Homework #3
Economics D11-1
Due Wednesday, October 11
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1. According to Theorem 4.15, the Euler and transversality conditions (equations (2) and (3) on page 98 of S-L) are sufficient for an interior sequence of x_t's to constitute an optimum of the SP problem. Necessity of the Euler equation is fairly obvious (we used a variational argument in class). Here is a sketch of an argument that establishes necessity of the transversality condition. A limitation of this argument is that, in addition to the usual assumptions, this argument also requires $x_t = 0 \in X$, and $0 \in \Gamma(x_t)$ for $x_t \in X$. I sketch this argument below. Convert this sketch into a rigorous proof. In your proof you may use, without proof, any results from the book that you wish. However, you must be absolutely clear always about which assumptions you are using.

   Let x_t^*, $t = 0, 1, 2, ...,$ denote a sequence of x_t's that solve the SP. Then,

$$v(x_t^*) = F(x_t^*, x_{t+1}^*) + \beta v(x_{t+1}^*), \; t = 0, 1, 2, ...,$$

   where, of course, $x_0^* = x_0$, the initial condition. Let x_t be any other sequence that satisfies $(x_t, x_{t+1}) \in A$, the domain of F, for all t (does this imply that x_t is in the domain of v?). Then,

$$v(x_t) - v(x_t^*) \geq F_1(x_t^*, x_{t+1}^*)(x_t - x_t^*).$$

   The proof follows in a straightforward (not *trivial*) way by replacing x_t with 0, for all t, by multiplying both sides of this expression by β^t, and driving $t \to \infty$.

   The assumption, $(0, 0) \in A$, is not one that we have made so far. Indeed, it is violated by the log utility example you have worked with in homework. Is there a simple adjustment to your proof that works for the case, $(0, 0) \notin A$?

2. Problems 5.17a and 5.17b in Stokey-Lucas, page 126-127 (the page numbers may be slightly off, since they are different in different printings). This studies a model that looks very much like the neoclassical

growth model, except that what is a variable marginal product of capital in the neoclassical model is replaced by a constant interest rate. The problem asks you to establish rigorously that when the interest rate is low, it is efficient to have a declining consumption profile over time, if feasible. In doing this question, you may assume the utility function is bounded.

3. Problem 6.3, page 139 in S-L.

4. Exercise 6.7a-e, pages 157-158 in S-L. As discussed in class, the efficient allocations reflect a balance between the properties of preferences and technology. In 6.7e you are asked to study a parameterization and set of initial capital stocks for the two-sector model in which it is efficient for consumption to cycle. Presumably, the technology implies that there are gains to be had in cycling. Otherwise it would not be efficient to cycle, given the preference for smooth consumption implied by concavity of utility. What are those gains? What is the intuition behind the result that it is desirable for consumption to cycle?