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Extensions on the Basic Model

- Open economy (based on work of Adolfson-Laséen-LindéVillani and Christiano-Trabandt-Walentin (CTW))

- Search and matching in labor market (based on work of Gertler-Sala-Trigari and Christiano-Ilut-Motto-Rostagno, CTW).

- Financial frictions (based on work of Bernanke-Gertler-Gilchrist and Christiano-Motto-Rostagno, CTW)

Basic Model

- Model of Example #5

  – Household preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ \left(u\left(C_t\right) - \exp\left(\tau_t\right) \frac{N_t^{1+\varphi}}{1+\varphi} \right),\; u\left(C_t\right) \equiv \log C_t$$

  – Aggregate resources and household intertemporal optimization:

$$Y_t = p_t^* A_t N_t,\; u_{c,t} = \beta E_t u_{c,t+1} \frac{R_t}{\bar{\pi}_{t+1}}$$

  – Law of motion of price distortion:

$$p_t^* = \left((1-\theta) \left(\frac{1-\theta\left(\bar{\pi}_t\right)^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right)^{-1} . \qquad (1)$$

– Equilibrium conditions associated with price setting:

$$1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta \theta F_{t+1} = F_t \tag{2}$$

$$F_t \left[\frac{1 - \theta \bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} = K_t \tag{3}$$

$$\frac{\varepsilon}{\varepsilon - 1}(1 - \nu_t) \overbrace{\overbrace{\frac{\exp(\tau_t) N_t^{\varphi}}{u_{c,t}}}^{=\frac{W_t}{P_t} \text{ by household optimization}} \frac{1 - \psi + \psi R_t}{A_t}}^{= \text{intermediate good firm marginal cost}} + E_t \beta \theta \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \\ = K_t \tag{4}$$

Extension to Small Open Economy

- Outline
  - the equilibrium conditions of the open economy model
    - system jumps from 5-6 equations in basic model to 16 equations in 16 variables!
    - additional variables:

$$\overbrace{s_t, x_t, a_t^f, p_t^x, q_t}^{\text{rate of depreciation, exports, real foreign assets, terms of trade, real exchange rate, respectively}},$$

$$\overbrace{p_t^c}^{\text{price of domestic consumption (now, } c \text{ is a composite of domestically produced goods and imports)}},$$

$$\overbrace{p_t^{m,c}}^{\text{price of imports}}, \quad \overbrace{\pi_t^c}^{\text{consumption price inflation}},$$

$$\overbrace{\Phi_t}^{\text{reduced form object to (i) achieve technical objective, (ii) correct a fundamental failing of open economy models}},$$

$$\overbrace{R_t, \bar{\pi}_t, N_t, c_t, K_t, F_t, p_t^*}^{\text{closed economy variables}}.$$

Extension to Small Open Economy ...

– computing the steady state

– the ‘uncovered interest parity puzzle’, and the role of Φ_t in addressing the puzzle.

– summary of the endogenous and exogenous variables of the model, as well as the equations.

– several computational experiments to illustrate the properties of the model.

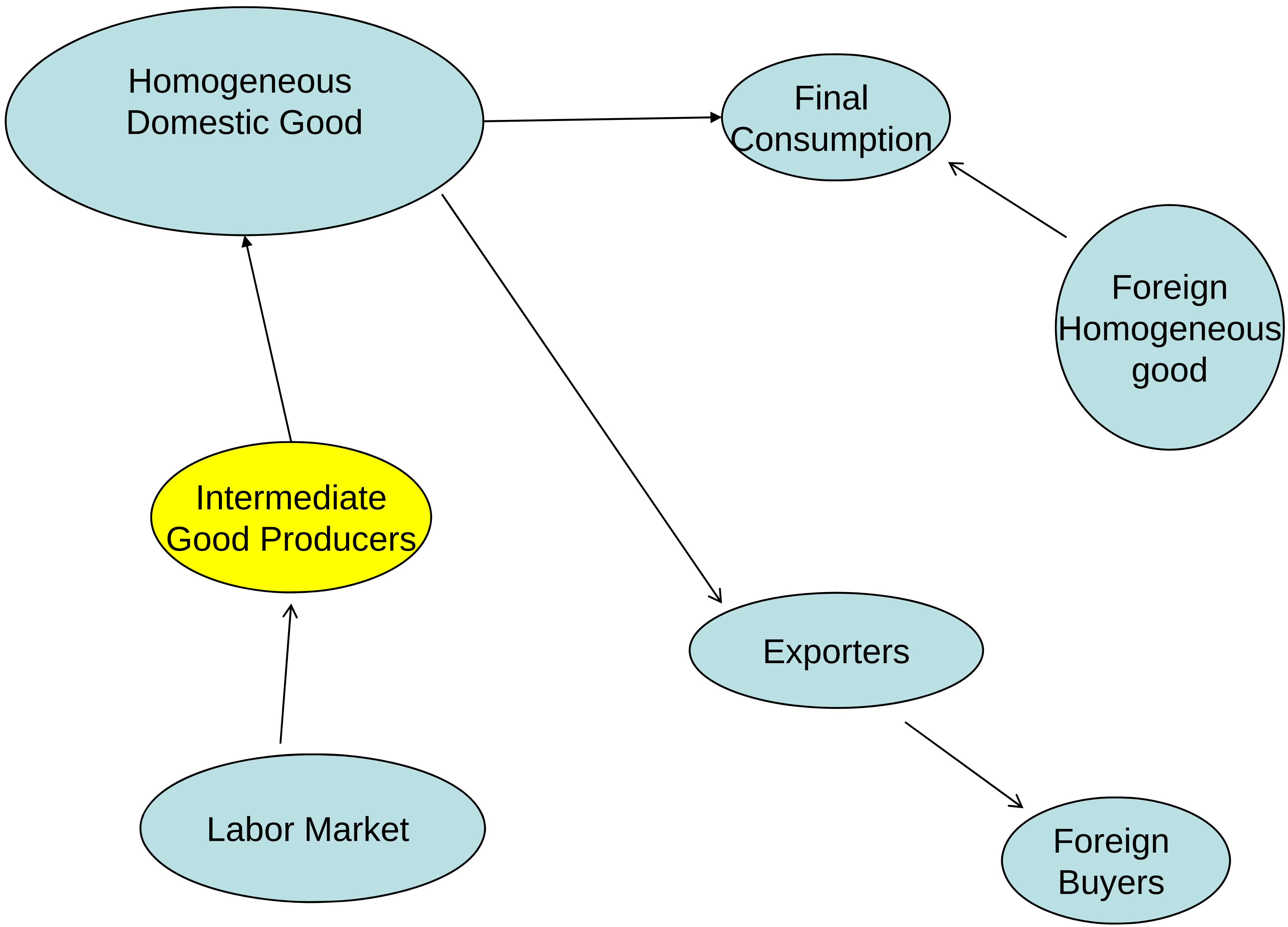
Extension to Small Open Economy ...

- Modifications to basic model to create open economy

  - unchanged:
    * household preferences
    * production of (domestic) homogeneous good, Y_t ($= A_t p_t^* N_t$)
    * three Calvo price friction equations

  - changes:
    * household budget constraint includes opportunity to acquire foreign assets/liabilities.
    * intertemporal Euler equation changed as a reduced form accommodation of evidence on uncovered interest parity.
    * $Y_t = C_t$ no longer true.
    * introduce exports, imports, current account.
    * exchange rate,

$$\boxed{S_t = \text{domestic currency price of one unit of foreign currency} = \frac{\text{domestic money}}{\text{foreign money}}.}$$



Homogeneous Domestic Good
Final Consumption
Foreign Homogeneous good
Intermediate Good Producers
Exporters
Labor Market
Foreign Buyers

- Monetary policy: three approaches
  - Taylor rule

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\, E_t[r_\pi \log\left(\frac{\pi^c_{t+1}}{\bar{\pi}^c}\right) + r_y \log\left(\frac{y_{t+1}}{y}\right)] + \varepsilon_{R,t}, \tag{5}$$

where (could also add exchange rate, real exchange rate and other things):

$\bar{\pi}^c$ ~target consumer price inflation

$\varepsilon_{R,t}$ ~iid, mean zero monetary policy shock

y_t ~Y_t/A_t

R_t ~'risk free' nominal rate of interest

  - Svensson-style policy that solves Ramsey problem with the following preferences:

$$E_t \sum_{j=0}^{\infty} \beta^j \{ \left(100\left[\pi^c_t \pi^c_{t-1} \pi^c_{t-2} \pi^c_{t-3} - (\bar{\pi}^c_t)^4\right]\right)^2 + \lambda_y \left(100 \log\left(\frac{y_t}{y}\right)\right)^2$$

$$+\lambda_{\Delta R} \left(400\left[R_t - R_{t-1}\right]\right)^2 + \lambda_s \left(S_t - \bar{S}\right)^2 \}$$

  - straight Ramsey policy that maximizes domestic social welfare.

Extension to Small Open Economy ...

- Household budget constraint

$$S_t A_{t+1}^f + P_t C_t + B_{t+1}$$
$$\leq \; B_t R_{t-1} + S_t \left[\Phi_{t-1} R_{t-1}^f\right] A_t^f + W_t N_t + Transfers\, and\, profits_t$$

- Domestic bonds

B_t ~beginning of period t stock of loans

R_t ~rate of return on bonds

- Foreign assets

A_t^f ~beginning-of-period t net stock of foreign assets
(liabilities, if negative) held by domestic residents.

$\Phi_t R_t^f$ ~rate of return on A_t^f

Φ_t ~premium on foreign asset returns, over foreign risk free rate, R_t^f

- optimality of household foreign asset decision (verify this by solving Lagrangian)

$$\frac{\overbrace{u_{c,t}S_t}^{\text{utility cost of one unit of foreign currency}=S_t \text{ units of domestic currency, which is } S_t/P_t^c \text{ units of } C_t}}{P_t^c}$$

$$= \beta E_t \overbrace{u_{c,t+1}}^{\text{conversion into utility units}} \times \overbrace{\frac{S_{t+1} \overbrace{R_t^f \Phi_t}^{\text{foreign currency payoff next period from one unit of foreign currency today}}}{P_{t+1}^c}}^{\text{quantity of domestic consumption goods that can be purchased from the payoff of one unit of foreign currency}}$$

or

$$\frac{S_t}{P_t^c C_t} = \beta E_t \frac{S_{t+1} R_t^f \Phi_t}{P_{t+1}^c C_{t+1}}$$

or

$$\frac{1}{c_t} = \beta E_t \frac{s_{t+1} R_t^f \Phi_t}{\pi_{t+1}^c c_{t+1} \exp\left(\Delta a_{t+1}\right)},\ s_t \equiv \frac{S_t}{S_{t-1}},\ c_t = \frac{C_t}{A_t}. \tag{6}$$

- Optimality of household domestic bond decision:

$$\frac{1}{P_t^c C_t} = \beta E_t \frac{R_t}{P_{t+1}^c C_{t+1}}$$

  – after scaling:

$$\frac{1}{c_t} = \beta E_t \frac{R_t}{\pi_{t+1}^c c_{t+1} \exp\left(\Delta a_{t+1}\right)}. \tag{7}$$

Extension to Small Open Economy ...

- Final domestic consumption goods, C_t
  - produced by representative, competitive firm using:

$$C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}} \left(C_t^d \right)^{\frac{(\eta_c-1)}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(C_t^m \right)^{\frac{(\eta_c-1)}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}$$

C_t^d ~one-for-one transformation on domestic homogeneous output good, price P_t

C_t^m ~imported good, with price $P_t^{m,c}$

C_t ~final consumption good, with price, P_t^c

η_c ~elasticity of substitution between domestic and foreign goods.

  - Profit maximization leads to:

$$\begin{aligned} C_t^d &= (1-\omega_c) \left(p_t^c \right)^{\eta_c} C_t \\ C_t^m &= \omega_c \left(\frac{p_t^c}{p_t^{m,c}} \right)^{\eta c} C_t. \\ p_t^c &= \left[(1-\omega_c) + \omega_c \left(p_t^{m,c} \right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \qquad (8) \\ p_t^c &\equiv \frac{P_t^c}{Pt},\ p_t^{m,c} \equiv \frac{P_t^{m,c}}{P_t} \end{aligned}$$

Extension to Small Open Economy ...

– C_t^m is produced by competitive firm, which converts foreign homogeneous output one-for-one into C_t^m.

$*$ Setting price equal to marginal cost:

$$P_t^{m,c} = S_t P_t^f \left(1 - \psi^f + \psi^f R_t^f\right), \; P_t^f$$ ~foreign currency price of foreign good.

or,

$$p_t^{m,c} = \frac{P_t^c}{P_t^c}\frac{S_t P_t^f}{P_t}\left(1 - \psi^f + \psi^f R_t^f\right) = p_t^c \overbrace{q_t}^{\text{real exchange rate } \equiv \frac{S_t P_t^f}{P_t^c}} \left(1 - \psi^f + \psi^f R_t^f\right). \tag{9}$$

– Consumption good inflation:

$$\pi_t^c \equiv \frac{P_t^c}{P_{t-1}^c} = \frac{P_t p_t^c}{P_{t-1} p_{t-1}^c} = \bar{\pi}_t \left[\frac{(1-\omega_c) + \omega_c \left(p_t^{m,c}\right)^{1-\eta_c}}{(1-\omega_c) + \omega_c \left(p_{t-1}^{m,c}\right)^{1-\eta_c}}\right]^{\frac{1}{1-\eta_c}}. \tag{10}$$

- Exports, X_t
  - foreign demand for exports

$$X_t = \left(\frac{P_t^x}{P_t^f}\right)^{-\eta_f} Y_t^f = \left(p_t^x\right)^{-\eta_f} Y_t^f \tag{11}$$

Y_t^f ~foreign output, P_t^f ~price of foreign good, P_t^x ~ price of export

  - X_t is produced one-for-one using the domestic homogeneous good by a representative, competitive producer. Equating price, $S_t P_t^x$, to marginal cost:

$$S_t P_t^x = P_t \left(\nu^x R_t + 1 - \nu^x\right),$$

where $\nu^x = 1$ if all inputs must be financed in advance. Rewriting

$$q_t p_t^x p_t^c = \nu^x R_t + 1 - \nu^x, \tag{12}$$

where

$$\overbrace{q_t}^{\text{real exchange rate}} \equiv \frac{S_t P_t^f}{P_t^c}, \quad \overbrace{p_t^x}^{\text{terms of trade}} = \frac{P_t^x}{P_t^f}$$

$$\frac{q_t}{q_{t-1}} = s_t \frac{\pi_t^f}{\pi_t^c}, \; s_t = \frac{S_t}{S_{t-1}}. \tag{13}$$

- Clearing in domestic homogeneous goods market:

$$\begin{aligned}
& \text{output of domestic homogeneous good, } Y_t \\
= & \text{ uses of domestic homogeneous goods} \\
= & \overbrace{C_t^d}^{\text{goods used in production of final consumption, } C_t} + \overbrace{X_t}^{\text{exports}} + \overbrace{G_t}^{\text{government}} \\
= & \left(1-\omega_c\right)\left(p_t^c\right)^{\eta_c} C_t + X_t + G_t .
\end{aligned}$$

- Substituting out for Y_t:

$$A_t p_t^* N_t = \left(1-\omega_c\right)\left(p_t^c\right)^{\eta_c} C_t + X_t + G_t,$$

or,

$$p_t^* N_t = \left(1-\omega_c\right)\left(p_t^c\right)^{\eta_c} c_t + x_t + g_t, \tag{14}$$

$$c_t \equiv \frac{C_t}{A_t},\ x_t \equiv \frac{X_t}{A_t},\ g_t \equiv \frac{G_t}{A_t}.$$

Extension to Small Open Economy ...

- Current Account
  - equality of international demand and supply for currency:

$$\overbrace{\underbrace{S_t A_{t+1}^f}_{\text{acquisition of new net foreign assets, in domestic currency units}} + \text{expenses on imports}_t}^{\text{currency flowing out of the country}}$$

$$= \overbrace{\text{receipts from exports}_t + \underbrace{S_t R_{t-1}^f \Phi_{t-1} A_t^f}_{\text{receipts from existing stock of net foreign assets}}}^{\text{currency flowing into the country}},$$

  - The pieces:

$$\text{expenses on imports}_t = S_t P_t^f \left(1 - \psi^f + \psi^f R_t^f\right) \omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta c} C_t$$

$$\text{receipts from exports}_t = S_t P_t^x X_t.$$

  - Current account:

$$S_t A_{t+1}^f + S_t P_t^f \left(1 - \psi^f + \psi^f R_t^f\right) \omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta_c} C_t = S_t P_t^x X_t + S_t R_{t-1}^f \Phi_{t-1} A_t^f.$$

– Divide current account by P_tA_t :

$$\frac{S_tA_{t+1}^f}{P_tA_t}+\frac{S_tP_t^f}{P_t}\left(1-\psi^f+\psi^fR_t^f\right)\omega_c\left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta_c}c_t=\frac{S_tP_t^x}{P_t}x_t+\frac{S_tR_{t-1}^f\Phi_{t-1}A_t^f}{P_tA_t},$$

or, using (9):

$$a_t^f+p_t^{m,c}\omega_c\left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta_c}c_t=p_t^cq_tp_t^xx_t+\frac{s_tR_{t-1}^f\Phi_{t-1}a_{t-1}^f}{\bar{\pi}_t\exp\left(\Delta a_t\right)}, \tag{15}$$

where a_t^f is ‘scaled real, domestic value of foreign assets’:

$$a_t^f=\frac{S_tA_{t+1}^f}{P_tA_t}$$

Extension to Small Open Economy ...

- ‘Risk’ adjustments

$$\Phi_t = \Phi\left(a_t^f, R_t^f, R_t, \tilde{\phi}_t\right) = \tag{16}$$
$$\exp\left(-\tilde{\phi}_a\left(a_t^f - \bar{a}\right) - \tilde{\phi}_s\left(R_t^f - R_t - \left(R^f - R\right)\right) + \tilde{\phi}_t\right)$$

$\tilde{\phi}_a > 0,$ small and not important for dynamics
$\tilde{\phi}_s > 0,$ important
$\tilde{\phi}_t$ ~mean zero, iid.

- Discussion of $\tilde{\phi}_a$.
  - $\tilde{\phi}_a > 0$ implies (i) if $a_t^f > \bar{a}$, then return on foreign assets low and $a_t^f \downarrow$; (ii) if $a_t^f < \bar{a}$, then return on foreign assets high and $a_t^f \uparrow$
  - implication: $\tilde{\phi}_a > 0$ is a force that drives $a_t^f \rightarrow \bar{a}$ in steady state, independent of initial conditions.
  - logic is same as reason why steady state stock of capital in neoclassical growth model is unique, independent of initial conditions.
  - in practice, $\tilde{\phi}_a$ is tiny, so that its only effect is to pin down a_t^f in steady state and not affect dynamics (see Schmitt-Grohe and Uribe).

- Discussion of $\tilde{\phi}_t$

  – Captures, informally, the possibility that there is a shock to the required return on domestic assets. Perhaps this could be a crude stand-in for a 'sub-prime mortgage crisis', because it implies that people require a higher return on domestic assets if they are to hold them.

- Discussion of $\tilde{\phi}_s$.

  – $\tilde{\phi}_s$ is an important reduced form feature, designed to correct an important flaw in models of international finance. It represents a quick fix for the problem, not a substitute for a longer-run solution.

  – to better explain this, it is convenient to first solve for the model's steady state.

- Steady state
  - household intertemporal efficiency conditions:

$$0 = E_t\left[\frac{1}{c_t} - \beta\frac{s_{t+1}R_t^f\Phi_t}{\pi_{t+1}^c c_{t+1}\exp\left(\Delta a_{t+1}\right)}\right], \text{ steady state: } 1 = \beta\frac{sR^f\Phi}{\pi^c} \quad (17)$$

$$0 = E_t\left[\frac{1}{c_t} - \beta\frac{1}{c_{t+1}}\frac{R_t}{\pi_{t+1}^c\exp\left(\Delta a_{t+1}\right)}\right], \text{ steady state: } 1 = \beta\frac{R}{\pi^c} \quad (18)$$

  - assumption about foreign households:

$$1 = \beta\frac{R^f}{\pi^f} \quad (19)$$

$$\pi_t^f \equiv \frac{P_t^f}{P_{t-1}^f} \text{ (exogenous)}$$

– Taylor rule:

$$\pi^c = \bar{\pi}^c \text{ (central bank's inflation target).} \tag{20}$$

– from (10):

$$\pi^c = \bar{\pi} \equiv \frac{P_t}{P_{t-1}}. \tag{21}$$

– using price friction equilibrium conditions:

$$p^* = \frac{\frac{1-\theta\bar{\pi}^{\varepsilon}}{1-\theta}}{\left(\frac{1-\theta(\bar{\pi})^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}}, \text{ (no distortion if } \bar{\pi} = 1, \text{)} \tag{22}$$

$$F = \frac{1}{1-\beta\theta\bar{\pi}^{\varepsilon-1}}, \text{ (don't allow } \bar{\pi}^{\varepsilon-1}\beta\theta < 1\text{)} \tag{23}$$

$$K = \frac{\frac{\varepsilon}{\varepsilon-1}(1-\nu) \ \exp(\tau) N^{\varphi+1} p^* (1-\psi+\psi R)}{1-\beta\theta\bar{\pi}^{\varepsilon}}, \ (\beta\theta\bar{\pi}^{\varepsilon} < 1) \tag{24}$$

$$K = F\left[\frac{1-\theta\bar{\pi}^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}. \tag{25}$$

– other steady state conditions:

$$p^c = \left[(1-\omega_c) + \omega_c \left(p^{m,c}\right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \tag{26}$$

$$p^{m,c} = p^c q \left(1 - \psi^f + \psi^f R^f\right) \tag{27}$$

$$q p^x p^c = \nu^x R + 1 - \nu^x \tag{28}$$

$$p^* N = (1-\omega_c)\left(p^c\right)^{\eta_c} c + x + g \tag{29}$$

$$a^f + p^c q \left(1 - \psi^f + \psi^f R^f\right) \omega_c \left(\frac{p^c}{p^{m,c}}\right)^{\eta_c} c \;=\; p^c q p^x x + \frac{s R^f \Phi a^f}{\bar{\pi}} \tag{30}$$

$$x = \left(p^x\right)^{-\eta_f} y^f \tag{31}$$

– 15 equations: (17)-(31), 15 unknowns:

$$p^c, p^{m,c}, q, p^x, c, x, a^f, R, \Phi, \bar{\pi}, K, F, p^*, \pi^c, s$$

– for convenience, set exogenous variables, g, $\bar{a}$,

$$g = \eta_g y, \;\; \bar{a} = \eta_a y, \;\; \text{where } y = p^* N.$$

– algorithm for solving for the steady state:

* p^*, F, K can be computed from (21), (22), (23) and (25).
* solve (24) for N.
* solve

$$\begin{aligned} g &= \eta_g p^* N \\ a^f &= \bar{a} = \eta_a p^* N. \end{aligned}$$

* (18), (19) imply

$$\frac{R^f}{\pi^f} = \frac{R}{\pi^c} \tag{32}$$

* steady state depreciation, s, can be computed from the inflation differential:

$$q_t \rightarrow q \text{ implies (see (13)) } s\pi^f = \pi^c.$$

* (17), (18) imply

$$sR^f \Phi = R, \tag{33}$$

or after multiplication by π^f and rearranging,

$$\frac{R^f}{\pi^f}\Phi = \frac{R}{\pi^c}, \text{ so (see (32)) } \Phi = 1 \text{ and } a_t = \bar{a} \text{ (see (16))}$$

– rest of the algorithm solves a single non-linear equation in a single unknown.

– set

$$\tilde{\varphi} = p^c q.$$

– use (27), (28), (26):

$$\begin{aligned}
p^{m,c} &= \tilde{\varphi} R^{\nu,*} \\
p^x &= \frac{R^x}{\tilde{\varphi}}, \\
p^c &= \left[(1-\omega_c) + \omega_c \left(p^{m,c}\right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \\
q &= \frac{\tilde{\varphi}}{p^c}.
\end{aligned}$$

– solve the resource constraint, (29), for c in terms of x :

$$c = \frac{p^c q p^x x + \frac{sR^f a^f}{\bar{\pi}} - a^f}{p^c q \left(1 - \psi^f + \psi^f R^f\right) \omega_c \left(\frac{p^c}{p^{m,c}}\right)^{\eta_c}}.$$

– use the latter to substitute out for c in the current account, (30):

$$
a^f + p^c q \left(1 - \psi^f + \psi^f R^f\right) \omega_c \left(\frac{p^c}{p^{m,c}}\right)^{\eta_c} \frac{p^c q p^x x + \frac{sR^f a^f}{\bar{\pi}} - a^f}{p^c q \left(1 - \psi^f + \psi^f R^f\right) \omega_c \left(\frac{p^c}{p^{m,c}}\right)^{\eta_c}}
$$
$$
= p^c q p^x x + \frac{sR^f a^f}{\bar{\pi}},
$$

which can be solved linearly for x.

– evaluate (31) and adjust $\tilde{\varphi}$ until it is satisfied. In practice, we set $\tilde{\varphi} = 1$ and used (31) to define y^f.

- Uncovered interest rate parity puzzle and Φ_t^b
  - subtract equations (17) and (18):

$$E_t\left[\frac{R_t - s_{t+1}R_t^f\Phi_t}{c_{t+1}\pi_{t+1}^c\exp\left(\Delta a_{t+1}\right)}\right] = 0. \tag{34}$$

  - totally differentiate the object in square brackets, and evaluate in steady state

$$d\frac{R_t - s_{t+1}R_t^f\Phi_t}{c_{t+1}\pi_{t+1}^c\exp\left(\Delta a_{t+1}\right)} = \frac{dR_t}{c\pi^c} - \frac{1}{c\pi^c}\left[sR^f d\Phi_t + sdR_t^f + R^f ds_{t+1}\right]$$
$$-\frac{R - sR^f}{\left[c\pi^c\right]^2}d\left[c_{t+1}\pi_{t+1}^c\exp\left(\Delta a_{t+1}\right)\right],$$

so that, taking into account (33), (34) is, to a first approximation:

$$\hat{R}_t = E_t\hat{s}_{t+1} + \hat{R}_t^f + \hat{\Phi}_t, \qquad \hat{x}_t = \log\left(x_t\right) - \log\left(x\right) = \frac{x_t - x}{x}.$$

– Note:

$$\hat{R}_t = \log R_t - \log(R) \simeq r_t - \log R, \qquad \hat{R}_t^f = \log R_t^f - \log\left(R^f\right) \simeq r_t^f - \log R^f$$
$$R_t \equiv 1 + r_t, \; R_t^f \equiv 1 + r_t^f,$$

so that:

$$r_t - \log(R) = \log S_{t+1} - \log S_t - \log s + r_t^f - \log R^f + \hat{\Phi}_t.$$

It follows from:

$$\log(R) - \log s - \log R^f = \log\left(\frac{R}{sR^f}\right) = 0,$$

that

$$\boxed{r_t = E_t \log S_{t+1} - \log S_t + r_t^f + \hat{\Phi}_t} \qquad (35)$$

$$\hat{\Phi}_t = \log \Phi_t = -\tilde{\phi}_a\left(a_t^f - \bar{a}\right) - \tilde{\phi}_s\left(r_t^f - r_t - \left(r^f - r\right)\right) + \tilde{\phi}_t$$

which is our log-linear expansion of (34).

Extension to Small Open Economy ...

– Uncovered Interest Parity (UIP)

  * Under UIP, $\hat{\Phi}_t \equiv 0$ and

$r_t > r_t^f \rightarrow$ there must be an anticipated depreciation (instantaneous appreciation) of the currency for people to be happy holding the existing stock of net foreign assets.

  * Consider the standard 'UIP regression' ($\tilde{\phi}_a \simeq 0$, $\tilde{\phi}_t = 0$), involving risk free rate differentials:

$$\log S_{t+1} - \log S_t = \alpha + \beta \left(r_t - r_t^f \right) + u_t.$$

  * Substitute out for $\log S_{t+1} - \log S_t$ from (35) and make use of the fact that a (rational expectations) forecast error is orthogonal to date t information, to obtain:

$$\hat{\beta} = \frac{cov\left(\log S_{t+1} - \log S_t, r_t - r_t^f \right)}{var\left(r_t - r_t^f \right)} = 1 - \tilde{\phi}_s,$$

Extension to Small Open Economy ...

$*$ In data,

- $\hat{\beta} \simeq -.75$, so UIP rejected (that's the *UIP puzzle*)
- $\tilde{\phi}_s = 1.75 \rightarrow \hat{\beta} = -0.75.$

$*$ VAR impulse responses by Eichenbaum and Evans (QJE, 1992)

- data: $r_t \uparrow$ after monetary policy shock $\rightarrow \log S_{t+j}$ falls slowly for $j = 1, 2, 3, ...$.

- UIP puzzle: $r_t \uparrow$ *and* expected appreciation of the currency represents a double-boost to the return on domestic assets. On the face of it, it appears that there is an irresistible profit opportunity. Why doesn't the double-boost to domestic returns launch an avalanche of pressure to buy the domestic currency? In standard models, this pressure produces a greater instantaneous appreciation in the exchange rate, until the familiar UIP overshooting result emerges - the pressure to buy the currency leads to such a large appreciation, that expectations of depreciation emerge. In this way, UIP leads to the counterfactual prediction that a higher r_t will be followed (after an instantaneous appreciation) by a period of time during which the currency depreciates.

· model's resolution of the UIP puzzle: when $r_t \uparrow$ the return required for people to hold domestic bonds rises. This is why the double-boost to domestic returns does not create an appetite to buy large amounts of domestic assets. Possibly this is a reduced form way to capture the notion that increases in r_t make the domestic economy more risky. (However, the precise mechanism by which the domestic required return rises - earnings on *foreign* assets go up - may be difficult to interpret. An alternative specification was explored, with risk-premia affecting domestic bonds, but this resulted in indeterminacy problems.)

- Model dynamics
  - 16 equations: price setting, (1), (2),(3) and (4); monetary policy rule, (5); household intertemporal Euler equations (6), (7); relative price equations (13), (8), (9), (10), (12); aggregate resource condition, (14); current account, (15); risk term, (16); demand for exports (11).
  - 16 endogenous variables: $p_t^c, p_t^{m,c}, q_t, R_t, \bar{\pi}_t, \pi_t^c, p_t^x, N_t, p_t^*, a_t^f, \hat{\Phi}_t, s_t, x_t, c_t, K_t, F_t.$
  - exogenous variables: $R_t^f, y_t^f, \tilde{\phi}_t, g_t, \varepsilon_{R,t}, \Delta a_t, \tau_t, \pi_t^f.$
    for the purpose of numerical calculations, these were modeled as independent scalar AR(1) processes.
  - the model was solved in the manner described above:
    * compute the steady state using the formulas described above
    * log-linearize the 16 equations about steady state
    * solve the log-linearized system
    * these calculations were made easy by implementing them in Dynare.

Extension to Small Open Economy ...

- Numerical examples
- Parameter values:

$$\overbrace{\bar{\pi}^c = \pi^f}^{\text{foreign and domestic inflation same}} = 1.005,\ \overbrace{\psi = \psi^f = \nu^x = 0}^{\text{no financial frictions}},\ \overbrace{\tilde{\phi}_a = 0.03}^{\text{small weight on } a_t^f \text{ in } \Phi_t},$$

$$\beta = 1.03^{-1/4},\ \overbrace{\theta = 3/4}^{\text{prices unchanged on average for 1 year}},\ \overbrace{\varphi = 1}^{1/\varphi \text{ Frisch elasticity}},$$

$$\overbrace{\varepsilon = 6}^{\text{modest elasticity of demand for domestic intermediate goods}},\ \overbrace{1 - \nu_t = \frac{\varepsilon - 1}{\varepsilon}}^{\text{subsidy extinguishes monopoly power in labor margin}},$$

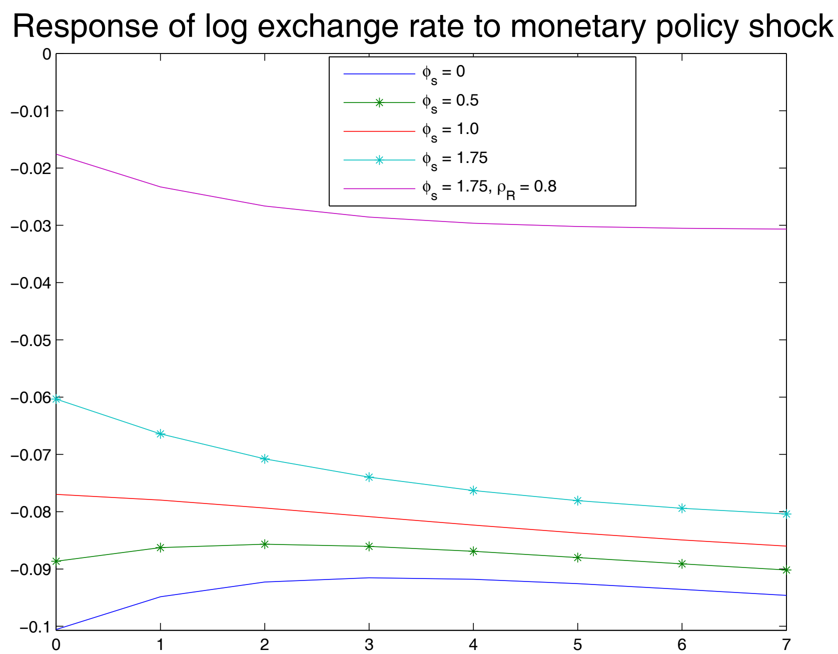
$$\overbrace{\eta_c = 5}^{\text{elasticity of substitution between domestic and foreign inputs in producing final consumption}}$$

$$\overbrace{\omega_c = 0.4}^{\text{roughly 60\% of domestic final consumption is composed of domestic content}},\ \overbrace{\eta_g = 0.3}^{\text{share of } g \text{ in } y},\ \overbrace{\eta_a = 0}^{\text{net foreign assets/y}},$$

$$\overbrace{\eta_f = 1.5}^{\text{elasticity of demand for exports as function of relative price paid by foreigners}},\ \overbrace{\rho_R = 0.9,\ r_\pi = 1.5, r_y = 0.15}^{\text{monetary policy rule parameters}}$$

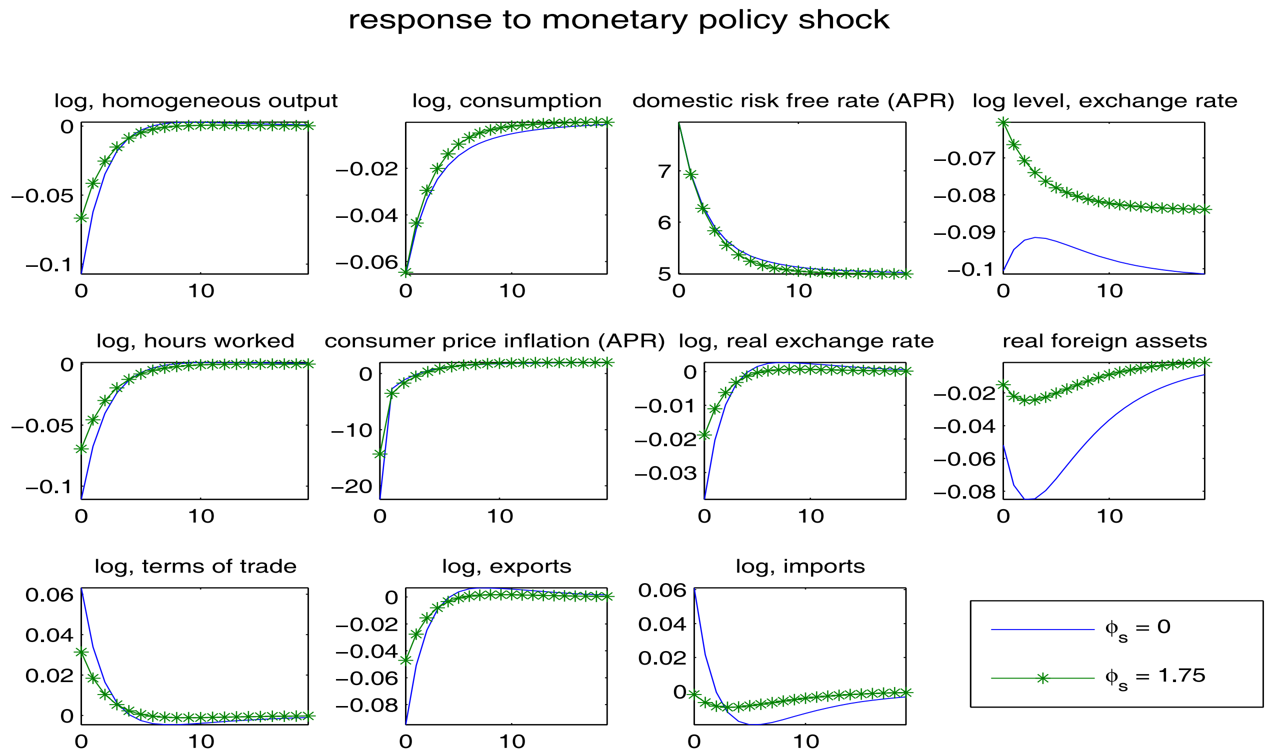
Extension to Small Open Economy ...

- iid shock, 0.01, to $\varepsilon_{R,t}$.
  - $\tilde{\phi}_s = 0 \longrightarrow$after instantaneous appreciation, positive $\varepsilon_{R,t}$ shock followed by depreciation.
  - for higher $\tilde{\phi}_s$, shock followed by appreciation.
  - long run appreciation is increasing function of persistence of ρ_R.



Extension to Small Open Economy ...

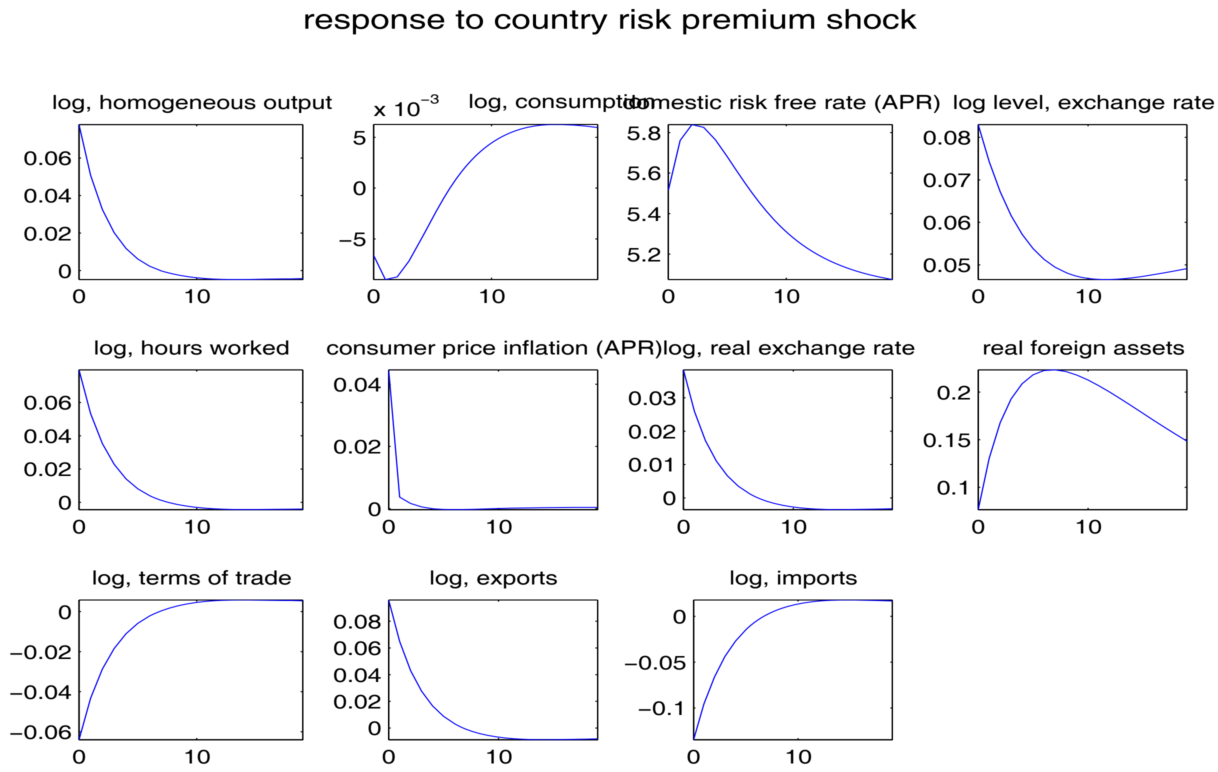
We now consider a monetary policy shock, $\varepsilon_{R,t} = 0.01$. According to (5), implies a four percentage point (at an annual rate) policy-induced jump in R_t. The dynamic effects are displayed in the following figure, for $\tilde{\phi}_s = 0$, $\tilde{\phi}_s = 1.75$.



Note: (i) appreciation smaller, though a more drawn out, when $\tilde{\phi}_s$ is big; (ii) smaller appreciation results in smaller drop in net exports, so less of a drop in demand, so less fall in output and inflation; (iii) smaller drop in net exports results in smaller drop in real foreign assets.

Extension to Small Open Economy ...

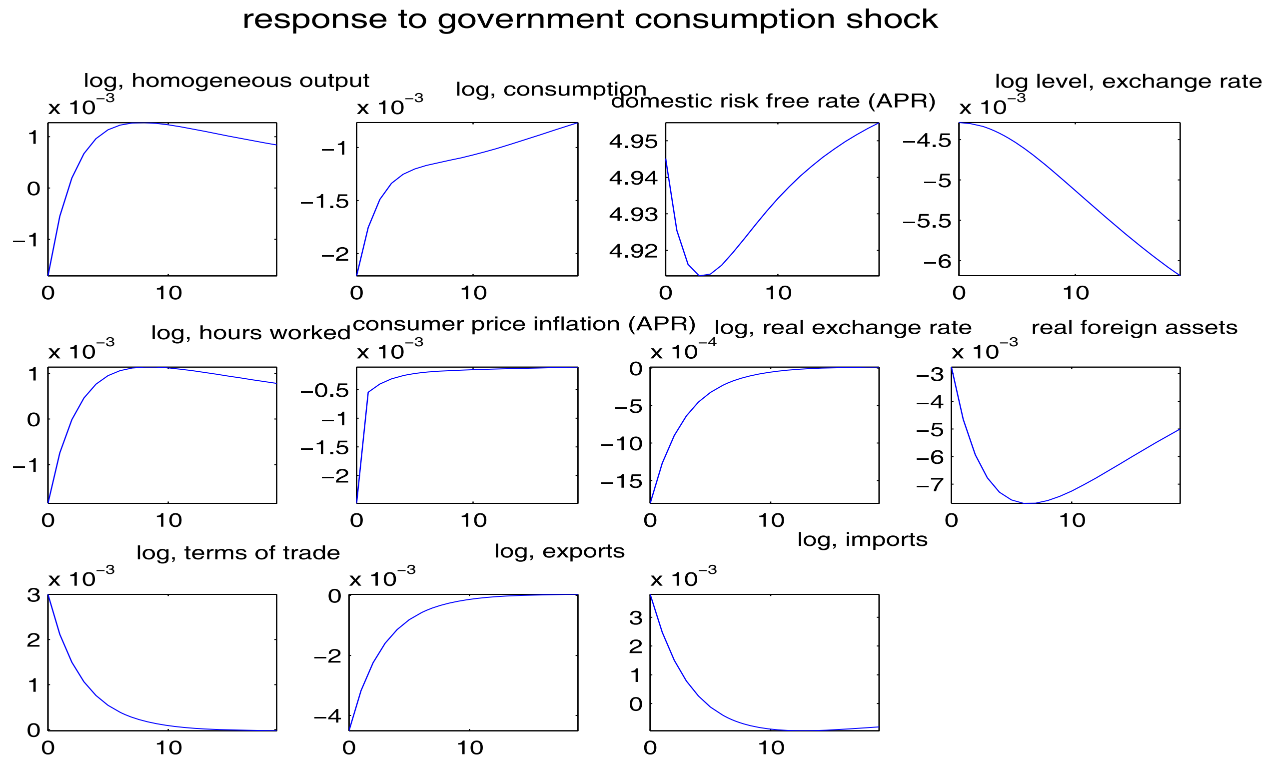
Consider now a domestic economy risk premium shock, a jump in the innovation to $\tilde{\phi}_t$ equal to 0.01.



With the reduced interest in domestic assets, (i) the currency depreciates, (ii) net exports rise, (iii) hours and output rise, (iv) the upward pressure on costs associated with higher output leads to a rise in prices.

Extension to Small Open Economy ...

Next we consider a 0.01 innovation in log, government consumption, g_t.



After a delay, the higher g_t leads to a rise in output. However, there is so much crowding out in the short run that output actually falls. There is crowding out of net exports and consumption because of the effects created by a higher interest rate. The higher interest rate directly reduces consumption, and by making the currency appreciate, it produces a fall in net exports. The initial drop in government spending in the wake of a rise in government spending is interesting.

Introducing Labor Market Search and Matching Frictions into Open Economy Model

- Change the interface between workers and intermediate good firms in the domestic homogeneous goods sector.

- Labor market in the previous model:

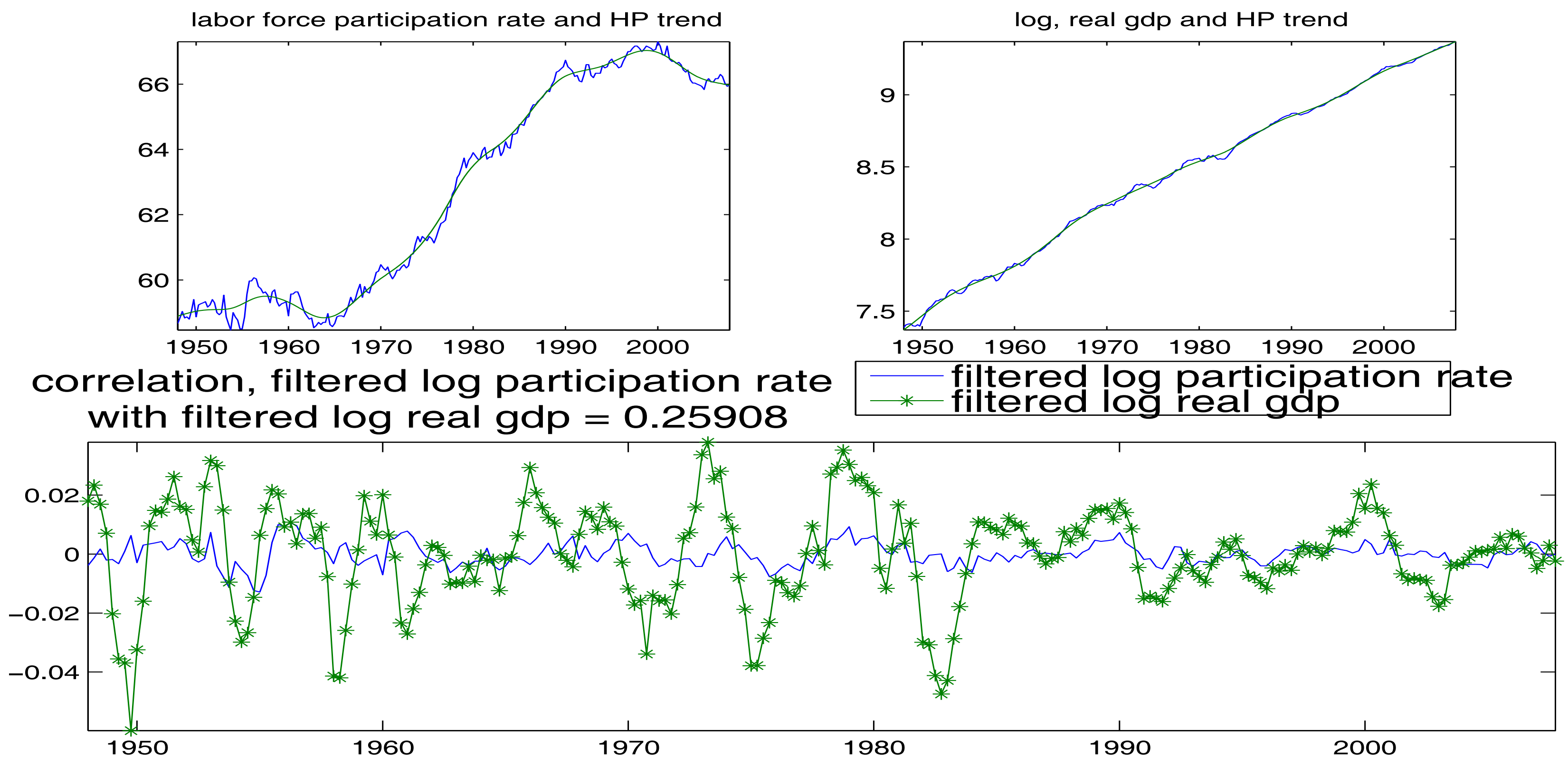
  - supply side (household):

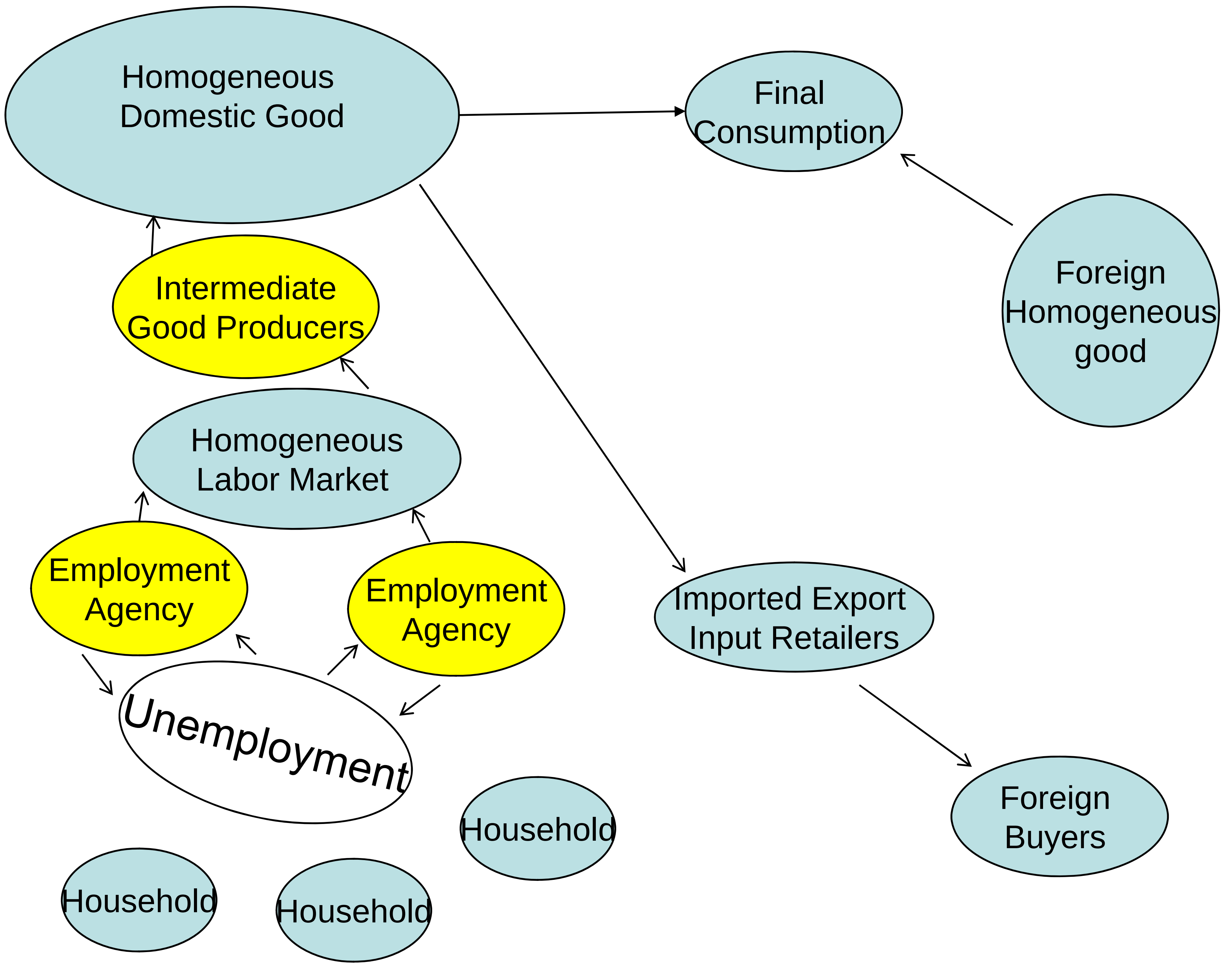
$$\exp(\tau_t) N_t^{\varphi} C_t = \frac{W_t}{P_t}.$$

  - demand side, i^{th} intermediate good firm enters homogeneous labor market and hires labor required to satisfy demand:

$$N_{i,t} = \frac{Y_t}{A_t} \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon}.$$

- New setup:
  - Each household has a large number of workers, and *all* are supplied inelastically to market.
    - No variation in labor force participation rate. Not a bad approximation, at least for business cycle variation:




Homogeneous Domestic Good
Final Consumption
Foreign Homogeneous good
Intermediate Good Producers
Homogeneous Labor Market
Employment Agency
Employment Agency
Imported Export Input Retailers
Unemployment
Household
Foreign Buyers
Household
Household

– Each worker is either unemployed and searching, or employed by an employment agency
  * the employment agency supplies ς_t units of intensity (hours) of labor services per worker to a competitive labor market
  * household utility:

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \{ \overbrace{\log(C_{t+l})}^{\text{workers share equally in household consumption}} - A_L \frac{\varsigma_{t+l}^{1+\sigma_L}}{1+\sigma_L} \overbrace{L_{t+l}}^{\text{each household has the same fraction, } L, \text{ of its workers employed}} \}$$

intensity of labor effort expended by each employed worker ~ς_t

– household intertemporal efficiency conditions, (6), (7) unaffected.
– worker search:
  * intensity of search is fixed and requires no effort or resources
  * search is ‘undirected’ (i.e., workers do not have useful information about where the highest wage jobs are).

- Employment agencies:
  - each agency has a large number of workers.
  - $1-\rho$ attached workers are randomly selected to separate and go into unemployment.[1]
  - new workers arrive from unemployment, in proportion to the number of vacancies posted by the agency in previous period.
  - wage paid by employment agency to workers determined.
  - intensity of labor effort per worker, ς_t, is set according to efficiency criterion:

$$\overbrace{W_t}^{\text{marginal benefit to agency of worker}} = \overbrace{\left[\frac{A_L \varsigma_t^{\sigma_L}}{\underbrace{\text{marginal utility of currency}}_{\upsilon_t}}\right]}^{\text{workers' marginal cost of work}} \overbrace{\rightarrow}^{\text{scaling}} \overbrace{\bar{w}_t}^{\equiv \frac{W_t}{P_t A_t}} = A_L \varsigma_t^{\sigma_L} \overbrace{c_t}^{\equiv \frac{C_t}{A_t}} \tag{36}$$

  - agency or worker unhappy with the match can choose to terminate it at this point.

[1] The assumed constancy of job separation is consistent with the findings reported in Hall (2005b,c) and Shimer (2005a,b), who report that the job separation rate is relatively acyclical.

– agencies are divided into M equal-sized cohorts:

  * each cohort has a large number of agencies
  * all agencies in each cohort negotiate the wage with existing workers (after all current period new arrivals and separations)
  * wage covers workers at the agency for the duration of the wage contract.

– to characterize what agencies do (bargain with workers and post vacancies), need:

  * 'technology' for acquiring new workers.
  * value (surplus) of a worker to the agency, J_t.
  * equilibrium conditions that characterize vacancy posting decisions.
  * marginal impact on J_t of a change in the worker wage, $J_{w,t}$.
  * must scale J_t, $J_{w,t}$ and vacancy posting equilibrium conditions, so that only variables that appear are those which have a steady state, independent of initial conditions.

– agency in i^{th} cohort posts vacancies, v_t^i, more workers in $t+1$

$*$ cost of vacancies:

$$\frac{\overbrace{\kappa A_t}^{\text{cost parameter that grows with } A_t}}{2} \left(\frac{Q_t^{\iota} v_t^i}{l_t^i}\right)^2 l_t^i$$

l_t^i ~quantity of workers in agency i at t

$$l_{t+1}^{i+1} = \left(\tilde{v}_t^i Q_t^{1-\iota} + \rho\right) l_t^i, \qquad \tilde{v}_t^i \equiv \frac{Q_t^{\iota} v_t^i}{l_t^i} \rightarrow \tilde{v}_t^i Q_t^{1-\iota} = \frac{\overbrace{Q_t v_t^i}^{\text{agency } i \text{ hiring rate}}}{l_t^i} \tag{37}$$

Q_t ~probability that a vacancy is filled

$$\iota = \begin{cases} 1 & \text{Gertler-Trigari specification, emphasizes cost of new hires} \\ 0 & \text{emphasizes costs of vacancies per se.} \end{cases}$$

$*$ costs of posting vacancies rise less rapidly in boom when $\iota = 1$.

- Employment Agency Vacancy Posting Problem

  – value function of employment agency in the period of contract negotiations ($i = 0$), which pays wage rate, ω_t :

$$F\left(l_t^0, \omega_t\right) = \text{present discounted value of profits}$$

$$F\left(l_t^0, \omega_t\right) = \sum_{j=0}^{M-1} \beta^j E_t \frac{\upsilon_{t+j}}{\upsilon_t} \max_{\tilde{v}_{t+j}^j} \left[\overbrace{\left(W_{t+j} - \omega_t\right) \varsigma_{t+j}}^{\text{period } t+j \text{ revenues per worker}} - \overbrace{P_{t+j} \frac{\kappa A_{t+j}}{2} \left(\tilde{v}_t^i\right)^2}^{\text{vacancy recruiting costs}} \right] l_{t+j}^j$$

$$+\beta^M E_t \frac{\upsilon_{t+M}}{\upsilon_t} F\left(l_{t+M}^0, \overbrace{\tilde{W}_{t+M}}^{\text{wage rate negotiated at next contract negotiation}} \right),$$

where,

$$\tilde{v}_t^i \equiv \frac{Q_t^\iota v_t^i}{l_t^i}$$

– Note:

$$F\left(l_t^0,\omega_t\right)=J\left(\omega_t\right)l_t^0$$

$J\left(\omega_t\right)$ ~ present discounted value of profits, per worker

$$\frac{F\left(l_t^0,\omega_t\right)}{l_t^0}\equiv J\left(\omega_t\right)=\max_{\left\{v_{t+j}^j\right\}_{j=0}^{M-1}}\{\left(W_t-\omega_t\right)\varsigma_t-P_tA_t\frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2$$

$$+\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}-\omega_t\right)\varsigma_{t+1}-P_{t+1}A_{t+1}\frac{\kappa}{2}\left(\tilde{v}_{t+1}^1\right)^2\right]\overbrace{\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)}^{=l_{t+1}^1/l_t^0}$$

$$+\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}-\omega_t\right)\varsigma_{t+2}-P_{t+2}A_{t+2}\frac{\kappa}{2}\left(\tilde{v}_{t+2}^2\right)^2\right]\overbrace{\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1Q_{t+1}^{1-\iota}+\rho\right)}^{=l_{t+2}^2/l_t^0}$$

$$+\ldots+$$

$$+\beta^M\frac{\upsilon_{t+M}}{\upsilon_t}J\left(\tilde{W}_{t+M}\right)\overbrace{\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+M-1}^{M-1}Q_{t+M-1}^{1-\iota}+\rho\right)}^{=l_{t+M}^0/l_t^0}\}.$$

– First order condition with respect to $\tilde{v}_t^0$ (after multiplying the result by $\left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right)/Q_t^{1-\iota}$):

$$
\begin{aligned}
0 = &-P_t A_t \kappa \tilde{v}_t^0 \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right)/Q_t^{1-\iota} \\
&+\beta \frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1} - \Gamma_{t,1}\omega_t\right)\varsigma_{t+1} - P_{t+1}A_{t+1}\frac{\kappa}{2}\left(\tilde{v}_{t+1}^1\right)^2\right]\left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \\
&+\beta^2 \frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2} - \Gamma_{t,2}\omega_t\right)\varsigma_{t+2} - P_{t+2}A_{t+2}\frac{\kappa}{2}\left(\tilde{v}_{t+2}^2\right)^2\right]\left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota} + \rho\right) \\
&+ \ldots + \\
&+\beta^M \frac{\upsilon_{t+M}}{\upsilon_t} J\left(\tilde{W}_{t+M}\right)\left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right)\left(\tilde{v}_{t+1}^1 Q_t^{1-\iota} + \rho\right)\cdots\left(\tilde{v}_{t+M-1}^{M-1} Q_{t+M-1}^{1-\iota} + \rho\right)\} \\
&= J\left(\omega_t\right) - \left(W_t - \omega_t\right)\varsigma_t + P_t A_t \frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2 - P_t A_t \kappa \tilde{v}_t^0 \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right)/Q_t^{1-\iota}
\end{aligned}
$$

or,

$$
J\left(\omega_t\right) = \left(W_t - \omega_t\right)\varsigma_t + P_t A_t \frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2 + P_t A_t \kappa \tilde{v}_t^0 \frac{\rho}{Q_t^{1-\iota}} \tag{38}
$$

– First order condition with respect to $\tilde{v}^1_{t+1}$ (after multiplication by $\left(\tilde{v}^1_{t+1}Q^{1-\iota}_{t+1}+\rho\right)/Q^{1-\iota}_{t+1}$):

$$\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[-P_{t+1}A_{t+1}\kappa\left(\tilde{v}^1_{t+1}\right)\right]\left(\tilde{v}^0_tQ^{1-\iota}_t+\rho\right)\left(\tilde{v}^1_{t+1}Q^{1-\iota}_{t+1}+\rho\right)/Q^{1-\iota}_{t+1}$$
$$+\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}-\omega_t\right)\varsigma_{t+2}-P_{t+2}A_{t+2}\frac{\kappa}{2}\left(\tilde{v}^2_{t+2}\right)^2\right]\left(\tilde{v}^0_tQ^{1-\iota}_t+\rho\right)\left(\tilde{v}^1_{t+1}Q^{1-\iota}_{t+1}+\rho\right)$$
$$+...+$$
$$+\beta^M\frac{\upsilon_{t+M}}{\upsilon_t}J\left(\tilde{W}_{t+M}\right)\left(\tilde{v}^0_tQ^{1-\iota}_t+\rho\right)\left(\tilde{v}^1_{t+1}Q^{1-\iota}_t+\rho\right)\cdots\left(\tilde{v}^{M-1}_{t+M-1}Q^{1-\iota}_{t+M-1}+\rho\right)=0,$$

which implies (using $\tilde{v}^0_t$ fonc to substitute out $t+2$ and later terms):

$$0=\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[-P_{t+1}A_{t+1}\kappa\left(\tilde{v}^1_{t+1}\right)\right]\left(\tilde{v}^0_tQ^{1-\iota}_t+\rho\right)\left(\tilde{v}^1_{t+1}Q^{1-\iota}_{t+1}+\rho\right)/Q^{1-\iota}_{t+1}$$

$$P_tA_t\kappa\tilde{v}^0_t\left(\tilde{v}^0_tQ^{1-\iota}_t+\rho\right)/Q^{1-\iota}_t$$

$$-\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}-\Gamma_{t,1}\omega_t\right)\varsigma_{t+1}-P_{t+1}A_{t+1}\frac{\kappa}{2}\left(\tilde{v}^1_{t+1}\right)^2\right]\left(\tilde{v}^0_tQ^{1-\iota}_t+\rho\right),$$

– divide the last term by $\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)$ and rearrange,

$$\frac{P_t A_t \kappa}{Q_t^{1-\iota}}\tilde{v}_t^0=\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}-\omega_t\right)\varsigma_{t+1}+P_{t+1}A_{t+1}\kappa\left(\frac{\left(\tilde{v}_{t+1}^1\right)^2}{2}+\frac{\tilde{v}_{t+1}^1\rho}{Q_{t+1}^{1-\iota}}\right)\right]$$

– to help understand latter first order condition, note:

$$\frac{\partial\overbrace{P_t\frac{\kappa A_t}{2}\left(\frac{\overbrace{Q_t^\iota v_t^0}^{\tilde{v}_t^0}}{l_t^0}\right)^2}^{\text{marginal cost (per } l_t^0\text{) of a vacancy}}}{\partial v_t^0}=P_t\kappa A_t\left(\frac{Q_t^\iota}{l_t^0}\right)^2 v_t^0=P_t\kappa A_t Q_t^\iota\tilde{v}_t^0=\frac{P_t A_t\kappa}{Q_t^{1-\iota}}\tilde{v}_t^0\frac{Q_t}{l_t^0}.$$

$$\frac{\overbrace{\partial\left(W_{t+1}-\omega_t\right)\varsigma_{t+1}\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)}^{\text{period } t+1 \text{ marginal revenue of a vacancy, per } l_t^0}}{\partial v_t^0}=\left(W_{t+1}-\omega_t\right)\varsigma_{t+1}\frac{Q_t}{l_t^0}$$

– continue, for periods $t+j$, $j=1,2,...,M-2$.

- $*$ multiply the first order necessary condition for $\tilde{v}_{t+j}^{j}$ by $\left(\tilde{v}_{t+j}^{j} Q_{t+j}^{1-\iota}+\rho\right)/Q_{t+j}^{1-\iota}$
- $*$ substitute out for the period $t+j+1$ and later terms using the first order condition for v_{t+j-1}^{j-1}

– for $i=M-1$, substitute out the period $t+M$ term using (38).

– result:

$$P_{t+j}A_{t+j}\kappa\tilde{v}_{t+j}^{j}\frac{1}{Q_{t+j}^{1-\iota}}$$

$$=\beta E_{t+j}\frac{\upsilon_{t+j+1}}{\upsilon_{t+j}}[(W_{t+j+1}-\omega_t)\varsigma_{t+j+1}+P_{t+j+1}A_{t+j+1}\kappa\left(\frac{\left(\tilde{v}_{t+j+1}^{j+1}\right)^2}{2}+\frac{\tilde{v}_{t+j+1}^{j+1}\rho}{Q_{t+j+1}^{1-\iota}}\right)],$$

$j=0,...,M-2,$

$$P_{t+M-1}A_{t+M-1}\kappa\tilde{v}_{t+M-1}^{M-1}\frac{1}{Q_{t+M-1}^{1-\iota}}$$

$$=\beta E_{t+M-1}\frac{\upsilon_{t+M}}{\upsilon_{t+M-1}}\left[\left(W_{t+M}-\tilde{W}_{t+M}\right)\varsigma_{t+M}+P_{t+M}A_{t+M}\kappa\left(\frac{\left(\tilde{v}_{t+M}^{0}\right)^2}{2}+\frac{\tilde{v}_{t+M}^{0}\rho}{Q_{t+M}^{1-\iota}}\right)\right]$$

– the previous results provide equilibrium expressions for vacancy choices over time by the cohort of employment agencies that bargain at time t.

– to solve the model, we require equilibrium conditions that hold in the cross-section of employment agencies at time t:

$$\frac{P_t A_t \kappa}{Q_t^{1-\iota}} \times \overbrace{\tilde{v}_t^j}^{\text{vacancy decision by agencies that most recently bargained } j \text{ periods in past, } j=0,1,\ldots,M-2}$$

$$= \beta E_t \frac{\upsilon_{t+1}}{\upsilon_t} [\left(W_{t+1} - \tilde{W}_{t-j}\right) \varsigma_{t+1} + P_{t+1} A_{t+1} \kappa \left(\frac{\left(\tilde{v}_{t+1}^{j+1}\right)^2}{2} + \frac{\tilde{v}_{t+1}^{j+1} \rho}{Q_{t+1}^{1-\iota}}\right)],$$

$$P_t A_t \kappa \tilde{v}_t^{M-1} \frac{1}{Q_t^{1-\iota}}$$

$$= \beta E_t \frac{\upsilon_{t+1}}{\upsilon_t} \left[\left(W_{t+1} - \tilde{W}_{t+1}\right) \varsigma_{t+1} + P_{t+1} A_{t+1} \kappa \left(\frac{\left(\tilde{v}_{t+1}^0\right)^2}{2} + \frac{\tilde{v}_{t+1}^0 \rho}{Q_{t+1}^{1-\iota}}\right)\right]$$

– next, scale these conditions.

– divide by $P_t A_t$ (use $\upsilon_t = 1/(P_t C_t)$, $C_t = A_t c_t$):

$$\frac{\kappa}{Q_t^{1-\iota}}\tilde{v}_t^j = \beta E_t \frac{c_t}{c_{t+1}}\left[\left(\bar{w}_{t+1} - \frac{\tilde{W}_{t-j}}{P_{t+1}A_{t+1}}\right)\varsigma_{t+1} + \kappa\left(\frac{\left(\tilde{v}_{t+1}^{j+1}\right)^2}{2} + \frac{\tilde{v}_{t+1}^{j+1}\rho}{Q_{t+1}^{1-\iota}}\right)\right],$$

$j = 0, 1, ..., M-2$, where

$$\bar{w}_t = \frac{W_t}{P_t A_t}.$$

– note:

$$\begin{aligned}
\frac{\tilde{W}_{t-j}}{P_{t+1}A_{t+1}} &= \frac{\tilde{W}_{t-j}}{P_{t-j}A_{t-j}\pi_{t-j+1}\cdots\pi_{t+1}\exp\left(\Delta a_{t-j+1}+\cdots+\Delta a_{t+1}\right)} \\
&= \frac{\left(\tilde{W}_{t-j}/W_{t-j}\right)\frac{W_{t-j}}{P_{t-j}A_{t-j}}}{\pi_{t-j+1}\cdots\pi_{t+1}\exp\left(\Delta a_{t-j+1}+\cdots+\Delta a_{t+1}\right)} \\
&= \frac{w_{t-j}\bar{w}_{t-j}}{\pi_{t-j+1}\cdots\pi_{t+1}\exp\left(\Delta a_{t-j+1}+\cdots+\Delta a_{t+1}\right)} \\
&= G_{t-j,j+1}w_{t-j}\bar{w}_{t-j}
\end{aligned}$$

where

$$\overbrace{G_{t-i,i+1}}^{\text{inflation and growth factor}} = \frac{1}{\pi_{t+1}\cdots\pi_{t-i+1}\exp\left(\Delta a_{t-j+1}+\cdots+\Delta a_{t+1}\right)},\ w_t = \frac{\tilde{W}_t}{W_t}.$$

– thus, for $j = 0, 1, ..., M-2,$

$$\frac{\kappa}{Q_t^{1-\iota}}\tilde{v}_t^j = \beta E_t \frac{c_t}{c_{t+1}}[(\bar{w}_{t+1} - G_{t-j,j+1} w_{t-j} \bar{w}_{t-j}) \varsigma_{t+1} + \kappa \left(\frac{\left(\tilde{v}_{t+1}^{j+1}\right)^2}{2} + \frac{\tilde{v}_{t+1}^{j+1}\rho}{Q_{t+1}^{1-\iota}} \right)]. \tag{39}$$

– for $j = M-1:$

$$\kappa \tilde{v}_t^{M-1} \frac{1}{Q_t^{1-\iota}} = \beta E_t \frac{c_t}{c_{t+1}} \left[(\bar{w}_{t+1} - w_{t+1} \bar{w}_{t+1}) \varsigma_{t+1} + \kappa \left(\frac{\left(\tilde{v}_{t+1}^0\right)^2}{2} + \frac{\tilde{v}_{t+1}^0 \rho}{Q_{t+1}^{1-\iota}} \right) \right] \tag{40}$$

$$w_t = \frac{\tilde{W}_t}{W_t}, \ \bar{w}_t = \frac{W_t}{P_t A_t}.$$

- Surplus Enjoyed by Employment Agency With Wage Rate, ω_t :

$$\overbrace{F\left(l_t^0, \omega_t\right)}^{\text{profits earned in employing the work force}} - \overbrace{0}^{\text{value of the firm in case the workers all leave (for unemployment)}}$$

- Surplus, divided by l_t^0 :

$$J\left(\omega_t\right).$$

• Expressions for scaled surplus and derivative of surplus with respect to wage

$$\frac{dJ\left(\omega_t\right)}{d\omega_t}=\frac{\partial J\left(\omega_t\right)}{\partial\omega_t}+\sum_{j=0}^{M-1}\overbrace{\frac{\partial J\left(\omega_t\right)}{\partial v_{t+j}^{j}}}^{=0\text{, by optimality of } v_{t+j}^{j}}\frac{dv_{t+j}^{j}}{d\omega_t}$$

$$\begin{aligned}= & -\varsigma_t-\beta\frac{\upsilon_{t+1}}{\upsilon_t}\varsigma_{t+1}\Omega_{t,1}-\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\varsigma_{t+2}\Omega_{t,2}\\ & -...-\beta^2\frac{\upsilon_{t+M-1}}{\upsilon_t}\varsigma_{t+M-1}\Omega_{t,M-1}.\end{aligned}$$

$$\overbrace{\Omega_{t,j}}^{\text{employment growth factor}}\equiv\begin{cases}\prod_{l=0}^{j-1}\overbrace{\left(\tilde{v}_{t+l}^{l}Q_{t+l}^{1-\iota}+\rho\right)}^{=\text{growth rate of stock of employees}} & j>0\\ 1 & j=0\end{cases}. \qquad (41)$$

– result:

$$\frac{dJ(\omega_t)}{d\omega_t} = -\varsigma_t - \beta\frac{\upsilon_{t+1}}{\upsilon_t}\varsigma_{t+1}\Omega_{t,1} - \beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\varsigma_{t+2}\Omega_{t,2}$$
$$-... - \beta^2\frac{\upsilon_{t+M-1}}{\upsilon_t}\varsigma_{t+M-1}\Omega_{t,M-1},$$

or,

$$\frac{dJ(\omega_t)}{d\omega_t} \equiv J_{w,t} = -\varsigma_t - \beta\frac{P_tA_tc_t}{P_{t+1}A_{t+1}c_{t+1}}\varsigma_{t+1}\Omega_{t,1} - \beta^2\frac{P_tA_tc_t}{P_{t+2}A_{t+2}c_{t+2}}\varsigma_{t+2}\Omega_{t,2}$$
$$-... - \beta^2\frac{P_tA_tc_t}{P_{t+M-1}A_{t+M-1}c_{t+M-1}}\varsigma_{t+M-1}\Omega_{t,M-1}$$

$$J_{w,t} = -\varsigma_t - \beta\varsigma_{t+1}\frac{c_t}{c_{t+1}}\overbrace{G_{t,1}}^{\text{inflation and growth factor}}\Omega_{t,1} - \beta^2\varsigma_{t+2}\frac{c_t}{c_{t+2}}G_{t,2}\Omega_{t,2} \quad (42)$$
$$-... - \beta^{M-1}\varsigma_{t+M-1}\frac{c_t}{c_{t+M-1}}G_{t,M-1}\Omega_{t,M-1}.$$

– agency surplus per worker:

$$J(\omega_t) = (W_t - \omega_t)\varsigma_t + P_t A_t \frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2 + P_t A_t \kappa \tilde{v}_t^0 \frac{\rho}{Q_t^{1-\iota}}$$

– scale by $P_t A_t$:

$$J_{A,t} \equiv \frac{J(\omega_t)}{P_t A_t} = \frac{W_t - \omega_t}{P_t A_t}\varsigma_t + \frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2 + \kappa \tilde{v}_t^0 \frac{\rho}{Q_t^{1-\iota}}$$

or

$$J_{A,t} = (\bar{w}_t - w_t \bar{w}_t)\varsigma_t + \frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2 + \kappa \tilde{v}_t^0 \frac{\rho}{Q_t^{1-\iota}} \tag{43}$$

- The Worker

  – V_t^i ~value function (in currency units) of a worker attached to an agency in cohort i, $i = 0, .., M-1$, at start of period t, after period t wage has been determined and all period t separations and job finding have occurred.

  – U_t ~value function of a worker (in currency units) at the start of period t, after all period t separations and job finding have occurred.

– value function, $i = 0, ..., M-1$:

$$V_t^i = \overbrace{\tilde{W}_{t-i}\varsigma_t}^{\text{current wage payments}} - \overbrace{A_L \frac{\varsigma_t^{1+\sigma_L}}{(1+\sigma_L)\upsilon_t}}^{\text{disutility of work, converted to currency units by } \upsilon_t}$$

$$+\beta E_t \frac{\upsilon_{t+1}}{\upsilon_t}[\rho \times \overbrace{V_{t+1}^{i+1}}^{\text{value in case of no separation next period}}$$

$$+(1-\rho) \times \overbrace{U_{t+1}}^{\text{value in case of separation (into unemployment)}}]$$

– scale by $P_t A_t$:

$$V_{A,t}^i = \overbrace{G_{t-i,i}}^{\text{inflation and growth factor}} w_{t-i}\bar{w}_{t-i}\varsigma_t - A_L \frac{\varsigma_t^{1+\sigma_L} c_t}{1+\sigma_L} + \beta E_t \frac{c_t}{c_{t+1}} [\rho V_{A,t+1}^{i+1} + (1-\rho) U_{A,t+1}] \tag{44}$$

$$V_{A,t}^i \equiv \frac{V_t^i}{P_t A_t},\; U_{A,t} \equiv \frac{U_t}{P_t A_t}.$$

– for Nash bargaining, it is useful to have the derivative of V_t^0 with respect to the wage rate negotiated at the start of period t by workers in agencies of cohort $i = 0$.

– denote this derivative, when evaluated at an arbitrary wage rate, ω_t, by $V_{wt}^0(\omega_t)$:

$$\begin{aligned} V_w^0(\omega_t) &= \sum_{j=0}^{M-1} (\beta\rho)^j E_t \varsigma_{t+j} \frac{\upsilon_{t+j}}{\upsilon_t} = \sum_{j=0}^{M-1} (\beta\rho)^j E_t \varsigma_{t+j} \frac{P_t A_t c_t}{P_{t+j} A_{t+j} c_{t+j}} \\ &= \sum_{j=0}^{M-1} (\beta\rho)^j E_t \varsigma_{t+j} \overbrace{G_{t,j}}^{\text{growth and inflation factor}} \frac{c_t}{c_{t+j}}. \end{aligned} \tag{45}$$

– note: effort, ς_t, is independent of ω_t. It is determined by the efficiency criterion.

– value function of unemployed worker, U_t :

$$U_t = \overbrace{P_t A_t b^u}^{\text{unemployment compensation}}$$

$$+\beta E_t \frac{\upsilon_{t+1}}{\upsilon_t}[\overbrace{f_t}^{\text{prob of finding a job}} \times \overbrace{V_{t+1}^x}^{\text{value function of worker that finds a job, before knowing what agency the job is in}}$$

$$+ (1 - f_t)\, U_{t+1}],$$

$$V_t^x = \sum_{i=0}^{M-1} \overbrace{\frac{\overbrace{\tilde{v}_{t-1}^i Q_{t-1}^{1-\iota}}^{\text{hiring rate}} l_{t-1}^i}{m_{t-1}}}^{\text{probability that a worker who finds a job, ends up in an agency that was in cohort } i \text{ in } t-1} \times V_t^{i+1} \quad (46)$$

$$\overbrace{m_t}^{\text{total number of matches}} = \sum_{j=0}^{M-1} \overbrace{\tilde{v}_t^j Q_t^{1-\iota} l_t^j}^{\text{job matches made by agencies in cohort } j} \quad (47)$$

– scaling by $P_t A_t$:

$$U_{A,t} = b^u + \beta E_t \frac{c_t}{c_{t+1}} \left[f_t V^x_{A,t+1} + (1 - f_t) U_{A,t+1} \right] \tag{48}$$

$$V^x_{A,t} \equiv \frac{V^x_t}{P_t A_t},\ U_{A,t} \equiv \frac{U_t}{P_t A_t}.$$

– other labor market relations:

$$\overbrace{N_t}^{\text{total hours worked}} = \varsigma_t \sum_{j=0}^{M-1} l^j_t, \tag{49}$$

$$\overbrace{L_t}^{\text{total employment}} = \sum_{j=0}^{M-1} l^j_t, \tag{50}$$

$$\overbrace{f_t}^{\text{worker job finding rate}} = \frac{m_t}{1 - L_t}, \tag{51}$$

$$\overbrace{m_t = \sigma_m (1 - L_t)^{\sigma} v_t^{1-\sigma}}^{\text{matching function}}, \qquad \overbrace{v_t}^{\text{total vacancies}} = \sum_{j=0}^{M-1} v^j_t,\ Q_t = \frac{m_t}{v_t}$$

- Nash bargaining problem

$$\max_{\omega_t}\left(\overbrace{\overbrace{V_t^0(\omega_t)}^{\text{value function of a worker, who receives wage, }\omega_t} - \overbrace{U_t}^{\text{the alternative to a successful bargain is unemployment}}}^{\text{surplus of a representative worker who bargains with firm on behalf of all workers, }l_t^0}\right)^{\eta}$$

$$\times\left(\overbrace{J(\omega_t)}^{\text{surplus of firm, divided by }l_t^0}\right)^{(1-\eta)}$$

– first order necessary condition for solution to Nash bargaining problem (after scaling):

$$\eta\frac{V_{w,t}^0}{V_{A,t}^0-U_{A,t}}+(1-\eta)\frac{J_{w,t}}{J_{A,t}}=0,\ \text{ evaluated at } \omega_t=\tilde{W}_t. \qquad (52)$$

- changes to previous version of the model:
  - clearing in homogeneous goods -

$$p_t^* N_t \;=\; g_t + \overbrace{\left(1-\omega_c\right)\left(p_t^c\right)^{\eta_c} c_t}^{\text{imports used in production of final consumption goods}} \tag{53}$$

$$+\overbrace{\left(p_t^x\right)^{-\eta_f} y_t^*}^{\text{exports}} + \overbrace{\frac{\kappa}{2}\sum_{j=0}^{M-1}\left(\tilde{v}_t^j\right)^2 l_t^j}^{\text{resources used up in producing vacancies}} .$$

  - price setting equation, (4) replaced by:

$$K_t = \frac{\varepsilon}{\varepsilon-1}\overbrace{\left(1-\nu_t\right)\bar{w}_t\left(1-\psi+\psi R_t\right)}^{=\text{ intermediate good firm marginal cost}} + E_t\beta\theta\bar{\pi}_{t+1}^{\varepsilon}K_{t+1} \tag{54}$$

- extra equations and variables that have been introduced because of the search and matching:

variable	equation(s)
$\bar{w}_t$	(36)
l_t^i	(37)
$\tilde{v}_t^i$	(39) , (40)
$J_{w,t}$	(42)
$J_{A,t}$	(43)
$V_{A,t}^i$	(44)
$V_{w,t}^0$	(45)
$V_{A,t}^x$	(46)
m_t	(47)
$U_{A,t}$	(48)
Q_t	(52)
ς_t	(49)
L_t	(50)
f_t	(51)

Financial Frictions

- Financial frictions occur when the people who implement projects are different from the people who have the surplus funds needed to finance those projects.
  - In practice, simple frictionless sharing rules are not feasible.
- Asymmetric information creates incentive problems in relationship between borrowers and lenders:
  - borrower might report ‘bad outcomes’, and pocket the difference between actual and reported outcomes.
    - borrowers might put in little effort, and then claim ex post that ‘bad outcomes’ were due to bad luck.
    - borrowers may undertake excessively risky investments, in case losses are bounded below and potential gains not bounded above.

Financial Frictions ...

- Arrangements between borrowers and lenders:
  - can be a source of shocks to the economy
  - can alter the propagation of standard shocks (e.g., government spending, technology, ...)
    - example: nominal interest is typically non-state contingent, so when price moves unexpectedly wealth is transferred between borrowers and lenders (Irving Fisher 'debt deflation'). Has real effects when these people are very different.
- We will review the financial frictions suggested by Bernanke, Gertler, Gilchrist (1999), as implemented by Christiano, Motto and Rostagno (2003, 2007), Christiano, Trabant and Walentin (2008).
- Other notes on financial frictions,

  see http://faculty.wcas.northwestern.edu/~lchrist/d16/d1608/syllabus.htm

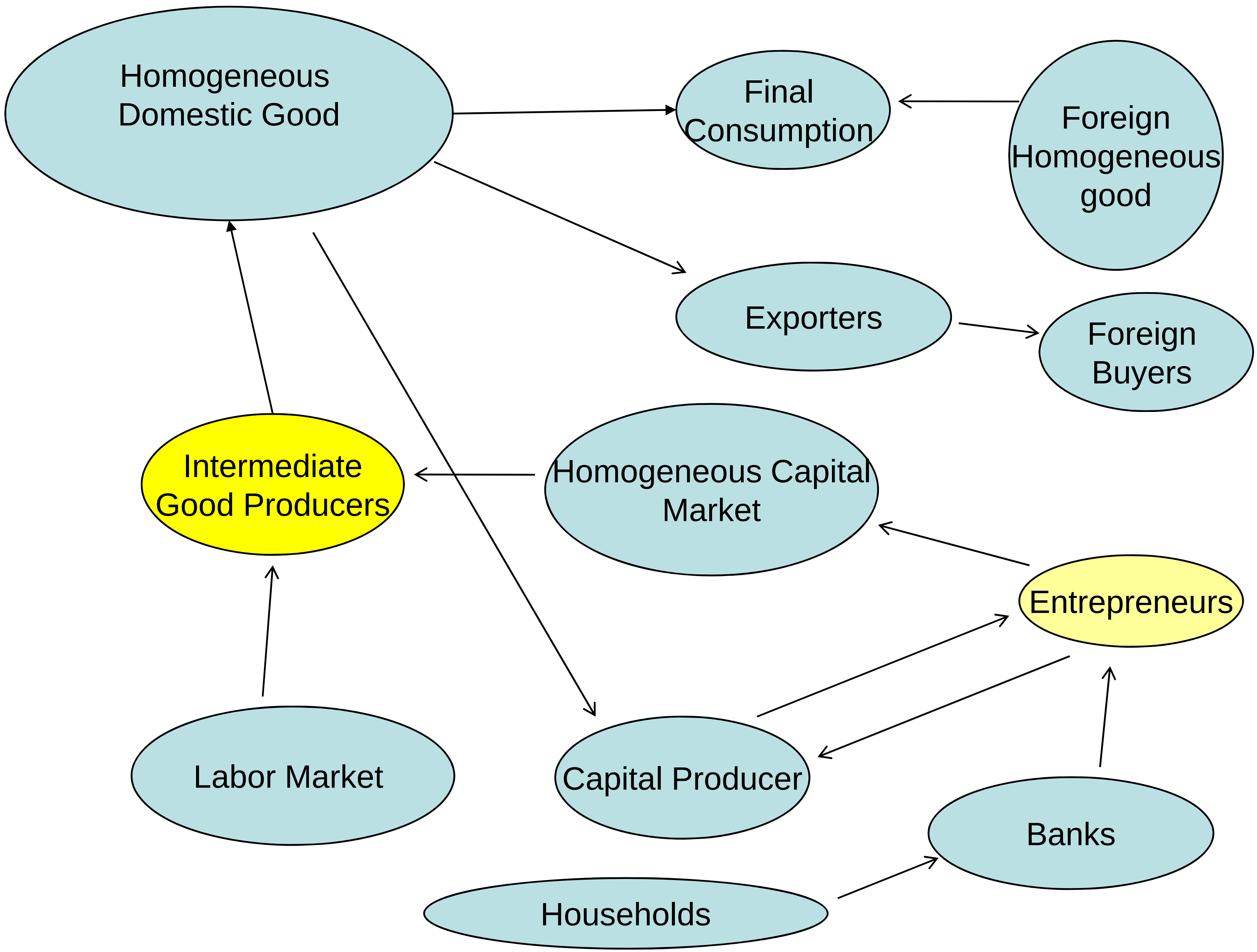
Financial Frictions ...

- We will stress financial frictions associated with the accumulation and management of the society's physical stock of capital (this involves long-term finance which surely is where to biggest frictions exist!)

- In RBC model, the household does the saving and also the investing. That model abstracts completely from financial frictions.

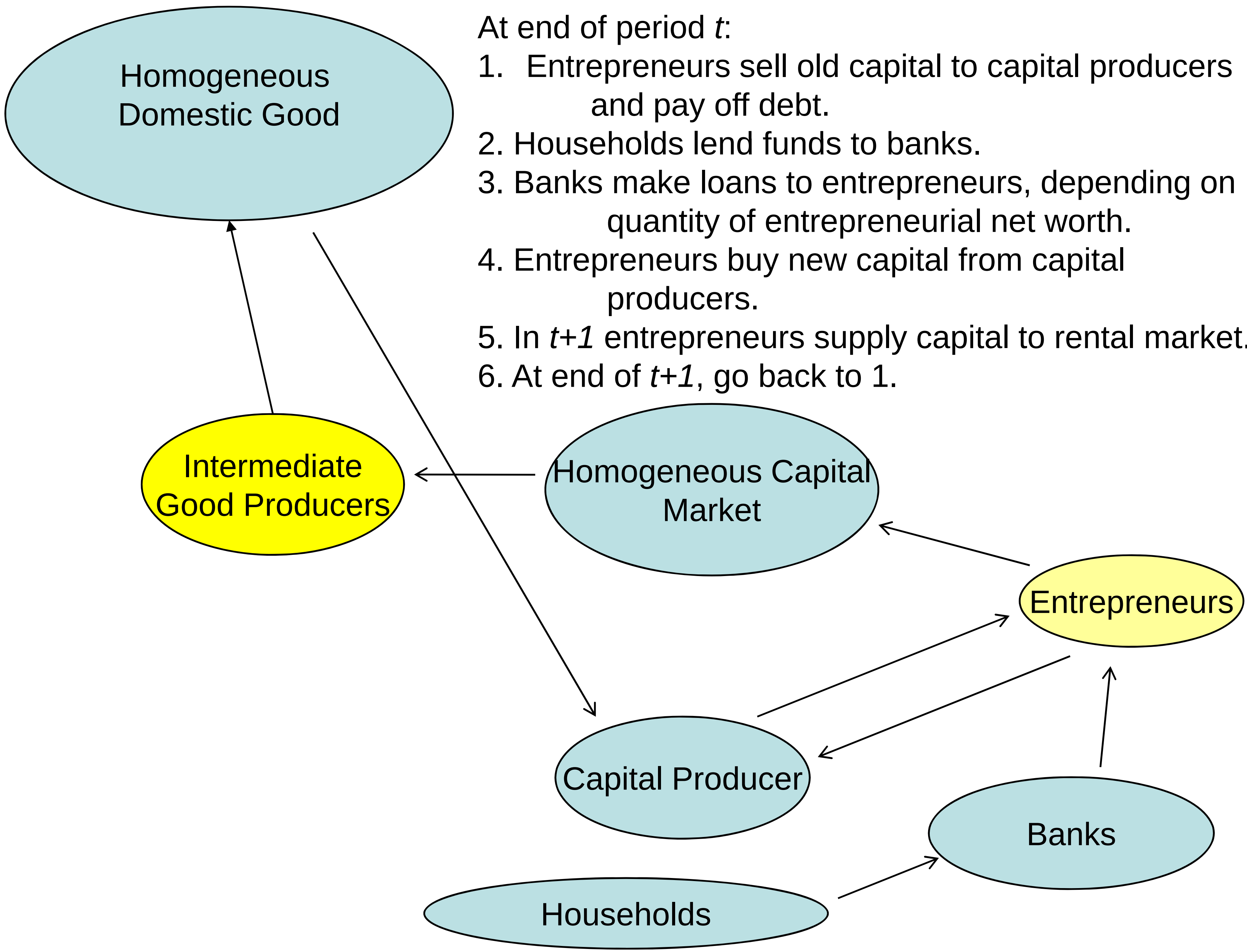
- We will assume that households save, but they do not own or manage the allocation of capital.

- First, we must describe the way capital is used in the model, and the technology for accumulating capital.


Homogeneous Domestic Good
Final Consumption
Foreign Homogeneous good
Exporters
Foreign Buyers
Intermediate Good Producers
Homogeneous Capital Market
Entrepreneurs
Labor Market
Capital Producer
Banks
Households

At end of period *t*:

1. Entrepreneurs sell old capital to capital producers and pay off debt.
2. Households lend funds to banks.
3. Banks make loans to entrepreneurs, depending on quantity of entrepreneurial net worth.
4. Entrepreneurs buy new capital from capital producers.
5. In *t+1* entrepreneurs supply capital to rental market.
6. At end of *t+1*, go back to 1.



- Intermediate good production function:

  – Previous specification:
$$Y_{i,t} = A_t N_{i,t}.$$

  – Now:
$$Y_{i,t} = (A_t N_{i,t})^{1-\alpha} K_{i,t}^{\alpha},\ 0 < \alpha < 1.$$

  – Capital used by i^{th} intermediate good producer, aggregate capital:
$$K_{i,t},\ K_t = \int_0^1 K_{i,t} di.$$

  – To avoid conflict with notation in price equation, we now replace K_t in with K_t^p.

- Optimization by intermediate good firms.

  – price-setting problem completely unchanged.

    * marginal cost still constant.

    * price-setting efficiency conditions, ((2), (3)), and (4) becomes

$$K_t^p = \frac{\varepsilon}{\varepsilon - 1} mc_t + E_t \beta \theta \bar{\pi}_{t+1}^{\varepsilon} K_{t+1}^p, \tag{55}$$

where mc_t denotes marginal cost in domestic currency units/P_t (see below for more).

Financial Frictions ...

– Production efficiency by intermediate good firms.

– firm enters in competitive factor markets and minimizes cost:

$$\min_{N_{i,t},K_{i,t}} \tau_t^d (1-\nu_t) \left[W_t (1-\psi+\psi R_t) N_{i,t} + P_t r_t^k K_{i,t}\right] + \lambda_t \left[Y_{i,t} - (A_t N_{i,t})^{1-\alpha} K_{i,t}^{\alpha}\right]$$

where

r_t^k	rental rate of capital, in units of domestic homogeneous good
τ_t^d	shock to marginal cost that does not affect production function
λ_t	multiplier, nominal marginal cost
ν_t	labor market subsidy

– foncs:

$$W_t [1-\psi+\psi R_t] = \lambda_t (1-\alpha) (A_t)^{1-\alpha} (N_{i,t})^{-\alpha} K_{i,t}^{\alpha}$$

$$P_t r_t^k = \lambda_t \alpha (A_t N_{i,t})^{1-\alpha} K_{i,t}^{\alpha-1}$$

– rewriting two foncs:

$$mc_t \equiv \frac{\lambda_t}{P_t} = \frac{\tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k P_t\right)^{\alpha} \left(W_t \left[1-\psi+\psi R_t\right]\right)^{1-\alpha}}{A_t^{1-\alpha} P_t}$$

$$= \tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k\right)^{\alpha} \left(\bar{w}_t \left[1-\psi+\psi R_t\right]\right)^{1-\alpha} \quad (56)$$

$$\bar{w}_t = \frac{W_t}{P_t A_t},$$

– also:

$$mc_t = \tau_t^d \frac{1}{P_t} \frac{W_t \left[1-\psi+\psi R_t\right]}{(1-\alpha)\left(A_t\right)^{1-\alpha} \left(N_{i,t}\right)^{-\alpha} K_{i,t}^{\alpha}}$$

$$= \tau_t^d \frac{\bar{w}_t \left[1-\psi+\psi R_t\right]}{(1-\alpha)\left(\frac{k_t}{\exp(\Delta a_t) N_t}\right)^{\alpha}} \quad (57)$$

- Domestic homogeneous good output:

$$Y_t = p_t^* \left(A_t N_t\right)^{1-\alpha} K_t^{\alpha}, \qquad p_t^* \equiv \left(\frac{P_t^*}{P_t}\right)^{\varepsilon}$$

$$P_t^* \equiv \left[\int_0^1 P_{i,t}^{-\varepsilon} di\right]^{\frac{-1}{\varepsilon}} = \left[(1-\theta)\,\tilde{P}_t^{-\varepsilon} + \theta \left(P_{t-1}^*\right)^{-\varepsilon}\right]^{\frac{-1}{\varepsilon}}$$

$$P_t \equiv \left[\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right]^{\frac{1}{1-\varepsilon}} = \left[(1-\theta)\,\tilde{P}_t^{(1-\varepsilon)} + \theta P_{t-1}^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}$$

– p_t^* has law of motion, (1).

– Scale K_{t+1} by A_t :

$$y_t = p_t^* \left(N_t\right)^{1-\alpha} \left(\frac{k_t}{\exp\left(\Delta a_t\right)}\right)^{\alpha}, \; y_t \equiv \frac{Y_t}{A_t}, \; k_t \equiv \frac{K_t}{A_{t-1}}, \; \exp\left(\Delta a_t\right) = \frac{A_t}{A_{t-1}}. \tag{58}$$

- Capital accumulation technology:

$$K_{t+1} = (1-\delta) K_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t.$$

$$\tilde{S}(x) = \frac{\tilde{S}''}{2}(x-1)^2, \qquad \tilde{S}(1) = \tilde{S}'(1) = 0.$$

- ‘Cost of change in investment’ specification:
  - Empirical motivations:
    - Christiano, Eichenbaum and Evans (*JPE*, 2005): helps account for hump-shaped response of investment to monetary policy shocks.
    - Sherwin Rose, *JPE*, empirical evidence from housing construction.
    - Matsuyama, 1984, ‘A Learning Effect of Model of Investment: An Alternative Interpretation of Tobin’s Q’, manuscript.
    - Conventional (‘static’) adjustment costs rejected by data (more below).
  - Economic motivations:
    - ‘learning-by-doing’ Matsuyama (1984)
    - D. Lucca, 2008, ‘Resuscitating Time-to-Build’, manuscript, US BOG.

- Capital production is operated by identical, competitive 'capital producers'
  - buy investment goods, I_t, for price P_t
  - buy 'old capital', x, for price $P_t P_{k',t}$
  - sell 'new capital', y, for price $P_t P_{k't}$
  - no finance required (simplification)
  - technology for building capital:

$$K_{t+1} = (1-\delta) \overbrace{K_t}^{\text{denote price by } P_t P_{k't}}$$

$$+ \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) \overbrace{I_t}^{\text{transformed one-for-one from homogeneous output, so price = } P_t}$$

  - Divide by A_t :

$$k_{t+1} = \frac{1-\delta}{\exp\left(\Delta a_t\right)} k_t + \left(1 - \tilde{S}\left(\frac{\exp\left(\Delta a_t\right) i_t}{i_{t-1}}\right)\right) i_t, \; i_t \equiv \frac{I_t}{A_t}. \qquad (59)$$

- Objective:

$$\max_{\{I_{t+j},x_{t+j}\}} E_t \sum_{j=0}^{\infty} \overbrace{\beta^j \upsilon_{t+j}}^{\text{marginal value (to household) of unit of currency}}$$

$$\times \overbrace{\left\{P_{t+j}P_{k',t+j}\left[x_{t+j}+\left(1-\tilde{S}\left(\frac{I_{t+j}}{I_{t+j-1}}\right)\right)I_{t+j}\right]-P_{t+j}P_{k',t+j}x_{t+j}-P_{t+j}I_{t+j}\right\}}^{\text{profits in currency units}}.$$

– Demand for x is indeterminate, so in equilibrium it is simply equal to supply:

$$x_t = (1-\delta)\,K_t.$$

– First order necessary condition for optimization of I_t (after rearranging):

$$P_{k',t} = \overbrace{\frac{1}{1-\tilde{S}\left(\frac{I_t}{I_{t-1}}\right)-\tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}}}^{\text{static marginal cost of capital = price of investment } \div \text{ marginal product of investment in producing new capital}}$$

$$-\frac{1}{1-\tilde{S}\left(\frac{I_t}{I_{t-1}}\right)-\tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}}\beta E_t\frac{\upsilon_{t+1}P_{t+1}}{\upsilon_t P_t}$$

$$\times \overbrace{P_{k',t+1}\tilde{S}'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2}^{>0 \text{ if planning } I_{t+1}/I_t>1\text{, so that } I_t\uparrow \text{ saves on } t+1 \text{ adjustment costs}}$$

– Compare to implication of ‘conventional’ adjustment costs:

$$K_{t+1} = (1-\delta)\, K_t + \kappa \left(\frac{I_t}{K_t} \right) K_t, \quad \kappa' > 0,\ \kappa'' < 0.$$

$$P_{k',t} = \frac{1}{\kappa' \left(\frac{I_t}{K_t} \right)}$$

– Conventional theory rejected because variables other than I_t enter.

– Adjustment cost in change of investment predicts this result.

– Connection of present theory to current literature needs additional exploration.

- Scaled equations associated with capital producer:

$$
\begin{aligned}
P_{k',t} \; = \; & \frac{1}{1-\tilde{S}\left(\frac{i_t \exp(\Delta a_t)}{i_{t-1}}\right)-\tilde{S}'\left(\frac{i_t \exp(\Delta a_t)}{i_{t-1}}\right)\frac{i_t \exp(\Delta a_t)}{i_{t-1}}} \\
& - \frac{1}{1-\tilde{S}\left(\frac{i_t \exp(\Delta a_t)}{i_{t-1}}\right)-\tilde{S}'\left(\frac{i_t \exp(\Delta a_t)}{i_{t-1}}\right)\frac{i_t \exp(\Delta a_t)}{i_{t-1}}} \qquad (60) \\
& \times \beta E_t \frac{c_t}{\exp\left(\Delta a_{t+1}\right) c_{t+1}} P_{k',t+1} \tilde{S}'\left(\frac{i_{t+1}\exp\left(\Delta a_{t+1}\right)}{i_t}\right)\left(\frac{i_{t+1}\exp\left(\Delta a_{t+1}\right)}{i_t}\right)^2
\end{aligned}
$$

Financial Frictions ...

- Entrepreneurs

  – the only people who can own and allocate (rent out) the economy's capital stock.

  – find it profitable to own more capital than they can afford (they borrow from banks).

  – asymmetric information creates financial frictions.

- The entrepreneur in t and $t+1$:

  – Period t :

    * State of entrepreneur: level of net worth, N_{t+1}

    * Entrepreneur with net worth, N ('$N-$type entrepreneur') borrows:

$$B_{t+1}^{N} = \overbrace{P_t P_{k',t} K_{t+1}^{N}}^{\text{purchase of capital by } N-\text{type entrepreneur}} - N_{t+1}.$$

$*$ After purchasing capital, $N-$type entrepreneur receives idiosyncratic shock:

$$K_{t+1}^{N} \rightarrow K_{t+1}^{N}\omega, \ \omega \sim iid \text{ cdf } F_t(\omega)$$

ω lognormal across entrepreneurs with $E\omega = 1,$ variance(log ω) $= \sigma_t^2.$

$*$ Entrepreneur sees ω, bank must pay a 'monitoring cost' to see it. Profit sharing too expensive for banks.

$*$ Good arrangement:

- Bank gives $N-$type entrepreneur 'standard debt contract', which specifies a level of borrowing, B_{t+1}^{N}, and an interest rate, Z_{t+1}^{N}.

- The bank only monitors the (bankrupt) entrepreneurs who say they cannot pay the stipulated interest rate.

– Period $t+1$:

* entrepreneur who purchased one unit of capital in t earns:

$$\frac{\overbrace{P_{t+1}r_{t+1}^{k}\omega}^{\text{rent}}+\overbrace{(1-\delta)P_{t+1}P_{k',t+1}\omega}^{\text{sale of undepreciated capital}}}{P_tP_{k',t}}\equiv\overbrace{R_{t+1}^{k}}^{\text{average return on capital}}\times\omega.$$

* resources available to $N-$type entrepreneur with shock, $\omega,$ for paying back debt:

$$P_tP_{k',t}R_{t+1}^{k}\omega K_{t+1}^{N}$$

* cutoff idiosyncratic shock, $\bar{\omega}_{t+1}^{N}$:

$$\bar{\omega}_{t+1}^{N}=Z_{t+1}\frac{1}{R_{t+1}^{k}}\frac{B_{t+1}^{N}}{P_tP_{k',t}K_{t+1}^{N}}$$

$\omega\ <\ \bar{\omega}_t^N\longrightarrow\ N-$ type entrepreneur declares bankruptcy,
is monitored, and hands over everything to bank
(monitoring cost, $\mu R_{t+1}^{k}\omega P_tP_{k',t}K_{t+1}^{N},$ is paid by bank)

$\omega\ \geq\ \bar{\omega}_t^N\rightarrow$ pays interest to bank, and is not bankrupt.

- Banks
  - zero profit condition for each type of entrepreneur:

$$\overbrace{\left[1 - F_t\left(\bar{\omega}_{t+1}^N\right)\right] Z_{t+1}^N B_{t+1}^N}^{\text{earnings from non-bankrupt entrepreneurs}} + \overbrace{(1-\mu)\int_0^{\bar{\omega}_{t+1}^N} \omega dF_t(\omega)\, R_{t+1}^k P_t P_{k',t} K_{t+1}^N}^{\text{net earnings from bankrupt entrepreneurs}}$$

$$= \overbrace{R_t B_{t+1}^N}^{\text{money owed by banks to households, not contingent on } t+1 \text{ shocks (Fisher deflation)}} .$$

  - the only source of funds available for banks to repay debt in $t+1$ is assumed to be their receipts from entrepreneurs.

– substitute out for $Z_{t+1}^N B_{t+1}^N$ in zero profit condition:

$$R_t B_{t+1}^N = \left\{ \left[1 - F_t\left(\bar{\omega}_{t+1}^N\right)\right] \bar{\omega}_{t+1}^N + (1-\mu) \int_0^{\bar{\omega}_{t+1}^N} \omega dF_t(\omega) \right\} R_{t+1}^k P_t P_{k',t} K_{t+1}^N$$

$$= \left[\overbrace{\Gamma_t(\bar{\omega}_{t+1}^N)}^{\text{share of entrepreneurial earnings received by banks}} - \overbrace{\mu G_t(\bar{\omega}_{t+1}^N)}^{\text{share of entrepreneurial earnings used up in monitoring}} \right]$$

$$\times \overbrace{R_{t+1}^k P_t P_{k',t} K_{t+1}^N}^{\text{entrepreneurial earnings}}$$

$$G_t(\bar{\omega}_{t+1}^N) \equiv \int_0^{\bar{\omega}_{t+1}^N} \omega dF_t(\omega),\ \Gamma_t(\bar{\omega}_{t+1}^N) \equiv \bar{\omega}_{t+1}^N \left[1 - F_t(\bar{\omega}_{t+1}^N)\right] + G_t(\bar{\omega}_{t+1}^N)$$

– can show: in neighborhood of equilibrium, object in square brackets is increasing in $\bar{\omega}_{t+1}^N$

$\Rightarrow$model has (sensible) implication that shock which drives up R_{t+1}^k, drives down bankruptcy rate

– Zero profit condition, divided by $R_t N_{t+1}$:

$$\varrho_t^N - 1 = \left[\Gamma_t(\bar{\omega}_{t+1}^N) - \mu G_t(\bar{\omega}_{t+1}^N)\right] \frac{R_{t+1}^k}{R_t} \varrho_t^N, \quad \varrho_t^N \equiv \frac{\overbrace{P_t P_{k',t} K_{t+1}^N}^{\text{assets to net worth ratio}}}{N_{t+1}}$$

– two parameters of the debt contract:

* loan amount, B_{t+1}^N, and interest rate, Z_{t+1}^N
* or, equivalently, $\bar{\omega}_{t+1}^N$ and ϱ_t^N

– in equilibrium, loan contract chosen to maximize (ex ante) entrepreneurial utility (scaled by entrepreneur's opportunity cost):

$$\frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty} \left[R_{t+1}^k \omega P_t P_{k',t} K_{t+1}^N - Z_{t+1}^N B_{t+1}^N\right] dF_t(\omega)}{R_t N_{t+1}} = E_t \left\{ \left[1 - \Gamma_t(\bar{\omega}_{t+1}^N)\right] \frac{R_{t+1}^k}{R_t} \right\} \varrho_t^N,$$

subject to bank zero profit condition.

– Lagrangian representation of problem:

$$\max_{\varrho_t^N, \{\bar{\omega}_{t+1}^N\}} E_t \Big\{ \frac{\left[1-\Gamma_t(\bar{\omega}_{t+1}^N)\right] R_{t+1}^k \varrho_t^N}{R_t}$$

$$+\lambda_{t+1}^N \overbrace{\left(\frac{\left[\Gamma_t(\bar{\omega}_{t+1}^N) - \mu G_t(\bar{\omega}_{t+1}^N)\right] R_{t+1}^k \varrho_t^N}{R_t} - \varrho_t^N + 1 \right)}^{\text{state-by-state zero profit condition}} \Big\}.$$

– First order necessary conditions for optimality:

$$\begin{aligned} E_t \left\{ \left[1-\Gamma_t(\bar{\omega}_{t+1}^N)\right] \frac{R_{t+1}^k}{R_t} + \lambda_{t+1}^N \left(\left[\Gamma_t(\bar{\omega}_{t+1}^N) - \mu G_t(\bar{\omega}_{t+1}^N)\right] \frac{R_{t+1}^k}{R_t} - 1 \right) \right\} &= 0 \\ -\Gamma_t'(\bar{\omega}_{t+1}^N) + \lambda_{t+1}^N \left[\Gamma_t'(\bar{\omega}_{t+1}^N) - \mu G_t'(\bar{\omega}_{t+1}^N)\right] &= 0 \\ \left[\Gamma_t(\bar{\omega}_{t+1}^N) - \mu G_t(\bar{\omega}_{t+1}^N)\right] \frac{R_{t+1}^k}{R_t} \varrho_t^N - \varrho_t^N + 1 &= 0. \end{aligned}$$

– Substituting out for λ_{t+1}^N:

$$\left[\Gamma_{t-1}(\bar{\omega}_t^N) - \mu G_{t-1}(\bar{\omega}_t^N)\right] \frac{R_t^k}{R_{t-1}} \varrho_{t-1}^N - \varrho_{t-1}^N + 1 = 0$$

$$E_t \left\{ \left[1 - \Gamma_t(\bar{\omega}_{t+1}^N)\right] \frac{R_{t+1}^k}{R_t} + \frac{\Gamma_t'(\bar{\omega}_{t+1}^N)\left(\left[\Gamma_t(\bar{\omega}_{t+1}^N) - \mu G_t(\bar{\omega}_{t+1}^N)\right] \frac{R_{t+1}^k}{R_t} - 1\right)}{\Gamma_t'(\bar{\omega}_{t+1}^N) - \mu G_t'(\bar{\omega}_{t+1}^N)} \right\} = 0.$$

– key properties of solution:

∗ $\varrho_t, \bar{\omega}_t$ independent of N (drop superscript, N)

∗ all entrepreneurs pay same interest rate, and $N-$type entrepreneur receives loans in proportion to his/her net worth.

$$B_{t+1} = N_{t+1}\left(\varrho_t - 1\right),\ Z_{t+1} = \frac{\varrho_t}{\varrho_t - 1} R_{t+1}^k \bar{\omega}_{t+1}.$$

- One more equilibrium condition: law of motion for aggregate net worth.

  – density of entrepreneurs with net worth N, $f_t\left(N\right)$, aggregate net worth, $\bar{N}_{t+1}$

$$\bar{N}_{t+1}=\int Nf_t\left(N\right)dN.$$

  – $N-$type entrepreneurs have, after paying back bank loans,

$$V_t^N=\left[1-\Gamma_{t-1}(\bar{\omega}_t)\right]R_t^kP_{t-1}P_{k',t-1}K_t^N$$

  – multiply by $f\left(N\right)$ and integate over all N, to obtain:

$$V_t=\left[1-\Gamma_{t-1}(\bar{\omega}_t)\right]R_t^kP_{t-1}P_{k',t-1}K_t,$$

  where

$$V_t=\int_N V_t^Nf_t\left(N\right)dN,\ K_t=\int_N K_t^Nf_t\left(N\right)dN.$$

– alternative representation of V_t

$$\begin{aligned}
V_t &= \overbrace{\left\{1-\bar{\omega}_t\left[1-F_{t-1}(\bar{\omega}_t)\right]-\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\right\}}^{=1-\Gamma_{t-1}(\bar{\omega}_t)} R_t^k P_{t-1}P_{k',t-1}K_t \\
&= R_t^k P_{t-1}P_{k',t-1}K_t \\
&\quad \overbrace{-\left(\bar{\omega}_t\left[1-F_{t-1}(\bar{\omega}_t)\right]+(1-\mu)\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\right)R_t^k P_{t-1}P_{k',t-1}K_t}^{\text{this represents earnings of banks, which must equal } B_t R_{t-1}=R_{t-1}(P_{t-1}P_{k',t-1}K_t-\bar{N}_t)} \\
&\quad -\mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)R_t^k P_{t-1}P_{k',t-1}K_t \\
&= R_t^k P_{t-1}P_{k',t-1}K_t - \overbrace{\left[R_{t-1}+\frac{\mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)R_t^k P_{t-1}P_{k',t-1}K_t}{P_{t-1}P_{k',t-1}K_t-\bar{N}_t}\right]}^{\text{gross interest paid by entrepreneurs as a whole on their loans}} B_t.
\end{aligned}$$

– after V_t is determined, entrepreneur exits with probability $1-\gamma_t$ and survives with probability γ_t, $1-\gamma_t$ new entrepreneurs 'born' without any wealth.

– all entrepreneurs who pass into the next period receive a transfer, W_t^e

– law of motion of aggregate net worth:

$$\bar{N}_{t+1} = \gamma_t \left[1 - \Gamma_{t-1}(\bar{\omega}_t)\right] R_t^k P_{t-1} P_{k',t-1} K_t + W_t^e.$$

– scaling by $P_t A_t$:

$$n_{t+1} = \gamma_t \left[1 - \Gamma_{t-1}(\bar{\omega}_t)\right] R_t^k \frac{P_{k',t-1} k_t}{\pi_t \exp\left(\Delta a_t\right)} + w_t^e$$

$$n_{t+1} \equiv \frac{\bar{N}_{t+1}}{P_t A_t},\ w_t^e \equiv \frac{W_t^e}{P_t A_t}.$$

- Summary of three equilibrium conditions associated with entrepreneur:

  – aggregate, scaled, entrepreneurial net worth:

$$n_{t+1} = \gamma_t \left[1 - \Gamma_{t-1}(\bar{\omega}_t)\right] R_t^k \frac{P_{k',t-1} k_t}{\pi_t \exp\left(\Delta a_t\right)} + w_t^e \tag{61}$$

  – zero profit condition for banks:

$$\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right] \frac{R_t^k}{R_{t-1}} \varrho_{t-1} - \varrho_{t-1} + 1 = 0 \tag{62}$$

  – optimality condition for entrepreneurial contract

$$E_t \left\{ \left[1 - \Gamma_t(\bar{\omega}_{t+1})\right] \frac{R_{t+1}^k}{R_t} + \frac{\Gamma_t'(\bar{\omega}_{t+1}) \left(\left[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right] \frac{R_{t+1}^k}{R_t} - 1 \right)}{\Gamma_t'(\bar{\omega}_{t+1}) - \mu G_t'(\bar{\omega}_{t+1})} \right\} = 0. \tag{63}$$

- Clearing of domestic homogeneous goods.

  – modification to (14):

$$y_t = (1 - \omega_c)\,(p_t^c)^{\eta_c}\,c_t + x_t + g_t + i_t + d_t, \tag{64}$$

  where y_t is defined in (58) and scaled monitoring costs, d_t, are:

$$d_t = \frac{\mu G_{t-1}(\bar{\omega}_t) R_t^k P_{t-1} P_{k',t-1} K_t}{P_t A_t} = \frac{\mu G_{t-1}(\bar{\omega}_t) R_t^k P_{k',t-1} k_t}{\pi_t \exp(\Delta a_t)}$$

  – net worth of exiting entrepreneurs:

$$(1 - \gamma_t)\,V_t.$$

  ∗ a fraction, $1 - \Theta$, of $(1 - \gamma_t)\,V_t$ is taxed and transferred lump sum to households, and Θ is consumed by entrepreneurs. We suppose Θ is so small that consumption by entrepreneurs can be ignored.

– Concluding observations on financial frictions

– New impulses:

  * Shock up in γ_{t+1} : raises amount of wealth in hands of entrepreneurs....model's way of capturing a 'bubble' jump in the stock market (i.e., one not obviously linked to fundamentals)

  * Shock up in σ_t : increases risk associated with entrepreneurial lending - resembles current subprime lending crisis.

  'don't know who is sitting on the bad mortgage loans and who is not'

- New propagation:
  - shock that drives down rental earnings or price of capital reduces net worth and restricts ability of entrepreneurs to borrow.
  - entrepreneurs' reduced ability to buy capital causes them to reduce purchases of goods by capital producers and induces a fall in output and employment
  - new friction: rigidity in nominal rate of interest affects propagation mechanism
    - acts as an accelerator for expansionary shocks that raise the price level
    - acts as a moderating factor for expansionary shocks that reduce the price level.

- Collecting the equations of the model, following 15:
  - price optimization:

$$K_t^p = \frac{\varepsilon}{\varepsilon - 1} mc_t + E_t \beta \theta \bar{\pi}_{t+1}^{\varepsilon} K_{t+1}^p, \tag{65}$$

$$F_t^p = 1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta \theta F_{t+1}^p \tag{66}$$

$$K_t^p = F_t^p \left[\frac{1 - \theta \bar{\pi}_t^{\varepsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} \tag{67}$$

  - investment efficiency condition:

$$\left[1 - \tilde{S} \left(\frac{i_t \exp\left(\Delta a_t\right)}{i_{t-1}} \right) - \tilde{S}' \left(\frac{i_t \exp\left(\Delta a_t\right)}{i_{t-1}} \right) \frac{i_t \exp\left(\Delta a_t\right)}{i_{t-1}} \right] P_{k',t}$$

$$= 1 - \beta E_t \frac{c_t}{\exp\left(\Delta a_{t+1}\right) c_{t+1}} P_{k',t+1} \tilde{S}' \left(\frac{i_{t+1} \exp\left(\Delta a_{t+1}\right)}{i_t} \right) \left(\frac{i_{t+1} \exp\left(\Delta a_{t+1}\right)}{i_t} \right)^2 \tag{68}$$

  - household labor optimization:

$$\exp\left(\tau_t\right) N_t^{\varphi} c_t = \bar{w}_t \tag{69}$$

– average rate of return on capital:

$$R_t^k = \bar{\pi}_t \frac{r_t^k + (1-\delta) P_{k',t}}{P_{k',t-1}} \tag{70}$$

– cost minimization:

$$mc_t \;=\; (1-\nu_t)\,\tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k\right)^{\alpha} \left(\bar{w}_t \left[1-\psi+\psi R_t\right]\right)^{1-\alpha} \tag{71}$$

$$mc_t \;=\; (1-\nu_t)\,\tau_t^d \frac{\bar{w}_t \left[1-\psi+\psi R_t\right]}{(1-\alpha)\left(\frac{k_t}{\exp(\Delta a_t) N_t}\right)^{\alpha}} \tag{72}$$

– domestic homogeneous output

$$y_t \;=\; p_t^* \left(N_t\right)^{1-\alpha} \left(\frac{k_t}{\exp\left(\Delta a_t\right)}\right)^{\alpha} \tag{73}$$

$$y_t \;=\; (1-\omega_c)\left(p_t^c\right)^{\eta_c} c_t + x_t + g_t + i_t + \frac{\mu G_{t-1}(\bar{\omega}_t) R_t^k P_{k',t-1} k_t}{\bar{\pi}_t \exp\left(\Delta a_t\right)} \tag{74}$$

– current account

$$a_t^f + p_t^{m,c}\omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta_c} c_t = p_t^c q_t p_t^x x_t + \frac{s_t R_{t-1}^f \Phi_{t-1} a_{t-1}^f}{\bar{\pi}_t \exp\left(\Delta a_t\right)} \tag{75}$$

– capital law of motion:

$$k_{t+1} = \frac{1-\delta}{\exp\left(\Delta a_t\right)} k_t + \left(1 - \tilde{S}\left(\frac{\exp\left(\Delta a_t\right) i_t}{i_{t-1}}\right)\right) i_t \tag{76}$$

– conditions related to entrepreneurs:

$$n_{t+1} = \gamma_t \left[1 - \Gamma_{t-1}(\bar{\omega}_t)\right] R_t^k \frac{P_{k',t-1} k_t}{\bar{\pi}_t \exp\left(\Delta a_t\right)} + w_t^e \tag{77}$$

$$0 = \left\{\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right] \frac{R_t^k}{R_{t-1}} - 1\right\} \frac{P_{k',t-1} k_t}{n_t} + 1 \tag{78}$$

$$0 = E_t \left\{ \left[1 - \Gamma_t(\bar{\omega}_{t+1})\right] \frac{R_{t+1}^k}{R_t} + \frac{\Gamma_t'(\bar{\omega}_{t+1}) \left(\left[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right] \frac{R_{t+1}^k}{R_t} - 1\right)}{\Gamma_t'(\bar{\omega}_{t+1}) - \mu G_t'(\bar{\omega}_{t+1})} \right\} \tag{79}$$

– 10 remaining equations:

monetary policy - (5); household intertemporal first order conditions - (6), (7); price equations - (8), (9), (10), (13), (1); demand for exports - (11); exporter equilibrium condition - (12).

– 25 variables:

$$s_t, c_t, \pi_t^c, p_t^c, p_t^{m,c}, q_t, \bar{\pi}_t, p_t^x, a_t^f, x_t, n_{t+1}, R_t^k, P_{k',t},$$

$$k_{t+1}, \bar{\omega}_t, N_t, i_t, p_t^*, K_t^p, F_t^p, mc_t, r_t^k, \bar{w}_t, y_t, R_t$$

– shocks:

$$\tau_t, \varepsilon_{R,t}, R_t^f, y_t^f, \pi_t^f, \Delta a_t, \sigma_t^2, \gamma_t, g_t, y_t^f, \tilde{\phi}_t, \tau_t^d.$$

- Steady state of the model
  - as before:

$$g = \eta_g y,\ \bar{a} = \eta_a y,$$

where $\bar{a}$ is the parameter controlling Φ_t (see (16)).

  - algorithm for solving for the steady state:
    * $\bar{\pi}^c$, $\bar{\pi}$, R, p^*, F^p can be computed from (20), (21), (18), (22), (23).

    * from (65),

$$K^p = \frac{\frac{\varepsilon}{\varepsilon-1}mc}{1-\beta\theta\bar{\pi}^{\varepsilon}}$$

    * combining this with (67),

$$mc = \frac{\varepsilon-1}{\varepsilon}F^p\left[1-\beta\theta\bar{\pi}^{\varepsilon}\right]\left[\frac{1-\theta\bar{\pi}^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon-1}{\varepsilon}\frac{1-\beta\theta\bar{\pi}^{\varepsilon}}{1-\beta\theta\bar{\pi}^{\varepsilon-1}}\left[\frac{1-\theta\bar{\pi}^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}$$

∗ (18), (19) imply

$$\frac{R^f}{\pi^f} = \frac{R}{\pi^c}$$

which can be used to solve for R^f given π^f.

∗ steady state depreciation, s, can be computed from the inflation differential:

$$q_t \rightarrow q \text{ implies (see (13)) } s\pi^f = \pi^c.$$

∗ (17), (18) imply

$$sR^f\Phi = R,$$

or after multiplication by π^f and rearranging,

$$\frac{R^f}{\pi^f}\Phi = \frac{R}{\pi^c}, \text{ so (see (32)) } \Phi = 1 \text{ and } a_t^f = \bar{a} \text{ (see (16))}$$

– equation (68), together with the facts, $S = S' = 0$ in steady state implies

$$P_{k'} = 1.$$

– rest of the algorithm solves a single non-linear equation, (11), in a single unknown, $\tilde{\varphi} = p^c q$.

– set

$$\tilde{\varphi} = p^c q.$$

– use (27), (28), (26):

$$\begin{aligned}
p^{m,c} &= \tilde{\varphi} R^{\nu,*} \\
p^x &= \frac{R^x}{\tilde{\varphi}}, \\
p^c &= \left[(1 - \omega_c) + \omega_c \left(p^{m,c} \right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \\
q &= \frac{\tilde{\varphi}}{p^c}.
\end{aligned}$$

– consider the following 11 unknowns:

$$n, c, \bar{w}, k, N, y, x, i, \bar{\omega}, r^k, R^k$$

in the following 11 equations:

$$\begin{aligned}
(1) R^k &= \bar{\pi}\left(r^k + 1 - \delta\right) \\
(2) y &= p^* (N)^{1-\alpha} \left(\frac{k}{\exp(\Delta a)}\right)^{\alpha} \\
(3) i &= \left[1 - \frac{1-\delta}{\exp(\Delta a)}\right] k \\
(4) \bar{w} &= \exp(\tau) N^{\varphi} c \\
(5) x &= \frac{a^f + p^{m,c} \omega_c \left(\frac{p^c}{p^{m,c}}\right)^{\eta_c} c - \frac{s R^f a^f}{\bar{\pi} \exp(\Delta a)}}{p^c q p^x} \\
(6) n &= \gamma \left[1 - \Gamma(\bar{\omega})\right] R^k \frac{k}{\bar{\pi} \exp(\Delta a)} + w^e,
\end{aligned}$$

$$
\begin{aligned}
(7) mc &= (1-\nu)\,\tau^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r^k\right)^{\alpha} \left(\bar{w}\left[1-\psi+\psi R\right]\right)^{1-\alpha} \\
(8) mc &= (1-\nu)\,\tau^d \frac{\bar{w}\left[1-\psi+\psi R\right]}{(1-\alpha)\left(\frac{k}{\exp(\Delta a) N}\right)^{\alpha}} \\
(9) y &= \frac{(1-\omega_c)\,(p^c)^{\eta_c}\, c + x + i + \frac{\mu G(\bar{\omega}) R^k k}{\bar{\pi}\exp(\Delta a)}}{1-\eta_g} \\
(10) 0 &= \left(\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right]\frac{R^k}{R} - 1\right)\frac{k}{n} + 1 \\
(11) 0 &= \left[1-\Gamma(\bar{\omega})\right]\frac{R^k}{R} + \frac{\Gamma'(\bar{\omega})\left(\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right]\frac{R^k}{R} - 1\right)}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})}
\end{aligned}
$$

– to solve these 11 equations, fix r^k. Compute R^k using (1). Solve for $\bar{\omega}$ using (11). Solve for k/n using (10). Solve for n, k by rewriting (6):

$$
n = \frac{w^e}{1 - \gamma\left[1-\Gamma(\bar{\omega})\right] R^k \frac{k/n}{\bar{\pi}\exp(\Delta a)}}.
$$

Solve for i using (3). Solve for $\bar{w}$ by rewriting (7):

$$\bar{w} = \left[\frac{mc}{(1-\nu)\,\tau^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r^k\right)^{\alpha} (1-\psi+\psi R)^{1-\alpha}} \right]^{\frac{1}{1-\alpha}}.$$

Solve for N by rewriting (8):

$$N = \frac{k}{\exp(\Delta a)} \left[(1-\nu)\,\tau^d \frac{\bar{w}\,[1-\psi+\psi R]}{(1-\alpha)\,mc} \right]^{-\frac{1}{\alpha}}$$

Solve for y using (2). Substitute out for x in (9) using (5). Solve the resulting version of (9) for c :

$$c = \frac{\left(1-\eta_g\right) y + \frac{a^f - \frac{sR^f a^f}{\bar{\pi}\exp(\Delta a)}}{p^c q p^x} - i - \frac{\mu G(\bar{\omega}) R^k k}{\bar{\pi}\exp(\Delta a)}}{(1-\omega_c)\,(p^c)^{\eta_c} + \frac{p^{m,c}\omega_c \left(\frac{p^c}{p^{m,c}}\right)^{\eta_c}}{p^c q p^x}}$$

Adjust r^k until (4) is satisfied.

– adjust $\tilde{\varphi}$ until the demand for exports, (11), is satisfied:

$$x = (p^x)^{-\eta_f} y^f.$$

– alternatively, one could simply fix $\tilde{\varphi}$ and let the previous equation define y^f.

– we evaluate $G(\bar{\omega})$ and $\Gamma(\bar{\omega})$.

  * the lognormal distribution has two parameters, a mean and a variance, σ_x^2.
  * the mean is determined by the requirement, $E\omega = 1$
  * the variance, σ_x^2, is determined by $F(\bar{\omega})$, which we fix a priori.

    for a given $\bar{\omega}$, we solve for σ_x using

$$F(\bar{\omega}) = prob\left[v < \frac{\log(\bar{\omega}) + \frac{1}{2}\sigma_x^2}{\sigma_x}\right]$$

$*$ given $\bar{\omega}$ and σ_x, we solve for $G(\bar{\omega})$, $\Gamma(\bar{\omega})$ using

$$
\begin{aligned}
G(\bar{\omega}) &= prob\left[v < \frac{\log(\bar{\omega}) + \frac{1}{2}\sigma_x^2}{\sigma_x} - \sigma_x\right] \\
\Gamma(\bar{\omega}) &= \bar{\omega}\left[1 - F(\bar{\omega})\right] + G(\bar{\omega}).
\end{aligned}
$$

$*$ also,

$$
\begin{aligned}
G'(\bar{\omega}) &= \bar{\omega}F'(\bar{\omega}) \\
\Gamma'(\bar{\omega}) &= 1 - F(\bar{\omega}) - \bar{\omega}F'(\bar{\omega}) + G'(\bar{\omega}) \\
&= 1 - F(\bar{\omega}) > 0,
\end{aligned}
$$

so that

$$
\begin{aligned}
&\frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})} \\
&= \frac{1 - F(\bar{\omega})}{1 - F(\bar{\omega}) - \mu\bar{\omega}F'(\bar{\omega})}
\end{aligned}
$$