

Economics 411-1
Christiano

Class Notes on Matsuyama, ‘Growing Through Cycles’, *Econometrica*

Here is the abstract of the paper: “This paper puts the neoclassical and neo-Schumpeterian growth models in a unified framework. In doing so, it is argued that these two views of growth, one based on factor accumulation and the other based on innovation, are complementary in that they may capture different phases of a single growth experience. In the models presented here, the economy achieves sustainable growth through cycles, perpetually moving back and forth between two phases, under an empirically plausible condition. One phase is characterized by higher output growth, higher investment, no innovation and a competitive market structure. The other phases is characterized by lower output growth, lower investment, high innovation, and a more monopolistic market structure. Both investment and innovation are essential in sustaining growth indefinitely, and yet only one of them appears to play a dominant role in each phase.”

The purpose of these notes is primarily to describe the model. The properties of equilibrium are studied in a homework. These notes briefly discuss the case where there is a random disturbance in the cost of introducing a new good. It defines the ‘Solow residual’, which is sometimes supposed to measure exogenous technical progress. We show that in the model studied here, the Solow residual is not completely exogenous. It is a function of the random cost disturbance and of the aggregate stock of capital.

1 Firms

Here, we take as given the total amount of labor, n , and capital, K , supplied by households.¹ In addition, we take as given the range of intermediate goods, $(0, M_{-1})$, that was in production in the previous period. Given n , K , M_{-1} , we compute the equilibrium prices and quantities for the firm sector. In the next section, we embed this into a general equilibrium with households and

¹These notes presume familiarity with Romer’s ‘Growth Based on Increasing Returns Based on Specialization,’ *American Economic Review*, vol. 77, issue 2, Papers and Proceedings, May 1987.

determine the firm sector variables and K , n . Since the partial equilibrium of the firm sector is static, in this part of these notes we do not use time subscripts.

Final good firms are competitive and make use of the linear homogeneous production function,

$$y = n^{1-\alpha} \int_0^M x(i)^\alpha di, \quad 0 < \alpha < 1,$$

where n is labor, $x(i)$ is the i^{th} intermediate input, and M indicates the range of intermediate goods that is available in the current period. Final good firms take the wage rate, w , the range of intermediate goods, $(0, M)$, and the i^{th} intermediate good price, $p(i)$, as given and beyond their control. Profit maximization leads to first order conditions:

$$(1 - \alpha)y/n = w, \quad \alpha n^{1-\alpha} x(i)^{\alpha-1} = p(i), \quad 0 \leq i \leq M. \quad (1)$$

There are two types of intermediate good firms: those that produce products that were previously introduced, $x(i)$, $0 \leq i \leq M_{-1}$, and those that produce new products, $x(i)$, $M_{-1} < i \leq M$. We suppose:

$$M \geq M_{-1}.$$

When $M = M_{-1}$, then there are no new goods introduced in the current period.

Intermediate good firms producing old products are competitive (i.e., they take their output price as invariant to their own decisions) and firms producing new products (if there are any) produce monopolistically (i.e., they set their price and take into account the demand curve for their product).

To produce $x(i)$ units of the intermediate good, $h(x(i); M_{-1})$ units of capital are needed, where

$$h(x(i); M_{-1}) = \begin{cases} x(i) + F, & \text{if } M_{-1} < i \leq M \text{ and } x(i) > 0 \\ 0, & \text{if } x(i) = 0, \quad 0 \leq i \leq M \\ x(i), & 0 \leq i \leq M_{-1}. \end{cases}$$

Thus, to produce a new good requires paying a fixed cost, $F > 0$. We can think of the introduction of a new good as an innovation, so that F is the cost of innovating. The given specification of h indicates that the fixed

cost only applies when a good is first introduced. As in the Romer model, monopoly power gives innovators the profits they need to cover the fixed costs of innovating.

We will consider a symmetric equilibrium, in which $x^c = x(i)$, $p^c = p(i)$ for $0 \leq i \leq M_{-1}$ and, *in case* $M > M_{-1}$, then $x^m = x(i)$, $p^m = p(i)$ for $M_{-1} < i \leq M$. Here, $x^m > 0$ is the profit maximizing level of output that a monopolist who ignores F would choose. The *actual* level of output that an innovator produces is either x^m or zero, whichever yields a higher level of profits. In the notation, m refers to ‘monopolist’, and c refers to ‘competitive’.

Conditional on M_{-1} , K , and n , the variables in the firm sector that are to be determined are

$$[p^m, x^m, p^c, x^c, y, w, r, M - M_{-1}].$$

We proceed now to determine these 8 variables. The final good production function is:

$$y = n^{1-\alpha} [(M - M_{-1}) (x^m)^\alpha + M_{-1} (x^c)^\alpha], \quad (2)$$

and the three first order conditions in (1) are:

$$(1 - \alpha)y/n = w, \quad \alpha n^{1-\alpha} (x^c)^{\alpha-1} = p^c, \quad \alpha n^{1-\alpha} (x^m)^{\alpha-1} = p^m.$$

The rental rate on capital is r . Since the firms that produce $x(i)$, $0 \leq i \leq M_{-1}$, are competitive, we know that the price is equal to their marginal cost of production. Given the form of the cost function, the marginal cost of production is simply the rental rate on capital:

$$p^c = r, \quad (3)$$

so that, after substituting into the demand curve:

$$x^c = \left[\frac{\alpha}{r} \right]^{\frac{1}{1-\alpha}} n \quad (4)$$

The monopolists who produce $x(i)$ for $M_{-1} < i \leq M$, also have marginal cost r . However, they set their price as a fixed markup, $1/\alpha$, over marginal cost:

$$p^m = \frac{r}{\alpha}. \quad (5)$$

Substituting this into the relevant demand curve, we obtain:

$$x^m = \left[\frac{\alpha^2}{r} \right]^{\frac{1}{1-\alpha}} n. \quad (6)$$

Because there is free entry of monopolists, monopoly profits cannot be positive in equilibrium. If a potential monopolist expects to make negative profits by innovating, then $M = M_{-1}$, i.e., there will be no innovation. If $M > M_{-1}$, so that there is innovation, then monopoly profits are zero because of free entry. Monopoly profits are revenues net of costs, $p^m x^m - r x^m - r F$, which can be written in the following form:

$$\begin{aligned} & p^m x^m - r x^m - r F \\ &= x^m (p^m - r) - r F \\ &= x^m \left(\frac{r}{\alpha} - r \right) - r F \\ &= r \left[x^m \left(\frac{1-\alpha}{\alpha} \right) - F \right] \\ &= r \left(\frac{1-\alpha}{\alpha} \right) \left[x^m - \left(\frac{\alpha}{1-\alpha} \right) F \right]. \end{aligned}$$

The implications of free entry and our restriction on innovation can be summarized as follows:

$$[M - M_{-1}] \geq 0, \quad x^m \leq \frac{\alpha}{1-\alpha} F, \quad \left[x^m - \frac{\alpha}{1-\alpha} F \right] [M - M_{-1}] = 0. \quad (7)$$

Clearing in the capital market requires that the supply of capital, K , equal the demand from competitors and from monopolists. After scaling by M_{-1} :

$$k = x^c + \frac{M - M_{-1}}{M_{-1}} (x^m + F). \quad (8)$$

where

$$k = \frac{K}{M_{-1}}.$$

It turns out that whether the firm equilibrium is characterized by $M - M_{-1}$ being zero (no innovation) or positive, depends on the relationship between k and $\theta F / (1 - \alpha)$, where

$$\alpha^{\frac{\alpha}{\alpha-1}} \equiv \theta.$$

Thus, suppose $k > \theta F/(1 - \alpha)$. To understand the term to the right of the inequality, recall first from (7) that $x^m \leq \alpha F/(1 - \alpha)$. In addition, by the ratio of (4) and (6) we obtain $x^c = \alpha^{\frac{1}{\alpha-1}} x^m$. As a result, we know that $\theta F/(1 - \alpha) \geq x^c$. Thus, $k > \theta F/(1 - \alpha)$ implies $k > x^c$. This in turn implies $M > M_{-1}$. Moreover, $x^m = \alpha F/(1 - \alpha)$ by (7), and $x^c = \theta F/(1 - \alpha)$.

Now suppose $k < \theta F/(1 - \alpha)$. We know that $x^c \leq k$. Suppose the inequality is strict, so that $x^c < k$. Then, $M - M_{-1} > 0$ and, by (7) it follows that $x^m = \alpha F/(1 - \alpha)$, or, $x^c = \theta F/(1 - \alpha)$. This implies $x^c > k$, a contradiction. Thus, it must be that $x^c = k$.

We summarize these observations as follows:

$$x^c = \alpha^{\frac{1}{\alpha-1}} x^m = \min \left\{ k, \frac{\theta}{1 - \alpha} F \right\}, \quad (9)$$

and

$$\frac{M - M_{-1}}{M_{-1}} = \begin{cases} 0, & \text{if } k < \frac{\theta F}{1 - \alpha} \\ \frac{1 - \alpha}{F} \left[k - \frac{\theta F}{1 - \alpha} \right], & \text{if } k > \frac{\theta F}{1 - \alpha} \end{cases}. \quad (10)$$

The last expression simply rearranges (8). It is easy to see that the previous results imply, after making use of (2):

$$\frac{y}{M_{-1}} = \begin{cases} n^{1-\alpha} k^\alpha, & \text{if } k < \frac{\theta F}{1 - \alpha} \\ n^{1-\alpha} A k, & \text{if } k > \frac{\theta F}{1 - \alpha} \end{cases} \quad (11)$$

where $A = (\theta F/(1 - \alpha))^{\alpha-1}$. The first expression corresponds to the case where $M - M_{-1} = 0$, so that $x^c = k$. The second expression corresponds to the case $M - M_{-1} > 0$, in which $x^m = \alpha F/(1 - \alpha)$. In this case, substituting into (2):

$$\begin{aligned} \frac{y}{M_{-1}} &= n^{1-\alpha} \left[\frac{(M - M_{-1})}{M_{-1}} + \theta \right] (x^m)^\alpha \\ &= n^{1-\alpha} \left[\frac{1 - \alpha}{F} \left(k - \frac{\theta F}{1 - \alpha} \right) + \theta \right] (x^m)^\alpha \\ &= n^{1-\alpha} \left(\frac{1 - \alpha}{F} \right) k (x^m)^\alpha \\ &= n^{1-\alpha} \left(\frac{1 - \alpha}{F} \right) k \left(\frac{\alpha F}{1 - \alpha} \right)^\alpha \\ &= n^{1-\alpha} k \left(\frac{\alpha^{\frac{\alpha}{\alpha-1}} F}{1 - \alpha} \right)^{\alpha-1}, \end{aligned}$$

which corresponds to the second term in (11). (Note how the aggregate production relation in (11) is continuous in k .)

The expression for the equilibrium rental rate on capital can be obtained by manipulating (4):

$$r = \begin{cases} \alpha n^{1-\alpha} k^{\alpha-1}, & \text{if } k < \frac{\theta F}{1-\alpha} \\ \alpha n^{1-\alpha} A, & \text{if } k > \frac{\theta F}{1-\alpha} \end{cases} \quad (12)$$

The first expression is obtained by setting $x^c = k$ in (4). The second expression is obtained by setting $x^c = \theta F / (1 - \alpha)$ in (4).

We have now obtained each of the 8 variables sought for the firm equilibrium, conditional on K , n , M_{-1} . From (12), we obtain r . From (11) we obtain y . From (10) we obtain M . From (9) we obtain x^c and x^m . From (3) and (5) we obtain p^c and p^m , respectively. Finally, we obtain w from $(1 - \alpha)y/n = w$.

A few observations are worth making about the firm sector when $k > \theta F / (1 - \alpha)$, so that there is innovation. First, in this case the share of total output paid to capital is rK/y , which, by combining (12) and (11), turns out to be α . Second, the share of output paid to labor, wn/y , is just $1 - \alpha$. So, if we estimate the Solow residual, S , in the usual way, we obtain:

$$S = \frac{y}{n^{1-\alpha} K^\alpha} = \frac{n^{1-\alpha} AK}{n^{1-\alpha} K^\alpha} = AK^{1-\alpha}.$$

Suppose that F is a random shock to ‘technology’. A drop in F encourages innovation by reducing the cost of developing new intermediate goods. Even if F is completely exogenous, S will not be. In particular, if there is some other shock which influences the households capital accumulation decision, then that shock will help predict S . For example, suppose there were a monetary disturbance which influenced investment. An RBC model researcher who believed that S measures an exogenous shock to technology would be surprised to find that the monetary disturbance seems to be helpful in predicting the future evolution of S . Interestingly, in a JME paper, Charlie Evans showed that in real world data, money growth does help predict the Solow residual.

2 Households

Suppose households supply $n_t = 1$ inelastically. Let their preferences be

$$\sum_{t=0}^{\infty} \beta^t \log(c_t),$$

and let their budget constraint be

$$c_t + I_t \leq w_t + r_t K_t,$$

where $K_{t+1} = (1 - \delta)K_t + I_t$. The resource constraint for this economy is:

$$c_t + I_t \leq y_t.$$

It is straightforward to define a standard sequence of markets equilibrium for this economy.