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Application of Log-linearization Methods: Optimal Policy

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- Optimal monetary policy addresses questions like:
  - ‘how much inflation volatility should we have?’
  - ‘should we have inflation targeting or should we have price level targeting?’
  - answer: depends on what gets you closest to the optimal policy benchmark.

- Actual policy seems to be gravitating in the direction of ‘optimal policy’.
  - In early days of ‘rational expectations revolution’, looked for simple rules with good operating characteristics to impose on central banks.
  - Perception (based on the 1970s performance) was that central banks had done a terrible job.
  - Simplicity of the rule supposed to help with monitoring central bank, which was viewed as always having an incentive to deviate from good policy and inflate (‘inflation bias’)

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- There is a widespread view that inflation bias is not a big problem.
  - The classic vehicle for the inflation bias idea, the Barro-Gordon model, appears to some to have been rejected by the data.
    - US seems to have tamed inflation without the institutional reform that BG said was required

    (for a challenge to the logic of this argument, see Chari, Christiano, Eichenbaum (JET), Albanesi, Chari, Christiano (RESTUD).)

  - It is thought that the inflation bias problem can be solved by central bankers' being sufficiently aware of the negative consequences of losing credibility. This, together with a good dose of self discipline, should be enough to ensure that inflation bias is not a problem, according to an emerging consensus.

  - Thus, the problem ('inflation bias') that simple rules are meant to solve, is no longer viewed as a serious problem.

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- Assessment of costs of simple rules were increased.
  - ‘Mother of all rules’, Friedman’s $k-$percent rule, viewed as positively dangerous in the presence of velocity shocks.
  - It is not hard to think of scenarios in which the Taylor rule could be destabilizing (see the end of these notes).
- For these reasons, central banks moving towards ‘look at everything, optimal’ policy.
  - Flexible inflation targeting.
  - Going back to simple rules is now unthinkable, especially in light of the recent financial crisis, which has required creativity and freedom of action on the part of central banks..

Log-linear Methods

- Equilibrium conditions:

$$v\left(k_t, k_{t+1}, k_{t+2}\right) = 0,$$

  $t = 0, 1, 2, \ldots$

- Solution:
  – compute steady state, k^* such that $v\left(k^*, k^*, k^*\right) = 0$.
  – expansion about steady state: $V_0\tilde{k}_t + V_1\tilde{k}_{t+1} + V_2\tilde{k}_{t+2} = 0$.
  – solve linearized system.

- Last time:
  – v equilibrium conditions of a monetary model.
  – included a monetary policy rule.

- This time:
  - what is optimal monetary policy?
  - drop monetary policy rule
  - now we’re short one equation!
  - system underdetermined....‘many solutions’
  - pick the best one.

- Potential problem: time inconsistency of optimal monetary policy:
  - period t announcement about period $t+1$ policy action, X, influenced in part by the impact of X on period t decisions by the public.
  - when $t+1$ occurs and it is time to actually implement X, period t decisions by public are past history.
    - temptation in $t+1$ to modify X since X no longer influences period t decisions of public.
  - temptation to modify X in $t+1$ must be avoided, if there is to be any hope to have optimal policy. Bad outcomes could occur otherwise.
    - discipline on the part of policy makers is required, if they are to avoid temptation to deviate.
- Technical implication of potential time inconsistency.
  - v equilibrium conditions seemingly not time invarient: apparently our log-linearization methods do not apply!
  - follow Kydland-Prescott 'trick' and put problem in Lagrangian form.
  - problem of avoiding temptation to deviate boils down to the admonition, 'remember your multipliers!'

Example #1: Optimal Monetary Policy - Toy Example

- Setup
  - Model
    - ∗ One equation characterizing private sector behavior:

$$\pi_t - \beta\pi_{t+1} - \gamma y_t = 0,\ t = 0, 1, 2, ... \tag{1}$$

    - ∗ Another equation characterizes policy.
  - Want to do *optimal* policy, so throw away policy equation.
  - System is now under-determined: one equation in two variables, π_t and y_t.

– Optimization delivers the other equations.

∗ optimize objective:

$$\sum_{t=0}^{\infty} \beta^t u\left(\pi_t, y_t\right)$$

subject to (1).

∗ If objective corresponds to social welfare function, this is called *Ramsey* optimal problem

∗ Objective may be preferences of policy maker.

- Lagrangian representation of problem:

$$\max_{\{\pi_t, y_t; t=0,1,...\}} \sum_{t=0}^{\infty} \beta^t \left\{u\left(\pi_t, y_t\right) + \lambda_t\left[\pi_t - \beta\pi_{t+1} - \gamma y_t\right]\right\}$$

$$= \max_{\{\pi_t, y_t; t=0,1,...\}} \{u\left(\pi_0, y_0\right) + \lambda_0\left[\pi_0 - \beta\pi_1 - \gamma y_0\right]$$

$$+\beta u\left(\pi_1, y_1\right) + \beta\lambda_1\left[\pi_1 - \beta\pi_2 - \gamma y_1\right] + ...\}$$

- First order necessary conditions for optimization:

$$\begin{aligned}
u_\pi\left(\pi_0, y_0\right) + \lambda_0 &= 0 \text{ (*)} \\
u_\pi\left(\pi_1, y_1\right) + \lambda_1 - \lambda_0 &= 0 \\
&\ldots \\
u_y(\pi_0, y_0) - \gamma\lambda_0 &= 0 \\
u_y(\pi_1, y_1) - \gamma\lambda_1 &= 0 \\
&\ldots \\
\pi_0 - \beta\pi_1 - \gamma y_0 &= 0 \\
\pi_1 - \beta\pi_2 - \gamma y_1 &= 0 \\
&\ldots
\end{aligned}$$

Example #1: Optimal Monetary Policy - Toy Example ...

- These equations 'look' different than the ones we've seen before
  - They are not stationary, (*) is different from the others.
    - reflects that at time 0 there is a constraint 'missing'
    - no need to respect what people were expecting you to do as of time -1
    - do need to respect what they expect you to do in the future, because that affects current behavior.
    - that's the source of the 'time inconsistency of optimal plans'.
- Can trick the problem into being stationary (see, e.g., Kydland and Prescott (JEDC, 1990s) and Levin, Onatski, Williams, and Williams, Macro Annual, 2005). Then, apply standard log-linearization solution method.

- Consider:

$$v(\pi_t, \pi_{t+1}, y_t, \lambda_t, \lambda_{t-1}) = \begin{bmatrix} u_\pi(\pi_t, y_t) + \lambda_t - \lambda_{t-1} \\ u_y(\pi_t, y_t) - \gamma\lambda_t \\ \pi_t - \beta\pi_{t+1} - \gamma y_t \end{bmatrix}, \text{ for all } t.$$

  – time t ‘endogenous variables’: λ_t, π_t, y_t

  – time t ‘state variable’: λ_{t-1}.

  – ‘solution’:

$$\lambda_t = \lambda(\lambda_{t-1}),\ \pi_t = \pi(\lambda_{t-1}),\ y_t = y(\lambda_{t-1}),$$

  such that

$$v(\pi(\lambda_{t-1}), \pi(\lambda(\lambda_{t-1})), y(\lambda_{t-1}), \lambda(\lambda_{t-1}), \lambda_{t-1}) = 0, \text{ for all possible } \lambda_{t-1}.$$

Example #1: Optimal Monetary Policy - Toy Example ...

- In general, solving this problem exactly is intractable.
- But, can log-linearize!

  – **Step 1**: find π^*, y^*, λ^* such that following three equations are satisfied:
  $$v\left(\pi^*, \pi^*, y^*, \lambda^*, \lambda^*\right) = \underbrace{0}_{3\times 1}.$$

  – **Step 2**: log-linearly expand v about steady state
  $$v\left(\pi_t, \pi_{t+1}, y_t, \lambda_t, \lambda_{t-1}\right) \simeq v_1\pi^*\hat{\pi}_t + v_2\pi^*\hat{\pi}_{t+1} + v_3 y^*\hat{y}_t + v_4\Delta\hat{\lambda}_t + v_5\Delta\hat{\lambda}_{t-1},$$
  where
  $$\Delta\hat{\lambda}_t \equiv \lambda_t - \lambda^*$$ (play it safe, don't divide by something that could be zero!)

  – **Step 3**: Posit
  $$\Delta\hat{\lambda}_t = A_\lambda\Delta\hat{\lambda}_{t-1},\ \hat{\pi}_t = A_\pi\Delta\hat{\lambda}_{t-1},\ \hat{y}_t = A_y\Delta\hat{\lambda}_{t-1},$$
  and find A_λ, A_π, A_y that solve
  $$\left[v_1\pi^* A_\pi + v_2\pi^* A_\pi A_\lambda + v_3 y^* A_y + v_4 A_\lambda + v_5\right]\Delta\hat{\lambda}_{t-1} = \underbrace{0}_{3\times 1}$$
  for all $\Delta\hat{\lambda}_{t-1}$.

- What does the stationary solution have to do with the original non-stationary problem?

  – Do we have a solution to the period 0 problem, (*)?

$$u_\pi\left(\pi_0, y_0\right) + \lambda_0 = 0.$$

  – Yes! Just pretend that this equation really has the following form:

$$u_\pi\left(\pi_0, y_0\right) + \lambda_0 - \lambda_{-1} = 0.$$

  Expression (*) does have this form, if we set $\lambda_{-1} = 0$. Then,

$$\pi_0 = \pi\left(0\right),\ y_0 = y\left(0\right),\ \lambda_0 = \lambda\left(0\right).$$

- The situation is exactly what it is in the neoclassical model when we want to know what happens when initial capital is away from steady state.

  – Plug k_0 into the stationary rule

$$k_1 = g\left(k_0\right).$$

- Possible computational pitfall: if $\lambda_{-1} = 0$ is far from λ^*, then linearized solution might be highly inaccurate (see LOWW).

Example #1: Optimal Monetary Policy - Toy Example ...

- Optimal policy in real time.

- Suppose today is date zero.

  – Solve for $\lambda(\cdot), y(\cdot), \pi(\cdot)$

  – set $\lambda_{-1} = 0$

  – Compute and present in charts:

$$\begin{aligned}
\lambda_0 &= \lambda(\lambda_{-1}),\ y_0 = y(\lambda_{-1}),\ \pi_0 = \pi(\lambda_{-1}) \\
\lambda_1 &= \lambda(\lambda_0),\ y_1 = y(\lambda_0),\ \pi_1 = \pi(\lambda_0) \\
&\ldots \\
\lambda_t &= \lambda(\lambda_{t-1}),\ y_t = y(\lambda_{t-1}),\ \pi_t = \pi(\lambda_0) \\
&\ldots.
\end{aligned}$$

- The optimal policy program may break down if policy makers succumb to the temptation to restart the Ramsey problem at a later date.

  - there is a temptation in period 1 when π_1 is determined, to ignore a constraint that went into determining the announcement made about π_1 in period 0:

$$\pi_0 - \beta\pi_1 - \gamma y_0 \text{ (*)}$$

  - If (*) is ignored at date 1, then π_1 computed in date 1 solves a different problem than π_1 computed at date 0 and there will be time inconsistency.

- Honoring past announcements is equivalent to 'always respect the past multipliers'.

  – 'Remembering λ_0' in period 1 ensures that constraint

  $$\pi_0 - \beta\pi_1 - \gamma y_0 \text{ (*)}$$

  is incorporated in period 1. In this case, π_1 solves the same problem in period 1 that it did in period 0.

- Practical implication of the admonition, 'always respect your multipliers':

  – Charts released after later meetings will be consistent with the continuation of charts released after later meetings.

– Example:

date 0 meeting : $y_0 = y(0)\,,\ y_1 = y(\lambda(\lambda_{-1}))\,, y_2 = y(\lambda(\lambda(\lambda_{-1})))\,, \ldots$

date 1 meeting : **YES -** $y_1 = y(\lambda(\lambda_{-1}))\,,\ y_2 = y(\lambda(\lambda(\lambda_{-1})))\,, \ldots$
NO - $y_1 = y(0)\,,\ y_2 = y(\lambda_1(0))\,, \ldots$

– If Central Bank selects the bad **(‘NO’)** option people will see the temporal inconsistency of policy, and CB will lose credibility.

– Any differences in charts from one meeting to the next must be fully explicable in terms of new information.

Example #2: Optimal Monetary Policy - More General Discussion

- The equilibrium conditions of a model

$$E_t \underbrace{f\left(z_{t-1}, z_t, z_{t+1}, s_t, s_{t+1}\right)}_{(N-1)\times 1} = 0, \text{ for all } \underbrace{z_{t-1}}_{N\times 1} \text{ (endogenous), } s_t \text{ (exogenous)}$$

$$s_t = P s_{t-1} + \varepsilon_t.$$

- Preferences:

$$E_t \sum_{t=0}^{\infty} \beta^t U\left(z_t, s_t\right).$$

  – Could include discounted utility in f :

$$v\left(z_{t-1}, z_t, s_t\right) = U\left(z_t, s_t\right) + \beta E_t v\left(z_t, z_{t+1}, s_{t+1}\right)$$

Example #2: Optimal Monetary Policy - More General Discussion ...

- Optimum problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(z_t, s_t) + \underbrace{\lambda_t'}_{1\times(N-1)} E_t \underbrace{f(z_{t-1}, z_t, z_{t+1}, s_t, s_{t+1})}_{(N-1)\times 1} \right\}.$$

- N first order conditions:

$$\underbrace{U_1(z_t, s_t)}_{1\times N} + \underbrace{\lambda_t'}_{1\times(N-1)} E_t \underbrace{f_2(z_{t-1}, z_t, z_{t+1}, s_t, s_{t+1})}_{(N-1)\times N}$$
$$+\beta^{-1} \underbrace{\lambda_{t-1}'}_{1\times(N-1)} \underbrace{f_3(z_{t-2}, z_{t-1}, z_t, s_{t-1}, s_t)}_{(N-1)\times N}$$
$$+\beta \underbrace{\lambda_{t+1}'}_{1\times(N-1)} E_t \underbrace{f_1(z_t, z_{t+1}, z_{t+2}, s_{t+1}, s_{t+2})}_{(N-1)\times N} = \underbrace{0}_{1\times N}$$

  – Endogenous variables: $z_t\ (N)$, $\lambda_t\ (N-1)$

  – Equations: Ramsey optimality conditions (N), equilibrium condition $(N-1)$

Example #2: Optimal Monetary Policy - More General Discussion ...

- First order conditions of optimum problem have exactly the same form as the type of problem we solved using linearization methods.

- Seem much more cumbersome:

  – must differentiate f (includes private first order conditions that have already involved differentiation!)

  – good news: LOWW wrote a program that takes U, f as input and writes Dynare code for solving the system

  – solving policy optimum problem is no harder than solving original problem.

Example #3: Optimal Monetary Policy - Rotemberg Model

- Household preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \frac{\chi}{2} h_t^2 \right].$$

- Household budget constraint:

$$P_t C_t + B_t = \left(1 + R_{t-1}\right) B_{t-1} + W_t h_t + \Pi_t.$$

- First order conditions:

$$\overbrace{\chi h_t C_t = \frac{W_t}{P_t}}^{\text{intratemporal fonc}}, \qquad \overbrace{\frac{1}{1+R_t} = \beta E_t \frac{P_t C_t}{P_{t+1} C_{t+1}}}^{\text{intertemporal fonc}}, \qquad t = 0, 1, 2, \ldots.$$

- Final good firms

$$\overbrace{Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}}^{\text{production function}}, \ \varepsilon \geq 1, \qquad \overbrace{Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} Y_t}^{\text{first order condition (demand curve for } j^{th} \text{ intermediate good producer)}}$$

- j^{th} intermediate good firm:

$$E_t \sum_{l=0}^{\infty} \beta^l \upsilon_{t+l} [(1+\nu) P_{j,t+l} Y_{j,t+l} - \overbrace{s_{t+l} P_{t+l} Y_{j,t+l}}^{\text{labor costs of production}}$$

$$- \overbrace{\frac{\phi}{2}\left(\frac{P_{j,t+l}}{P_{j,t+l-1}} - 1\right)^2 P_{t+l} C_{t+l}}^{\text{cost (in terms of final goods) of adjusting prices}}],$$

$$Y_{j,t} = \overbrace{A_t h_{j,t}}^{\text{production function}}, \ a_t \equiv \log(A_t), \ s_t = \overbrace{\frac{W_t}{P_t A_t}}^{\text{real marginal cost}} = \overbrace{\frac{\chi h_t C_t}{A_t}}^{\text{substitute out household fonc}}.$$

- Substitute out j^{th} firm's demand curve $\upsilon_t = 1/(P_t C_t)$:

$$\max_{\{P_{j,l}\}_{l=0}^{\infty}} E_t \sum_{l=0}^{\infty} \beta^l \frac{1}{P_{t+l} C_{t+l}} [(1+\nu) P_{j,t+l} \left(\frac{P_{j,t+l}}{P_{t+l}} \right)^{-\varepsilon} Y_{t+l}$$
$$-P_{t+l} s_{t+l} \left(\frac{P_{j,t+l}}{P_{t+l}} \right)^{-\varepsilon} Y_{t+l} - \frac{\phi}{2} \left(\frac{P_{j,t+l}}{P_{j,t+l-1}} - 1 \right)^2 P_{t+l} C_{t+l}]$$

- Differentiate with respect to $P_{j,t}$:

$$\left[(1+\nu)(1-\varepsilon) \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} \frac{1}{P_t} + s_t \varepsilon \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon-1} \frac{1}{P_t} \right] \frac{Y_t}{C_t} - \phi \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right) \frac{1}{P_{j,t-1}}$$
$$+\beta \phi E_t \left(\frac{P_{j,t+1}}{P_{j,t}} - 1 \right) \frac{P_{j,t+1}}{(P_{j,t})^2} = 0.$$

- Rearrange firm efficiency condition:

$$\overbrace{(1+\nu)\frac{P_{j,t}}{P_t}}^{\text{real price received}} = \overbrace{\frac{\varepsilon}{\varepsilon-1}}^{\text{markup}} \times \overbrace{s_t}^{\text{real marginal cost (exclusive of price adjustment costs)}}$$

$$+\frac{1}{\varepsilon-1}\phi\left(\frac{P_{j,t}}{P_t}\right)^{\varepsilon}\frac{C_t}{Y_t}[\quad -\overbrace{\left(\frac{P_{j,t}}{P_{j,t-1}}-1\right)\frac{P_{j,t}}{P_{j,t-1}}}^{\text{if static considerations entail a rise in price, adj costs imply rising by less}}$$

$$+\beta E_t \quad \overbrace{\left(\frac{P_{j,t+1}}{P_{j,t}}-1\right)\frac{P_{j,t+1}}{P_{j,t}}}^{\text{if contemplating a rise in future price, then raise price today by more}} \quad].$$

- When $\phi = 0$:
  - get ‘normal’ efficiency condition, ‘price equals markup over marginal cost’
  - get ‘normal’ monopoly power correction: $1+\nu = \varepsilon/(\varepsilon-1)$.

- Impose $P_{j,t} = P_{i,t} = P_t$ for all i, j :

$$(\pi_t - 1)\,\pi_t = \frac{1}{\phi}\left(1 + \nu - \frac{\varepsilon}{\varepsilon - 1}\right)(1 - \varepsilon)\,\frac{Y_t}{C_t} + \frac{\varepsilon}{\phi}\,(s_t - 1)\,\frac{Y_t}{C_t} + \beta E_t\,(\pi_{t+1} - 1)\,\pi_{t+1}.$$

- Resource constraint:

$$C_t\left[\overbrace{1}^{\text{consumption}} + \overbrace{\frac{\phi}{2}\,(\pi_t - 1)^2}^{\text{price adjustment costs}}\right] = \overbrace{A_t h_t}^{\text{total output}} = Y_t,\;\; a_t = \rho a_{t-1} + u_t,\;\; a_t \equiv \log A_t$$

- Substitute out the resource constraint:

$$\begin{aligned}(\pi_t - 1)\,\pi_t \;&=\; \frac{1}{\phi}\left[\left(1 + \nu - \frac{\varepsilon}{\varepsilon - 1}\right)(1 - \varepsilon) + \varepsilon\,(s_t - 1)\right]\left[1 + \frac{\phi}{2}\,(\pi_t - 1)^2\right] \\ &\quad + \beta E_t\,(\pi_{t+1} - 1)\,\pi_{t+1}.\end{aligned}$$

- Looks ‘sort of’ like Calvo equilibrium relation for inflation.

- Log-linearize around 'efficient steady state' $(\pi_t = 1, 1+\nu = \varepsilon/(\varepsilon-1),$ $s_t = 1)$:

$$(\pi_t - 1)\pi_t = \frac{1}{\phi}\left[\left(1+\nu-\frac{\varepsilon}{\varepsilon-1}\right)(1-\varepsilon)+\varepsilon(s_t-1)\right]\left[1+\frac{\phi}{2}(\pi_t-1)^2\right]$$
$$+\beta E_t(\pi_{t+1}-1)\pi_{t+1}.$$

$$d[(\pi_t-1)\pi_t] \overbrace{=}^{\text{totally differentiate}} \pi_t d\pi_t + (\pi_t-1)d\pi_t \overbrace{=}^{\text{evaluate in steady state, } \pi=1} \pi d\pi_t \equiv \pi^2\hat{\pi}_t = \hat{\pi}_t.$$

- Doing the log-linearization:

$$\hat{\pi}_t = \frac{\varepsilon}{\phi}\hat{s}_t + \beta E_t\hat{\pi}_{t+1}.$$

'marginal cost affects price less the bigger is ϕ'

so Rotemberg *IS* Calvo, up to linear approximation and with a different interpretation of slope on marginal cost.

- Summarizing equilibrium conditions:

  – household intertemporal efficiency condition:

$$\frac{1}{1+R_t} = \beta E_t \frac{P_t C_t}{P_{t+1} C_{t+1}}$$

  – firm efficiency condition for prices (after rearranging) -

$$\left[\left(1+\nu-\frac{\varepsilon}{\varepsilon-1}\right)(1-\varepsilon)+\varepsilon\left(s_t-1\right)\right]\left(1+\frac{\phi}{2}\left(\pi_t-1\right)^2\right)-\phi\left(\pi_t-1\right)\pi_t$$

$$+\beta\phi E_t\left(\pi_{t+1}-1\right)\pi_{t+1} \;=\; 0$$

  – marginal cost:

$$s_t = \frac{\overbrace{\chi h_t C_t}^{\text{uses household intratemporal efficiency condition}}}{A_t}$$

  – Resource constraint:

$$C_t\left[1+\frac{\phi}{2}\left(\pi_t-1\right)^2\right] = A_t h_t,\ a_t = \rho a_{t-1} + u_t$$

- Ramsey optimal problem:

$$\max_{\{\nu, C_t, h_t, \pi_t, R_t\}} E_0 \sum_{t=0}^{\infty} \beta_n^t \{ \left(\log\left(C_t\right) - \frac{\chi}{2} h_t^2 \right) +$$

$$\lambda_{1t} \left(\frac{1}{1+R_t} - \beta E_t \frac{C_t}{\pi_{t+1} C_{t+1}} \right)$$

$$+\lambda_{2t} [\left(\left(1 + \nu - \frac{\varepsilon}{\varepsilon - 1} \right) (1-\varepsilon) + \varepsilon \left(\frac{\chi h_t C_t}{A_t} - 1 \right) \right) \left(1 + \frac{\phi}{2} \left(\pi_t - 1\right)^2 \right)$$

$$-\phi \left(\pi_t - 1\right) \pi_t + \beta \phi E_t \left(\pi_{t+1} - 1\right) \pi_{t+1}]$$

$$+\lambda_{3t} \left[C_t \left(1 + \frac{\phi}{2} \left(\pi_t - 1\right)^2 \right) - A_t h_t \right] \}$$

- Conjecture about solution to Ramsey problem:

$$\lambda_{1t} = \lambda_{2t} = 0.$$

- We will implement, and then verify the conjecture formally.

- Intuition:

  – intertemporal equation non-binding from the point of view of maximizing utility, because R_t (a variable of no direct interest in utility) can always be chosen to enforce intertemporal Euler equation).

  – price equation non-binding from the point of view of maximizing utility, because ν (a variable of no direct interest in utility) can always be chosen to enforce the price equation.

- Simplified Ramsey problem (ν, R_t are a matter of indifference here)

$$\max_{\{C_t,h_t,\pi_t\}} E_0 \sum_{t=0}^{\infty} \beta_n^t \{ \left(\log \left(C_t \right) - \frac{\chi}{2} h_t^2 \right) + \lambda_{3t} \left[C_t \left(1 + \frac{\phi}{2} \left(\pi_t - 1 \right)^2 \right) - A_t h_t \right] \}.$$

- – first order necessary condition for π_t:

$\lambda_{3t} C_t \phi \left(\pi_t - 1 \right) = 0 \rightarrow \pi_t = 1$ (obviously, $\lambda_{3t} = 0,$ or $C_t = 0$ is not the solution!)

– first order condition for h_t, C_t :

$$-\chi h_t - \lambda_{3t} A_t = 0,\ \frac{1}{C_t} + \lambda_{3t} \left(1 + \frac{\phi}{2} \left(\pi_t - 1 \right)^2 \right) = 0 \rightarrow \lambda_{3t} = -\frac{1}{C_t}.$$

or

$$\chi h_t = \frac{1}{C_t} A_t,\ \rightarrow \overbrace{\chi h_t C_t}^{MRS} = \overbrace{A_t}^{MPL}$$

so solution to Ramsey problem achieves ‘first best’:

$$\max_{C_t,h_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\log \left(C_t \right) - \frac{\chi}{2} h_t^2 \right],\ \text{subject to } C_t = A_t h_t.$$

- Solution to Ramsey problem:

$$
\begin{aligned}
C_t &= A_t h_t \\
\chi h_t C_t &= A_t \rightarrow h_t^2 = \left[\frac{1}{\chi}\right]^{1/2}, \\
\pi_t &= 1, \\
R_t &= \frac{1}{\beta E_t \frac{P_t C_t}{P_{t+1} C_{t+1}}} - 1, \\
1 + \nu &= \frac{\varepsilon}{\varepsilon - 1}.
\end{aligned}
$$

- Notice:
  - this is first best, and there is no time inconsistency problem!
  - treatment of ν crucial here (reason that price equation is non-binding)

Example #3: Optimal Monetary Policy - Rotemberg Model ...

- Ramsey problem with ν fixed, $\neq \varepsilon / (\varepsilon - 1)$

$$\max_{\{C_t, h_t, \pi_t, R_t\}} E_0 \sum_{t=0}^{\infty} \beta_n^t \{ \left(\log (C_t) - \frac{\chi}{2} h_t^2 \right) +$$

$$\lambda_{1t} \left(\frac{1}{1+R_t} - \beta E_t \frac{C_t}{\pi_{t+1} C_{t+1}} \right)$$

$$+\lambda_{2t} [\left(\left(1 + \nu - \frac{\varepsilon}{\varepsilon - 1} \right) (1 - \varepsilon) + \varepsilon \left(\frac{\chi h_t C_t}{A_t} - 1 \right) \right) \left(1 + \frac{\phi}{2} (\pi_t - 1)^2 \right)$$

$$-\phi (\pi_t - 1) \pi_t + \beta \phi E_t (\pi_{t+1} - 1) \pi_{t+1}]$$

$$+\lambda_{3t} \left[C_t \left(1 + \frac{\phi}{2} (\pi_t - 1)^2 \right) - A_t h_t \right] \},$$

- Note: intertemporal condition still non-binding.

- First order conditions for Ramsey problem (impose $\lambda_{1t} \equiv 0$, β_n is discount rate in planner objective, $\lambda_{i,-1} = 0$) for h_t, π_t, C_t :

$$\boxed{-\chi h_t + \lambda_{2t}\varepsilon\frac{\chi C_t}{A_t}\left(1 + \frac{\phi}{2}\left(\pi_t - 1\right)^2\right) - \lambda_{3,t}A_t = 0.}$$

$$\boxed{\begin{array}{l} \lambda_{2t}\left[\left(\left(1 + \nu - \frac{\varepsilon}{\varepsilon - 1}\right)\left(1 - \varepsilon\right) + \varepsilon\left(\frac{\chi h_t C_t}{A_t} - 1\right) - 1\right)\phi\left(\pi_t - 1\right) - \phi\pi_t\right] \\ \\ +\lambda_{2,t-1}\beta_n^{-1}\beta\phi\left[\left(\pi_t - 1\right) + \pi_t\right] + \lambda_{3t}C_t\phi\left(\pi_t - 1\right) = 0 \end{array}}$$

$$\boxed{\frac{1}{C_t} + \lambda_{2t}\varepsilon\frac{\chi h_t}{A_t}\left(1 + \frac{\phi}{2}\left(\pi_t - 1\right)^2\right) + \lambda_{3t}\left(1 + \frac{\phi}{2}\left(\pi_t - 1\right)^2\right) = 0}$$

- Plus, intertemporal household equation, price equation and resource constraint yields 6 equations in $R_t, h_t, C_t, \pi_t, \lambda_{2t}, \lambda_{3t}$.

- It is instructive to study the steady state of the previous equations.

- Let

$$\omega = \frac{1+\nu}{\frac{\varepsilon}{\varepsilon-1}} = \frac{\text{subsidy}}{\text{steady state markup in the absence of a subsidy}}$$

Then, in steady state with $\beta_n = \beta$:

$$\pi = 1,\ C = h = \left[\frac{\omega}{\chi}\right]^{1/2},\ \lambda_2 = \frac{\omega-1}{2\varepsilon\omega},\ \lambda_3 = -\chi h \frac{\omega+1}{2\omega}$$

- Notes on Ramsey steady state....

$$\pi = 1,\ C = h = \left[\frac{\omega}{\chi}\right]^{1/2},\ \lambda_2 = \frac{\omega - 1}{2\varepsilon\omega},\ \lambda_3 = -\chi h \frac{\omega + 1}{2\omega}$$

  – Although Ramsey has only one degree of freedom (i.e., an excess of only one variable over equations) and two barriers to first best (i.e., monopoly power and inflation), it chooses to neutralize only inflation (i.e., $\pi = 1$).

  – Steady state hours is bigger than first best if subsidy is too high (i.e., $\omega > 1$) and smaller in case the subsidy is too low.

  – Sign of λ_2 depends on whether the subsidy is too high (then, $\lambda_2 > 0$) or too low.

  – $\lambda_3 < 0$ always, as expected.

  – Can verify numerically that $\lambda_{2,t-1}$ enters policy rules for h_t, C_t, R_t.
    * So, when $\lambda_2 \neq 0$ the Ramsey problem is not time consistent.

Example 3: Time inconsistency and 'inflation bias'

- Consider the following model parameter values:

$$\beta = 0.99,\ \varepsilon = 5,\ \phi = 100,\ \rho = 0.9,\ \nu = 0,\ \chi = 0.8.$$

- Note that the subsidy rate has been set to zero.

- The model solution, after linearizing the equilibrium conditions about steady state:

$$\begin{aligned}
h_t &= 1 + 2.4\left(\lambda_{2,t-1} + 0.025\right) \\
R_t &= 0.01 - 0.71\left(\lambda_{2,t-1} + 0.025\right) - 0.10\left(A_t - 1\right) \\
C_t &= 1 + 2.4\left(\lambda_{2,t-1} + 0.025\right) + \left(A_t - 1\right) \\
\pi_t &= 1 + 0.45\left(\lambda_{2,t-1} + 0.025\right) \\
\lambda_{2,t} &= -0.025 + 0.59\left(\lambda_{2,t-1} + 0.025\right)
\end{aligned}$$

Example 3: Time inconsistency and 'inflation bias' ...

- Suppose the system is in a Ramsey steady state, with $A_t = 1$ in each t.

  – with no deviation, obtain:
  $$C_t = h_t = 1,\ R_t = 0.01,\ \pi_t = 1,\ \lambda_{2t} = -0.025.$$
  – suppose the monetary authority deviates in period, t, by restarting the Ramsey program. The values of the variables in period t :
  $$C_t = h_t = 1 + 2.4 \times 0.025 = 1.06,\ R_t = -0.0078,\ \pi_t = 1.01.$$
  – the deviation pushes consumption, hours worked and inflation up, and the interest rate down. This pushes employment in the direction of first best: $h_t = 1.12.$

  – in effect, the 'surprise inflation deviation' is the only way monetary policy can address the monopoly power problem. The problem is that if there is a deviation, then credibility breaks down and the Ramsey optimal plan is not implemented. What occurs instead may be very bad.

  – the nature of the temptation to deviate under this parameterization corresponds to the famous 'inflation bias' studied by Kydland-Prescott and Barro-Gordon and many others.

Example 3: Time inconsistency and '*de*flation bias'

- Suppose that $\nu = 1$, so that ν *over* stimulates the economy (this seems implausible to me...)

- Otherwise, all parameter values are unchanged.

- The linear approximation is now:

$$
\begin{aligned}
h_t &= 1.41 + 3.4\,(\lambda_{2,t-1} - 0.0375) \\
R_t &= 0.01 - 1.53\,(\lambda_{2,t-1} - 0.0375) - 0.10\,(A_t - 1) \\
C_t &= 1.41 + 3.4\,(\lambda_{2,t-1} - 0.0375) + 1.41\,(A_t - 1) \\
\pi_t &= 1 + 0.54\,(\lambda_{2,t-1} - 0.0375) \\
\lambda_{2,t} &= 0.0375 + 0.30\,(\lambda_{2,t-1} - 0.0375)
\end{aligned}
$$

- Note how much higher steady hours worked and consumption are.

Example 3: Time inconsistency and '*de*flation bias' ...

- Suppose the system is in a Ramsey steady state, with $A_t = 1$ in each t.
  - with no deviation, obtain:
  $$C_t = h_t = 1.41,\ R_t = 0.01,\ \pi_t = 1,\ \lambda_{2t} = 0.0375.$$
  - suppose the monetary authority deviates in period, t, by restarting the Ramsey program. The values of the variables in period t :
  $$C_t = h_t = 1.41 - 3.4 \times 0.0375 = 1.28,\ R_t = 0.07,\ \pi_t = 0.98.$$
  - the deviation pushes consumption, hours worked and inflation *down*, and the interest rate *up*. This pushes employment in the direction of first best: $h_t = 1.12$.
  - in effect, the 'surprise deflation deviation' is the only way monetary policy can address the monopoly power problem. Again, if there is a deviation, then credibility breaks down and the Ramsey optimal plan is not implemented. What occurs instead may be very bad.
  - the nature of the temptation to deviate under this parameterization can be called a 'deflation bias'.

Example 3: Conclusion

- When labor market is treated optimally, policy achieves first best (i.e., it solves both the monopoly and inflation problems) and there is no time consistency problem.

- When labor market not treated optimally, Ramsey optimal policy solves the inflation problem, but does not touch the monopoly power problem. There is now a time consistency problem. One interpretation is that surprise deviations from Ramsey policy are the only way to address the monopoly problem.

- Good news: linearization methods apply.

- Bad news: Ramsey first order conditions painful to compute in practice.

- Good news: there exists computer code for deriving the Ramsey first order conditions symbolically (this is explored in a homework with Dynare).

Example #4: Optimal Monetary Policy - CGG

$$
\begin{aligned}
&\max_{\nu_t, p_t^*, N_t, R_t, \bar{\pi}_t, F_t, K_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \left(\log N_t + \log p_t^* - \exp\left(\tau_t\right) \frac{N_t^{1+\varphi}}{1+\varphi} \right) \\
&+\lambda_{1t} \left[\frac{1}{p_t^* N_t} - E_t \frac{A_t \beta}{p_{t+1}^* A_{t+1} N_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \right] \\
&+\lambda_{2t} \left[\frac{1}{p_t^*} - \left(\left(1-\theta\right) \left(\frac{1-\theta\left(\bar{\pi}_t\right)^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right) \right] \\
&+\lambda_{3t} \left[1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta \theta F_{t+1} - F_t \right] \\
&+\lambda_{4t} \left[\left(1-\nu_t\right) \frac{\varepsilon}{\varepsilon-1} \exp\left(\tau_t\right) N_t^{1+\varphi} p_t^* \left(1-\psi+\psi R_t\right) + E_t \bar{\pi}_{t+1}^{\varepsilon} \beta \theta K_{t+1} - K_t \right] \\
&+\lambda_{5t} \left[F_t \left[\frac{1-\theta \bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} - K_t \right] \}
\end{aligned}
$$

- ‘two degree of freedom’ 7 variables, 5 equilibrium conditions

- Law of motion of technology:
$$A_t = \rho A_{t-1} + u_t.$$
- We only consider the case,
$$(1-\nu)\frac{\varepsilon}{\varepsilon - 1} = 1.$$
- First consider the case, $\psi = 0$
  - Conjecture: restrictions 1, 3, 4, 5 nonbinding (i.e., $\lambda_{1t} = \lambda_{3t} = \lambda_{4t} = \lambda_{5t} = 0$)
    - Step 1: Optimize w.r.t. p_t^*, $\bar{\pi}_t$, N_t ignoring restrictions 1, 3, 4, 5.
    - Step 2: Solve for ν_t, R_t, F_t, K_t, to satisfy restrictions 1, 3, 4, 5.
  - If this can be done, then the conjecture is verified.

- Simplified problem under conjecture:

$$\max_{\bar{\pi}_t, p_t^*, N_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \left(\log N_t + \log p_t^* - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

$$+\lambda_{2t} \left[\frac{1}{p_t^*} - \left((1-\theta) \left(\frac{1-\theta(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right) \right] \}$$

first order conditions with respect to p_t^*, $\bar{\pi}_t$, N_t (after rearranging):

$$p_t^* + \beta \lambda_{2,t+1} \theta \bar{\pi}_{t+1}^{\varepsilon} = \lambda_{2t}, \ \bar{\pi}_t = \left[\frac{\left(p_{t-1}^*\right)^{\varepsilon-1}}{1-\theta+\theta\left(p_{t-1}^*\right)^{\varepsilon-1}} \right]^{\frac{1}{\varepsilon-1}}, \ N_t = \exp\left(-\frac{\tau_t}{\varphi+1}\right)$$

– Substituting the solution for $\bar{\pi}_t$ into the law of motion for p_t^* :

$$p_t^* = \left[(1-\theta) + \theta \left(p_{t-1}^*\right)^{(\varepsilon-1)} \right]^{\frac{1}{(\varepsilon-1)}}.$$

- Can the other constraints be satisfied?

  – Choose R_t so the intertemporal constraint is satisfied:

$$R_t = \frac{\frac{1}{p_t^* N_t}}{E_t \frac{A_t \beta}{p_{t+1}^* A_{t+1} N_{t+1} \bar{\pi}_{t+1}}}.$$

  – Remaining constraints: three price-setting conditions.

Example #4: Optimal Monetary Policy - CGG ...

- Price setting conditions:

$$1 + E_t\bar{\pi}_{t+1}^{\varepsilon-1}\beta\theta F_{t+1} = F_t \text{ (1)}$$

$$(1-\nu_t)\frac{\varepsilon}{\varepsilon-1}\exp(\tau_t)N_t^{1+\varphi}p_t^* + E_t\bar{\pi}_{t+1}^{\varepsilon}\beta\theta K_{t+1} = K_t \text{ (2)}$$

$$F_t\left[\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \overbrace{=}^{\text{(making use of the expression for optimal inflation)}} F_t p_t^* = K_t \text{ (3)}$$

- Divide (2) by p_t^*, impose (3) and use $\bar{\pi}_{t+1} = p_t^*/p_{t+1}^*$:

$$\overbrace{\frac{\varepsilon}{\varepsilon-1}}^{\text{markup}} \times \overbrace{(1-\nu_t)\times \overbrace{\exp(\tau_t)N_t^{1+\varphi}}^{=\frac{W_t}{P_tA_t}=\frac{MRS}{A}\overbrace{=}^{\text{by optimality of } N_t}1}}^{\text{=firm marginal cost}/P_t} + E_t\bar{\pi}_{t+1}^{\varepsilon-1}\beta\theta F_{t+1} = F_t$$

- Subtract from (1) (subsidy must cancel markup and interest rate distortion):

$$(1-\nu)\frac{\varepsilon}{\varepsilon-1} = 1.$$

- Bottom line. Optimality under state-contingent ν_t implies:

$$
\begin{aligned}
p_t^* &= \left[(1-\theta)+\theta\left(p_{t-1}^*\right)^{(\varepsilon-1)}\right]^{\frac{1}{(\varepsilon-1)}} \\
\bar{\pi}_t &= \frac{p_{t-1}^*}{p_t^*} \\
N_t &= \exp\left(-\frac{\tau_t}{1+\varphi}\right) \\
1-\nu &= \frac{\varepsilon-1}{\varepsilon} \\
C_t &= p_t^* A_t N_t.
\end{aligned}
$$

- Ramsey-optimal policy is time consistent (no forward-looking constraints on core problem).
- If $\psi > 0$ and ν_t not state-contingent must work out Ramsey solution numerically.

  (For further discussion, see Christiano-Motto-Rostagno, ‘Two Reasons Why Money Might be Useful in Monetary Policy’, 2007 NBER WP.)

- Example - no working capital channel ($\psi = 0$):

$$\theta = 0.75,\ \varepsilon = 2,\ \beta = 0.99,\ \rho = 0.5,\ \varphi = 1.$$

- In this case:

$$\begin{aligned}
N_t &= 1 + 0.45(\lambda_{1t-1} - \lambda_1) + .06(\lambda_{3,t-1} - \lambda_3) + 0.63(\lambda_{4,t-1} - \lambda_4) \\
r_t &= 0.01 - 0.50(\lambda_{1t-1} - \lambda_1) + 0.10(\lambda_{3,t-1} - \lambda_3) - 0.02(\lambda_{4,t-1} - \lambda_4) - 0.25a_{t-1} \\
&\quad -0.51u_t \\
\pi_t &= 1 + 0.07(\lambda_{1t-1} - \lambda_1) + 0.09(\lambda_{3,t-1} - \lambda_3) + 0.31(\lambda_{4,t-1} - \lambda_4) + 0.25(p^*_{t-1} - 1) \\
\lambda_{1t} &= 0, \\
\lambda_{2,t} &= 3.88 + 0.82(\lambda_{1t-1} - \lambda_1) + 1.46(\lambda_{3,t-1} - \lambda_3) + 3.65(\lambda_{4,t-1} - \lambda_4) \\
&\quad +4.13(p^*_{t-1} - 1) \\
\lambda_{3,t} &= 0.05(\lambda_{1t-1} - \lambda_1) + 0.69(\lambda_{3,t-1} - \lambda_3) + 0.12(\lambda_{4,t-1} - \lambda_4) \\
\lambda_{4,t} &= -0.05(\lambda_{1t-1} - \lambda_1) + 0.06(\lambda_{3,t-1} - \lambda_3) + 0.63(\lambda_{4,t-1} - \lambda_4) \\
\lambda_{5,t} &= 0.05(\lambda_{1t-1} - \lambda_1) - 0.06(\lambda_{3,t-1} - \lambda_3) + 0.12(\lambda_{4,t-1} - \lambda_4)
\end{aligned}$$

$$\lambda_1 = \lambda_3 = \lambda_4 = \lambda_5 = 0, \lambda_2 = 3.88$$

- ‘Resetting multipliers’ makes no difference: *no* time inconsistency problem.

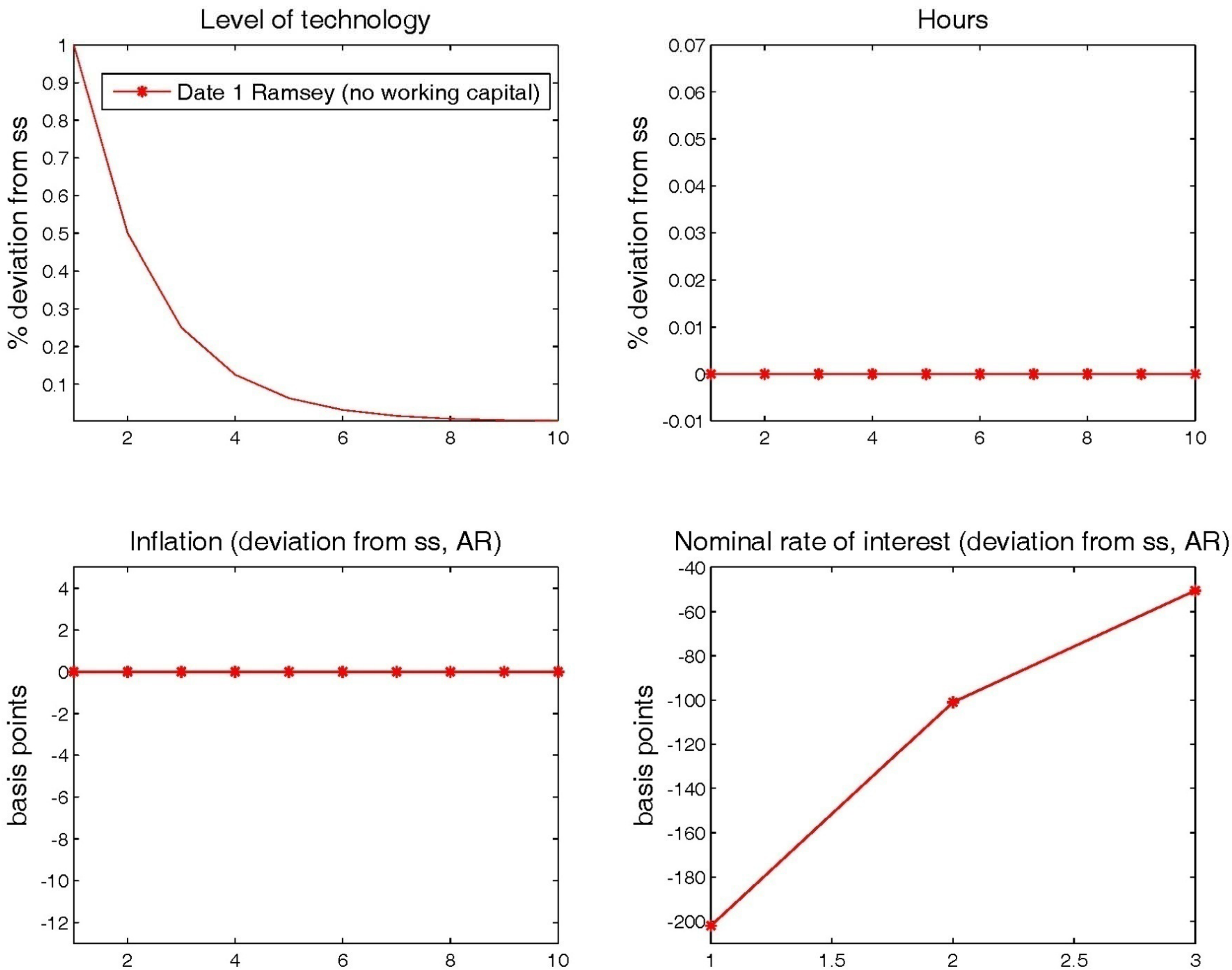
- Example with $\psi = 0.7$:

$$
\begin{aligned}
N_t &= 1 + 0.50\lambda_{1t-1} + .03\lambda_{3,t-1} + 0.40\lambda_{4,t-1} + 0.02a_{t-1} + 0.03u_t \\
r_t &= 0.01 - 0.51\lambda_{1t-1} + 0.12\lambda_{3,t-1} + 0.30\lambda_{4,t-1} - 0.24a_{t-1} - 0.49u_t \\
\pi_t &= 1 + 0.05\lambda_{1t-1} + 0.10\lambda_{3,t-1} + 0.31\lambda_{4,t-1} - 0.01a_{t-1} + 0.25(p^*_{t-1} - 1) - 0.02u_t \\
p^*_t &= 1 + .75(p^*_{t-1} - 1) \\
\lambda_{1t} &= -0.01\lambda_{1t-1} + 0.04\lambda_{3,t-1} + 0.44\lambda_{4,t-1} + 0.02A_{t-1} + 0.03u_t \\
\lambda_{2,t} &= 3.88 + 0.95\lambda_{1t-1} + 1.42\lambda_{3,t-1} + 3.63\lambda_{4,t-1} + 0.09A_{t-1} + 0.18u_t + 4.13(p^*_{t-1} - 1) \\
\lambda_{3,t} &= 0.01\lambda_{1t-1} + 0.70\lambda_{3,t-1} + 0.13\lambda_{4,t-1} - 0.02a_{t-1} - 0.05u_t \\
\lambda_{4,t} &= -0.01\lambda_{1t-1} + 0.05\lambda_{3,t-1} + 0.62\lambda_{4,t-1} + 0.02a_{t-1} + 0.05u_t \\
\lambda_{5,t} &= 0.015\lambda_{1t-1} - 0.05\lambda_{3,t-1} + 0.13\lambda_{4,t-1} - .02a_{t-1} - 0.05u_t
\end{aligned}
$$

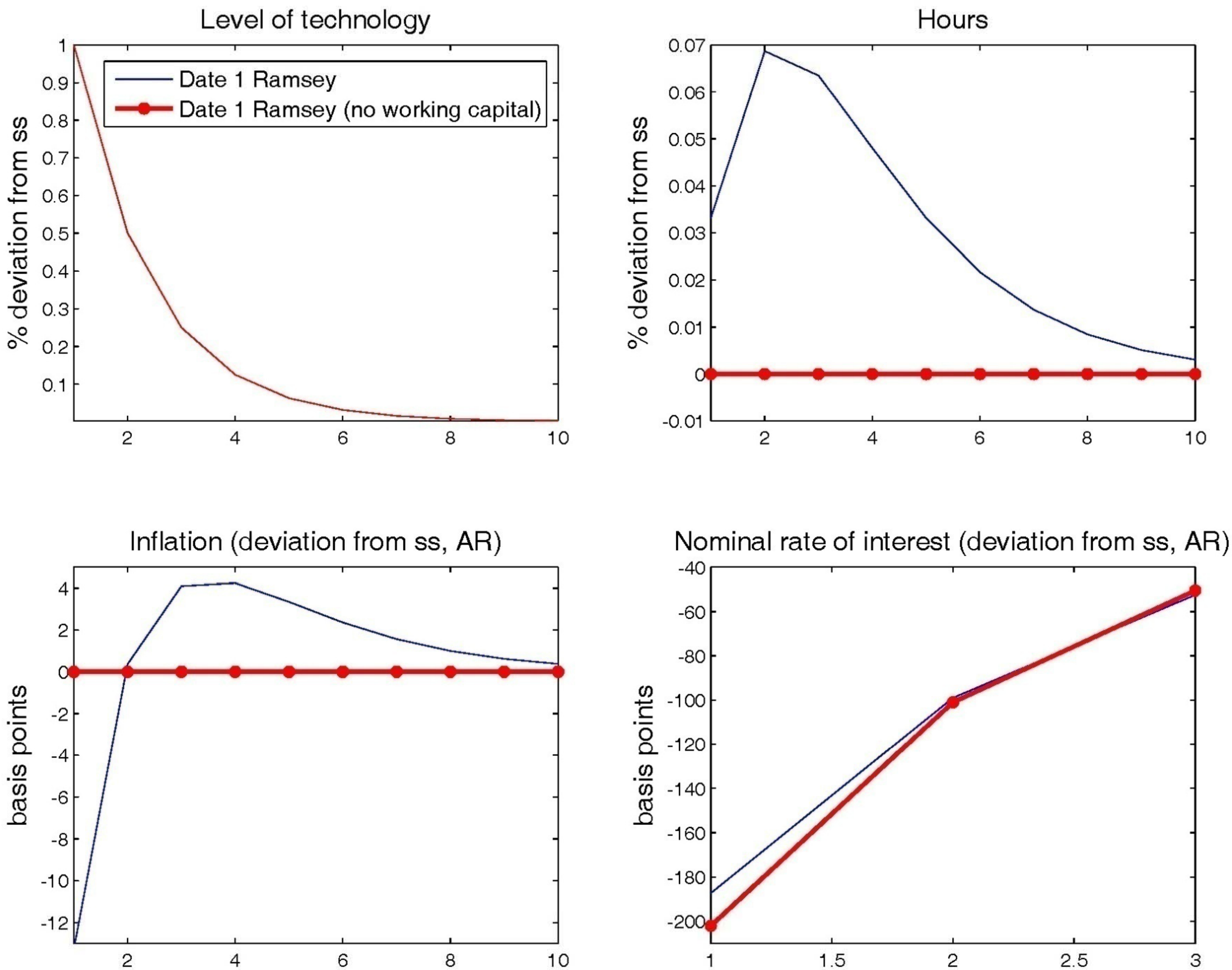
$$\lambda_1 = \lambda_3 = \lambda_4 = \lambda_5 = 0, \lambda_2 = 3.88$$

- Properties: all multipliers respond to u_t; optimal plan not time consistent; employment and inflation respond to u_t; r_t drops a little less than before (it's a tax now); N_t falls somewhat because of the interest rate 'tax'.

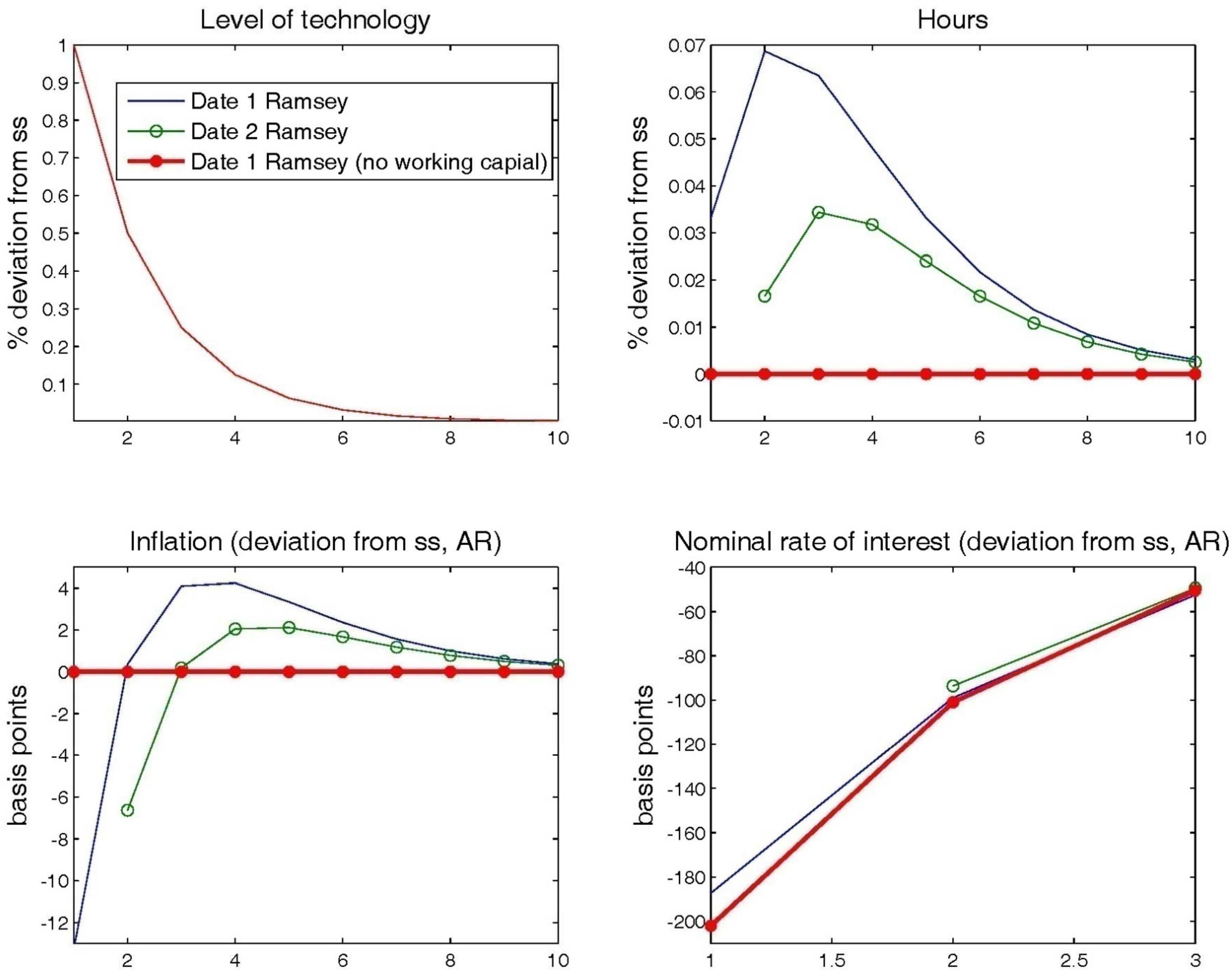
- Experiment:
  - Economy is in steady state of optimal plan up to period t.
  - A positive shock to technology occurs.
  - Monetary authority computes optimal policy and displays it in a set of charts.
  - Redo charts one period later.



Level of technology
Date 1 Ramsey (no working capital)
% deviation from ss
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
2
4
6
8
10
Hours
% deviation from ss
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
2
4
6
8
10
Inflation (deviation from ss, AR)
basis points
4
2
0
-2
-4
-6
-8
-10
-12
2
4
6
8
10
Nominal rate of interest (deviation from ss, AR)
basis points
-40
-60
-80
-100
-120
-140
-160
-180
-200
1
1.5
2
2.5
3



Level of technology
Date 1 Ramsey
Date 1 Ramsey (no working capital)
% deviation from ss
Hours
% deviation from ss
Inflation (deviation from ss, AR)
basis points
Nominal rate of interest (deviation from ss, AR)
basis points


Level of technology
% deviation from ss
Date 1 Ramsey
Date 2 Ramsey
Date 1 Ramsey (no working capial)
Hours
% deviation from ss
Inflation (deviation from ss, AR)
basis points
Nominal rate of interest (deviation from ss, AR)
basis points

Example #4: Optimal Monetary Policy - Clarida-Gali-Gertler Model ...

- Discussion of the results
  - In the absence of a working capital channel (i.e., $\psi = 0$) it is optimal to cut the interest rate, to encourage households not smooth consumption away from what is optimal.
  - In the presence of a working capital channel, (i.e., $\psi > 0$), the cut in the interest rate reduces the marginal cost of labor and expands output and employment. By reducing marginal cost, inflation drops.
  - The rise in employment and fall in inflation are both costly, and so:
    - it is optimal when $\psi > 0$ to cut the interest rate by less.
    - it is optimal to manage expectations so that the incentive to cut prices in the present is reduced.
      - announce inflation close to zero in the next period
      - announce relatively small interest rate drop in the next period.

Example #4: Optimal Monetary Policy - Clarida-Gali-Gertler Model ...

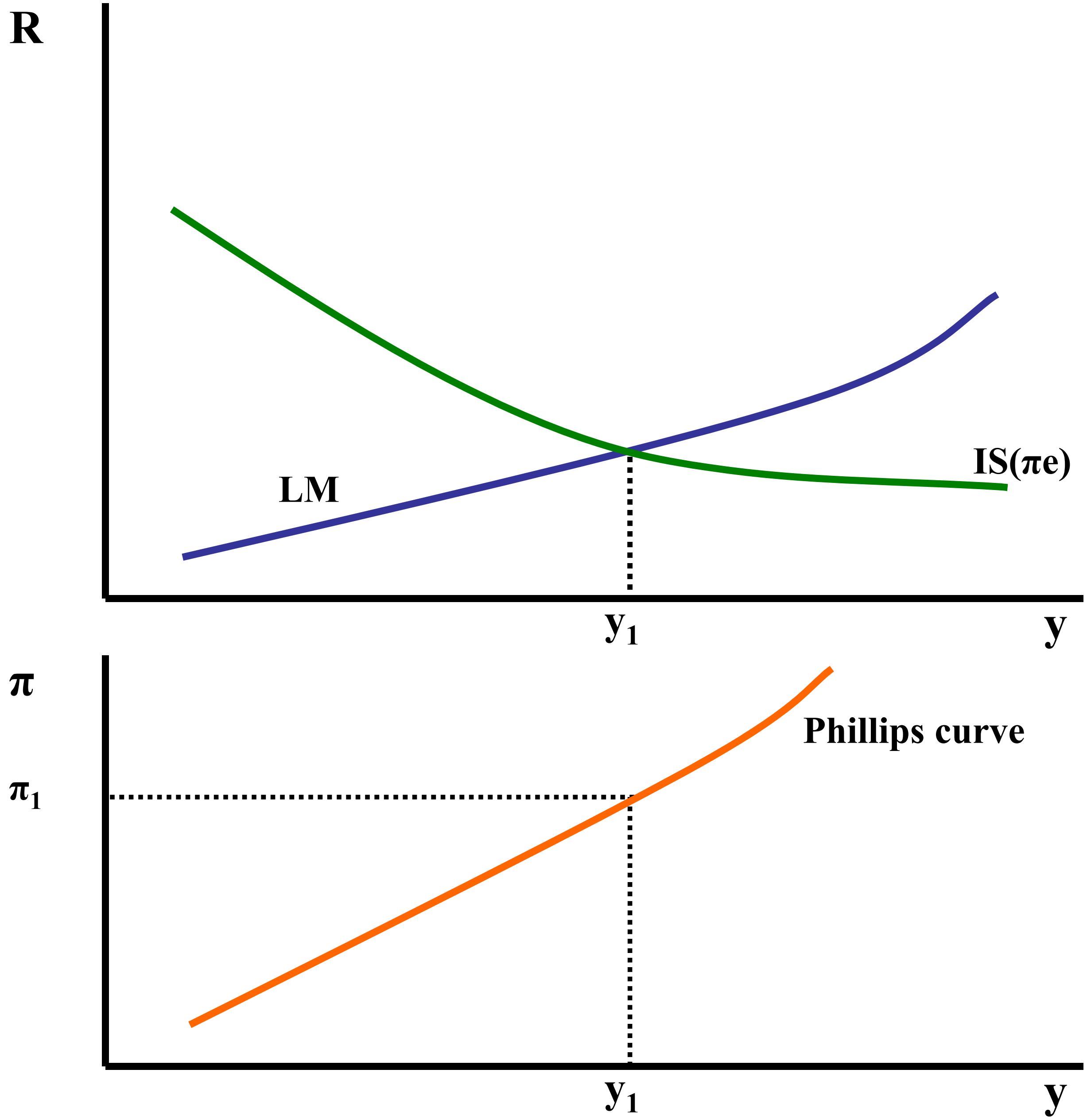
- Previous example illustrates importance of working capital channel ('lending channel')

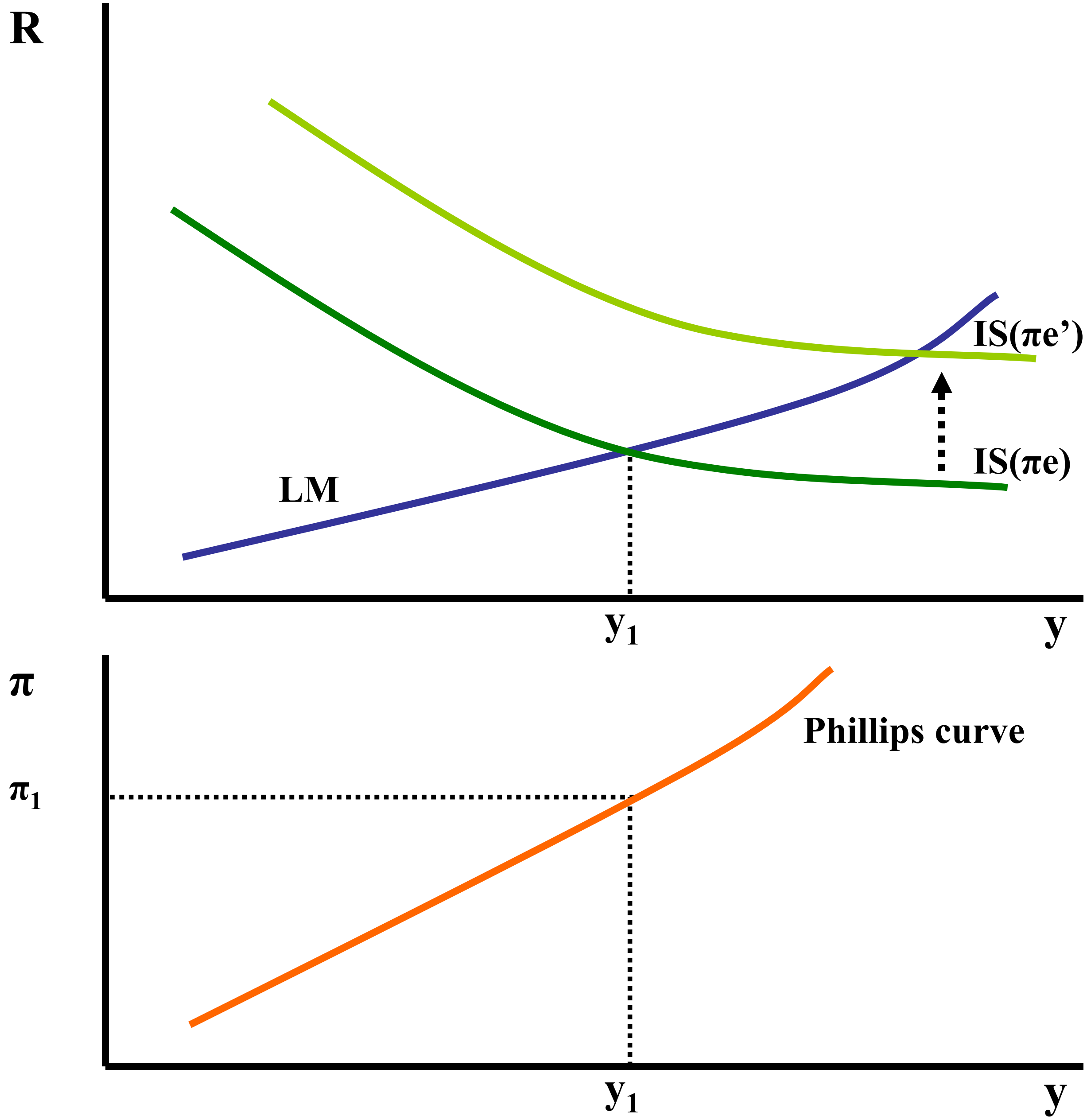
  – CCG model with Taylor rule illustrates this too

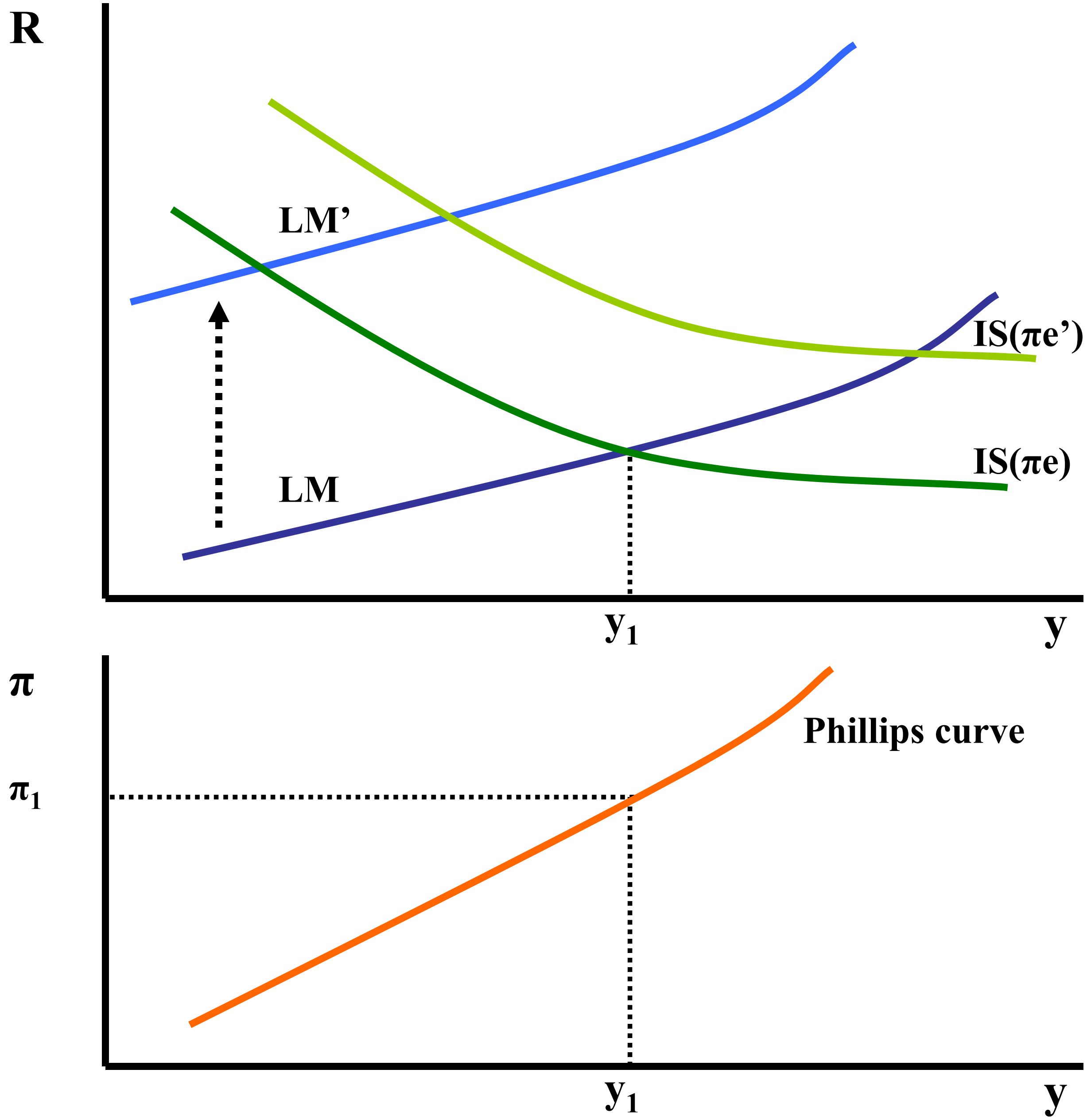
$$R_t = 1.5\pi^e_{t+1}$$

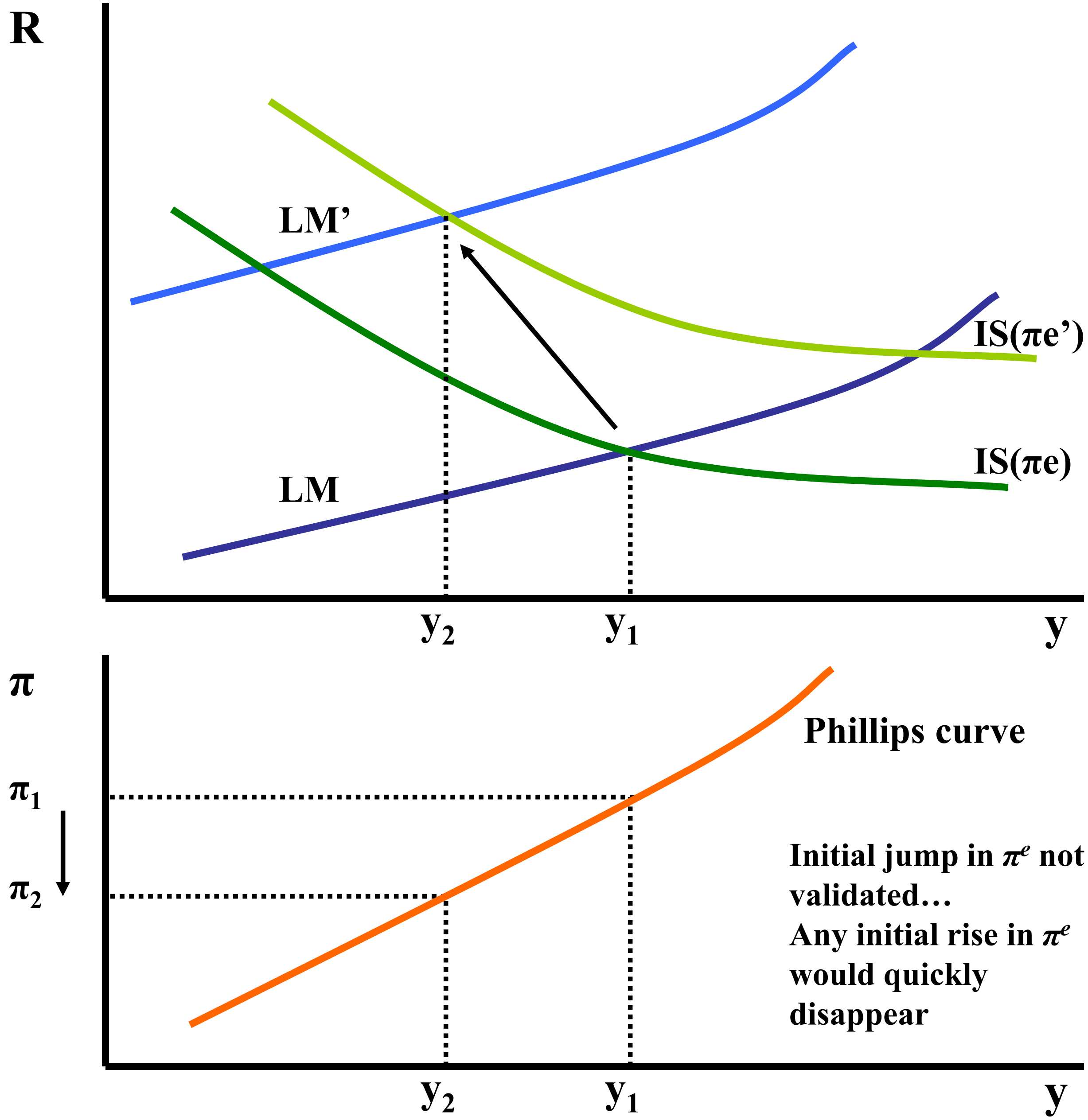
  – With $\psi = 0$, obtain standard result that inflation expectations cannot be self-fulfilling.

  – With $\psi > 0$, results turn upside down (see CMR, 'Two Reasons...')


R
LM
IS(πe)
y1
y
π
Phillips curve
π1
y1
y

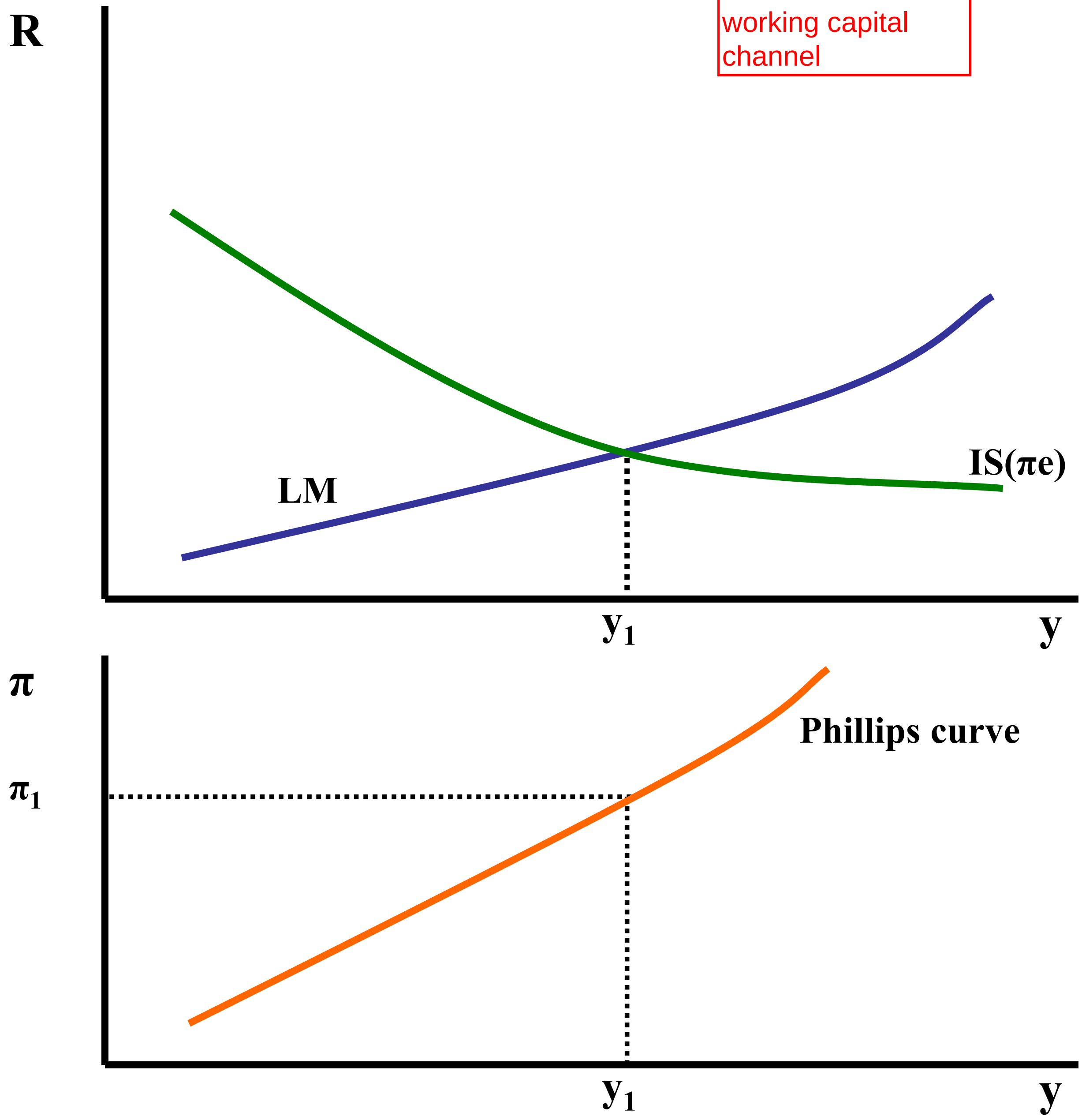

R
IS(πe')
IS(πe)
LM
y_1
y
π
Phillips curve
π_1
y_1
y

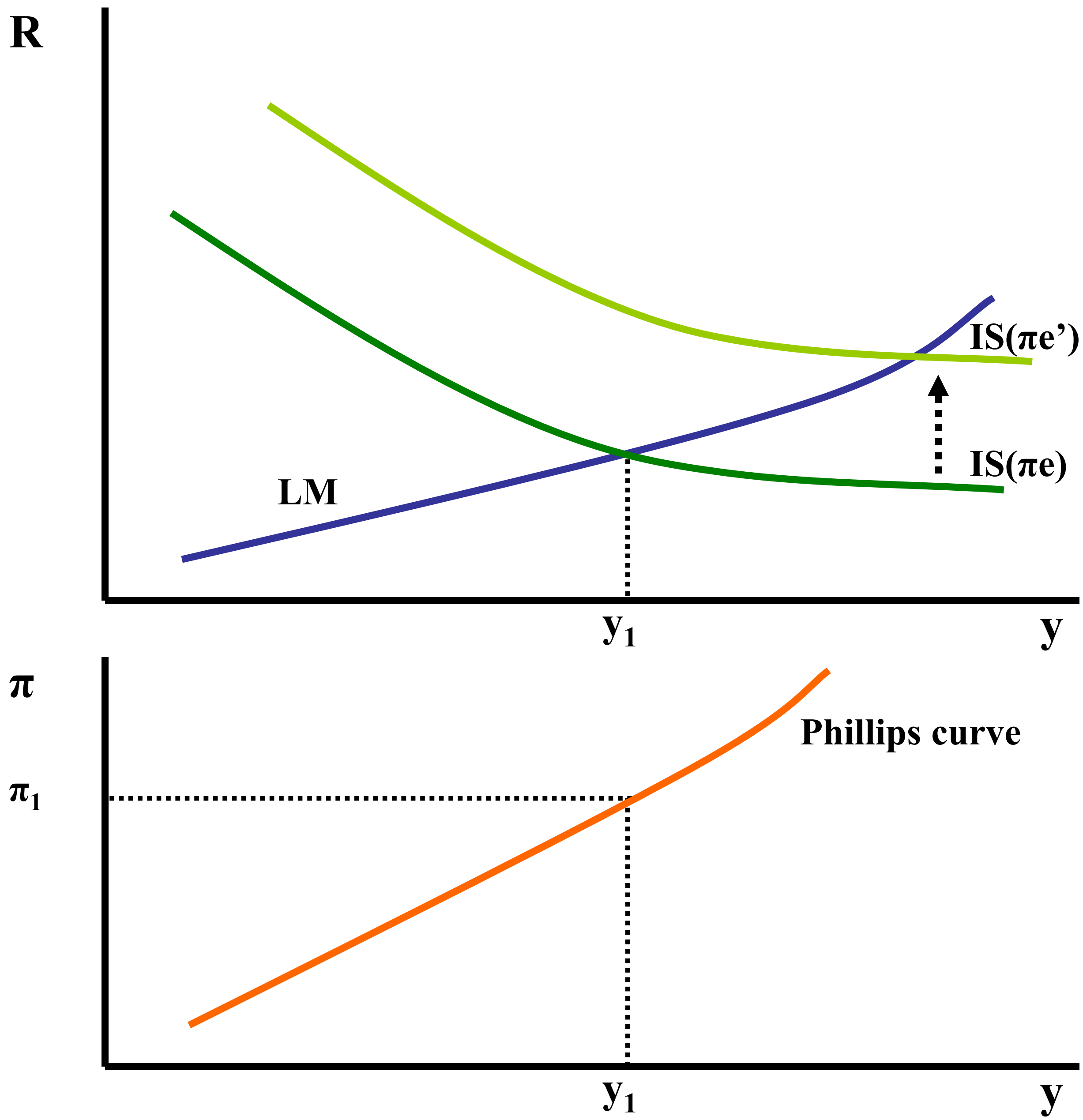

R
LM'
IS(πe')
IS(πe)
LM
y1
y
π
Phillips curve
π1
y1
y

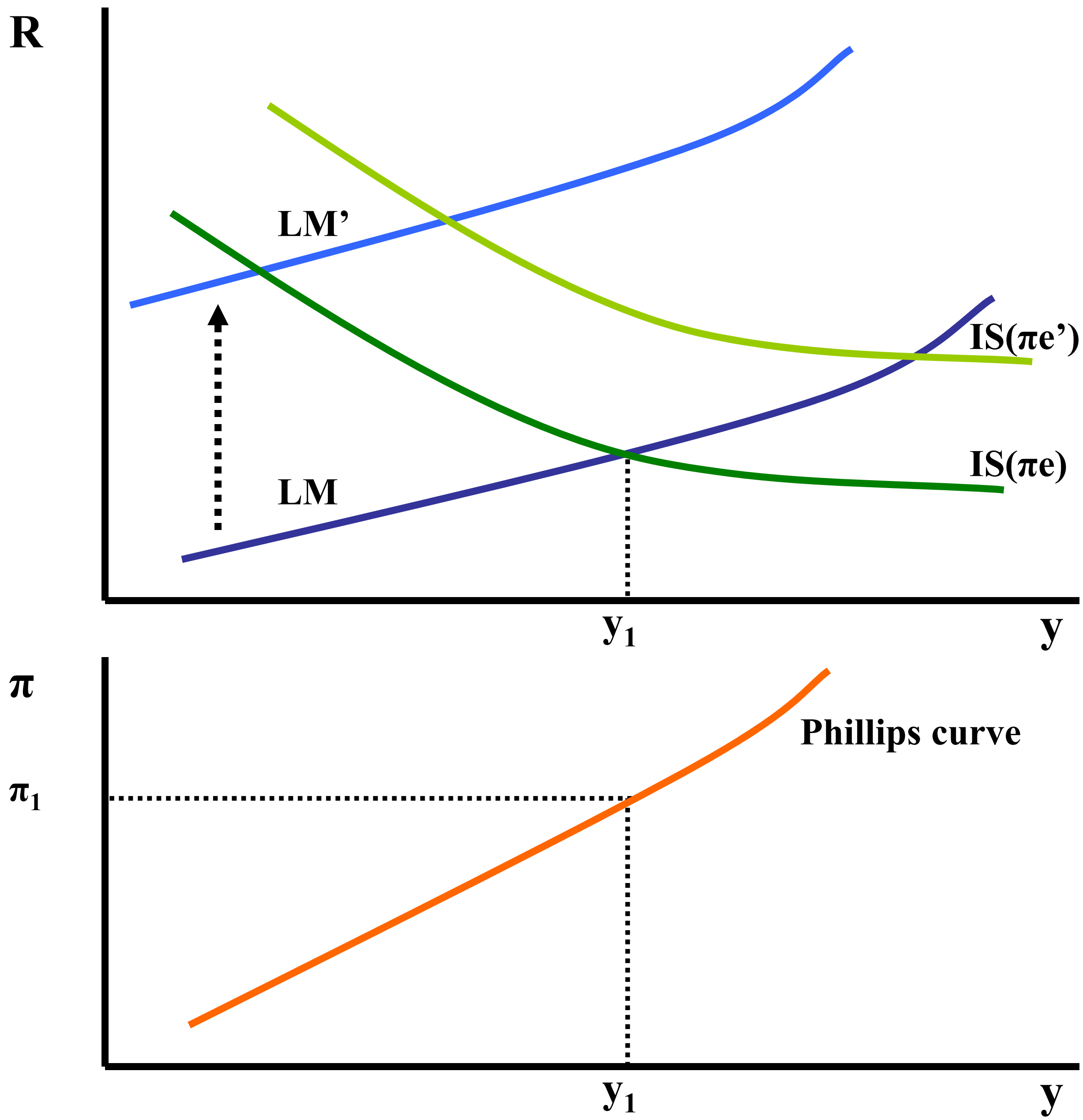


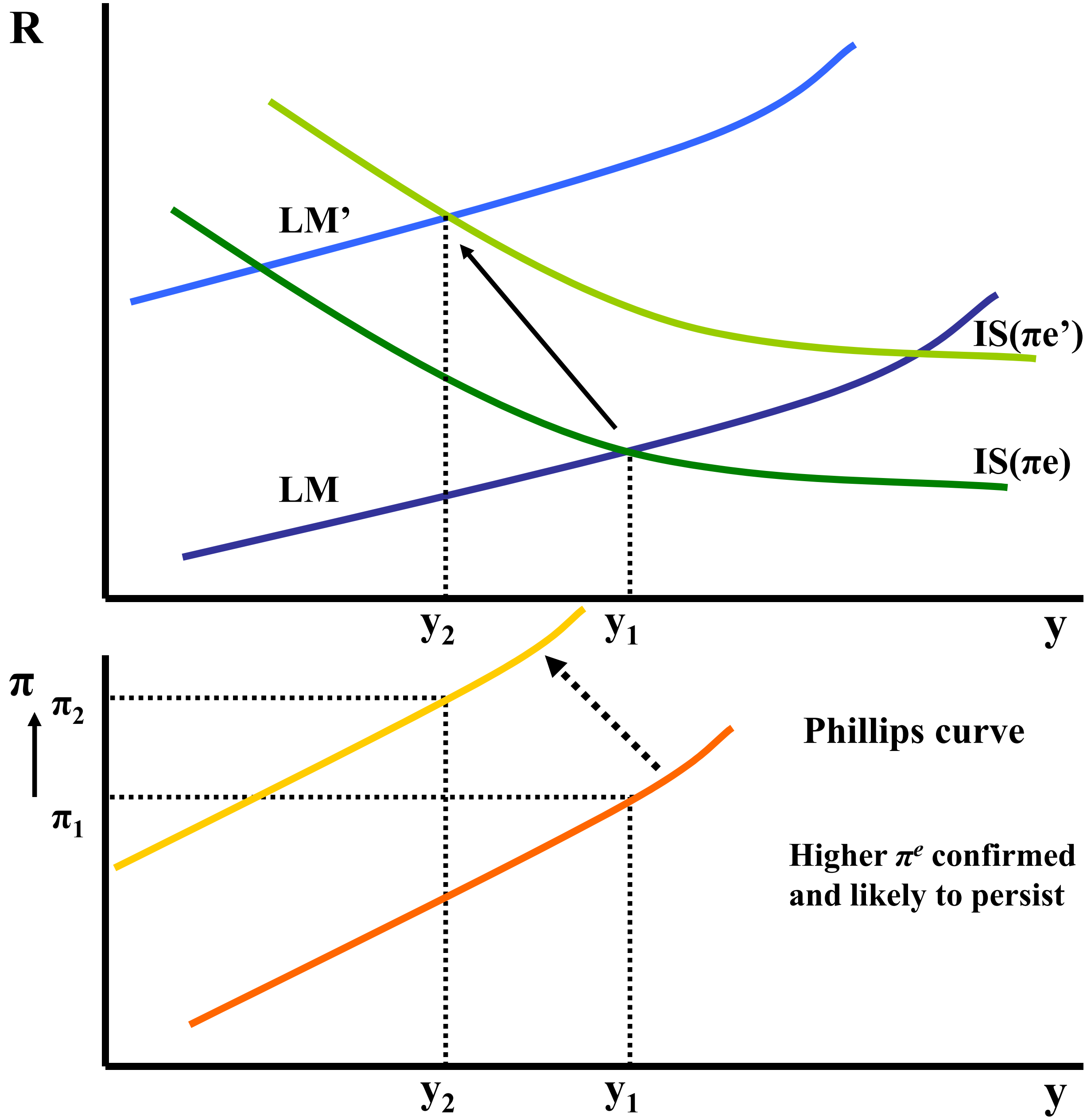
R
LM'
IS(πe')
LM
IS(πe)
y2
y1
y
π
Phillips curve
π1
π2
Initial jump in π^e not validated…
Any initial rise in π^e would quickly disappear
y2
y1
y

now consider the case with a working capital channel




R
IS(πe')
IS(πe)
LM
y1
y
π
Phillips curve
π1
y1
y


R
LM'
IS(πe')
IS(πe)
LM
y1
y
π
Phillips curve
π1
y1
y


R
LM'
LM
IS(πe')
IS(πe)
y₂
y₁
y
π
π₂
π₁
Phillips curve
Higher π^e confirmed
and likely to persist
y₂
y₁
y