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The Lower Bound in DSGE Models

by

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Background

- Countries Have Fought and Won a Tough Battle Against Inflation.
  - Problem Now is to Figure Out How to Keep Inflation Low.
  - One Possibility is to Target a Low Inflation Rate!
  - Recent Literature (Krugman, Eggertsson-Woodford) Suggests This Exposes An Economy to Risk of Economic Collapse When the Lower Bound on the Nominal Interest Rate Binds
  - Some Argue that Japan's 'Lost Decade' is a Consequence of Hitting the Lower Bound, and that Japan Therefore Illustrates the Real Danger Associated with Low Inflation.

Background ...

- Eggertson-Woodford Construct a Simple Model Environment Which Potentially Rationalizes the Concerns.
  - Example is Dramatic: Things Can Go *Badly* Wrong.
  - Simple: You Can Work it Out on a Napkin Over Beer.
- Model Suggests a Solution to the Problem: Price *Level* Targetting
  - Interestingly, Does not Require High Inflation.
  - Need to Inject a Small Amount of Inflation After Certain Shocks.

Questions

- Is the Lower Bound Still a Matter of Concern In Models that Incorporate Investment And Open Economy Considerations?
  - Answer: Lower Bound is Much Less Likely With Investment and Open Economy
- What Does Lower Bound Imply for Effects of Fiscal Shocks?
  - Answer: Predicts that Government Spending Multiplier Huge

Outline

- Simple Intuition of E-W Example
- Introducing Capital into E-W Model, Rexamining the Likelihood of Hitting the Lower Bound.
- The Output Effects of Government Spending in the Lower Bound

Model

- Household Preferences:

$$E_0 \sum_{t=0}^{\infty} \beta_t \left[u(C_t, M_t/P_t) - v(H_t) \right],$$

- Discount Rate:

$$\begin{aligned} \beta_t &= \frac{1}{\left(1+r_0^n\right)\left(1+r_1^n\right)\cdots\left(1+r_{t-1}^n\right)}, \\ \frac{\beta_{t+1}}{\beta_t} &= \frac{1}{1+r_t^n}. \end{aligned}$$

Model ...

- Experiment:

  r_0^n low, and remains low with probability p

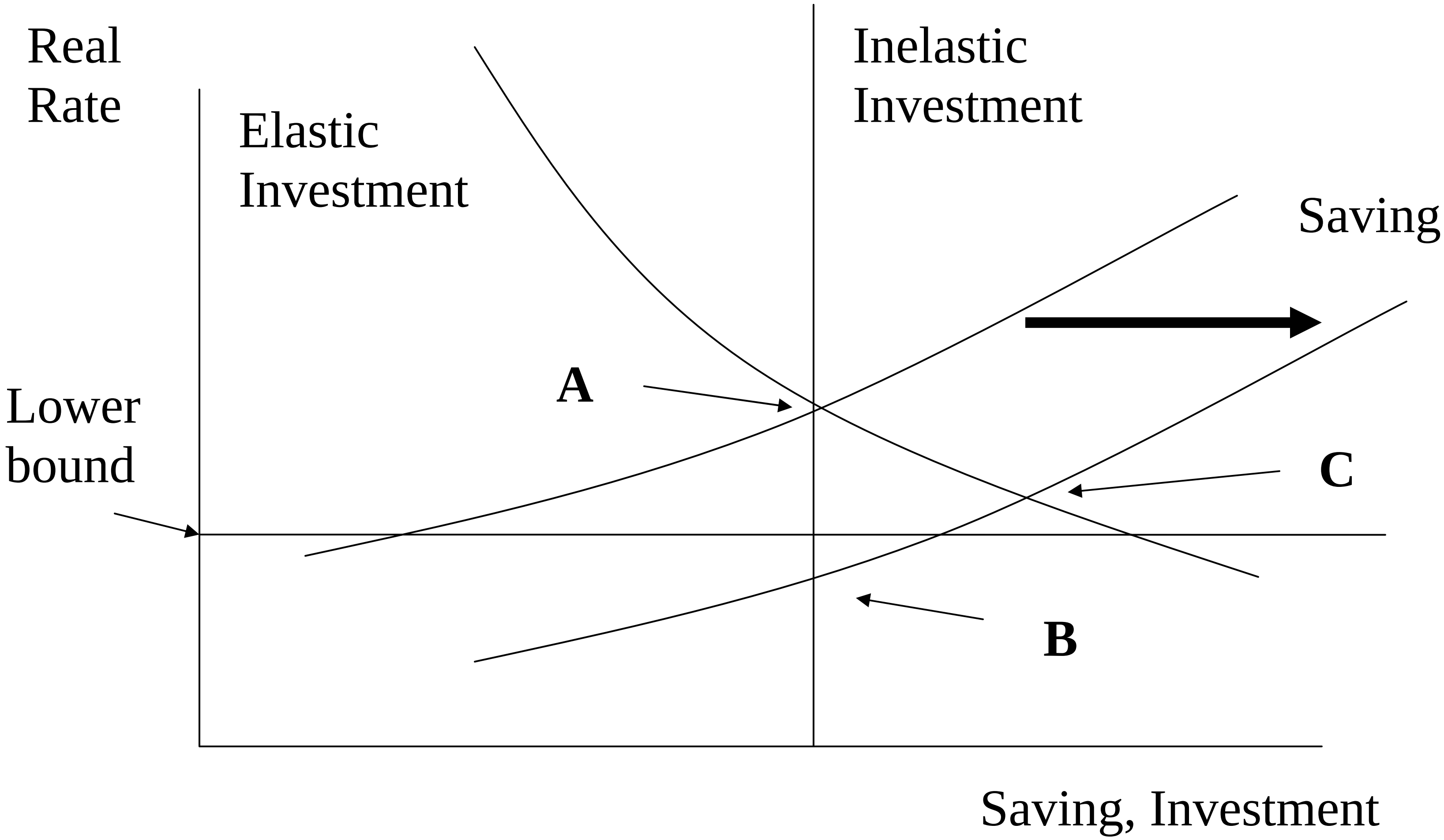
  with probability $1-p,$ it jumps back up to its steady state and remains there

- Monetary Policy:

  Set Nominal Interest Rate, $i_t,$ so that $\pi_t = P_t/P_{t-1} = 1,$ if possible

  otherwise, set $i_t = 0$ and let market forces determine π_t

Figure 1: Consequence of Increase in Saving When there is Lower Bound on Real Interest Rate. For Two Investment Elasticities



Simple Algebra of Eggertsson-Woodford

- Linearized Intertemporal Euler Equation ('IS Curve')

$$x_t = E_t x_{t+1} - \sigma \left(\hat{\imath}_t - E_t \hat{\pi}_{t+1} - \hat{r}_t^n \right)$$

  – Here:

$$\begin{aligned} x_t &= \frac{c_t - c}{c} \\ \hat{\imath}_t &= \frac{i_t - i}{1+i} & \left(\frac{1}{1+i} = \beta\right) \\ \hat{\pi}_t &= \frac{\pi_t - \pi}{\pi} & (\pi = 1) \\ \hat{r}_t^n &= \frac{r_t^n - r^n}{1+r^n} & \left(\frac{1}{1+r^n} = \beta\right). \end{aligned}$$

- Linearized Calvo Equation:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa x_t.$$

Simple Algebra of Eggertsson-Woodford ...

- Implications of Zero Bound For $\hat{\imath}_t$:

$$\hat{\imath}_t \equiv \frac{i_t - i}{1+i} = \beta \left(i_t + 1 \right) - 1$$

so, $\hat{\imath}_t \geq \beta - 1$

- Monetary Policy:

Set $\hat{\pi}_t = 0,$ unless this Implies $\hat{\imath}_t < \beta - 1$

If $\hat{\pi}_t = 0$ Implies $\hat{\imath}_t < \beta - 1,$ Set $\hat{\imath}_t = \beta - 1$ and Let $\hat{\pi}_t$ Be Determined Endogenously

Simple Algebra of Eggertsson-Woodford ...

- Equations of Model:

$$x_t = E_t x_{t+1} - \sigma \left(\hat{\imath}_t - E_t \hat{\pi}_{t+1} - \hat{r}_t^n \right)$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa x_t.$$

- In Steady State:

$$x_t = \hat{\pi}_t = \hat{\imath}_t = \hat{r}_t^n = 0$$

- Suppose $\hat{r}_t^n < 0$
  - If $\hat{r}_t^n \geq \beta - 1$, Set $\hat{\imath}_t = \hat{r}_t^n$, And $x_t = \hat{\pi}_t = 0$ Is Still Equilibrium
  - If $\hat{r}_t^n < \beta - 1$, $\hat{\imath}_t = \beta - 1$, $\hat{\pi}_t$ is Free

Simple Algebra of Eggertsson-Woodford ...

- What Happens if $\hat{r}_t^n < \beta - 1$?
- Depends on Expectations About the Future!
- Here is the E-W Setup:
  - In Period 0 and 1 :

$$\hat{r}_0^n \; < \; \beta - 1 = \hat{r}_l^n$$

$$\hat{r}_1^n \; = \; \begin{cases} \hat{r}_l^n & \text{probability } p \\ 0 & \text{probability } 1-p \end{cases}$$

  - In Period t :

$$\text{if } \hat{r}_{t-1}^n \; = \; 0, \; \hat{r}_t^n = 0$$

$$\text{or, } \hat{r}_t^n \; = \; \begin{cases} \hat{r}_l^n & \text{probability } p \\ 0 & \text{probability } 1-p \end{cases}$$

Simple Algebra of Eggertsson-Woodford ...

- Equilibrium is Simple to Compute!

- In Low State,

$$\hat{\pi}_t = \hat{\pi}_l,\ x_t = x_l$$

- Find These Variables by Solving:

$$x_l \ = \ px_l - \sigma((\beta - 1) - p\pi_l - \hat{r}_l^n)$$

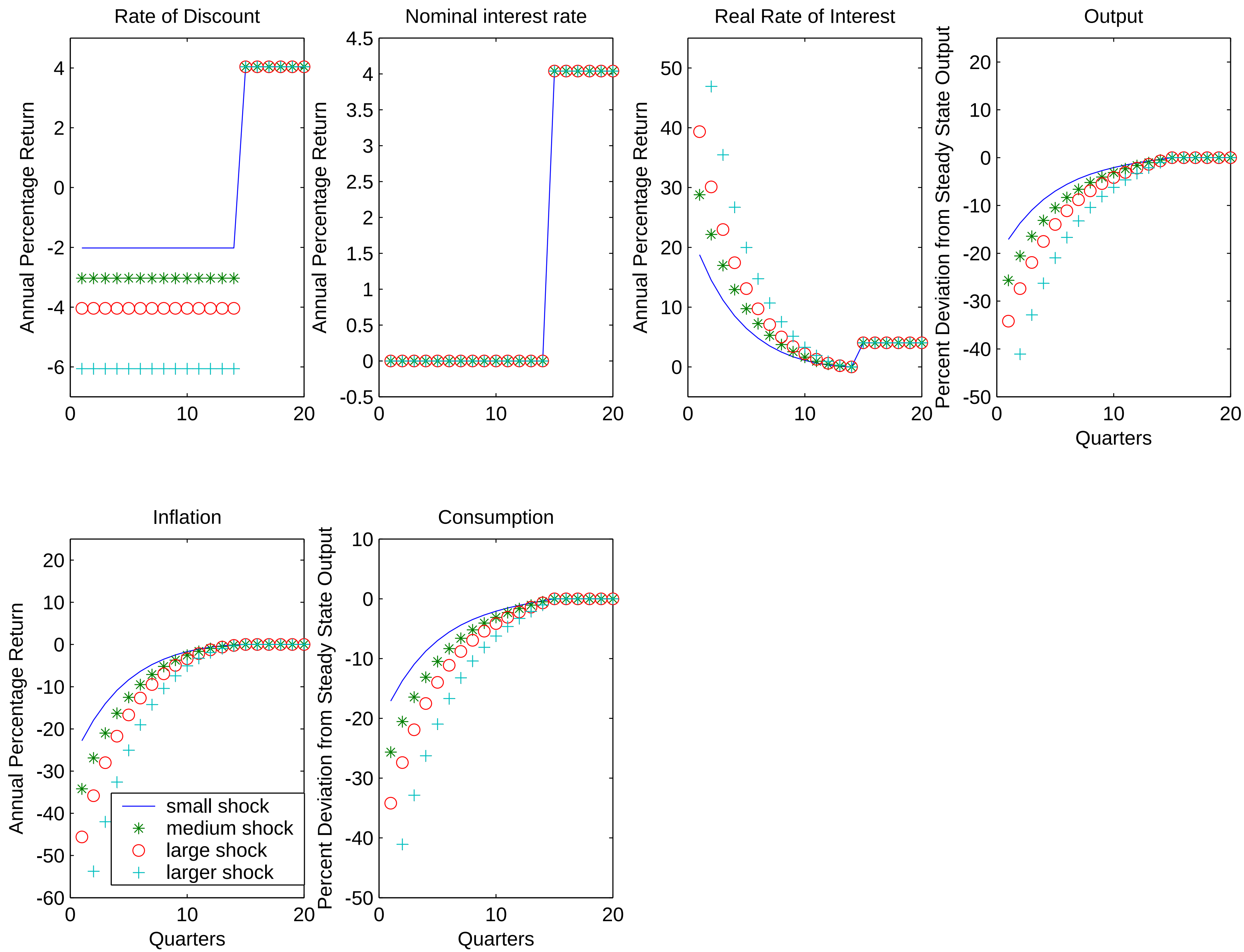
$$\pi_l \ = \ \beta p\pi_l + \kappa x_l$$

- Parameterization:

$$p = 0.9,\ \sigma = 0.5,\ \kappa = 0.02,\ \beta = 0.99,\ r_l^n = -.02/4.$$

$$x_l = -0.14,\ \pi_l = -0.0263.$$

Figure 3: Discount Rate Shock in Model without Investment, Three Discount Rate Shocks



Model With Investment

- Household Preferences:

$$E_0 \sum_{t=0}^{\infty} \beta_t \left[u(C_t, M_t/P_t) - v(H_t) \right],$$

- Discount Rate:

$$\begin{aligned} \beta_t &= \frac{1}{(1+r_0^n)(1+r_1^n)\cdots\left(1+r_{t-1}^n\right)}, \\ \frac{\beta_{t+1}}{\beta_t} &= \frac{1}{1+r_t^n}. \end{aligned}$$

- Household Budget Constraint:

$$P_t C_t + M_t + B_{t+1} \le M_{t-1} + B_t(1+i_{t+1}) + \int_0^1 P_t w_t(j) H_t(j) dj + T_t$$

Model With Investment ...

- Final Goods Production Function:

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \; \theta > 1.$$

- Intermediate Goods Production (Capital is firm-specific)

$$y_t(i) = K_t(i) f\left(\frac{h_t(i)}{K_t(i)}\right).$$

- Intermediate Goods Investment Technology:

$$I_t(i) = I\left(\frac{k_{t+1}(i)}{k_t(i)}\right) k_t(i)$$

Model With Investment ...

- Objective of Firms:

$$E_t \sum_{j=0}^{\infty} \beta_{t+j} \Lambda_{t+j} \left\{ (1+\tau) P_{t+j}(i) y_{t+j}(i) - P_{t+j} w_{t+j}(i) h_{t+j}(i) - P_{t+j} I_{t+j}(i) \right\}.$$

- Subsidy Eliminates Monopoly Power Distortions

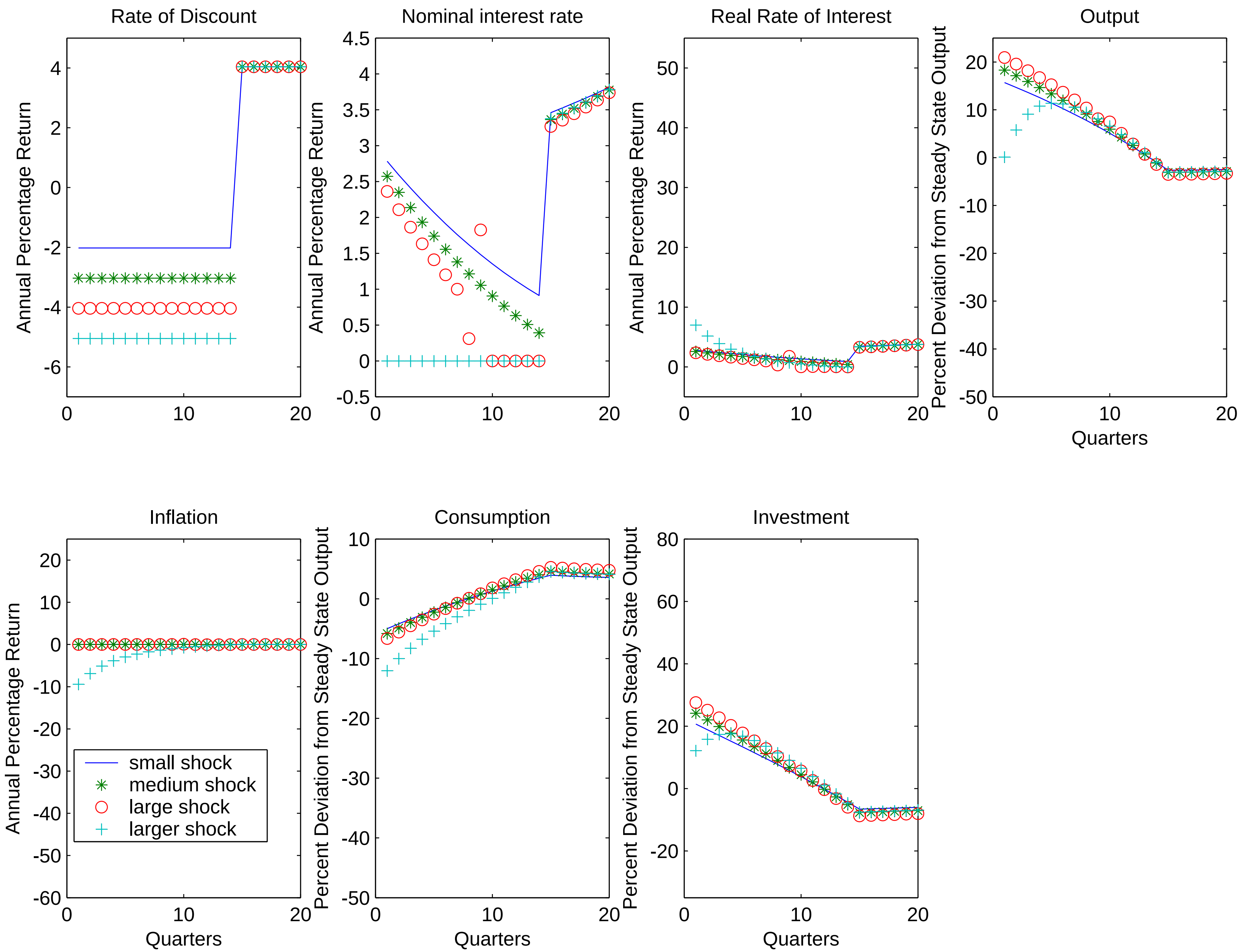
$$1+\tau \ = \ \frac{\theta}{\theta-1}$$

- Resource Constraint and Production Technology:

$$C_t + I_t + G_t \ = \ Y_t$$

$$I_t \ = \ \int_0^1 I_t(i) di.$$

Figure 4: Discount Rate Shock in Model with Investment, Three Discount Rate Shocks



Increasing Government Spending When the Lower Bound Binds

- In Steady State, $G = 0.18 \times Y^{steady\ state}$.

- I Set $G = .1925 \times Y^{steady\ state}$, for $t = 1, 2, ..., 14$

- With Small Preference Shock:
  - Lower Bound Not Binding and Multiplier Small (0.76 initially, and 0.41 eventually)
  - This is the Normal Government Spending Multiplier in DSGE Models.

- With Largest Preference Shock, Government Spending Has Huge Impact.
  - This is What Happens in Textbook ‘Paradox of Thrift’ Analysis.

Figure 7: Dynamic Response to Small Shock, With (*) and Without (-) Increase in Gov't Spending

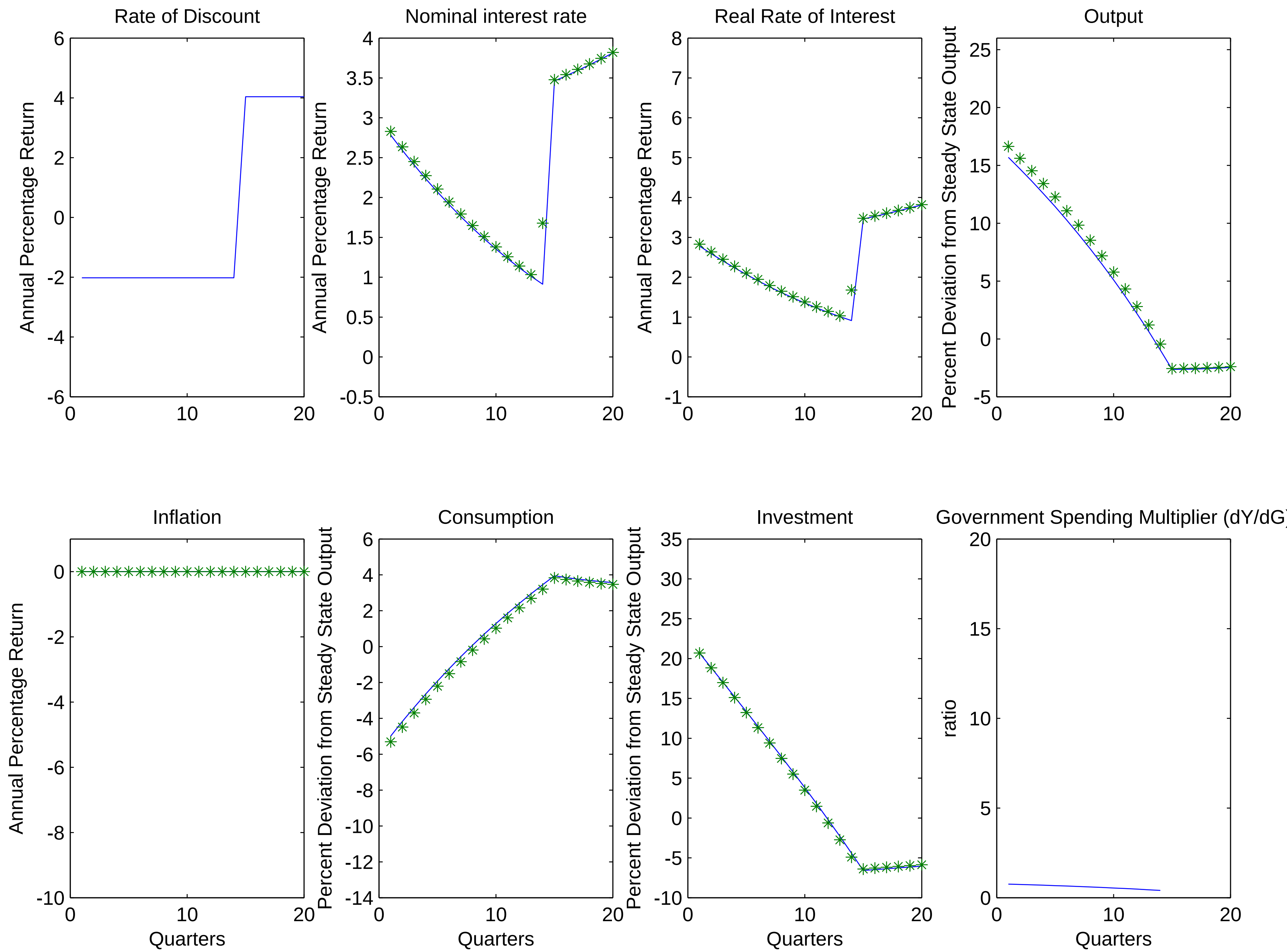
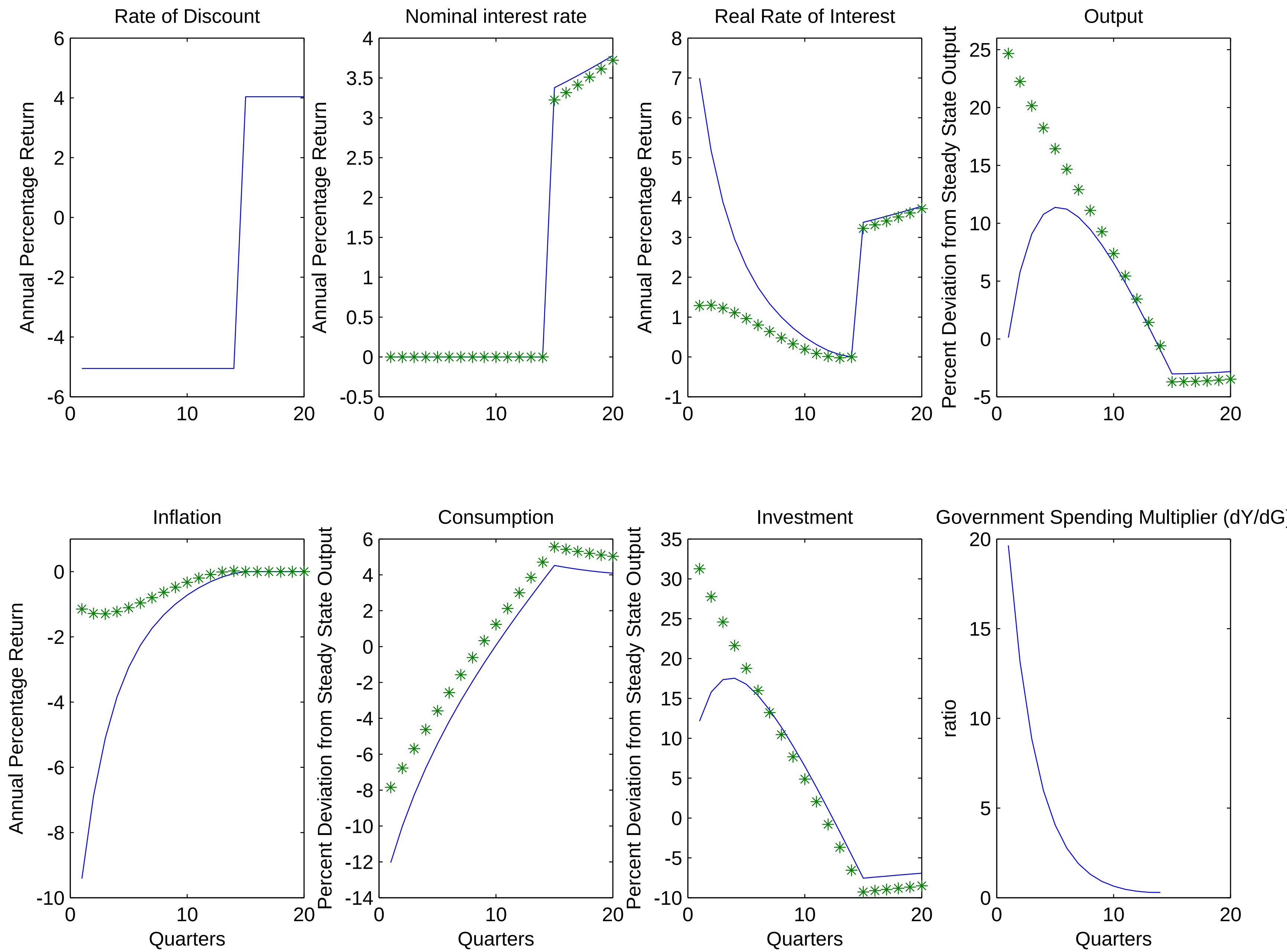


Figure 8: Dynamic Response to Larger Shock, With (*) and Without (-) Increase in Gov't Spending



Conclusion

- E-W Have Produced a Very Sharp Example of the Sort of Things People Might Have in Mind When they Worry About Low Inflation.
- It is Interesting to Investigate Robustness to:
  - Presence of Investment
  - Other types of Shocks, other Frictions
- Analysis Suggests that DSGE Models Do Form a Case that Inflation Targetting in a Low Inflation Environment Exposes an Economy To Risks Due to Lower Bound Considerations.
  - In Worst Case Scenario, Can Expand Fiscal Policy
- Is Japan in a Low r^n E-W Trap?
  - Conjecture: Model Predicts Large Y Effect From Positive G.
  - Japan did Increase G, So What Is Happening in Japan Must Not Reflect the Lower Bound Considerations Raised by Eggertsson and Woodford.