1. Consider the Ramsey optimal taxation model studied in class. Prove that the government has an incentive to deviate from the Ramsey policies in the second subperiod, after saving has occurred and the capital stock has been determined.

2. Suppose a typical household solves

\[
\max_{c,n} u(c, n), \text{ subject to } c \leq (1 - \tau)wn, \ c, n \geq 0,
\]

where \(w\) is the wage rate, \(\tau\) is the labor tax rate, \(n\) is hours worked, and \(c\) is consumption. Suppose the production function is \(f(n) = n\), so that the equilibrium wage rate is \(w = 1\). Also, suppose the utility function has the following form:

\[
u(c, n) = c - \frac{n^2}{2}
\]

Suppose there is a government which has to finance a fixed level of expenditures, \(g \leq \frac{1}{4}\). The government's task is to choose \(0 \leq \tau \leq 1\) to maximize the utility of the representative agent, subject to its budget constraint:

\[g \leq \tau wn.\]

(a) Suppose the government commits itself to a value for \(\tau\) before the households make their decisions (i.e., it solves the 'Ramsey problem'). What is the set of \(\tau\) consistent with the government satisfying its budget constraint? What range of household utilities is associated with the elements in this set? What is the tax rate and utility level that solves the Ramsey problem?

(b) Suppose the government selects a value for \(\tau\) after households have committed themselves to a level of employment. How many sustainable equilibria are there? (Sustainable equilibria will be discussed in class on November 29.) What are the associated utility levels?
(c) Replace the government budget constraint by $g = T$, where $T$ represents a lump-sum tax. Also, replace the household’s budget constraint by

$$c \leq wn - T.$$ 

What is the welfare gain from going to the lump sum tax economy from the Ramsey problem?