Homework #2
Economics 411-1, fall 2003
Due Friday, in recitation, October 10
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1. (Boldrin-Montrucchio 1986). Consider the policy rule, $g : [0,1] \to [0,1]$:

$$g(x) = 4x(1-x).$$

Find an economy, (F, Γ, β, X), for which the above function is the policy rule, where the economy satisfies *all* of our assumptions (i.e., A4.3-A4.9).

Some hints: Recall, $\Gamma, \beta \in (0,1)$ and F satisfy

$$g(x) = \max_{y \in \Gamma(x)} F(x,y) + \beta v(y),$$

where

$$v(x) = F(x, g(x)) + \beta v(g(x)).$$

Recall that defining a function or correspondence requires not just the functional forms, but also domains and ranges. Recall too that a function is strictly concave iff the second derivative of the function w.r.t. each argument is negative everywhere, and its Hessian is positive everywhere. Also, note that

$$g(x) = \arg \max_{y \in \Gamma(x)} \Psi(x,y),$$

where

$$\Psi(x,y) = -\frac{1}{2}y^2 + yg(x) - \frac{1}{2}Lx^2 + ax$$

Finally, note that $v(x) = \Psi(x, g(x))$, and use this to back out F. You can think of your task as having to identify values of a and L that ensure A4.3-A4.9 are satisfied. How many such values are there?

2. (This is a continuation of question 1.) Let

$$\begin{aligned} a &= 150 \\ L &= 48 \\ \beta &= 0.01072 \\ d &< 0.0062 \end{aligned}$$

Consider the following two-sector economy: in the final good sector the production function is $c \leq f(k_c, l_c)$, while the capital good is produced according to the Leontieff production function $k' \leq \min\{k_k/d, l_k\}$, where d is a positive real constant. Utility from consumption is linear $u(c) = c$ and the aggregate constraints are $l_c + l_k \leq 1$, $k_c + k_k \leq k$. Let $f(k_c, l_c) = F(k_c + d(1-l_c), 1-l_c)$ from the previous question, and argue that the g function of the previous question is the policy rule for this economy. Find the economic intuition for the fact that for low values of k, g is increasing in k, whereas it is decreasing in k for larger values of k (hint: recall our discussion of this point in class).

3. Plot the sequence of efficient x's for the above economy, for $x_0 = 1$, 0.5, 0.75 and 0.70. Do you observe monotone convergence to a unique steady state?

4. In lecture on Monday, we showed that if there is a steady state, k^*, for the neoclassical growth model, then $f'(k^*) = 1/\beta$. Now suppose that you have a k^* such that $f'(k^*) = 1/\beta$. Prove that k^* has the property, $g(k^*) = k^*$. (Hint: you can adapt another argument used in the lecture for this.)