

Homework #9
Economics 4-11
Christiano
Due Friday, December 5, 2003.

1. Consider the taxation example discussed in class. Consider, as a benchmark, the Ramsey equilibrium. Suppose a policymaker has an opportunity to reoptimize the labor and capital tax rate at the beginning of the second subperiod, when the first period saving decision is determined but second period labor has not yet been determined. Compute the utility of the Ramsey equilibrium. Compute the utility to the reoptimizing planner. These calculations should show why a planner who reoptimizes at the beginning of the second subperiod would always choose to deviate from the Ramsey policies. They show why the Ramsey equilibrium is in general not attainable in the absence of commitment.
2. (Fall Prelim Question, see Cole and Kocherlakota Paper on Class Website) Consider an economy in which a continuum of infinitely-lived identical households with the following preferences over consumption, c_t , and labor, l_t :

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, l_t).$$

Time starts at $t = 0$, when the representative household is in possession of m_0 units of cash, k_0 units of physical capital and b_0 units of bonds. Each period is divided into two subperiods. In the first subperiod goods and factor markets are open, and in the second subperiod asset markets are open. In the first subperiod, the household supplies labor and capital for wage rate, w_t , and capital rental rate, r_t , respectively. In addition, it purchases consumption goods and investment goods, x_t . Investment goods augment the stock of capital according to the following technology:

$$k_{t+1} = (1 - \delta)k_t + x_t, \quad \delta \in (0, 1).$$

To purchase c_t units of consumption goods, the household is required to have sufficient cash on hand. Since monetary payments from wages and

other sources are not received until the asset markets open, the only cash the household has available to satisfy this constraint is whatever it has at the beginning of the period. The restriction on consumption is summarized by the following cash in advance constraint:

$$P_t c_t \leq m_t,$$

where m_t is the representative household's beginning of period t stock of money.

Asset markets open next. This is when financial claims arising from previous goods and asset markets are settled, and the household makes a portfolio decision. Sources of cash include wage earnings, $w_t l_t$, capital rental, $r_t k_t$, profits, π_t , a money transfer from the government, X_t , and interest earned on bonds, $(1 + R_t)b_t$, where R_t is the nominal rate of interest. Uses of cash are payments for consumption and investment goods purchased in the previous goods market, $P_t(c_t + x_t)$. The household allocates its net financial assets at the end of the goods market between money, m_{t+1} , and bonds, b_{t+1} . This gives rise to the following budget constraint:

$$m_{t+1} + b_{t+1} \leq (1 + R_t)b_t + m_t + X_t + w_t l_t + r_t k_t + \pi_t - P_t(c_t + x_t)$$

To ensure that the household's intertemporal consumption opportunities are bounded, we suppose that $b_{t+1} \geq -b$, for some finite positive b which is sufficiently large that it is not binding. The household takes b_0 , k_0 and m_0 , as well as prices and rates of return, as given and chooses c_t , k_{t+1} , l_t , $t = 0, 1, 2, \dots$ to maximize utility subject to its two constraints. Firms operate the following constant returns to scale production technology, to produce output, y_t :

$$y_t = f(k_t, l_t).$$

They are competitive in goods and factor markets and choose y_t , k_t , l_t to maximize profits.

The economy-wide average stock of money at the beginning of period t is M_t . Its law of motion is

$$M_{t+1} = M_t + X_t.$$

In answering this question, you can ignore transversality conditions.

- (a) Define an equilibrium for this economy. What is the value of b_t in equilibrium? Explain.
- (b) Set up the Lagrangian representation of the household's optimization problem and display the first order conditions. Substitute out the multipliers and reduce the household's first order conditions to three expressions: two corresponding to the intertemporal decisions and one corresponding to the labor decision. Discuss the intuition behind each equation, by working out the implication that feasible deviations from an optimum cannot produce an increase in utility.
- (c) Derive the first order conditions for optimization by firms, and use these to substitute out for factor prices in the household's first order conditions.
- (d) Display the first order conditions associated with the efficient allocations in the version of the above economy in which there is no money or cash in advance constraint (this is just a standard neoclassical growth model with utility function, u , and production function, f .)
- (e) Suppose policy manages to set $R_t = 0$, for $t = 0, 1, 2, \dots$. Show that the intertemporal Euler equation associated with capital and the intratemporal first order condition associated with labor coincide with those in (d) for $t = 1, 2, 3, \dots$.
- (f) Is there an equilibrium for the $R_t = 0$, $t = 0, 1, 2, \dots$ economy in which the allocations are identical to the efficient allocations alluded to in (d)? Explain carefully. Are there other equilibria? Explain.
- (g) Consider the steady state equilibrium for the monetary economy (hint: a steady state equilibrium is one in which c_t , k_{t+1} , l_t , R_t , P_t/P_{t-1} , w_t/P_t , r_t^k/P_t are constant). Is the steady state capital-labor ratio distorted when R_t is large? What about the steady state capital shock (here, it may be useful to assume something about the utility function. You may assume there is no income effect on leisure, so that $-u_l/u_c$ is not a function of c . You may also assume that $-u_l/u_c$ is increasing in l . If you use assumptions about the utility function, make clear how and why.)