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411-1, Fall 2003

FINAL EXAM

There are four questions. The number of points available for each question is indicated. If a question seems ambiguous, state why, sharpen it up and answer the revised question. You have 1 hour and 55 minutes. Good luck!

1. (15) Consider an economy in which a representative final good producer uses the following technology:

$$y = [\theta V^\rho + (1 - \theta)x^\rho]^{\frac{1}{\rho}}, \quad \rho \leq 1, \quad 0 \leq \theta \leq 1,$$

where x denotes the quantity of oil imported from abroad and V is a function of capital and labor:

$$V = k^\alpha l^{1-\alpha}$$

Suppose that the price of oil, denoted in units of y , is p . That is, if one unit of oil is purchased from abroad, then p units of y must be paid for it. Suppose p is determined in world markets, and has nothing to do with anything in the domestic economy.

- (a) Consider an econometrician with only data on y , k , and l . Show that the econometrician will have the impression that this is an economy in which y is a Cobb-Douglas function of k and l , disturbed by exogenous, disembodied technical change.
 - (b) Show that a jump in p will appear to this econometrician as an exogenous shift down in technology.
2. (15) Consider an economy in which the representative household has preferences,

$$u(c, l) = \log(c) - \frac{\zeta}{2}l^2.$$

Here, l denotes employment and a higher value of ζ indicates an increased aversion to labor effort (it can be shown that an increase in

ζ is a reduced form way to model a rise in the labor market power of workers). Suppose that the resource constraint has the following form:

$$c_t + k_{t+1} - (1 - \delta)k_t \leq k_t^\alpha l_t^{1-\alpha}.$$

Suppose there is a 20 percent rise in ζ .

- (a) What happens to the steady state value of the capital-labor ratio?
 - (b) What happens to the steady state value of k and l ?
3. (35) Consider an economy in which there are two types of households, capitalists and workers. Capitalists accumulate capital, but have no labor power. Workers supply labor, but have no access to capital markets. There are two periods. In the first period, capitalists have an endowment, ω , which they can accumulate in the form of capital, k , or they can consume, c_1 . Their first period budget constraint is:

$$c_1 + k \leq \omega.$$

Their second period budget constraint is:

$$c_2^k \leq R(1 - \delta)k,$$

where R is the rental rate of capital (an exogenous parameter of the model) and δ is the capital income tax rate. The lifetime utility of capitalists is:

$$u^k(c_1, c_2^k) = c_1 + c_2^k$$

We assume that if the capitalist is indifferent between consuming in periods 1 or 2, then they choose to do all their consumption in period 2.

Consider the workers. In period 1, they have no utility. In period 2, their budget constraint is:

$$c_2^w \leq (1 - \tau)l,$$

where τ is the labor tax rate and the wage rate is set to unity. Workers' utility function is

$$u^w(c_2^w, l) = c_2^w - \frac{1}{2}l^2.$$

The social welfare function in this society is:

$$u(u^w, u^k) = u^w + u^k.$$

Suppose the government faces an exogenously determined required level of spending, g , where

$$(R - 1)\omega < g < (R - 1)\omega + \frac{1}{4}$$

The government's budget constraint is:

$$g \leq \tau l + \delta Rk.$$

- (a) Consider the best equilibrium, relative to the given social welfare function. Explain carefully why the labor tax rate in the best equilibrium must be positive. Call the policies in the best equilibrium, the Ramsey policies.
- (b) Suppose the task of administering government policy is given to an administrator who is benevolent in the sense that he is interested in maximizing the social welfare function. Suppose this person must impose the policy during an 'administration period', which occurs at the beginning of period 2, before the labor decision has been taken. Prove that this administrator will deviate from the Ramsey policies. Provide intuition.
- (c) Suppose everyone understands that policy will be implemented by the benevolent administrator in (b). Define a sustainable equilibrium for this economy, and explain the outcomes that occur in sustainable equilibrium.
- (d) Suppose the task of administering government policy in the administration period is given to an administrator who is *not* benevolent. The administrator has preferences:

$$u(u^w, u^k; \lambda) = u^w + \lambda u^k.$$

An administrator with preferences $\lambda > 1$ is partial towards capitalists. Explain why it is that there is a $\lambda > 1$, such that an administrator with this value of λ would choose, in the administration period, not to deviate from the Ramsey policies.

- (e) Given a choice between the sustainable equilibrium outcomes described in (c) and the outcomes in (d), is it possible that workers might prefer an administrator that is partial to capitalists, if they were asked at the beginning of period 1? Explain.
4. (35) Consider an economy in which household preferences have the following form:

$$\sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_t),$$

where c_{1t} denotes cash goods, c_{2t} denotes credit goods and n_t denotes employment. Each period of time is broken into two subperiods, with the first being an asset market trading period and the second, a goods market trading period. In the asset market period, the household constraint is:

$$M_t + B_t \leq W_{t-1}n_{t-1} - P_{2t-1}c_{2t-1} - P_{1t-1}c_{1t-1} + M_{t-1} + R_{t-1}B_{t-1} + T_{t-1} + D_{t-1}.$$

Here, M_t and B_t denote the bonds held by the household at the end of the asset market period. The terms on the right side of the inequality indicate credits and debits arising from activities in the previous period's asset and goods markets. Here, W_{t-1} denotes the nominal wage rate, R_{t-1} denotes the nominal gross rate of interest on bonds held from $t-1$ to t , T_{t-1} denotes transfer payments received from the government and D_{t-1} denotes profits arising from households' ownership of firms. To purchase cash goods in the period t goods market, the household must satisfy the following cash in advance constraint:

$$P_{1t}c_{1t} \leq M_t.$$

A perfectly competitive, representative firm produces output, y_t , using the production function, $y_t = n_t$. The monetary authority faces the constraint:

$$T_t = (g_t - 1)M_t,$$

where g_t is the gross growth rate of the money stock, $M_{t+1} = g_t M_t$. The resource constraint for the economy is:

$$c_{1t} + c_{2t} \leq y_t.$$

- (a) Explain why it is that in equilibrium, $P_{1t} = P_{2t}$.
- (b) Set up the household problem as a Lagrangian problem. Derive the necessary conditions for household optimality, after substituting out the multipliers. Provide an intuitive explanation of these conditions. Provide intuition into why it is that the Ramsey equilibrium is the one associated with $R_t = 1$.

From here on, you may assume that the utility function has the form,

$$u(c_1, c_2, n) = \log(c_1) + \log(c_2) + \log(1 - n)$$

- (c) Prove that if policy sets $g_t = g < \beta$, for all t then there exists no equilibrium (hint: the cash in advance constraint plays an important role here).
- (d) Prove that if $g_t = g$ for all t , and $\beta < g < 1$, then Ramsey is an equilibrium.
- (e) Under the policy in (d), there are other equilibria as well. Here is a countable set of equilibria: $R_t = g/\beta$ for $t \leq t^*$ and $R_t = 1$ for $t > t^*$, for $t^* = 1, 2, \dots$. Prove that these are equilibria. What are the values of c_{1t} , c_{2t} , P_{t+1}/P_t in each of these equilibria?