Optimal Fiscal and Monetary Policy

Background

- We Have Discussed the Construction and Estimation of DSGE Models
- Next, We Turn to Analysis
- Most Basic Policy Question:
  - How Should the Policy Variables of the Government be Set?
  - What is *Optimal* Policy? What Should R Be, How Volatile Should P Be?
- In Past 10 Years, Profession Has Explored Operating Characteristics of Simple Policy Rules
  - One Finding: A Taylor Rule with High Weight on Inflation Works Well in New-Keynesian Models
- Recent Development:
  - Increasingly, Analysts Studying *Optimal* Policy
  - Perhaps Because there is a Perception that Current DSGE Models Fit Data Well
- We Will Review Some of this Work.

Modern Quantitative Analysis of Optimal Policy

- Case Where Intertemporal Government Budget Constraint Does Not Bind
  - Example - Current Generation of Monetary Models
    - Assume Presence of Lump-Sum Taxes Used to Ensure Government Budget Constraint is Satisfied
  - Optimal Policy Studied, Among Others, By Schmitt-Grohe and Uribe (2004), Levin, Onatski, Williams, Williams (2005), and References They Cite.
- Case Where Intertemporal Government Budget Constraint Binds
  - Example - When the Government Does not Have Access to Distorting Taxes
  - Chari-Christiano-Kehoe (1991, 1994), Schmitt-Grohe and Uribe (2001), Siu (2001), Benigno-Woodford (2003, 2005), Others.

Outline

- Optimal Monetary and Fiscal Policy When the Intertemporal Budget Constraint Binds
  - Analyze the Friedman-Phelps Debate over the Optimal Nominal Rate of Interest.
  - What is the Optimal Degree of Price Variability?
  - How Should Policy React to a Sudden Jump in G?
  - Log-Linearization as a Solution Strategy
  - Woodford's Timeless Perspective
- Optimal Monetary Policy When the Intertemporal Budget Constraint Can be Ignored.
  - Log-Linearization as a Solution Strategy

Optimal Policy in the Presence of a Budget Constraint

- Sketch of Phelps-Friedman Debate
- Some Ideas from Public Finance - Primal Problem
- Simple One-Period Example
- Determining Who is Right, Friedman or Phelps, Using Lucas-Stokey Cash-Credit Good Model
- Financing a Sudden Expenditure (Natural Disaster): Barro versus Ramsey.

Friedman-Phelps Debate

- Money Demand:

$$\frac{M}{P} = \exp[-\alpha R]$$

- Friedman:

a. Efforts to Economize Cash Balances when R High are Socially Wasteful

b. Set R as Low As Possible: $R = 1$.

c. Since $R = 1 + r + \pi$, Friedman Recommends $\pi = -r$.

i. $r \sim$ exogenous (net) real interest rate rate

ii. $\pi \sim$ inflation rate, $\pi = (P - P_{-1})/P_{-1}$

- Phelps:
  a. Inflation Acts Like a Tax on Cash Balances -

$$\begin{aligned} \text{Seigniorage} &= \frac{M_t - M_{t-1}}{P_t} = \frac{M_t}{P_t} - \frac{P_{t-1}}{P_t}\frac{M_{t-1}}{P_{t-1}} \\ &\approx \frac{M}{P}\frac{\pi}{1+\pi} \end{aligned}$$

  b. Use of Inflation Tax Permits Reducing Some Other Tax Rate

  c. Extra Distortion in Economizing Cash Balances Compensated by Reduced Distortion Elsewhere.

  d. With Distortions a Convex Function of Tax Rates, Would Always Want to Tax All Goods (Including Money) At Least A Little.

  e. Inflation Tax Particularly Attractive if Interest Elasticity of Money Demand Low.

Question: Who is Right, Friedman or Phelps?

- Answer: Friedman Right Surprisingly Often

- Depends on Income Elasticity of Demand for Money

- Will Address the Issue From a Straight Public Finance Perspective, In the Spirit of Phelps.

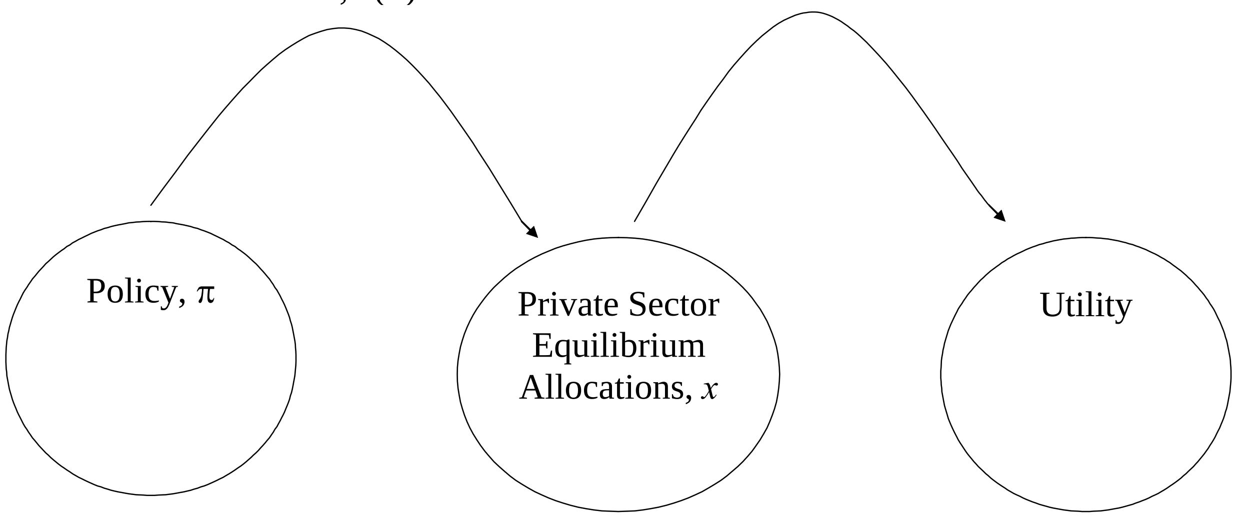
- Easy to Develop an Answer, Exploiting a Basic Insight From Public Finance.

Some Basic Ideas from Ramsey Theory

- **Policy**, π, Belonging to the Set of 'Budget Feasible' Policies, A.
- **Private Sector Equilibrium Allocations**, Equilibrium Allocations, x, Associated with a Given π; $x \in B$.
- **Private Sector Allocation Rule**, mapping from π to x (i.e., $\pi : A \rightarrow B$).
- **Ramsey Problem**: Maximize, w.r.t. π, $U(x(\pi))$.
- **Ramsey Equilibrium**: $\pi^* \in A$ and x^*, such that π^* solves Ramsey Problem and $x^* = x(\pi^*)$. 'Best Private Sector Equilibrium'.
- **Ramsey Allocation Problem**: Solve, $\tilde{x} = \arg max\, U(x)$ for $x \in B$
- Alternative Strategy for Solving the Ramsey Problem:
  a. Solve Ramsey Allocation Problem, to Find $\tilde{x}$.
  b. Execute the Inverse Mapping, $\tilde{\pi} = x^{-1}(\tilde{x})$.
  c. $\tilde{\pi}$ and $\tilde{x}$ Represent a Ramsey Equilibrium.
- **Implementability Constraint**: Equations that Summarize Restrictions on Achievable Allocations, B, Due to Distortionary Tax System.

Question: Who is Right, Friedman or Phelps? ...

Private sector Allocation
Rule, $x(\pi)$



Set, A, of Budget-Feasible Policies

Set, B, of Private Sector Allocations Achievable by Some Budget-Feasible Policy

Example

- Households:

$$\max_{c,l} u(c, l)$$

$$c \leq z(1-\tau)l,$$

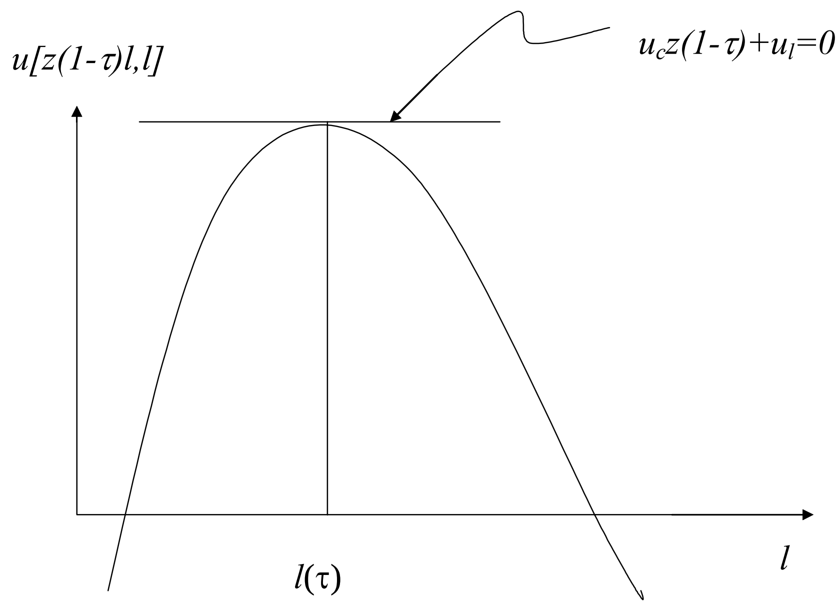
$z \sim$ wage rate

$\tau \sim$ labor tax rate

Example ...

- Household Problem Implies Private Sector Allocation Rules, $l(\tau)$, $c(\tau)$, defined by:

$$u_c z(1-l) + u_l = 0,\ c = (1-\tau)zl$$



Private Sector Allocation Rules:
$l(\tau)$, $c(\tau) = z(1-\tau)l$

Example ...

- **Ramsey Problem**:

$$\max_{\tau} u(c(\tau), l(\tau))$$

$$\text{subject to } g \leq zl(\tau)\tau$$

- **Ramsey Equilibrium**: τ^*, c^*, l^* such that

  a. $c^* = c(\tau^*)$, $l^* = l(\tau^*)$

  $*$ 'Private Sector Allocations are a Private Sector Equilibrium'

  b. τ^* Solves Ramsey Problem

  $*$ 'Best Private Sector Equilibrium'

Analysis of Ramsey Equilibrium

- Simple Utility Specification:

$$u(c,l) \;=\; c - \frac{1}{2}l^2$$

- Two Ways to Compute the Ramsey Equilibrium

  a. Direct Way: Solve Ramsey Problem (In Practice, Hard)

  b. Indirect Way: Solve Ramsey Allocation Problem, or Primal Problem (Can Be Easy)

Direct Approach

- Private Sector Allocation Rules:

$$c(\tau) = z^2(1-\tau)^2,\ l(\tau) = z(1-\tau)$$

- ‘Utility Function’ for Ramsey Problem:

$$u(c(\tau), l(\tau)) = \frac{1}{2}z^2(1-\tau)^2$$

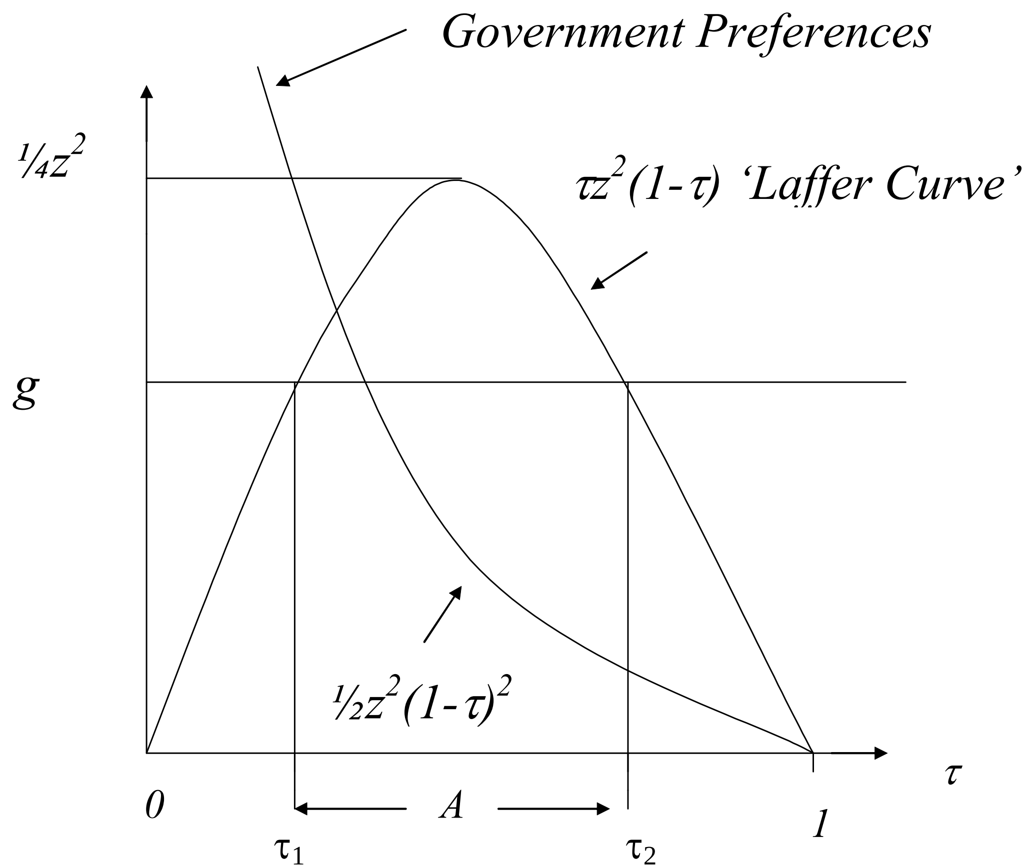
- Constraint on Ramsey Problem:

$$g \leq zl(\tau)\tau = z^2(1-\tau)\tau$$

- Ramsey Problem:

$$\max_{\tau} \frac{1}{2}z^2(1-\tau)^2$$
$$\text{subject to}:\ g \leq \tau z^2(1-\tau).$$

Analysis of Ramsey Equilibrium ...



$$\tau^* = \tau_1 = \tfrac{1}{2} - \tfrac{1}{2}[\,1 - 4\,g/z^2\,]^{1/2} \qquad \tau_2 = \tfrac{1}{2} + \tfrac{1}{2}[\,1 - 4\,g/z^2\,]^{1/2}$$

$$\boxed{l(\tau^*) = \tfrac{1}{2}\{\,z + [\,z^2 - 4\,g\,]^{1/2}\,\}}$$

Indirect Approach

- Approach: Solve Ramsey Allocation Problem, Then 'Inverse Map' Back into Policies
- Problem: Would Like a Characterization of B that Only Has (c, l), Not the Policies

$$
\begin{aligned}
B &= \{c, l : \exists \tau, \text{ with } u_c z(1-\tau) + u_l = 0, \\
c &= (1-\tau) z l,\ g \leq \tau z l\}
\end{aligned}
$$

- Solution: Rearrange Equations in B, So That Only (c, l) Appears

$$
(*)\ u_c c + u_l l = 0,\ (**)\ c + g \leq zl.
$$

- Conclude: $B = D$, where:

$$
D = \left\{ (c, l) : \underbrace{c + g \leq zl}_{\text{resource constraint}},\quad \underbrace{u_c c + u_l l = 0}_{\text{implementability constraint}} \right\}
$$

- Express Ramsey Allocation Problem:

$$\max_{c,l} u(c,l), \text{ subject to } (c,l) \in D$$

- Alternatively:

$$\max_{c,l} u(c,l),$$

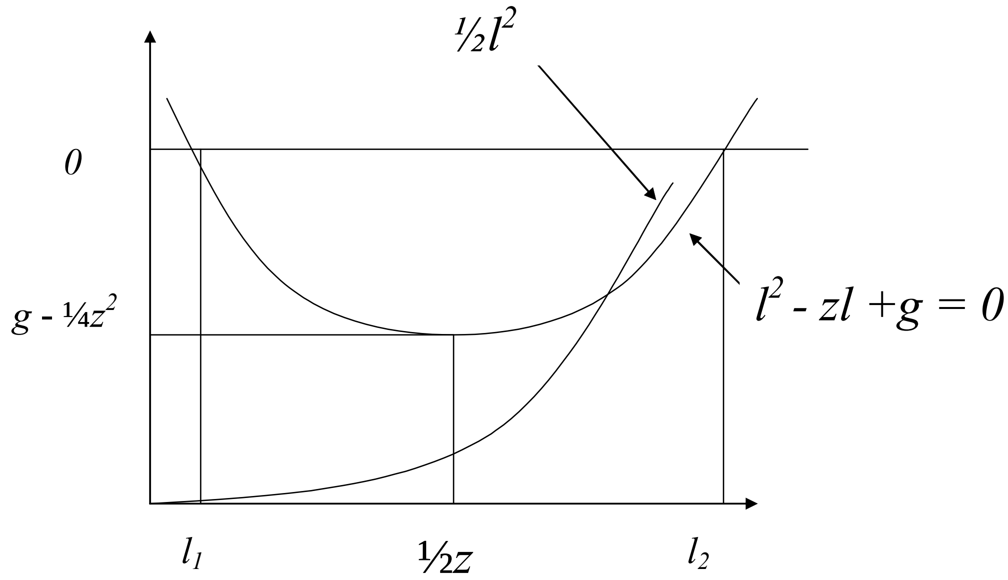
$$\text{s.t. } u_c c + u_l l \ = \ 0, \ c + g \leq zl$$

- Or,

$$\max_{l} \frac{1}{2} l^2$$

$$\text{s.t. } l^2 + g \ \leq \ zl$$

Analysis of Ramsey Equilibrium ...



Ramsey Allocation Problem:

Max $\frac{1}{2}l^2$

Subject to $l^2 + g \leq zl$

Solution:

$l_2 = \frac{1}{2}\{ z + [z^2 - 4g]^{\frac{1}{2}} \}$

Same Result as Before!

Lucas-Stokey Cash-Credit Good Model

- Households
- Firms
- Government

Households

- Household Preferences:

$$\sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, l_t),$$

c_{1t} ~ cash goods, c_{2t} ~ credit goods, l_t ~ labor

- Distinction Between Cash and Credit Goods:
  - All Goods Paid With Cash At the Same Time, After Goods Market, in Asset Market
  - Cash Good: Must Carry Cash In Pocket Before Consuming It

$$M_t \geq P_t c_{1t}$$

  - Credit Good: No Need to Carry Cash Before Purchase.

Household Participation in Asset and Good Markets

- Asset Market: First Half of Period, When Household Settles Financial Claims Arising From Activities in Previous Asset Market and in Previous Goods Market.

- Goods Market: Second Half of Period, Goods are Consumed, Labor Effort is Applied, Production Occurs.

Analysis of Ramsey Equilibrium ...

Asset Market | Goods Market

t | $t+1$

Sources of Cash for Household:

- $M^d_{t-1} - P_{t-1}c_{1,t-1} - P_{t-1}c_{2,t-1}$
- $R_{t-1}B^d_{t-1}$
- $(1-\tau_{t-1})zl_{t-1}$

Uses of Cash

- Bonds, B^d_t
- Cash, M^d_t

- $c_{1,t}$, $c_{2,t}$ Purchased
- l_t Supplied
- Production Occurs
- M^d_t Not Less Than $P_t c_{1,t}$

- Constraint On Households in Asset Market (Budget Constraint)

$$\begin{aligned} & M_t^d + B_t^d \\ \leq\ & M_{t-1}^d - P_{t-1}c_{1t-1} - P_{t-1}c_{2t-1} \\ & + R_{t-1}B_{t-1}^d + (1-\tau_{t-1})zl_{t-1} \end{aligned}$$

Household First Order Conditions

- Cash versus Credit Goods:

$$\frac{u_{1t}}{u_{2t}} = R_t$$

- Cash Goods Today versus Cash Goods Tomorrow:

$$u_{1t} = \beta u_{1t+1} R_t \frac{P_t}{P_{t+1}}$$

- Credit Goods versus Leisure:

$$u_{3t} + (1 - \tau_t) z u_{2t} = 0.$$

Firms

- Technology: $y = zl$
- Competition Guarantees Real Wage = z.

Government

- Inflows and Outflows in Asset Market (Budget Constraint):

$$\underbrace{M_t^s - M_{t-1}^s + B_t^s}_{\text{Sources of Funds}} \geq \underbrace{R_{t-1}B_{t-1}^s + P_{t-1}g_{t-1} - P_{t-1}\tau_{t-1}zl_{t-1}}_{\text{Uses of Funds}}$$

- Policy:

$$\pi = \left(M_0^s, M_1^s, ..., B_0^s, B_1^s, ..., \tau_0, \tau_1, ...\right)$$

Ramsey Equilibrium

- Private Sector Allocation Rule:

For each policy, $\pi \in A,$ there is a Private Sector Equilibrium:

$$x = (\{c_{1t}\}, \{c_{2t}\}, \{l_t\}, \{M_t\}, \{B_t\})$$
$$p = (\{P_t\}, \{R_t\})$$
$$M_t = M_t^s = M_t^d$$
$$B_t = B_t^s = B_t^d$$
$$R_t \geq 1 \text{ (i.e., } u_{1t}/u_{2t} \geq 1)$$

- Ramsey Problem:

$$\max_{\pi \in A} U(x(\pi))$$

- Ramsey Equilibrium:

$$\pi^*, x(\pi^*), p(\pi^*),$$

Such that π^* Solves Ramsey Problem.

Finding The Ramsey Equilibrium By Solving the Ramsey Allocation Problem

$$\max_{\{c_{1t},c_{2t},l_t\}\in D} \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, l_t),$$

where D is the set of allocations, c_{1t}, c_{2t}, l_t, $t = 0, 1, 2, ...$, such that

$$\sum_{t=0}^{\infty} \beta^t [u_{1t}c_{1t} + u_{2t}c_{2t} + u_{3t}l_t] \;=\; u_{2,0}a_0,$$

$$c_{1t} + c_{2t} + g \;\leq\; zl_t,\; \frac{u_{1t}}{u_{2t}} \geq 1,$$

$$a_0 = \frac{R_{-1}B_{-1}}{P_0} \sim \text{ real value of initial government debt}$$

Lagrangian Representation of Ramsey Allocation Problem

- There is a $\lambda \geq 0$, s. t. Solution to R A Problem Also Solves:

$$\max_{\{c_{1t},c_{2t},l_t\}} \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, l_t) + \lambda \left(\sum_{t=0}^{\infty} \beta^t [u_{1t}c_{1t} + u_{2t}c_{2t} + u_{3t}l_t] - u_{2,0}a_0 \right)$$

subject to $c_{1t} + c_{2t} + g \leq zl_t,\ \dfrac{u_{1t}}{u_{2t}} \geq 1,$

or,

$$\max_{\{c_{1t},c_{2t},l_t\}} \bar{W}(c_{10}, c_{20}, l_0; \lambda) + \sum_{t=1}^{\infty} \beta^t W_t(c_{1t}, c_{2t}, l_t; \lambda)$$

subject to : $c_{1t} + c_{2t} + g \leq zl_t,\ \dfrac{u_{1t}}{u_{2t}} \geq 1,$

$$\bar{W}(c_{10}, c_{20}, l_0; \lambda) = u(c_{1,0}, c_{2,0}, l_0) + \lambda \left([u_{1,0}c_{1,0} + u_{2,0}c_{2,0} + u_{3,0}l_0] - u_{2,0}a_0 \right)$$

$$W(c_{1,t}, c_{2,t}, l_t; \lambda) = u(c_{1,t}, c_{2,t}, l_t) + \lambda \left([u_{1,t}c_{1,t} + u_{2,t}c_{2,t} + u_{3,t}l_t \right)$$

Ramsey Allocation Problem

- Lagrangian:

$$\max_{\{c_{1t},c_{2t},l_t\}} \bar{W}(c_{10}, c_{20}, l_0; \lambda) + \sum_{t=1}^{\infty} \beta^t W(c_{1t}, c_{2t}, l_t; \lambda)$$

$$\text{subject to} \quad : \quad c_{1t} + c_{2t} + g \leq z l_t, \ \frac{u_{1t}}{u_{2t}} \geq 1,$$

$$\bar{W}(c_{10}, c_{20}, l_0; \lambda) \ = \ u(c_{1,0}, c_{2,0}, l_0) + \lambda\left([u_{1,0}c_{1,0} + u_{2,0}c_{2,0} + u_{3,0}l_0] - u_{2,0}a_0\right)$$

$$W(c_{1,t}, c_{2,t}, l_t; \lambda) \ = \ u(c_{1,t}, c_{2,t}, l_t) + \lambda\left([u_{1,t}c_{1,t} + u_{2,t}c_{2,t} + u_{3,t}l_t\right)$$

- How to Solve this?
  - Fix $\lambda \geq 0$, Solve The Above Problem
  - Evaluate Implementability Constraint
  - Adjust λ Until Implemetability Constraint is Satisfied

Special Structure of Ramsey Allocation Problem

- Given λ (If we Ignore $\frac{u_{1t}}{u_{2t}} \geq 1$), Looks Like Standard Optimization Problem:

$$\max_{\{c_{1t}, c_{2t}, l_t\}} \bar{W}(c_{10}, c_{20}, l_0; \lambda) + \sum_{t=1}^{\infty} \beta^t W(c_{1t}, c_{2t}, l_t; \lambda)$$

$$\text{s.t. } c_{1t} + c_{2t} + g \leq z l_t.$$

- After First Period, 'Utility Function' Constant

- Problem: For Exact Solution, Need λ...Not Easy to Compute!

- But,
  - Can Say Much Without Knowing Exact Value of λ (Will Pursue this Idea Now)
  - Under Certain Conditions, Can Infer Value of λ From Data (Will Pursue this Idea Later)

Special Structure of Ramsey Allocation Problem ...

- Ignoring $\frac{u_{1t}}{u_{2t}} \geq 1$, after Period 1 :

$$\frac{W_1(c_1, c_2, l; \lambda)}{W_2(c_1, c_2, l; \lambda)} = 1$$

- 'Planner' Equates Marginal Rate of Substitution Between Cash and Credit Good to Associated Marginal Rate of Technical Substitution

Restricting the Utility Function

- Utility Function:

$$u(c_1, c_2, l) = h(c_1, c_2)v(l),$$

$$h \;\sim\; \text{homogeneous of degree } k,\;\; v \;\sim\; \text{strictly decreasing.}$$

- Then, $u_1c_1 + u_2c_2 + u_3l = h\left[kv + v'\right]$, so

$$W(c_1, c_2, l; \lambda) = hv + \lambda h\left[kv + v'\right] = h(c_1, c_2)Q(l, \lambda).$$

- Conclude - Homogeneity and Separability Imply:

$$1 = \frac{W_1(c_1, c_2, l; \lambda)}{W_2(c_1, c_2, l; \lambda)} = \frac{h_1(c_1, c_2, l)Q(l, \lambda)}{h_1(c_1, c_2, l)Q(l, \lambda)} = \frac{h_1(c_1, c_2, l)}{h_1(c_1, c_2, l)} = \frac{u_1(c_1, c_2, l)}{u_2(c_1, c_2, l)}.$$

Surprising Result: Friedman is Right More Often Than You Might Expect

- Suppose You Can Ignore $u_{1t}/u_{2t} \geq 1$ Constraint. Then, Necessary Condition of Solution to Ramsey Allocation Problem:

$$\frac{W_1(c_1, c_2, l; \lambda)}{W_2(c_1, c_2, l; \lambda)} = 1.$$

- This, In Conjunction with Homogeneity and Separability, Implies:

$$\frac{u_1(c_1, c_2, l)}{u_2(c_1, c_2, l)} = 1.$$

- Note: $u_{1t}/u_{2t} \geq 1$ is Satisfied, So Restriction is Redundant Under Homogeneity and Separability.

- Conclude: $R = 1$, So Friedman Right!

Generality of the Result

- Result is True for the Following More General Class of Utility Functions:

$$u(c_1, c_2, l) = V(h(c_1, c_2), l),$$

  where h is homothetic.

- Analogous Result Holds in ‘Money in Utility Function’ Models and ‘Transactions Cost’ Models (Chari-Christiano-Kehoe, *Journal of Monetary Economics*, 1996.)

- Actually, strict homotheticity and separability are not necessary.

Interpretation of the Result

- 'Looking Beyond the Monetary Veil' -

  – The Connection Between The $R = 1$ Result and the Uniform Taxation Result for Non-Monetary Economies

- The Importance of Homotheticity

  – The Link Between Homotheticity and Separability, and The Consumption Elasticity of Money Demand.

Uniform Taxation Result from Public Finance For Non-Monetary Economies

- Households:

$$\max_{c_1,c_2,l} u(c_1, c_2, l) \text{ s.t. } zl \geq c_1(1+\tau_1) + c_2(1+\tau_2)$$

$$\Rightarrow \; c_1 = c_1(\tau_1, \tau_2), \; c_2 = c_2(\tau_1, \tau_2), \; l = l(\tau_1, \tau_2).$$

- Ramsey Problem:

$$\max_{\tau_1,\tau_2} u(c_1(\tau_1, \tau_2), c_2(\tau_1, \tau_2), l(\tau_1, \tau_2))$$

$$\text{s.t. } g \; \geq \; c_1(\tau_1, \tau_2)\tau_1 + c_2(\tau_1, \tau_2)\tau_2$$

- *Uniform Taxation Result*:

$$\text{if } u \; = \; V(h(c_1, c_2), l), \; h \sim \text{homothetic}$$

$$\text{then } \tau_1 \; = \; \tau_2.$$

$$\text{Proof} \; : \; \text{trivial! (just study Ramsey Allocation Problem)}$$

Similarities to Monetary Economy

- Rewrite Budget Constraint:

$$\frac{zl}{1+\tau_2} \geq c_1 \frac{1+\tau_1}{1+\tau_2} + c_2.$$

- Similarities:

$$\frac{1}{1+\tau_2} \sim 1-\tau,\ \frac{1+\tau_1}{1+\tau_2} \sim R.$$

- Positive Interest Rate 'Looks' Like a Differential Tax Rate on Cash and Credit Goods.

- **Have the Same Ramsey Allocation Problem**, Except Monetary Economy Also Has:

$$\frac{u_1}{u_2} \geq 1.$$

What Happens if You Don't Have Homotheticity?

- Utility Function:

$$u(c_1, c_2, l) = \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\delta}}{1-\delta} + v(l)$$

- 'Utility Function' in Ramsey Allocation Problem:

$$\begin{aligned} W(c_1, c_2, l) \ &= \ [1 + (1-\sigma)\lambda]\,\frac{c_1^{1-\sigma}}{1-\sigma} \\ &\quad + [1 + (1-\delta)\lambda]\,\frac{c_2^{1-\delta}}{1-\delta} + v(l) + \lambda v'(l) l \end{aligned}$$

What Happens if You Don't Have Homotheticity? ...

- Marginal Rate of Substitution in Ramsey Allocation Problem That Ignores $u_1/u_2 \geq 1$ Condition:

$$1 = \frac{W_1(c_1, c_2, l; \lambda)}{W_2(c_1, c_2, l; \lambda)} = \frac{1 + (1-\sigma)\lambda}{1 + (1-\delta)\lambda} \times \frac{u_1}{u_2},$$

or, since $u_1/u_2 = R$:

$$R = \frac{1 + (1-\delta)\lambda}{1 + (1-\sigma)\lambda}$$

- Finding:

$$\begin{aligned}
\delta &= \sigma \Rightarrow R = 1 \text{ (homotheticity case)} \\
\delta &> \sigma \Rightarrow R \geq 1 \text{ Binds, so } R = 1 \\
\delta &< \sigma \Rightarrow R > 1.
\end{aligned}$$

Note: Friedman Right More Often Than Uniform Taxation Result, Because $u_1/u_2 \geq 1$ is a Restriction on the Monetary Economy, Not the Barter Economy.

Consumption Elasticity of Demand

- Homotheticity and Separability Correspond to Unit Consumption Elasticity of Money Demand.

- Money Demand:

$$\begin{aligned} R &= \frac{u_1}{u_2} = \frac{h_1}{h_2} = f\left(\frac{c_2}{c_1}\right) \\ &= f\left(\frac{c - \frac{M}{P}}{\frac{M}{P}}\right) \\ &= \tilde{f}\left(\frac{c}{M/P}\right). \end{aligned}$$

- Note: Holding R Fixed, Doubling c Implies Doubling M/P

Money Demand and Failure of Homotheticity

- Money Demand:

$$R = \frac{u_1}{u_2} = \frac{c_1^{-\sigma}}{c_2^{-\delta}} = \frac{\left(\frac{M}{P}\right)^{-\sigma}}{\left(c - \frac{M}{P}\right)^{-\delta}}$$

- Taylor Series Approximation About Steady State ($m \equiv M/P$ in steady state) :

$$\hat{m} = \underbrace{\frac{1}{\frac{m}{c} + \frac{\sigma}{\delta}\left(1 - \frac{m}{c}\right)}}_{\text{Consumption Money Demand Elasticity, } \varepsilon_M} \times \hat{c} - \underbrace{\frac{1}{\delta \frac{m}{c-m} + \sigma}}_{\text{Interest Elasticity}} \times \hat{R}$$

- Can Verify:

Utility Function Parameters	ε_M	Non-Monetary Economy	Monetary Economy
$\delta > \sigma$	$\varepsilon_M > 1$	$\tau_2 \geq \tau_1$	$R = 1$
$\delta < \sigma$	$\varepsilon_M < 1$	$\tau_2 < \tau_1$	$R > 1$
$\delta = \sigma$	$\varepsilon_M = 1$	$\tau_1 = \tau_2$	$R = 1$

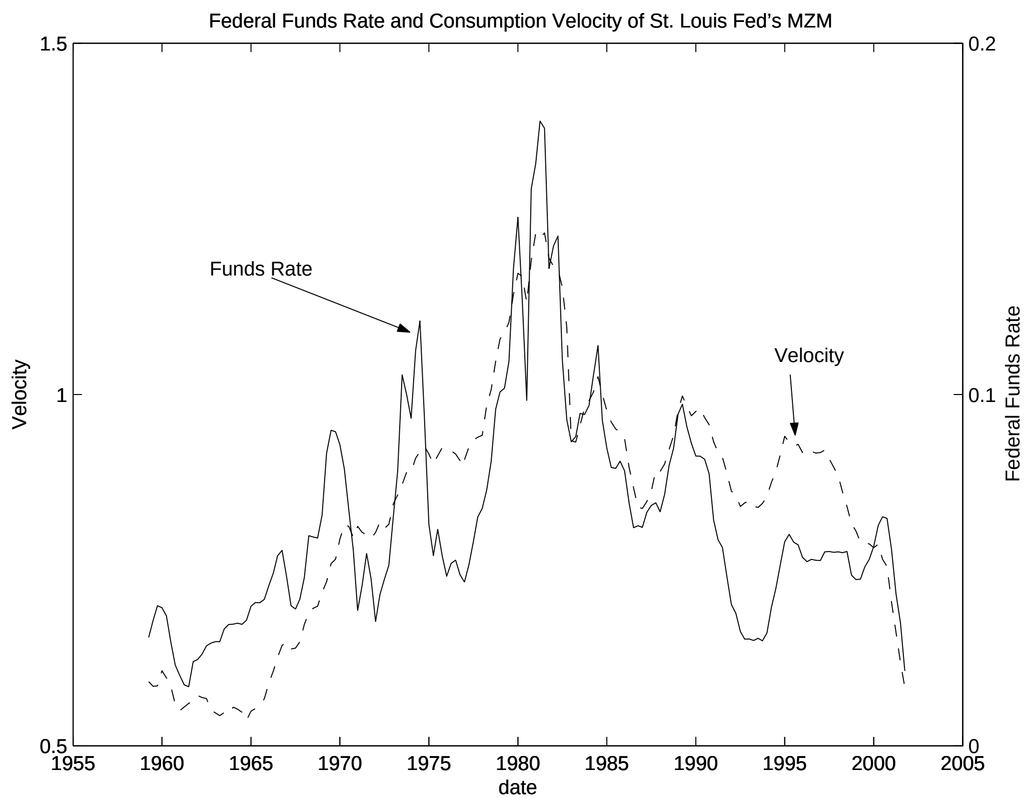
Bottom Line:

- Friedman is Right ($R = 1$) When Consumption Elasticity of Money Demand is Unity or Greater

- Implicitly, High Interest Rates Tax Some Goods More Heavily that Others. Under Homotheticity and Separability Conditions, Want to Tax Goods at Same Rate.

Bottom Line: ...

- What is Consumption Elasticity in the Data?



- Answer: Not Far From Unity - Velocity and the Interest Rate Are Both Roughly Where they Were in the 1960, Though Consumption is Higher.

Interest Rate and Velocity Data for Euro Area

What To Do, When g, z Are Random?

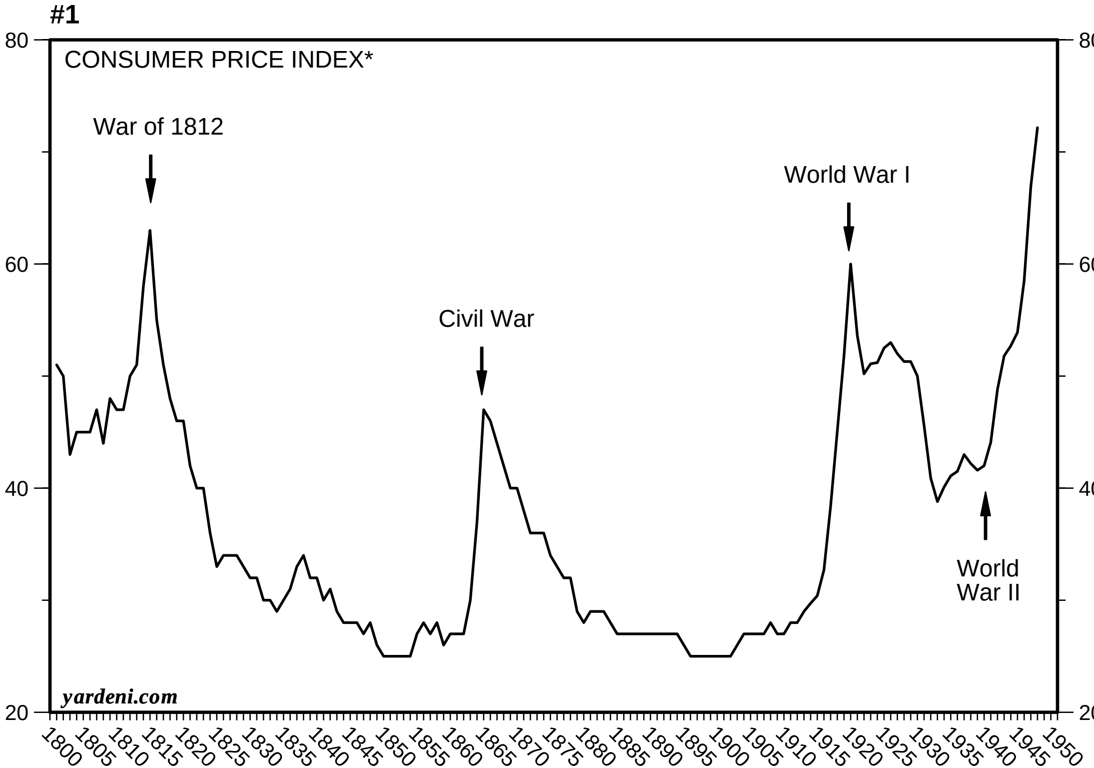
- Results for Optimal R Completely Unaffected

- Ramsey Principle: Minimize Tax Distortions
  - After Bad Shock to Government Constraint:
    - Tax Capital
    - Raise Price Level to Reduce Value of Government Debt
  - After Good Shock To Government Budget Constraint
    - Subsidize Capital
    - Reduce Price Level to Reduce Value of Government Debt

What To Do, When g, z Are Random? ...

- If there is Staggered Pricing in the Economy, Desirability of Price Volatility Depends on Two Forces
  - Fiscal Force Just Discussed, Which Implies the Price Level Should Be Volatile
  - Relative Price Dispersion Considerations Which Suggest that Prices Should Not Be Volatile
- Schmitt-Grohe/Uribe and Henry Siu Find:
  - For Shocks of the Size of Business Cycles, the Relative Price Dispersion Considerations Dominate
- Henry Siu Finds:
  - For War-Size Shocks, Fiscal Considerations Dominate.
  - Some Evidence for this in the Data

- Inflation, War, & Peace -



Wars are inflationary. Peace times are deflationary. Historically, prices soared during wars, plunged during peace times. Wars are trade barriers. There is more competition and technological innovation during peace times.

* Base index from 1800 to 1947 is 1967 = 100.
Source: US Department of Commerce, Bureau of the Census, Historical Statistics of the US.

Financing War: Barro versus Ramsey

When War (or Other Large Financing Need) Suddenly Strikes:

- Barro:
  - Raise Labor and Other Tax Rates a Small Amount So That When Held Constant at That Level, Expected Value of War is Financed
  - This Minimizes Intertemporal Substitution Distortions
  - Involves a Big *Increase* in Debt in Short Run
  - Prediction for Labor Tax Rate: Random Walk.

- Ramsey:
  - Tax Existing Capital Assets (Human, Physical, etc) For Full Amount of Expected Value of War. Do This at the First Sign of War.
  - This Minimizes Intertemporal *and* Intratemporal Distortions (Don't Change Tax Rates on Income at all).
  - *Reduce* Outstanding Debt
  - Make Essentially No Change Ever to Labor Tax Rate

– Example:

  - $*$ Suppose War is Expected to Last Two Periods, Cost: \$1 Per Period

  - $*$ Suppose Gross Rate of Interest is 1.05 (i.e., 5%)

  - $*$ Tax Capital $1 + 1/1.05 = 1.95$ Right Away.

  - $*$ Debt Falls \$0.95 in Period When War Strikes.

– Involves a *Reduction* of Outstanding Debt in Short Run.

– Prediction for Labor Tax Rate: Roughly Constant.

A Computational Issue

- *Conditional* On a Value for λ, Finding Ramsey Allocations Easy (Can Use Simple Linearization Procedures!)

- Policies Can Then Be Computed From Ramsey Allocations.

  – Example: Labor Tax Rate Can Be Computed from Ramsey Allocations By Solving for τ_t :

$$u_l\left(c_t, l_t\right) + u_c\left(c_t, l_t\right) \times f_n\left(k_t, l_t\right) \times \left(1 - \tau_t\right) = 0$$

- But, How To Get λ?

  – Get it the Hard Way, Outlined Above

  – Under Very Limited Conditions, can Calibrate λ

Calibrating the Multiplier, λ

- Conditional on λ :
  - Nonstochastic Steady State Consumption, Capital Stock, Labor, Labor Tax Rate Functions of λ :

$$c = c(\lambda), \; l = l(\lambda)$$

  - Steady State Policy Variable (debt, labor tax, capital tax rate) Can Be Computed:

$$\tau(\lambda) = 1 + \frac{u_l(c, l)}{u_c(c, l)\, f_n(k, l)}$$

- In Practice, $\tau(\lambda)$ is a Monotone Function of λ. Choose $\hat{\lambda}$ So That

$$\hat{\tau} = \tau\left(\hat{\lambda}\right), \; \hat{\tau} \sim \text{Sample Average of Labor Tax Rate}$$

Problem With Calibrating Multiplier

- Implicitly, this Assumes the Economy Was in an Optimal Policy Regime in the Historical Sample

- Problem

  – When People Compute Optimal Policy, they Want to be Open to the Possibility that Policy Outcomes are *Not* Optimal

  – Want to Use the Ramsey-Optimal Policies as a Basis For Recommending *Better* Policies

- Still, Calibration of λ Works for an Analyst Who Seriously Entertains the Hypothesis that Policy in the Sample Was Optimal

- Related to Woodford's Idea of the *Timeless Perspective*

Optimal Monetary Policy When the Intertemporal Budget Constraint Does Not Bind

- Current Generation of Monetary Models Put Government Budget Constraint in Background by Assuming Presence of Lump Sum Taxes to Balance Budget.

- Ramsey Optimal Policies in These Models Easy to Compute.

- Suppose:
  - You Have a Very Simple Model, With One Equation Characterizing the Equilibrium of the Private Economy, and One For the Policy Rule.
  - The Private Economy Equation is:

$$\pi_t - \beta\pi_{t+1} - \gamma y_t = 0, \ t = 0, 1, ... \tag{1}$$

  - You Want to Do Optimal Policy. So You Threw Away the Policy Rule.
  - The Setup At this Point Has One Equation, (1) in Two Unknowns, π_t, y_t. Need More Equations!
  - The Additional Equations Come In When We Optimize.

Optimal Monetary Policy When the Intertemporal Budget Constraint Does Not Bind ...

– Lagrangian Problem:

$$\max_{\{\pi_t, y_t; t=0,1,...\}} \sum_{t=0}^{\infty} \beta^t \{u(\pi_t, y_t) + \lambda_t [\pi_t - \beta\pi_{t+1} - \gamma y_t]\}$$

– Equations that Characterize the Optimum: (1), and

$$u_\pi(\pi_t, y_t) + \lambda_t - \beta\lambda_{t-1} = 0$$

$$u_y(\pi_t, y_t) - \gamma\lambda_t = 0, \ t = 0, 1, ...$$

– We Made Up for the One Missing Equation, By Adding Two Equations and One New Unknown.