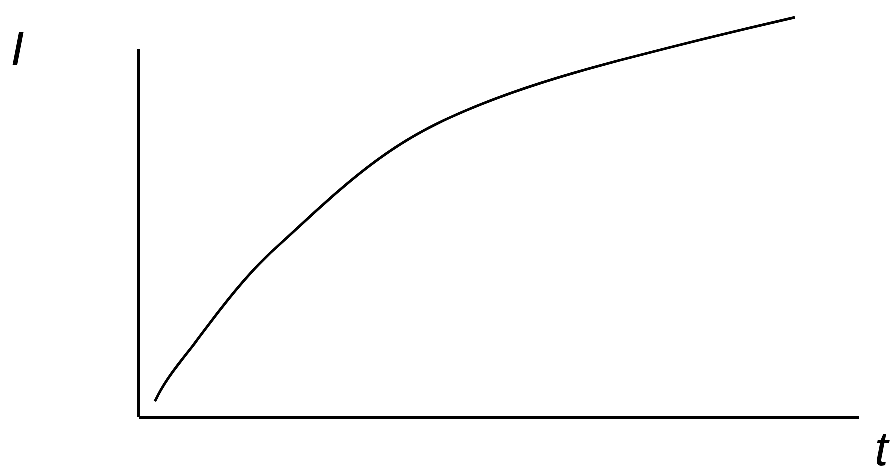
- Top Graph: Velocity of Money
- Bottom Graph: Actual and Predicted Interest Rate



- Findings: Static Money Demand Equation Fits the Data Well!

Dynamic Response of Investment to Monetary Policy Shock

- In Estimated Impulse Responses:
  - Investment Rises in Hump-Shaped Pattern:



Investment ‘Puzzle’

- Rate of Return on Capital

$$R_t^k = \frac{MP_{t+1}^k + P_{k',t+1}(1-\delta)}{P_{k',t}},$$

$P_{k',t}$ ~ consumption price of installed capital

MP_t^k ~marginal product of capital

$\delta \in (0,1)$ ~depreciation rate.

- Rough ‘Arbitrage’ Condition:

$$\frac{R_t}{\pi_{t+1}} \approx R_t^k.$$

- Positive Money Shock Drives Real Rate:

$$R_t^k \downarrow$$

- Problem: Burst of Investment!

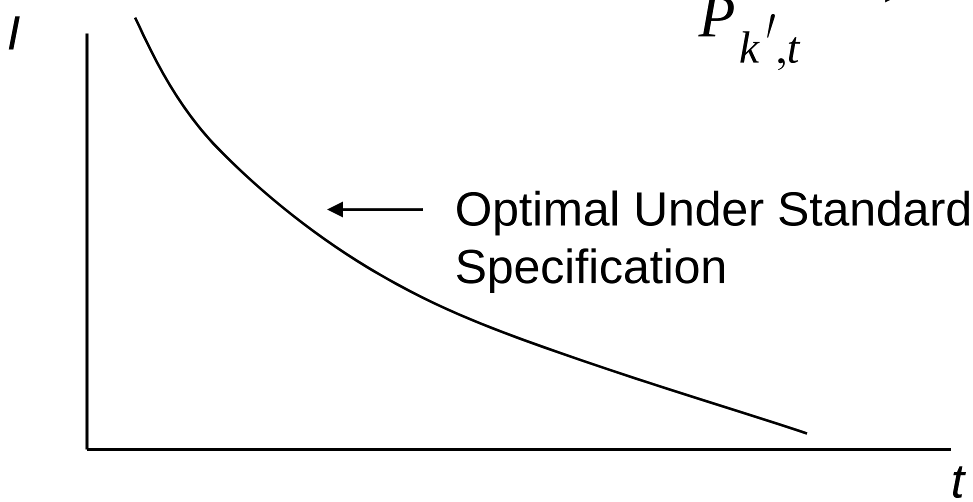
One Solution to Investment Puzzle

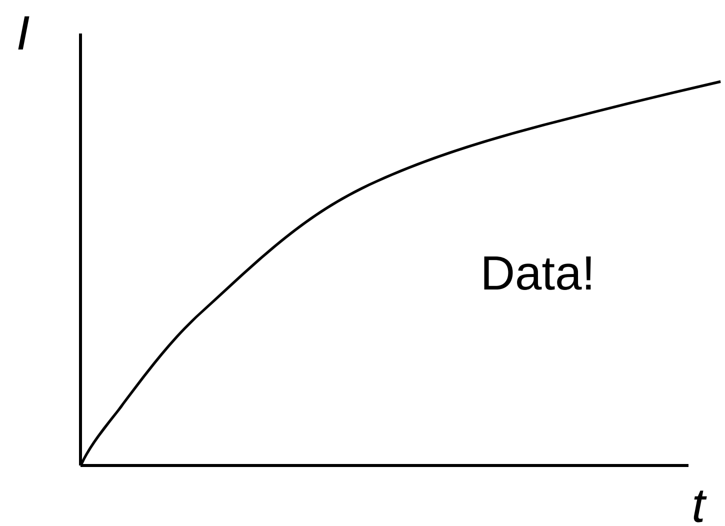
- Adjustment Costs in Investment
  - Standard Model (Lucas-Prescott)

$$k' = (1-\delta)k + F(\frac{I}{k})I.$$

  - Problem:
    - Hump-Shape Response Creates Anticipated Capital Gains

$$\frac{P_{k',t+1}}{P_{k',t}} > 1$$





One Solution to Investment Puzzle…

- Cost-of-Change Adjustment Costs:

$$k' = (1 - \delta)k + F\left(\frac{I}{I_{-1}}\right)I$$

- This Does Produce a Hump-Shape Investment Response
  - – Other Evidence Favors This Specification
  - – Empirical: Matsuyama, Smets-Wouters.
  - – Theoretical: Matsuyama, David Lucca

Financial Frictions

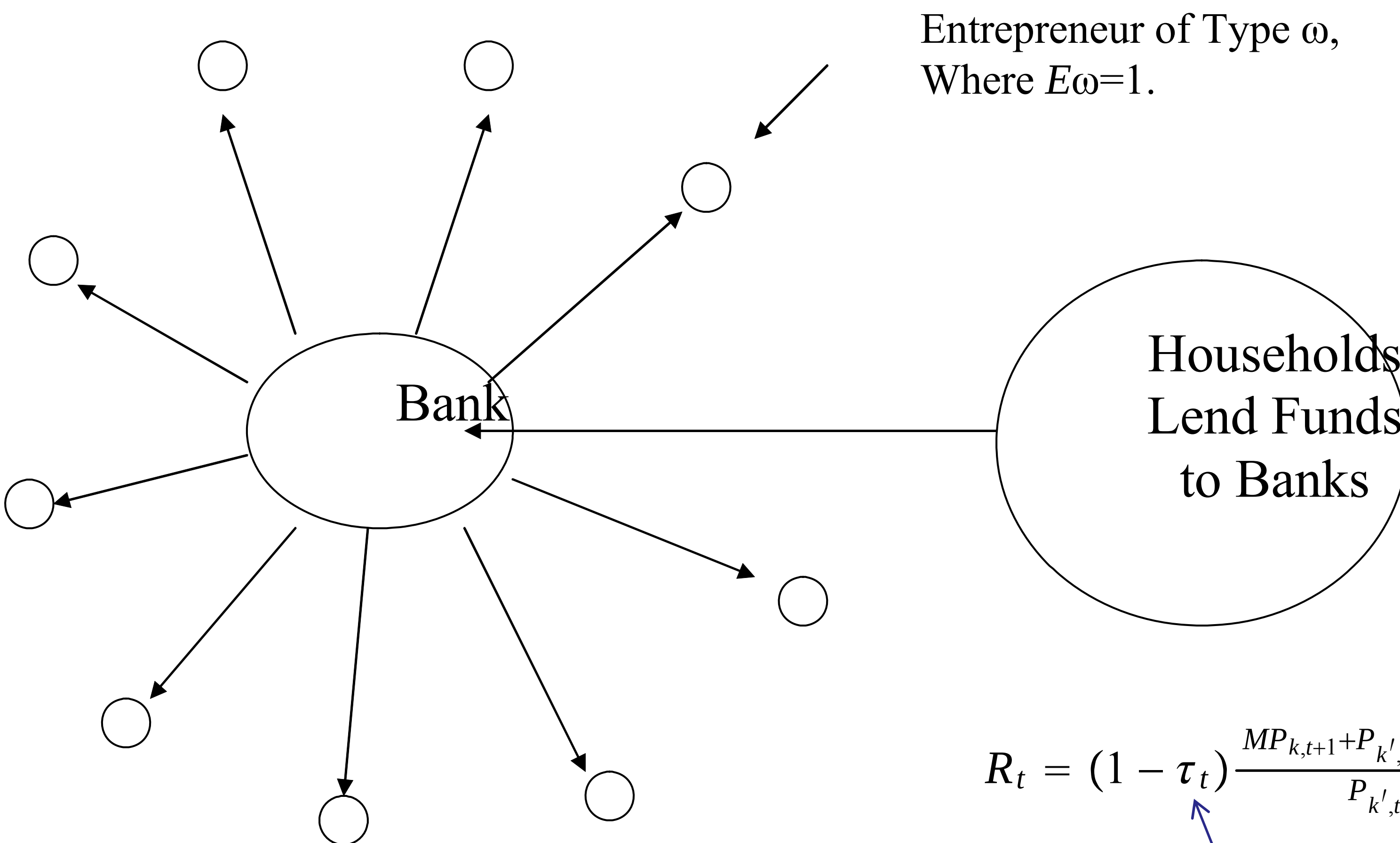
- In the standard model, household does the saving needed to produce capital, and the household is also the one that puts the capital to work (i.e., rents it out and decides its utilization rate).

- In practice, the saving decision and the operation of capital is done by different people. These different people may have a conflict of interest.

- Conflict of interest leads to ‘frictions’ (a loss of resources)

Bernanke-Gertler-Gilchrist model

- Source of friction:
  - People who provide funds and people who put funds to work are different people
  - When funds are put to work, idiosyncratic things happen that are known only to the people putting the funds to work.
  - Savers and borrowers can’t just share the output, because borrowers have an incentive to misreport earnings.



$$R_t = (1 - \tau_t)\frac{MP_{k,t+1} + P_{k',t+1}(1-\delta)}{P_{k',t}}$$

Financial friction

1. **Entrepreneurs receive standard debt contract**
2. **If productivity high enough, pay fixed rate**
3. **If bankrupt, turn over all assets and monitoring occurs**

Wage Decisions

- Households supply differentiated labor.
- Standard Calvo set up as in Erceg, Henderson and Levin and CEE.

Monetary and Fiscal Policy

$$x_t = M_t/M_{t-1}$$

$$\begin{aligned}
\hat{x}_{M,t} &= \rho_M \hat{x}_{M,t-1} + \varepsilon_{M,t} \\
\hat{x}_{z,t} &= \rho_{xz} \hat{x}_{z,t-1} + c_z \varepsilon_{z,t} + c_z^p \varepsilon_{z,t-1} \\
\hat{x}_{\Upsilon,t} &= \rho_{x\Upsilon} \hat{x}_{\Upsilon,t-1} + c_\Upsilon \varepsilon_{\Upsilon,t} + c_\Upsilon^p \varepsilon_{\Upsilon,t-1}
\end{aligned}$$

- $\hat{x}_{M,t}$: response of monetary policy to a monetary policy shock, $\varepsilon_{M,t}/$
- $\hat{x}_{z,t}$: response of monetary policy to an innovation in neutral technology, $\varepsilon_{z,t}$.
- $\hat{x}_{\Upsilon,t}$: response of monetary policy to an innovation in capital embodied technology, $\varepsilon_{\Upsilon,t}$.
- Government has access to lump sum taxes, pursues a Ricardian fiscal policy.

Loan Market and Final Good Market Clearing Conditions, Equilibrium

- Financial intermediaries receive $M_t - Q_t + (x_t - 1) M_t$ from the household.
  - Lend all of their money to intermediate good firms, which use the funds to pay for H_t.
- Loan market clearing
$$W_t H_t = x_t M_t - Q_t.$$
- The aggregate resource constraint is
$$(1 + \eta(V_t))C_t + \Upsilon_t^{-1} \left[I_t + a(u_t)\bar{K}_t \right] \ \le \ Y_t.$$
- We adopt a standard sequence-of-markets equilibrium concept.

Econometric Methodology

- Variant of limited information strategy used in CEE (2004).
  - Impose a subset of assumptions made in equilibrium model to estimate impulse response functions of ten key macroeconomic variables to the three shocks in our model.
  - Neutral technology shocks, capital embodied technology shocks and monetary policy shocks.
- Choose values for key parameters of structural model to minimize difference between estimated impulse response functions and analogous objects in model.

Estimating Parameters in the Model

- Partition Parameters into Three Groups.
  - Parameters set a priori (e.g., β, δ,...)
  - ζ_1: remaining parameters pertaining to the nonstochastic part of model

$$\zeta_2 = [\xi_w, \gamma, \sigma_a, b, S'', \epsilon]$$

  - ζ_2: parameters pertaining to stochastic part of the model
- Number of parameters, $\zeta = (\zeta_1, \zeta_2)$, to be estimated - 18
- Estimation Criterion
  - $\Psi(\zeta)$: mapping from ζ to model impulse responses
  - $\hat{\Psi}$: 592 impulse responses estimated using VAR
  - Estimation Strategy:

$$\hat{\zeta} = \arg\min_{\zeta} \left(\hat{\Psi} - \Psi(\zeta)\right)' V^{-1} \left(\hat{\Psi} - \Psi(\zeta)\right).$$

  - V: diagonal matrix with sample variances of $\hat{\Psi}$ along the diagonal.

- Parameter estimates

TABLE 2: ESTIMATED PARAMETER VALUES ζ_1							
Model	λ_f	ξ_w	γ	σ_a	b	S''	ϵ
Benchmark	1.35 (0.17)	.75 (0.06)	.32 (0.32)	0.06 (0.18)	0.80 (0.04)	4.85 (2.15)	0.77 (0.27)

- Parameters are surprisingly consistent with estimates reported in JPE (2005) based on studying only monetary policy shocks

- Note slope of Phillips curve is fairly large, but standard error is large too!
  - At point estimates: $\xi_p = 0.58, \ \frac{1}{1-\xi_p} = 2.38$ quarters

- Other parameters ‘reasonable’: estimation results really want sticky wages!

- Parameters of exogenous shocks:

TABLE 3: ESTIMATED PARAMETER VALUES ζ_2											
ρ_M	σ_M	ρ_{μ_z}	σ_{μ_z}	ρ_{xz}	c_z	c_z^p	ρ_{μ_Υ}	σ_{μ_Υ}	$\rho_{x\Upsilon}$	c_Υ	c_Υ^p
Benchmark Model											
-0.10 (0.12)	0.31 (0.10)	$.91$ (0.03)	0.05 (0.02)	0.36 (0.22)	3.68 (1.55)	2.49 (1.22)	-0.24 (0.52)	0.17 (0.06)	0.91 (0.07)	-0.10 (0.57)	0.63 (0.65)

- Neutral technology shock, ρ_{μ_z} ,is highly persistent.

Figure 1: Response to a monetary policy shock (o - Model, - VAR, grey area - 95 % Confidence Interval)

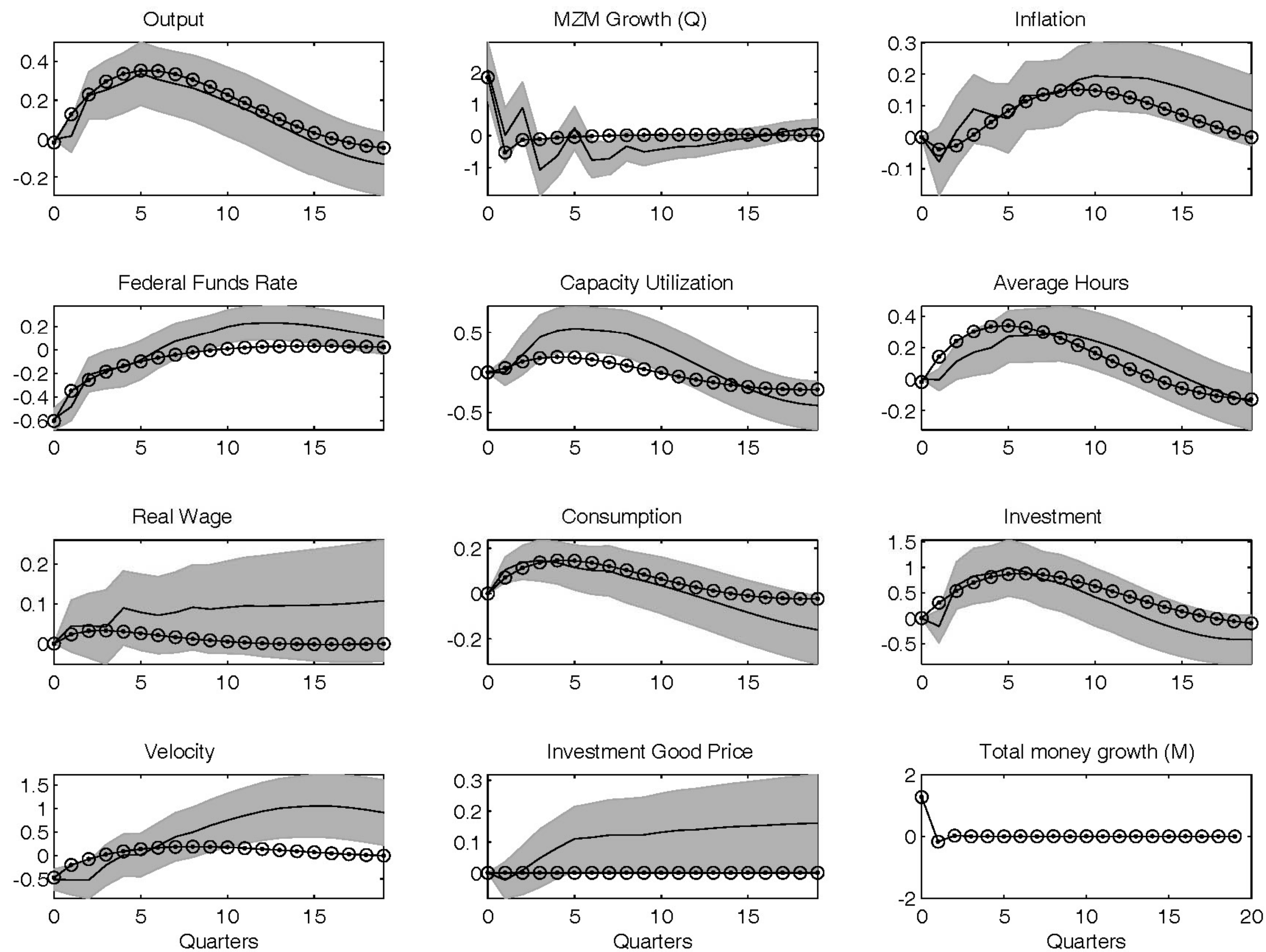


Figure 2: Response to a neutral technology shock (o - Model, - VAR, grey area - 95 % Confidence Interval)

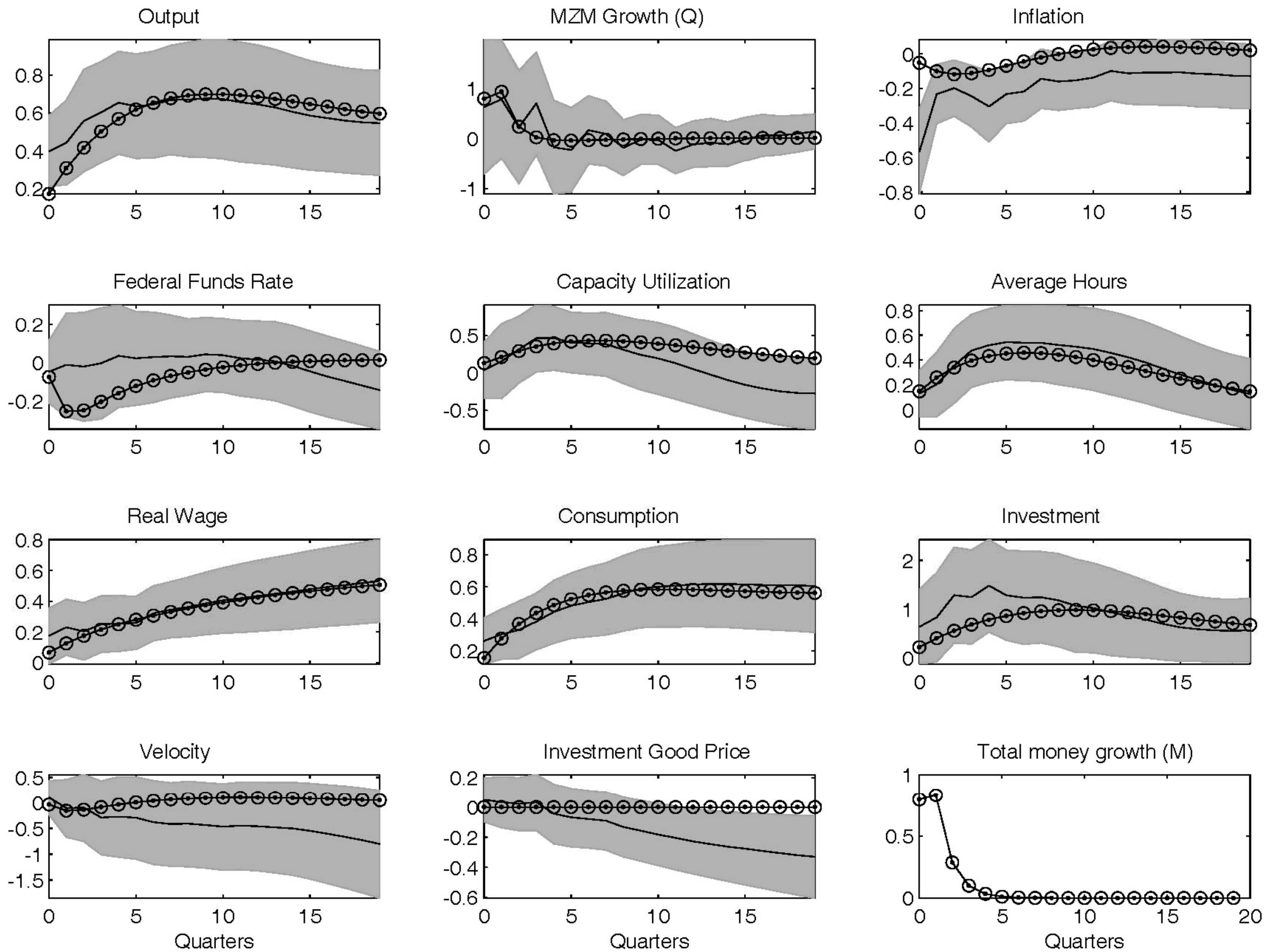
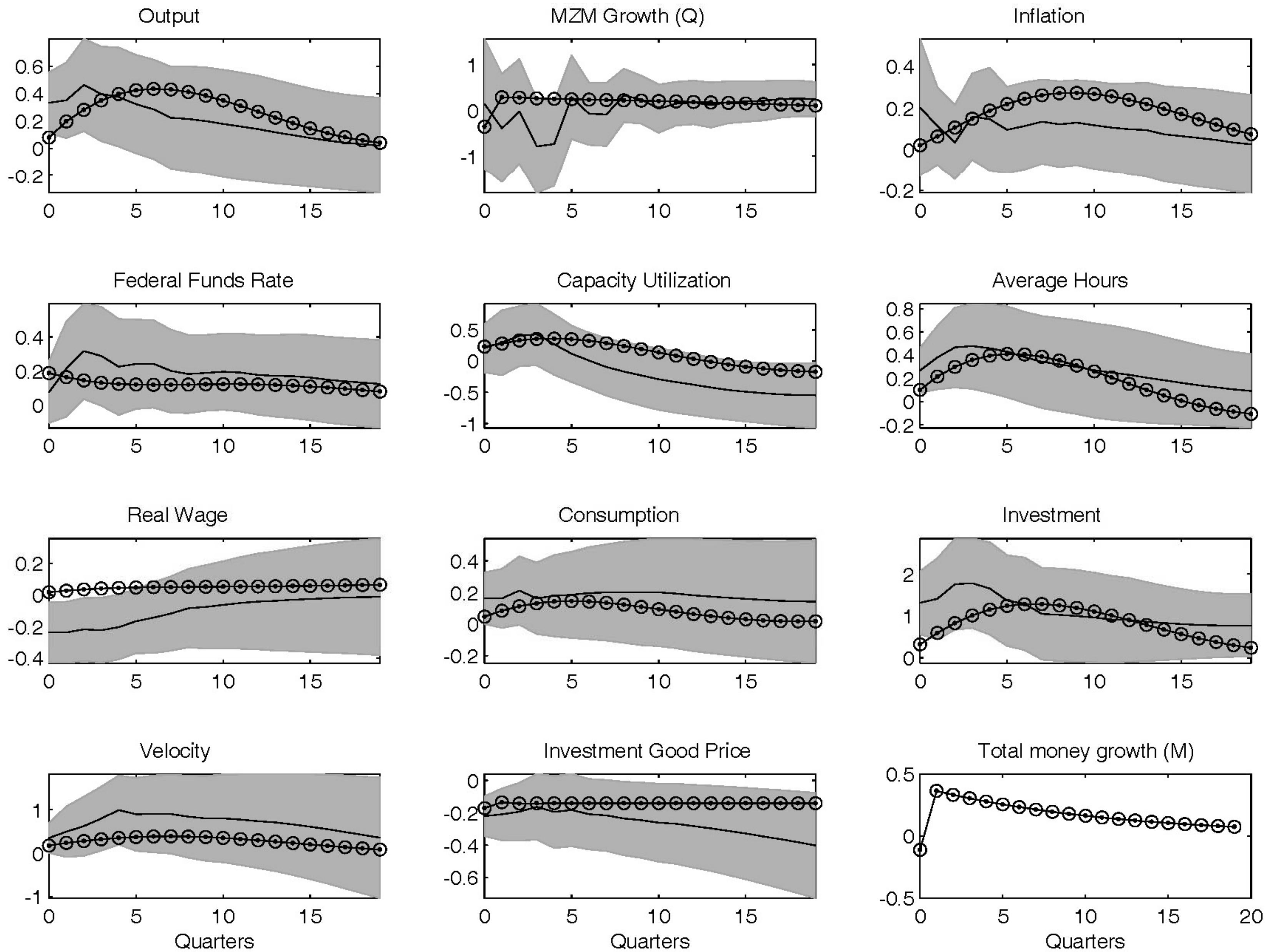


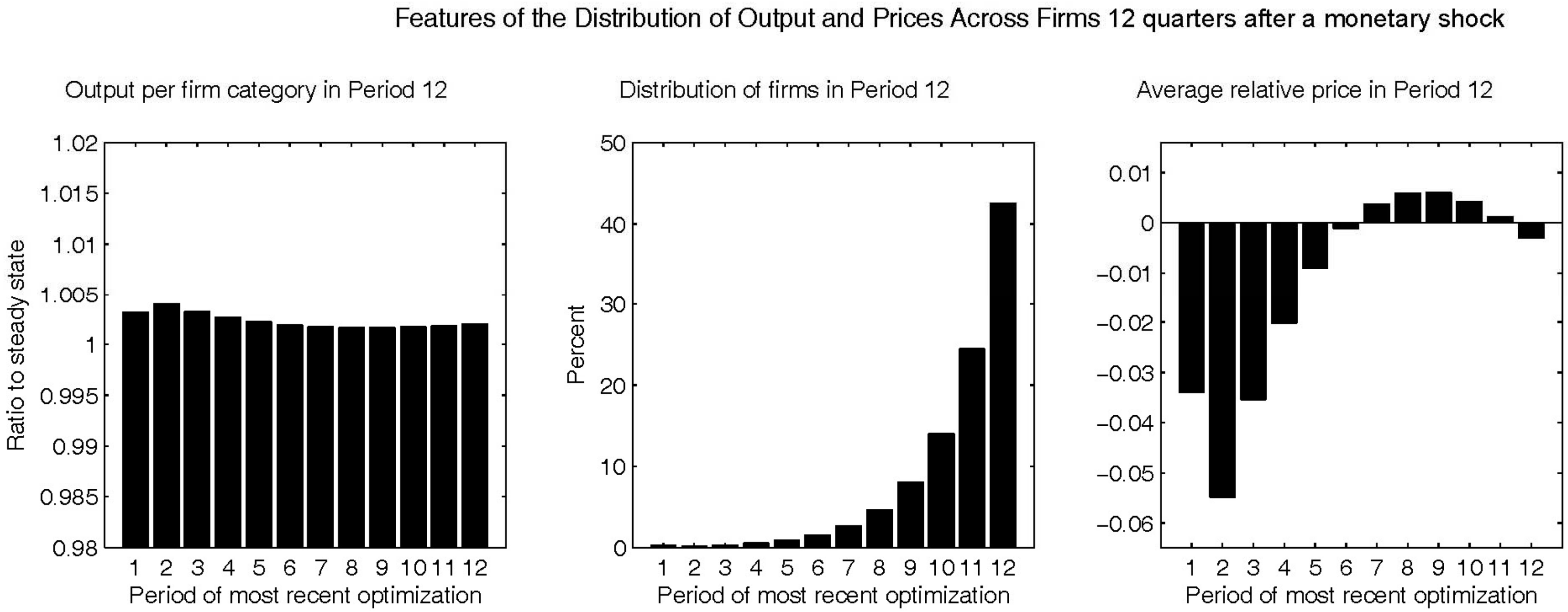
Figure 3: Response to an embodied technology shock (o - Model, - VAR, grey area - 95 % Confidence Interval)



The Experiment

- Begin at steady state and assume there is an expansionary monetary policy shock in period 1.
- Period 1
  - Prices and output is the same for all firms.
- Period 2
  - $(1-\xi_p)$ firms re-optimize and implement new price, ξ_p do not.
- Period 3: there are 4 types of firms.
  - $(1-\xi_p)^2$ re-optimize in period 2 and 3,
  - ξ_p^2 don't re-optimize in either period 2 or period 3.
  - $(1-\xi_p)\xi_p$ re-optimized in period 2 but not in period 3.
  - $\xi_p(1-\xi_p)$ did not re-optimize in period 2 but did re-optimize in period 3
- In period s there are 2^{s-1} different firms.
- For each period s we calculated the distribution of output and prices across firms.

Implications of the Estimated Model for the Distribution of Production Across Firms



Summary

- We constructed a dynamic GE model of cyclical fluctuations.
- Given assumptions satisfied by our model, we identified dynamic response of key US economic aggregates to 3 shocks
  - Monetary Policy Shocks
  - Neutral Technology Shocks
  - Capital Embodied Technology Shocks
- These shocks account for substantial cyclical variation in output.
- Estimated GE model does a good job of accounting for response functions (However, Misses on Inflation Response to Neutral Shock)
- Our point estimates suggest slope of Phillips curve steep, so there is no micro-macro price puzzle. However, large standard error.

Summary…

- Calvo Sticky Prices and Wages Seems Like Good Reduced Form
  - What is the Underlying Structure?
  - Is it information frictions?