

Homework #3
Economics 411-1
Due Wednesday, October 17
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1. Consider the neoclassical growth model studied in class, with $\beta = 1/1.03$, $\alpha = 1/3$, $\delta = 0.10$, $\gamma = 1$, where preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

and the aggregate resource constraint is given by:

$$c_t + k_{t+1} - (1 - \delta)k_t \leq k_t^\alpha.$$

What is the steady state value of k ? How long does it take to close 95 percent of a gap between an initial value of the capital stock, k_0 , and the steady state value?

Consider the Solow model. This assumes that people save and invest a fixed fraction, s , of gross output, k^α :

$$k_{t+1} - (1 - \delta)k_t = sk_t^\alpha$$

What value of s is required in order for the steady states of the neoclassical and Solow models to coincide? How much time does it take for 95 percent of the gap between k_0 and steady state capital to be closed in the Solow model?

2. According to Theorem 4.15, the Euler and transversality conditions (equations (2) and (3) on page 98 of S-L) are sufficient for an interior sequence of x_t 's to constitute an optimum of the SP problem. Necessity of the Euler equation is fairly obvious (we used a variational argument in class). Here is a sketch of an argument that establishes necessity of the transversality condition too. The argument requires, in addition to the usual assumptions, that $0 \in X$, and $0 \in \Gamma(x_t)$ for all $x_t \in X$. I sketch this argument below. Convert this crude sketch into a rigorous proof. In your proof you may use, without proof, any results from the

book that you wish. However, you must be absolutely clear always about which assumptions you are using.

Let x_t^* , $t = 0, 1, 2, \dots$, denote a sequence of x_t 's that solve the SP. Then,

$$v(x_t^*) = F(x_t^*, x_{t+1}^*) + \beta v(x_{t+1}^*), \quad t = 0, 1, 2, \dots,$$

where, of course, $x_0^* = x_0$, the initial condition. Let x_t be any other sequence that satisfies $(x_t, x_{t+1}) \in A$ (recall, A is the domain of F) for all t . Then,

$$v(x_t) - v(x_t^*) \leq F_1(x_t^*, x_{t+1}^*)(x_t - x_t^*).$$

The proof follows in a straightforward (not *trivial*) way by replacing x_t with 0, for all t , by multiplying both sides of this expression by β^t , and driving $t \rightarrow \infty$.

3. Problems 5.17a and 5.17b in Stokey-Lucas, page 126-127 (the page numbers may be slightly off, since they are different in different printings). This studies a model that looks very much like the neoclassical growth model, except that what is a variable marginal product of capital in the neoclassical model is replaced by a constant interest rate. The problem asks you to establish rigorously that when the interest rate is low, it is efficient to have a declining consumption profile over time, if feasible. In doing this question, you may assume the utility function is bounded.
4. Exercise 6.7a-f, pages 157-158 in S-L. Ex. 6.7d refers to a situation in which the steady state capital stock is 'unstable'. What they mean by this is that if the initial stock of capital, k_0 , is even only slightly different from the steady state value, k_t , $t = 1, 2, 3, \dots$ diverges away from the steady state. The question asks you to verify the existence of a two period cycle. For extra credit, do exercise 6.7g, which explores the 'stability' of that cycle.