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D11-1, Fall 2000

FINAL EXAM

There are three questions. The total number of possible points is 100, and each question is worth the same number of points. If a question seems ambiguous, state why, sharpen it up and answer the revised question. You have 2 hours. Good luck!

1. Consider a variant of the setup studied by Shleifer. Final goods are produced by a representative, competitive firm that makes use of the following production function:

$$y_t = \left[\int_0^1 x_t(i)^{\frac{1}{\mu}} di \right]^{\mu}, \ 1 < \mu < \infty,$$

where $x_t(i)$ is an intermediate good produced in industry $i \in [0,1]$. Let $p_t(i)$ denote the price of the i^{th} intermediate good, and normalize the price of y_t to unity. There is a large number of potential producers in each industry and each is free to enter and produce in that industry. In equilibrium, only one producer is active. That producer is competitive in the factor market.

   (a) Carefully, derive the demand curve for $x_t(i)$.

The intermediate good is produced using labor only. The wage rate is denoted w_t. There is a technology available to everyone in $t = 1$, under which it takes c units of labor to produce one unit of $x_1(i)$, each i.

Suppose that for some particular industry in period 1 exactly one firm is selected to receive an idea about how to produce more efficiently. That idea allows the firm to produce one unit of the intermediate input using only c/λ units of labor, $\lambda > 1$. The idea is private information to the firm that receives it. Once the idea has been implemented for one period, it is known to all the other firms in the industry. You may suppose that the only relevant alternatives for the firm receiving an idea in period 1 is whether to implement in period 1 or period 2. No additional ideas will ever arrive in this industry again.

The firm seeks to maximize the present discounted value of profits:

$$\sum_{t=1}^{\infty} \frac{1}{\prod_{j=1}^{t-1} R_j} \pi_t,$$

where, as usual, $\prod_{j=1}^{t-1} R_j \equiv 1$ for $t = 1$, and π_t denotes time t profits. Firms treat R_t as beyond their control.

(b) Suppose the firm receiving the idea implements it in period $t = 1$. What price will it set? Explain carefully. What role does the free entry assumption play here?

(c) What is the present discounted value of profits for the firm in part (b)? Explain.

(d) Suppose the firm receiving the new idea decides to implement in period $t = 2$. What is its discounted value of profits? Explain.

(e) In Shleifer's model, firms receiving new ideas decide when to implement depending on a comparison of current aggregate output, y_1, with discounted expected future output. Using your findings in (c) and (d), comment on the generality of this result.

2. Consider the problem of the representative household:

$$\max_{\{c_t, B_{t+1}; t \geq 0\}} \sum_{t=0}^{\infty} \beta^t u(c_t), \; 0 < \beta < 1,$$

where $u : R_+ \to R$ is strictly concave, increasing, and differentiable. The budget constraint, for $t \geq 0$, is:

$$c_t + \frac{B_{t+1}}{1+R} \leq y - \tau + B_t, \; B_0 > 0, \text{ given.}$$

Here, B_t, c_t, y, τ, R denote, respectively, bond holdings, consumption, income, taxes, and the rate of interest. As the notation indicates, y, τ, and R are assumed to be constant. They are beyond the control of the household. Also, suppose that $1 + R = 1/\beta$. The nonnegativity constraints are c_t, $B_{t+1} \geq 0$ for all t.

There is a government which issues bonds, b_t. Its budget constraint, in per capita terms, is:

$$\frac{b_{t+1}}{1+R} + \tau = b_t,$$

where b_0 is given and equal to B_0. The left side of the above equality represents the government's source of funds - from issuance of new debt and taxes - and the right side represents the sole use of funds: paying off old debt.

Clearing in the goods and bonds market requires:

$$c_t = y, \; b_{t+1} = B_{t+1}, \; t = 0, 1, 2, ...$$

(a) Show that if $b_0 > b^*$, where $b^* = (1+R)\tau/R$, then $b_t/[(1+R)^t] \to \Delta$ as $t \to \infty$, where $\Delta > 0$. Display an explicit formula for Δ.

(b) Write down a sequence of Euler equations and a transversality condition, and show that if a sequence, $c_t, B_{t+1}, \; t > 0$, solves these, then they solve the household problem.

(c) Suppose $b_0 = B_0 > b^*$. Consider the following candidate solution to the household problem: $c_t = y$, $B_{t+1} = b_{t+1}$ for all t.

   i. Show that this sequence satisfies the budget and Euler equations, but *not* the transversality condition.

   ii. Prove that the candidate solution is in fact not optimal. (For example, you could identify another sequence that is feasible - i.e., consistent with the household's budget constraint and nonnegativity constraints - that generates higher utility.)

(d) Define a sequence of markets equilibrium for this economy. Explain why it is that, if there is an equilibrium, $b_0 = b^*$.

3. Consider the following exogenous growth model. Households are all identical and the representative household has the following preferences:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}, \; \gamma > 0.$$

The household supplies its one unit of labor inelastically to the labor market. In addition, it uses its wage and profit income to purchase consumption, c_t, and investment goods, i_t. The household owns the economy's stock of capital and it accumulates according to the following technology:

$$k_{t+1} = (1-\delta)k_t + i_t.$$

The initial level of capital, k_0, is given.

A representative firm produces final output using the following linear homogeneous production function:

$$y_t = n_t^{1-\alpha} \int_0^{M_t} x_t(i)^\alpha di, \;\; 0 < \alpha < 1.$$

This firm takes M_t to be exogenous, as well as the wage rate, w_t, and the price of the intermediate inputs, $p_t(i)$, $i \in [0, M_t]$.

The i^{th} intermediate input, $x_t(i)$, is produced by a monopolist using this technology:

$$x_t(i) = k_t(i),$$

where $k_t(i)$ is physical capital. Entry into the production of the i^{th} intermediate input is forbidden. The producer of the i^{th} intermediate input is competitive in the market for renting capital and takes its rental rate, r_t, as given.

The allocation of capital across intermediate goods must satisfy the following constraint:

$$k_t = \int_0^{M_t} k_t(i) di,$$

where k_t is the aggregate stock of capital, which is given at the beginning of time t. The aggregate resource constraint is:

$$c_t + k_{t+1} - (1-\delta)k_t \leq y_t, \;\; 0 < \delta < 1.$$

Finally, the number of varieties of goods evolves exogenously over time as follows:

$$M_t = M_{t-1}(1+\mu), \;\; \mu > 0,$$

where M_0 is given.

(a) Write down a budget constraint for the household and define a sequence of markets equilibrium, for $t = 0, 1, 2, \ldots$.

(b) Prove that, in equilibrium, $x_t(i) = x_t$ for all $i \in [0, M_t]$. Develop a formula relating the rental rate of capital to the aggregate stock of capital and M_t.

(c) Prove that there exists a balanced growth equilibrium, i.e., there exist values for M_0, k_0 such that output, consumption and the stock of capital all grow at the same rate for each $t = 0, 1, 2, ...$. What is that growth rate? Show that there does not exist a balanced growth path for all possible M_0, k_0.

(d) Show that the efficient allocations are characterized by the same growth rate as the equilibrium allocations, but that the levels of c_t, y_t and k_{t+1} in the efficient allocations are higher. Provide explicit formulas to support your answer.

(e) Explain why, in the efficient allocations, the economy converges monotonically to its balanced growth equilibrium path. Sketch the outlines of a rigorous argument.

(f) "Market signals about the rate of return on capital understate the true return. This is why the level of consumption, capital and output in equilibrium is too low." Explain carefully.