

Homework #8
Economics 4-11
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Econ 4-11
Due Friday, November 28, 2003.

1. Prove that the capital tax rate in the Ramsey problem defined in class on Wednesday, November 19, is optimally set to $(R - 1)/R$.
2. Consider the Shleifer model. Construct a numerical example illustrating an equilibrium in which the South always implements immediately, while the North always implements with a one period delay. Write in the numbers for output, profits and wage rate for each period.
3. Consider an economy in which a large number of identical households have preferences:

$$u(C, l) = \frac{[C(1 - l)^\psi]^{1-\gamma}}{1 - \gamma}, \quad \psi > 0, \quad \gamma > 0.$$

The resource constraint is

$$Y_t = C_t + I_t = K_t^\theta (z_t l_t)^{1-\theta},$$

and the capital accumulation technology is:

$$K_{t+1} - (1 - \delta)K_t = q_t I_t.$$

Here, z_t and q_t grow exogenously over time

$$z_t = \mu_z z_{t-1}, \quad q_t = \Upsilon q_{t-1}.$$

- (a) Define prices and rates of return for this economy. Let households own and accumulate capital, and rent it in a market for capital services. Let households supply labor to homogeneous labor markets and let them purchase investment and consumption goods in a competitive market for Y_t . Set up the household and firm maximization problems. Define a competitive equilibrium.

(b) Let a steady state competitive equilibrium be one in which l_t , C_t/Y_t , I_t/Y_t and the value of capital relative to output are constant for all t . Show that in a steady state competitive equilibrium,

- i. the growth rate of C_t , Y_t , I_t , and the real wage coincide with the growth rate of z_t^* , where

$$z_t^* = \mu_z^t \Upsilon^{\left(\frac{\alpha}{1-\alpha}t\right)}.$$

- ii. the price of capital, in units of the consumption good, $P_{k't}$, grows at the same rate as Υ^{-t} , while the capital stock grows at the same rate as the growth rate of $z_t^* \Upsilon^t$. Note this implies that the value of capital, $P_{k',t} K_t$, grows at the same rate as output, Y_t .

- iii. the rental rate of capital, r_t^k , grows at the same rate as Υ^{-t} .

(c) How can it be that households have a sufficient incentive to accumulate capital in a steady state equilibrium, given that the rental rate on capital is falling over time?

(d) Show that the allocations in the competitive equilibrium are efficient.